VISUAL WALKTHROUGH

Each chapter begins with a set of learning objectives. Learning Objectives describe what the reader should be able to do after participating in the learning activity. Learning Objectives give learners a clear picture of what to expect and what is expected of them.

2

Learning Objectives

This chapter deals with 2D signals and systems. The transform which is widely used in system study is the Z-transform. An introduction to 2D Z-transforms is given in this chapter. After reading this chapter, the reader should be familiar with the following concepts:

2D signals
Periodic, aperiodic signals
2D linear shift Invariant systems
Properties of 2D systems
Forward and inverse 2D Z-transforms

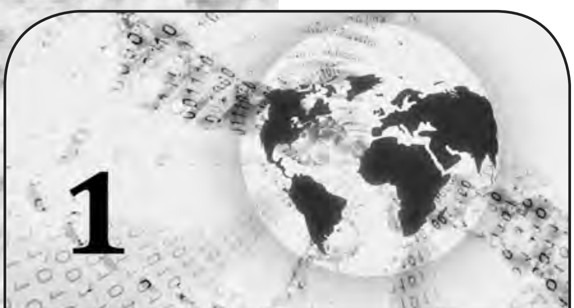
2D Signals and Systems

2.1 INTRODUCTION

Signals are variables that carry information. A signal is a function of one or more variables. An example of a one-dimensional signal is an ECG signal. An example of a 2D signal is a still image. The intensity of the image can vary along the 'x' and 'y' directions, and hence it is a 2D signal. Here 'x' and 'y' are called *spatial variables*.

A system basically processes the input signal and produces output signals. Thus, a system can be viewed as an operation or a set of operations performed on the input signal to produce an output signal.

Each chapter begins with an Introduction that gives a brief summary of the background and contents of the chapter.



1

Learning Objectives

This chapter provides an overview of the image-processing system which includes various elements like image sampling, quantisation, processing, storage and display. On completion of this chapter, the reader is expected to be familiar with the following concepts:

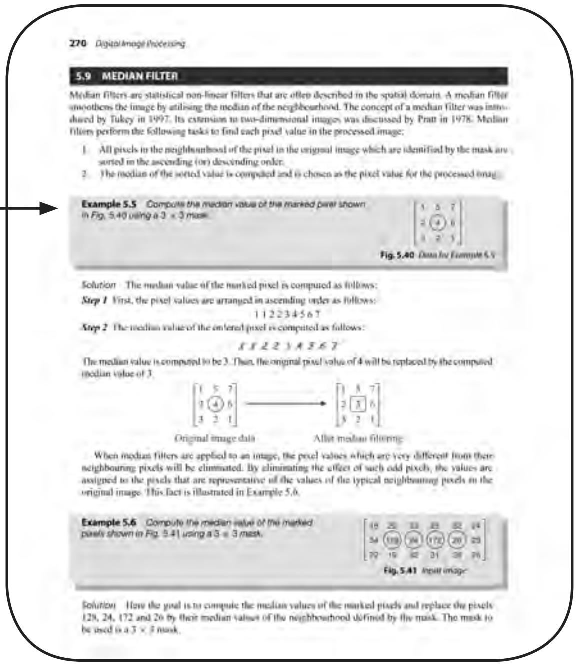
Image sampling
Image types
Image sensors
Image storage
Image processing
Image display

Introduction to Image-processing System

1.1 INTRODUCTION

Digital images play an important role, both in daily-life applications such as satellite television, magnetic resonance imaging, computer tomography as well as in areas of research and technology such as geographical information systems and astronomy. An image is a 2D representation of a three-dimensional scene. A digital image is basically a numerical representation of an object. The term *digital image processing* refers to the manipulation of an image by means of a processor. The different elements of an image-processing system include image acquisition, image storage, image processing and display. This chapter begins with the basic definition of a digital image and is followed by a detailed discussion on two-dimensional sampling. This is followed by different elements of image processing systems.

Worked Examples are provided in sufficient number in each chapter and at appropriate locations, to aid the understanding of the text material.



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5.9 MEDIAN FILTER

Median filters are statistical non-linear filters that are often described in the spatial domain. A median filter smoothens the image by utilising the median of the neighbourhood. The concept of a median filter was introduced by Tukey in 1997. Its extension to two-dimensional images was discussed by Pratt in 1978. Median filters perform the following tasks to find each pixel value in the processed image:

1. All pixels in the neighbourhood of the pixel in the original image which are identified by the mask are sorted in the ascending (or) descending order.
2. The median of the sorted value is computed and is chosen as the pixel value for the processed image.

Example 5.5 *Compute the median value of the marked pixel shown in Fig. 5.40 using a 3 × 3 mask.*

$$\begin{bmatrix} 1 & 5 & 7 \\ 2 & \textcircled{4} & 6 \\ 3 & 2 & 1 \end{bmatrix}$$

Fig. 5.40 *Data for Example 5.5*

Solution The median value of the marked pixel is computed as follows:

Step 1 First, the pixel values are arranged in ascending order as follows:

1 1 2 2 3 4 5 6 7

Step 2 The median value of the ordered pixel is computed as follows:

1 1 2 2 3 4 5 6 7

The median value is computed to be 3. Then, the original pixel value of 4 will be replaced by the computed median value of 3.

$$\begin{bmatrix} 1 & 5 & 7 \\ 2 & \textcircled{4} & 6 \\ 3 & 2 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 5 & 7 \\ 2 & \boxed{3} & 6 \\ 3 & 2 & 1 \end{bmatrix}$$

Original image data — After median filtering

When median filters are applied to an image, the pixel values which are very different from their neighbouring pixels will be eliminated. By eliminating the effect of such odd pixels, the values are assigned to the pixels that are representative of the values of the typical neighbouring pixels in the original image. This fact is illustrated in Example 5.6.

Example 5.6 *Compute the median value of the marked pixels shown in Fig. 5.41 using a 3 × 3 mask.*

Fig. 5.41 *Input image*

Solution Here the goal is to compute the median values of the marked pixels and replace the pixels 128, 24, 172 and 26 by their median values of the neighbourhood defined by the mask. The mask to be used is a 3 × 3 mask.

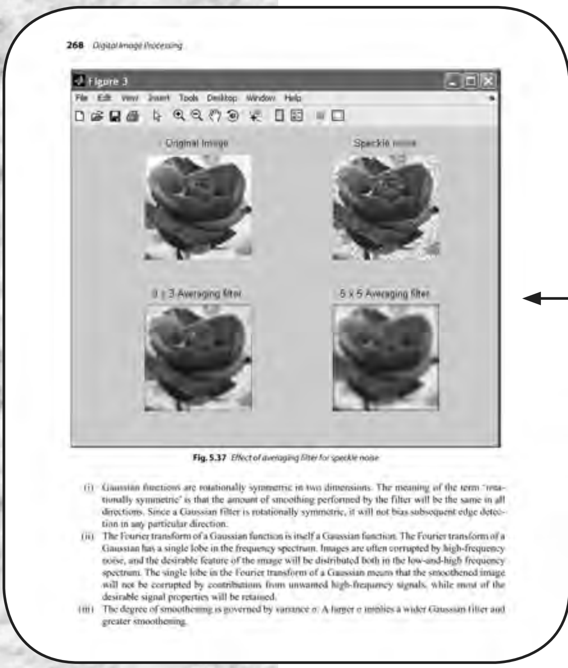

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Fig. 5.37 *Effect of averaging filter for speckle noise*

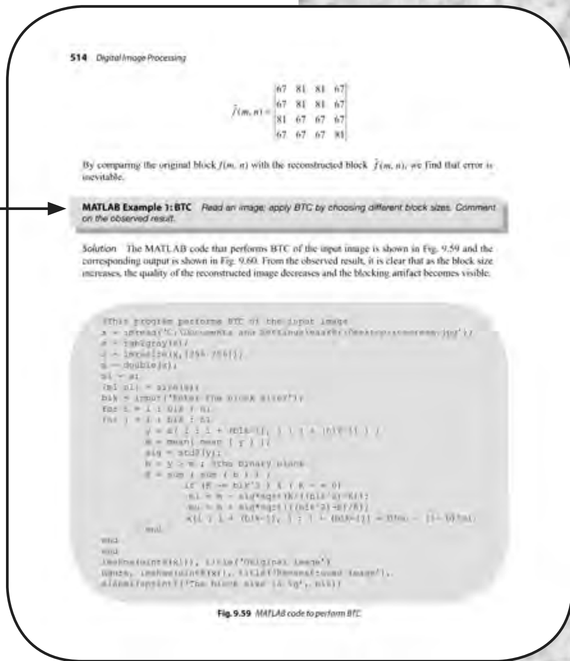
(i) Gaussian functions are rotationally symmetric in two dimensions. The meaning of the term 'rotationally symmetric' is that the amount of smoothing performed by the filter will be the same in all directions. Since a Gaussian filter is rotationally symmetric, it will not bias subsequent edge detection in any particular direction.

(ii) The Fourier transform of a Gaussian function is itself a Gaussian function. The Fourier transform of a Gaussian has a single lobe in the frequency spectrum. Images are often corrupted by high-frequency noise, and the desirable feature of the image will be distributed both in the low-and-high frequency spectrum. The single lobe in the Fourier transform of a Gaussian means that the smoothened image will not be corrupted by contributions from unwanted high-frequency signals, while most of the desirable signal properties will be retained.

(iii) The degree of smoothening is governed by variance σ. A larger σ implies a wider Gaussian filter and greater smoothening.

Other than conventional images like Lena and Barbara, natural images are used to illustrate the concept.

MATLAB is a scientific programming language that provides strong mathematical and numerical support for the implementation of algorithms. MATLAB examples related to image-processing concepts are given wherever necessary in the text.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$$\hat{f}(m,n) = \begin{bmatrix} 67 & 81 & 81 & 67 \\ 67 & 81 & 81 & 67 \\ 81 & 67 & 67 & 67 \\ 67 & 67 & 67 & 81 \end{bmatrix}$$

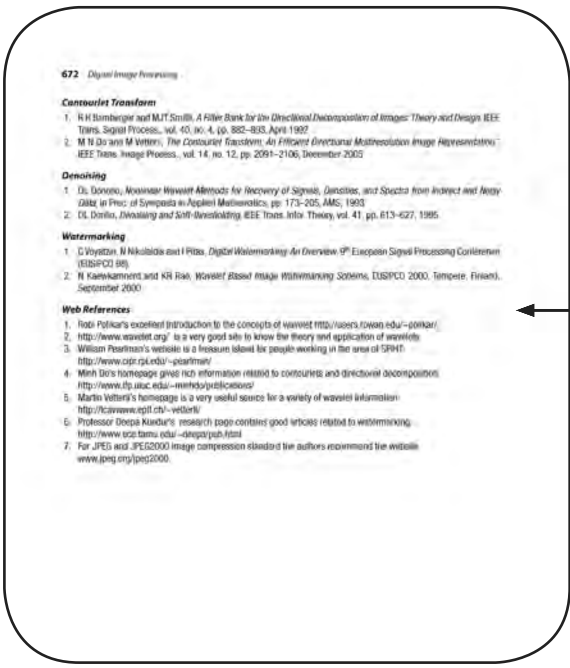
By comparing the original block $f(m, n)$ with the reconstructed block $\hat{f}(m, n)$, we find that error is inevitable.

MATLAB Example 1: BTC *Read an image; apply BTC by choosing different block sizes. Comment on the observed result.*

Solution The MATLAB code that performs BTC of the input image is shown in Fig. 9.59 and the corresponding output is shown in Fig. 9.60. From the observed result, it is clear that as the block size increases, the quality of the reconstructed image decreases and the blocking artifact becomes visible.

Fig. 9.59 *MATLAB code to perform BTC*

To effectively use the internet resources, references to relevant web addresses are provided at the end of each chapter.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Contourlet Transform

1. R H Bamberger and MJT Smith, *A Filter Bank for the Directional Decomposition of Images: Theory and Design* IEEE Trans. Signal Process., vol. 40, no. 4, pp. 882–893, April 1992
2. M N Do and M Vetterli, *The Contourlet Transform: An Efficient Directional Multiresolution Image Representation* IEEE Trans. Image Process., vol. 14, no. 12, pp. 2091–2106, December 2005

Denoising

1. DL Donoho, *Nonlinear Wavelet Methods for Recovery of Signals, Densities, and Spectra from Indirect and Noisy Data*, in Proc. of Symposia in Applied Mathematics, pp. 173–205, AMS, 1993
2. DL Donoho, *Denoising and Soft-thresholding* IEEE Trans. Infor. Theory, vol. 41, pp. 613–627, 1995

Watermarking

1. C Voyatzis, N Nikolaidis and I Pitas, *Digital Watermarking: An Overview* 9th European Signal Processing Conference (EUSIPCO 98)
2. N Kaewkamnerd and KR Rao, *Wavelet Based Image Watermarking Scheme*, EUSIPCO 2000, Tampere, Finland, September 2000

Web References

1. Robi Polikar's excellent introduction to the concepts of wavelet http://users.rowan.edu/~polikar/
2. http://www.wavelet.org/ is a very good site to know the theory and application of wavelets
3. William Pearlman's website is a treasure island for people working in the area of SPIHT http://www.cipr.rpi.edu/~pearlman/
4. Minh Do's homepage gives rich information related to contourlets and directional decomposition http://www.ifp.uiuc.edu/~minhdo/publications/
5. Martin Vetterli's homepage is a very useful source for a variety of wavelet information http://lcavwww.epfl.ch/~vetterli/
6. Professor Deepa Kundur's research page contains good articles related to watermarking http://www.ece.tamu.edu/~deepa/pub.html
7. For JPEG and JPEG2000 image compression standard the authors recommend the website www.jpeg.org/jpeg2000.

At the end of each chapter, a comprehensive list of book and journal references are provided.

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4.15 The 4 × 4 Hadamard matrix is given by $H = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$. Check whether premultiplication of the matrix H by the matrix $S = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$ puts the row in the Walsh sequency order.

References

Books

1. K R Rao and P C Yip, *The Transform and Data Compression Handbook*, CRC Press, 2001
2. Alexander D Poularikas, *The Transforms and Applications Handbook*, CRC Press, 1996
3. R N Bracewell, *The Fourier Transform and its Application*, McGraw-Hill, New York
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2. Hotelling, Harold, *Analysis of a Complex of Statistical Variables into Principal Components*, Journal of Educational Psychology, vol. 24, pp. 498–520, 1933
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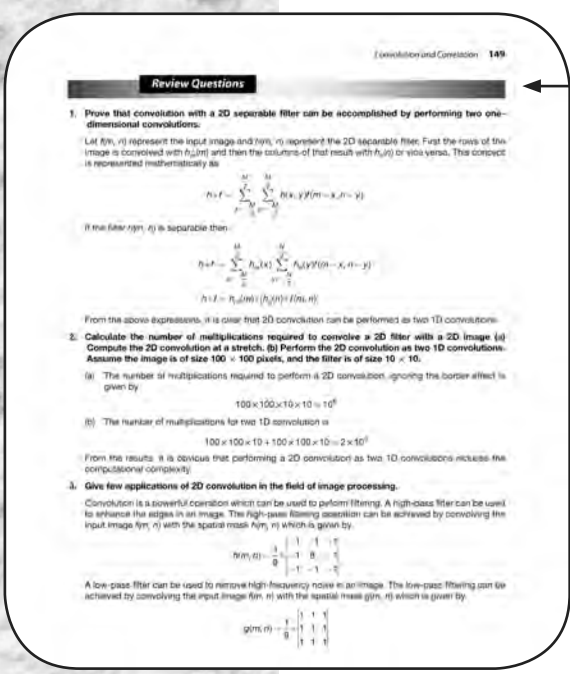
5. Frantisek Matus and Jan Flusser, *Image Representations via a Finite Radon Transform*, IEEE Transaction on Pattern Analysis and Machine Intelligence, vol. 15, no. 10, pp. 996–1006, October 1993
6. V C Klema and A J Laub, *The Singular Value Decomposition: its Computation and Some Applications*, IEEE Trans. Autom Control, vol. 25, pp. 164–176, April 1980

Web References

1. Dr K R Rao's teaching material: http://www-ee.uta.edu/dip/
2. Professor Min Wu's lecture material: http://www.ece.umd.edu/class/enee631/



Convolution and Correlation 149

Review Questions

1. **Prove that convolution with a 2D separable filter can be accomplished by performing two one-dimensional convolutions.**

Let $f(m, n)$ represent the input image and $h(m, n)$ represent the 2D separable filter. First the rows of the image is convolved with $h_m(m)$ and then the columns of that result with $h_n(n)$ or vice versa. This concept is represented mathematically as

$$h * f = \sum_{x=-\frac{M}{2}}^{\frac{M}{2}} \sum_{y=-\frac{N}{2}}^{\frac{N}{2}} h(x, y) f(m-x, n-y)$$

If the filter $h(m, n)$ is separable then

$$h * f = \sum_{x=-\frac{M}{2}}^{\frac{M}{2}} h_m(x) \sum_{y=-\frac{N}{2}}^{\frac{N}{2}} h_n(y) f(m-x, n-y)$$

$$h * f = h_m(m) * (h_n(n) * f(m, n))$$

From the above expressions, it is clear that 2D convolution can be performed as two 1D convolutions.

2. **Calculate the number of multiplications required to convolve a 2D filter with a 2D image (a) Compute the 2D convolution at a stretch. (b) Perform the 2D convolution as two 1D convolutions. Assume the image is of size 100 × 100 pixels, and the filter is of size 10 × 10.**

(a) The number of multiplications required to perform a 2D convolution, ignoring the border effect is given by

$$100 \times 100 \times 10 \times 10 = 10^6$$

(b) The number of multiplications for two 1D convolution is

$$100 \times 100 \times 10 + 100 \times 100 \times 10 = 2 \times 10^5$$

From the results, it is obvious that performing a 2D convolution as two 1D convolutions reduces the computational complexity.

3. **Give few applications of 2D convolution in the field of image processing.**

Convolution is a powerful operation which can be used to perform filtering. A high-pass filter can be used to enhance the edges in an image. The high-pass filtering operation can be achieved by convolving the input image $f(m, n)$ with the spatial mask $h(m, n)$ which is given by

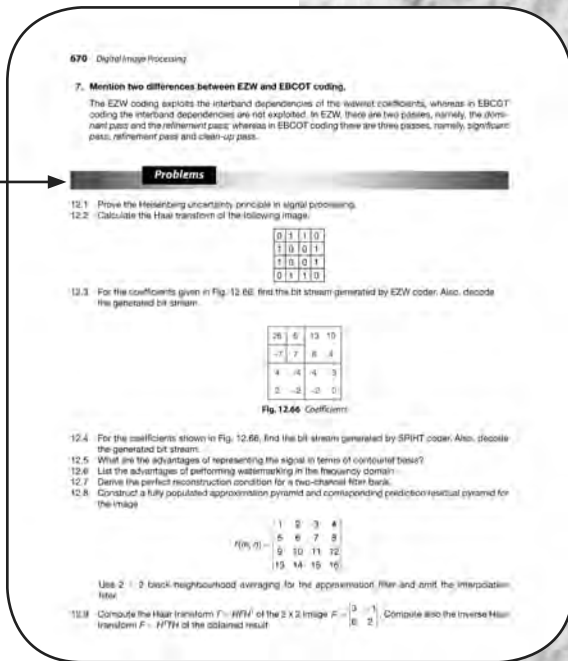
$$h(m, n) = \frac{1}{9} \times \begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

A low-pass filter can be used to remove high-frequency noise in an image. The low-pass filtering can be achieved by convolving the input image $f(m, n)$ with the spatial mask $g(m, n)$ which is given by

$$g(m, n) = \frac{1}{9} \times \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Each chapter contains a set of Review Questions with answers. These review questions provide the essence of the concepts discussed in each chapter.

A set of problems are given as exercise to the students. These are very helpful to teachers in setting class work, assignments, quizzes and examinations.



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7. Mention two differences between EZW and EBCOT coding.

The EZW coding exploits the interband dependencies of the wavelet coefficients, whereas in EBCOT coding the interband dependencies are not exploited. In EZW, there are two passes, namely, the *dominant pass* and the *refinement pass*; whereas in EBCOT coding there are three passes, namely, *significant pass, refinement pass* and *clean-up pass*.

Problems

12.1 Prove the Heisenberg uncertainty principle in signal processing.

12.2 Calculate the Haar transform of the following image.

0	1	1	0
1	0	0	1
1	0	0	1
0	1	1	0

12.3 For the coefficients given in Fig. 12.66, find the bit stream generated by EZW coder. Also, decode the generated bit stream.

26	6	13	10
−7	7	6	4
4	−4	4	−3
2	−2	−2	0

Fig. 12.66 *Coefficients*

12.4 For the coefficients shown in Fig. 12.66, find the bit stream generated by SPIHT coder. Also, decode the generated bit stream.

12.5 What are the advantages of representing the signal in terms of contourlet basis?

12.6 List the advantages of performing watermarking in the frequency domain.

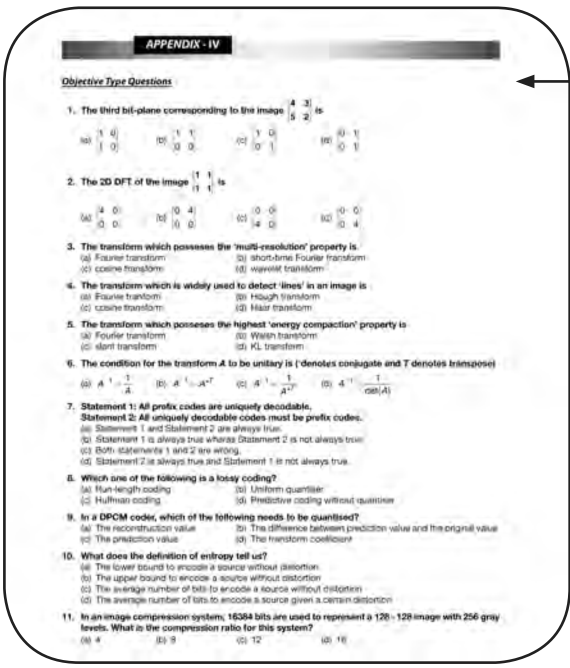
12.7 Derive the perfect reconstruction condition for a two-channel filter bank.

12.8 Construct a fully populated approximation pyramid and corresponding prediction residual pyramid for the image

$$f(m, n) = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$$

Use 2 × 2 block neighbourhood averaging for the approximation filter and omit the interpolation filter.

12.9 Compute the Haar transform $T = HFH^T$ of the 2 × 2 image $F = \begin{bmatrix} 3 & -1 \\ 6 & 2 \end{bmatrix}$. Compute also the inverse Haar transform $F = H^T T H$ of the obtained result.



APPENDIX - IV

Objective Type Questions

1. The third bit-plane corresponding to the image $\begin{bmatrix} 4 & 3 \\ 5 & 2 \end{bmatrix}$ is

(a) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$

2. The 2D DFT of the image $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ is

(a) $\begin{bmatrix} 4 & 0 \\ 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 4 \\ 0 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 0 \\ 4 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 0 \\ 0 & 4 \end{bmatrix}$

3. The transform which possesses the 'multi-resolution' property is

(a) Fourier transform (b) short-time Fourier transform
(c) cosine transform (d) wavelet transform

4. The transform which is widely used to detect 'lines' in an image is

(a) Fourier tranform (b) Hough transform
(c) cosine transform (d) Haar transform

5. The transform which posseses the highest 'energy compaction' property is

(a) Fourier transform (b) Walsh transform
(c) slant transform (d) KL transform

6. The condition for the transform A to be unitary is (* denotes conjugate and T denotes transpose)

(a) $A^{-1} = \frac{1}{A}$ (b) $A^{-1} = A^{*T}$ (c) $A^{-1} = \frac{1}{A^{*T}}$ (d) $A^{-1} = \frac{1}{\det(A)}$

7. Statement 1: All prefix codes are uniquely decodable.
Statement 2: All uniquely decodable codes must be prefix codes.

(a) Statement 1 and Statement 2 are always true.
(b) Statement 1 is always true wheras Statement 2 is not always true
(c) Both statements 1 and 2 are wrong.
(d) Statement 2 is always true and Statement 1 is not always true.

8. Which one of the following is a lossy coding?

(a) Run-length coding (b) Uniform quantiser
(c) Huffman coding (d) Predictive coding without quantiser

9. In a DPCM coder, which of the following needs to be quantised?

(a) The reconstruction value (b) The difference between prediction value and the original value
(c) The prediction value (d) The transform coefficient

10. What does the definition of entropy tell us?

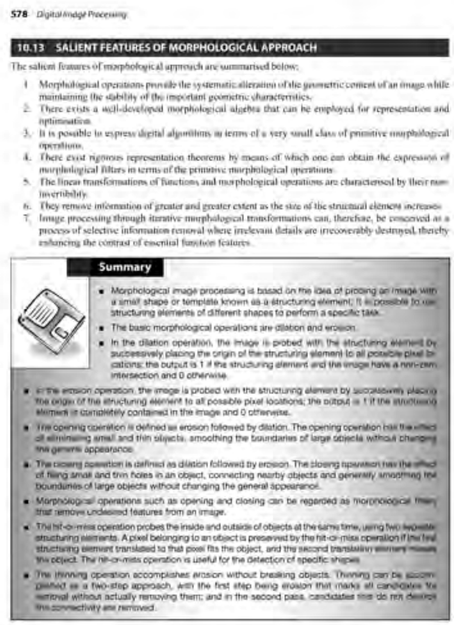
(a) The lower bound to encode a source without distortion
(b) The upper bound to encode a source without distortion
(c) The average number of bits to encode a source without distortion
(d) The average number of bits to encode a source given a certain distortion

11. In an image compression system, 16384 bits are used to represent a 128 × 128 image with 256 gray levels. What is the compression ratio for this system?

(a) 4 (b) 8 (c) 12 (d) 16

Objective questions enable the user to have a clear comprehension of the subject matter. Answers to all the objective questions are provided.

A bulleted summary gives the essence of each chapter in brief.



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10.13 SALIENT FEATURES OF MORPHOLOGICAL APPROACH

The salient features of morphological approach are summarised below:

1. Morphological operations provide the systematic alteration of the geometric content of an image while maintaining the stability of the important geometric characteristics.
2. There exists a well-developed morphological algebra that can be employed for representation and optimisation.
3. It is possible to express digital algorithms in terms of a very small class of primitive morphological operations.
4. There exist rigorous representation theorems by means of which one can obtain the expression of morphological filters in terms of the primitive morphological operations.
5. The linear transformations of functions and morphological operations are characterised by their non-invertibility.
6. They remove information of greater and greater extent as the size of the structural element increases.
7. Image processing through iterative morphological transformations can, therefore, be conceived as a process of selective information removal where irrelevant details are irrecoverably destroyed, thereby enhancing the contrast of essential function features.

Summary

- Morphological image processing is based on the idea of probing an image with a small shape or template known as a structuring element. It is possible to use structuring elements of different shapes to perform a specific task.
- The basic morphological operations are dilation and erosion.
- In the dilation operation, the image is probed with the structuring element by successively placing the origin of the structuring element to all possible pixel locations; the output is 1 if the structuring element and the image have a non-zero intersection and 0 otherwise.
- In the erosion operation, the image is probed with the structuring element by successively placing the origin of the structuring element to all possible pixel locations; the output is 1 if the structuring element is completely contained in the image and 0 otherwise.
- The opening operation is defined as erosion followed by dilation. The opening operation has the effect of eliminating small and thin objects, smoothing the boundaries of large objects without changing the general appearance.
- The closing operation is defined as dilation followed by erosion. The closing operation has the effect of filling small and thin holes in an object, connecting nearby objects and generally smoothing the boundaries of large objects without changing the general appearance.
- Morphological operations such as opening and closing can be regarded as morphological filters that remove undesired features from an image.
- The hit-or-miss operation probes the inside and outside of objects at the same time, using two separate structuring elements. A pixel belonging to an object is preserved by the hit-or-miss operation if the first structuring element translated to that pixel fits the object, and the second translation element misses the object. The hit-or-miss operation is useful for the detection of specific shapes.
- The thinning operation accomplishes erosion without breaking objects. Thinning can be accomplished as a two-step approach, with the first step being erosion that marks all candidates for removal without actually removing them; and in the second pass, candidates that do not destroy the connectivity are removed.

Glossary of Image Processing Terms

Archival image A digital image taken at the highest practicable resolution and stored securely

Access images A term used to denote low-resolution images (thumbnails, ½ screen images) that are made available (usually at no cost) through the Internet

Adaptive filter A filter whose behaviour changes in response to variations in local image properties

Additive primary colour The colours red, green and blue which, when added in different combinations, can produce all the colours

Affine transformation A first-order geometric transformation that involves a combination of translation, scaling, rotation and skewing

Algorithm A computer program that will perform tasks, e.g., compression of file sizes

Aliasing A phenomenon which occurs when an image is undersampled; due to aliasing, the information with a high spatial frequency is incorrectly represented, manifesting itself as an artifact with a lower spatial frequency

Alpha-trimmed mean filter A filter that sorts pixel values from the neighbourhood into ascending order, discards a certain number of values at either end of the list and then outputs the mean of the remaining values

Amplitude spectrum A measure of how much of each frequency component is present in an image

Analog image An image characterised by a physical magnitude varying continuously in space

Analog-to-digital converter Used to convert an analog signal into digital form

Anti-aliasing The technique of minimising the distortion artifacts, known as aliasing, when representing a high-resolution signal at a lower resolution

Area array A common type of detector arrangement within a digital camera containing a fixed number of horizontal and vertical pixels

Artifacts Unwanted blemishes, which may have been introduced to an image by electrical noise during scanning or compression

Bandwidth A measure of data speed in bits per second in digital systems; a high bandwidth network is required for fast transfer of image files as they typically contain large amounts of data

Basis function Used to represent an image; the basis function should be linearly independent and it should span the space

Binary Computer data made up of a series of 0s or 1s; each individual character is referred as a bit

Binary image Binary image takes only two pixel values which are either '0' or '1'

Byte A collection of eight bits

Bit Depth Number of bits used to describe the colour of each pixel; greater bit depth allows more colours to be used in the colour palette for the image—8-bits per pixel will allow 256 colours; 8-bits per colour component in a RGB image will allow 16777216 colours (256 × 256 × 256)

A list of commonly used terms in digital image processing and their meanings are given in the form of a glossary. The glossary will serve as a dictionary for the students.