Chapter 3

Forces and Motion Along a Line

A sailplane (or "glider") is a small, unpowered, high-performance aircraft. A sailplane must be initially towed a few thousand feet into the air by a small airplane, after which it relies on regions of upward-moving air such as thermals and ridge currents to ascend farther. Suppose a small plane requires about 120 m of runway to take off by itself. When it is towing a sailplane, how much more runway does it need?

Concepts & Skills to Review

- net force: vector addition (Section 2.4)
- free-body diagrams (Section 2.4)
- gravitational force (Section 2.5)
- internal and external forces (Section 2.4)

Making The Connection: **motion of a train**

3.1 POSITION AND DISPLACEMENT

A nonzero net force acting on an object changes the object's velocity. According to Newton's second law, the net force determines the *acceleration* of the object. In order to see how interactions affect motion, we first need to carefully define the quantities used to describe motion (position, displacement, velocity, and acceleration) and we must understand the relationships among them. The description of motion using these quantities is called **kinematics**. When we combine kinematics with Newton's laws, we are in the realm of **dynamics**. Chapter 3 considers motion only along a straight line.

To describe motion clearly, we first need a way to say *where* an object is located. Suppose that at 3:00 P.M. a train stops on an east-west track as a result of an engine problem. The engineer wants to call the railroad office to report the problem. How can he tell them where to find the train? He might say something like "three kilometers east of the old trestle bridge." Notice that he uses a point of reference: the old trestle bridge. Then he states how far the train is from that point and in what direction. If he omits any of the three pieces (the reference point, the distance, and the direction) then his description of the train's whereabouts is ambiguous.

The same thing is done in physics. First, we choose a reference point, called the **origin**. Then, to describe the location of something, we give its distance from the origin and the direction. These two quantities, direction and distance, together describe the **position** of the object. Position is a vector quantity; the direction is just as important as the distance. The position vector is written symbolically as $\vec{\mathbf{r}}$. Graphically, the position vector can be drawn as an arrow starting at the origin and ending with the arrowhead on the object. When more than one position vector is to be drawn, a scale is chosen so that the length of the vector is proportional to the distance between the object and the origin.

Once the train's engine is repaired and it gets on its way, we might want to describe its motion. At 3:14 P.M. it leaves its initial position, 3 km east of the origin (Fig. 3.1). At 3:56 P.M. the train is 26 km west of the origin, which is 29 km to the west of its initial position. **Displacement** is defined as the change of position; it is written $\Delta\vec{\mathbf{r}}$, where the

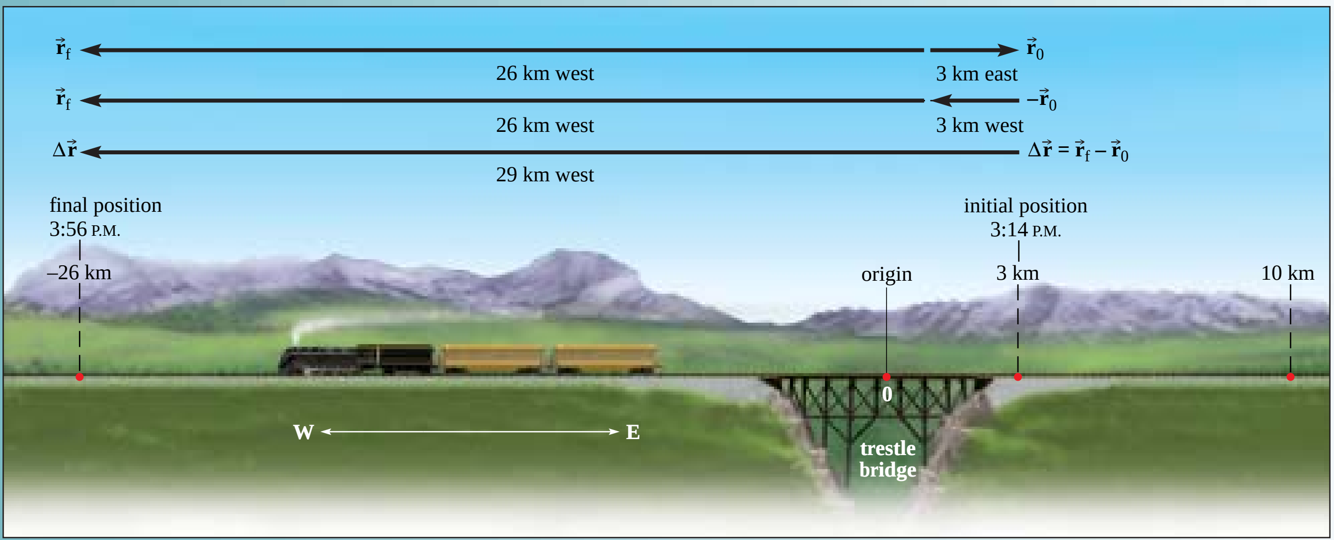

Figure 3.1 Initial ($\vec{\mathbf{r}}_0$) and final ($\vec{\mathbf{r}}_f$) position vectors for a train; the displacement vector $\Delta\vec{\mathbf{r}}$ is found by subtracting the vector for the initial position from the vector for the final position.

symbol Δ means *the change in*, or the final value minus the initial value. If the initial and final positions are $\vec{\mathbf{r}}_0$ and $\vec{\mathbf{r}}_f$, then

Displacement

$$\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_f - \vec{\mathbf{r}}_0 \qquad (3\text{-}1)$$

The symbol Δ stands for *the change in*. If the initial value of a quantity Q is Q_0 and the final value is Q_f, then $\Delta Q = Q_f - Q_0$. ΔQ is read "delta Q."

Since the positions are vector quantities, the operation indicated in Eq. (3-1) is a vector subtraction. To subtract a vector is to add its opposite, so $\vec{\mathbf{r}}_f - \vec{\mathbf{r}}_0 = \vec{\mathbf{r}}_f + (-\vec{\mathbf{r}}_0)$. Vectors representing the initial and final positions are shown as arrows in Fig. 3.1 and the vector subtraction is indicated. The displacement or change of position $\Delta\vec{\mathbf{r}}$ is shown; it is a vector of magnitude 29 km pointing west.

As a shortcut to subtract $\vec{\mathbf{r}}_0$ from $\vec{\mathbf{r}}_f$, instead of adding $-\vec{\mathbf{r}}_0$ to $\vec{\mathbf{r}}_f$, we can simply draw an arrow from the tip of $\vec{\mathbf{r}}_0$ to the tip of $\vec{\mathbf{r}}_f$ when the two vectors are drawn starting from the same point, as in the uppermost vector diagram of Fig. 3.1. This shortcut works because the displacement vector is the change of position; it takes us from the initial position to the final position. Since all vectors add and subtract according to the same rules, this same method can be used to subtract *any* kind of vector—as long as the vectors are drawn starting with their tails at the same point.

Notice that the magnitude of the displacement vector is not necessarily equal to the *distance traveled* by the train. Perhaps the train first travels 7 km to the east, putting it 10 km east of the origin, and then reverses direction and travels 36 km to the west. The total distance traveled in that case is (7 km + 36 km) = 43 km, but the magnitude of the displacement—which is the distance between the initial and final positions—is 29 km. The displacement, since it is a vector quantity, must include the direction; it is 29 km to the west.

Displacement depends only on the starting and ending positions, not on the path taken.

Conceptual Example 3.1

A Relay Race

A relay race is run along a straight line track of length L running south to north. The first runner starts at the south end of the track and passes the baton to a teammate at the north end of the track. The teammate races back to the start line and passes the baton to a third teammate who races 1/3 of the way northward to a finish line. What is the baton's displacement during the race?

Strategy The displacement of the baton is the vector from the starting point to the finish point; it is the *vector sum* of the *displacements* of each runner.

Solution The displacement is the vector sum of the three separate displacements that occurred during the race (Fig. 3.2). Since the first two displacements are equal in magnitude but in opposite directions, their vector sum is zero. The total displacement is then equal to the 3rd displacement, from the origin to the finish line, located 1/3 of the way along the track north of the starting point.

$$\Delta\vec{\mathbf{r}} = (\Delta\vec{\mathbf{r}}_1 + \Delta\vec{\mathbf{r}}_2) + \Delta\vec{\mathbf{r}}_3$$
$$= 0 + \Delta\vec{\mathbf{r}}_3 = \tfrac{1}{3}L, \text{ directed north}$$

Discussion Note that the displacement magnitude is less than the total distance traveled by the baton ($2\tfrac{1}{3}L$). The distance

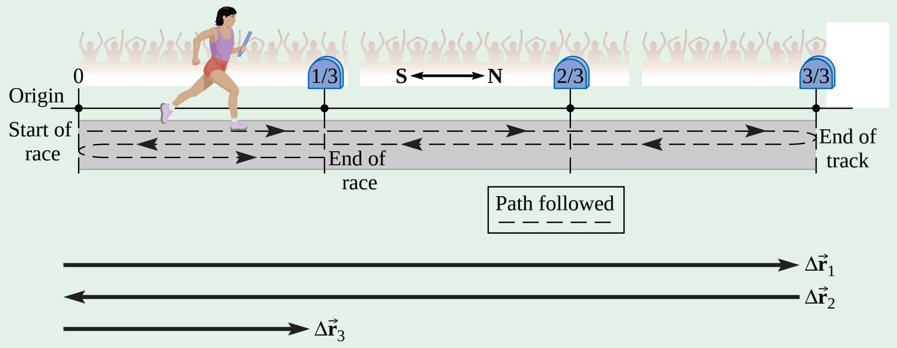

Figure 3.2
Baton displacements during race; vector sum of displacements

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Conceptual Example 3.1 *continued*

traveled depends on the path followed, while the magnitude of the displacement is always the shortest distance between the two points of interest. The displacement vector shows the straight line path from starting point to finish point.

Conceptual Practice Problem 3.1 Around the bases

Casey hits a long fly ball over the heads of the outfielders. He runs from home plate to first base, to second base, to third base, and slides back to home plate safely (an inside-the-park home run). What is Casey's total displacement?

The sign of a vector component indicates the direction.

Vector Components

Since our universe has three spatial dimensions, positions in space are specified using a three-dimensional coordinate system, with x-, y-, and z-axes that are mutually perpendicular. Any vector can be specified either by its magnitude and direction or by its **components** along the x-, y-, and z-axes. The x-, y-, and z-components of vector $\vec{\mathbf{A}}$ are written with subscripts as follows: A_x, A_y, and A_z. One exception to this otherwise consistent notation is that the components of the position vector $\vec{\mathbf{r}}$ are usually written simply as x, y, and z rather than r_x, r_y, and r_z. The components themselves are scalars, which have a magnitude, units, and an algebraic sign. The sign indicates the direction; a positive x-component indicates the direction of the positive x-axis, while a negative x-component indicates the opposite direction (the negative x-axis). Specifying the components of a vector is as complete a description of the vector as is giving the magnitude and direction.

This chapter concentrates on motion along a straight line. If we choose one of the axes along that line, then the position, displacement, velocity, and acceleration vectors each have only one nonzero component. For horizontal linear motion, we usually choose the x-axis to lie along the direction of motion. For objects that move vertically, it is conventional to use the y-axis instead. One direction along the axis is chosen to be positive. Customarily the direction to the right of the origin is called the positive direction for a horizontal axis and upward is called the positive direction for a vertical axis. If vector $\vec{\mathbf{A}}$ points along the x-axis in the direction of $+x$ (to the right), then $A_x = +|\vec{\mathbf{A}}|$; if $\vec{\mathbf{A}}$ points in the opposite direction—in the direction of $-x$ (to the left)—then $A_x = -|\vec{\mathbf{A}}|$.

In Fig. 3.1 the compass arrows indicate that east is chosen as the direction for the positive x-axis. Using components, the train of Fig. 3.1 starts at $x = +3$ km and is later found at $x = -26$ km. The displacement (in component form) is $\Delta x = x_f - x_0 = -29$ km. The sign of the component indicates the direction. Since the positive x-axis is east, a negative x-component means that the displacement is to the west.

The rules for adding vectors translate into adding components with their algebraic signs. If $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$, then $C_x = A_x + B_x$, $C_y = A_y + B_y$, and $C_z = A_z + B_z$. Components become increasingly useful when adding vectors having more than one nonzero component. We can also subtract vectors by subtracting their components, since reversing the direction of a vector changes the sign of each of its components.

Example 3.2

A Stubborn Mule

In an attempt to get a mule moving in a horizontal direction, a farmer stands in front of the mule and pulls on the reins with a force of 320 N to the east while his son pushes on the mule's hindquarters with a force of 110 N to the east. In the meantime the mule pushes on the ground with a force of 430 N to the east (Fig. 3.3a). Use vector components to find the sum of the horizontal forces acting on the mule. Draw a vector diagram to verify the result.

Strategy We are interested in the forces acting *on the mule*. One force given in the problem statement is a force exerted *on the ground* (by the mule). From Newton's third law, the ground pushes back on the mule with a force of equal magnitude and opposite direction; therefore, we needed to know this force to determine one of the horizontal forces acting on the mule. Other forces acting on the mule that have not been explicitly stated are the mule's weight and the normal force on the mule due to the ground. These two are vertical forces: the weight is downward and the normal force is upward; these forces do not concern us since they have no effect on the horizontal motion of the mule. Therefore, we are left with three horizontal forces to sum. When finding vector components, we must first choose a direction for the positive *x*-axis. Here we choose east as the +*x*-direction.

Given: Force on mule by farmer: $\vec{\mathbf{F}}_{\mathrm{mf}}$ = 320 N to the east; force on mule by boy: $\vec{\mathbf{F}}_{\mathrm{mb}}$ = 110 N to the east; horizontal force on ground by mule: $\vec{\mathbf{F}}_{\mathrm{gm}}$ = 430 N to the east.

To find: Sum of horizontal forces on mule

Solution From Newton's third law, the ground pushes on the mule with a force equal and opposite to that of the mule on the ground; therefore, the ground pushes on the mule with a force $\vec{\mathbf{F}}_{\mathrm{mg}}$ of 430 N to the west. The other two horizontal forces acting on the mule are directed to the east. Now we can drop the "m" subscripts since we deal only with the three forces acting on the mule ($\vec{\mathbf{F}}_{\mathrm{mf}}$ becomes $\vec{\mathbf{F}}_{\mathrm{f}}$, etc.). The *x*-components of forces directed to the east ($F_{\mathrm{f}x}$ and $F_{\mathrm{b}x}$) are positive, since we chose east as the +*x*-direction. The *x*-component of the one force directed to the west ($F_{\mathrm{g}x}$) is negative. Notice that the *F*'s are not in bold-faced type because we are now talking about *components* of vectors; components are scalars. Then $F_{\mathrm{f}x} = +320$ N, $F_{\mathrm{b}x} = +110$ N, and $F_{\mathrm{g}x} = -430$ N.

$$\Sigma F_x = F_{\mathrm{f}x} + F_{\mathrm{b}x} + F_{\mathrm{g}x}$$

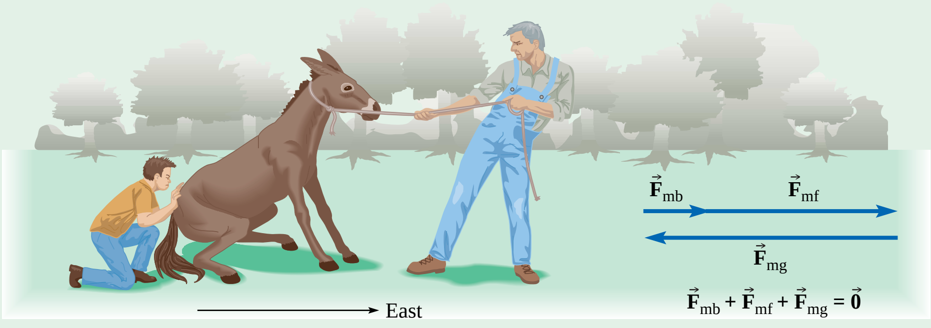
$$\Sigma F_x = 320\ \mathrm{N} + 110\ \mathrm{N} + (-430\ \mathrm{N}) = 0$$

The sum of the horizontal forces is zero. Drawing the vectors to scale (Fig. 3.3b) verifies that the vector sum is zero: $\vec{\mathbf{F}}_{\mathrm{f}} + \vec{\mathbf{F}}_{\mathrm{b}} + \vec{\mathbf{F}}_{\mathrm{g}} = \vec{\mathbf{0}}$.

Discussion Each force component has a sign that indicates the direction along the *x*-axis in which the force acts. Adding the three *x*-components with their algebraic signs gives the *x*-component of the sum. If the sum of the *x*-components had been positive, the sum of the vectors would be to the right (east); if it had been negative, the sum of the forces would be to the left (west).

Practice Problem 3.2 At last, the mule cooperates

The mule finally starts moving and hauls the farmer's wagon along a straight road for 4.3 km directly east to the neighboring farm, where a few bushels of corn are loaded onto the wagon. Then the farmer drives the mule back along the same straight road, heading west for 7.2 km to the market. Use vector components to find the displacement of the mule from the starting point.



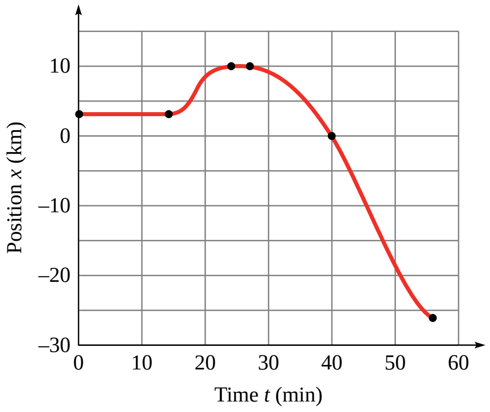
(a) (b)

Figure 3.3 (a) A farmer and his son encouraging their mule to move and (b) the vector sum of the horizontal forces on the mule

3.2 VELOCITY

A displacement vector indicates by how much and in what direction the position has changed, but implies nothing about how long it took to move from one point to the other. If we want to describe how fast something moves, we would give its speed. But we find that a more useful quantity than speed is **velocity**. Speed is a measure of "how fast": distance traveled divided by elapsed time (distance traveled *per unit time*). Velocity includes the direction of motion as well as the speed: it is defined as *displacement*

Velocity: a vector that measures how fast and in what direction something moves



x (km)	+3	+3	+10	+10	0	–26
t (min)	0	14	23	28	40	56

Figure 3.4 Graph of position x versus time t for the train.

divided by elapsed time (displacement per unit time). Speed, the magnitude of the velocity vector, is a scalar.

Average Velocity

Making The Connection: motion of a train

Figure 3.4 is a graph showing the position of the train considered in Section 3.1 as a function of time. The positions of the train at various times are marked with a dot. The position of the train would have to be measured at more frequent time intervals in order to accurately trace out the shape of the graph.

The graph of position versus time shows a curving line, but that does not mean the train travels along a curved path. The motion of the train is constrained along a straight line since the track runs in an east-west direction. The graph shows the x-component of the train's position as a function of time.

A horizontal portion of the graph (as from $t = 0$ to $t = 14$ min) indicates that the position is not changing during that time interval. Sloping portions of the graph indicate that the train is moving. The steeper the graph, the faster its position changes. The slope is a measure of the rate of change of position. A positive slope ($t = 14$ min to $t = 23$ min) indicates motion in the $+x$-direction while a negative slope ($t = 28$ min to $t = 56$ min) indicates motion in the $-x$-direction.

When a displacement $\Delta\vec{\mathbf{r}}$ occurs during a time interval Δt, the **average velocity** $\vec{\mathbf{v}}_{av}$ during that time interval is:

Average velocity is written $\vec{\mathbf{v}}_{av}$. 1

Average velocity

$$\vec{\mathbf{v}}_{av} = \frac{\Delta\vec{\mathbf{r}}}{\Delta t} \qquad (3\text{-}2)$$

Average velocity is a vector because it is the product of a vector, the displacement ($\Delta\vec{\mathbf{r}}$), and a scalar, the inverse of the time($1/\Delta t$). Since Δt is always positive, the direction of the average velocity vector must be the same as the direction of the displacement vector. In terms of x-components,

$$v_{av,x} = \frac{\Delta x}{\Delta t} \qquad (3\text{-}3)$$

The symbol Δ does not stand alone and cannot be canceled in equations because it modifies the quantity that follows it; $\Delta x/\Delta t$ is *not* the same as x/t.

The *average* velocity does not convey detailed information about the motion during Δt, just the net effect. During the time interval in question, the object's motion could change direction and speed in many ways and still have the same *average* velocity. For any given displacement, the average velocity does indicate a constant speed and a straight line direction that would result in the same displacement during the same amount of time.

Suppose a butterfly flutters from a point O on one flower to a point D on another flower along the dashed line path shown in Fig. 3.5 (path *OABCD*). The displacement of the butterfly is the vector arrow $\Delta\vec{\mathbf{r}}$ from O to D. The average velocity of the butterfly is the displacement per unit time, $\Delta\vec{\mathbf{r}}/\Delta t$; the direction of the average velocity is the same as the direction of the displacement vector $\Delta\vec{\mathbf{r}}$. The average velocity does not distinguish the actual path of the butterfly from any other path that begins at O and ends at D over the same time interval Δt.

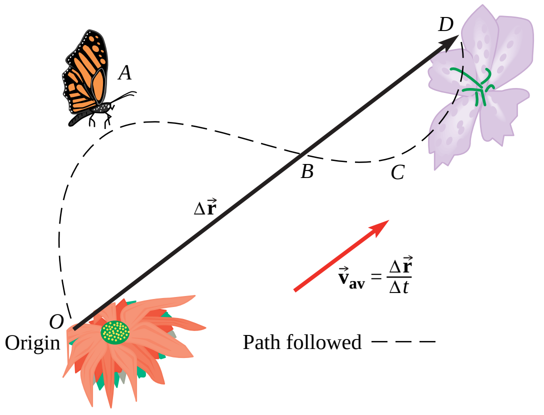


Figure 3.5 Path followed by a butterfly, fluttering from one flower to another.

Example 3.3

Average Velocity of Train

Find the average velocity of the train shown in Fig. 3.1 during the time interval between 3:14 P.M., when the train is 3 km east of the origin, and 3:56 P.M., when it is 26 km west of the origin.

Strategy We already know the displacement $\Delta\vec{\mathbf{r}}$ from Fig. 3.1. The direction of the average velocity is the direction of the displacement.

Known: Displacement $\Delta\vec{\mathbf{r}}$ = 29 km west; start time = 3:14 P.M.; finish time = 3:56 P.M.

To find: $\vec{\mathbf{v}}_{av}$

Solution From Section 3.1, the displacement is 29 km to the west:

$$\Delta\vec{\mathbf{r}} = 29 \text{ km west}$$

The time interval is

$$\Delta t = 56 \text{ min} - 14 \text{ min} = 42 \text{ min}$$

We convert the time interval to hours, so that we can use units of km/h.

$$\Delta t = 42 \text{ min} \times \frac{1\text{h}}{60 \text{ min}} = 0.70 \text{ h}$$

The average velocity is

$$\vec{\mathbf{v}}_{av} = \frac{\text{displacement}}{\text{time interval}} = \frac{\Delta\vec{\mathbf{r}}}{\Delta t}$$

$$\vec{\mathbf{v}}_{av} = \frac{29 \text{ km west}}{0.70 \text{ h}} = 41 \text{ km/h to the west}$$

We can also use components to express the answer to this question. Assuming a positive x-axis pointing east, the x-component of the displacement vector is $\Delta x = -29$ km, where the negative sign indicates that the vector points to the west. Then

$$v_{av,x} = \frac{\Delta x}{\Delta t} = \frac{-29 \text{ km}}{0.70 \text{ h}} = -41 \text{ km/h}$$

The negative sign shows that the average velocity is directed along the negative x-axis, or to the west.

Discussion If the train had started at the same instant of time, 3:14 P.M., and had traveled directly west at a constant 41 km/h, it would have ended up in exactly the same place—26 km west of the trestle bridge—at 3:56 P.M.

Had we started measuring time from when we first spotted the motionless train at 3:00 P.M., instead of 3:14 P.M., we would have found the average velocity over a different time interval, changing the average velocity.

The average velocity depends on the time interval considered.

Practice Problem 3.3 Average velocity of the baton from the relay race

Consider the baton from the relay race of Conceptual Example 3.1. For the winning team, the magnitudes of the average velocities of the first, second, and third runners are 7.30 m/s, 7.20 m/s, and 7.80 m/s, respectively. If the length of the track is 3.00×10^2 m, what is the average velocity of the baton for the entire race? [*Hint:* Find the time spent by each runner in completing her portion of the race.]

In Example 3.3, the magnitude of the train's average velocity is *not* equal to the total distance traveled divided by the time interval for the complete trip. That quantity is called the average speed:

$$\text{average speed} = \frac{\text{distance traveled}}{\text{total time}} = \frac{43\text{ km}}{0.70\text{ h}} = 61\text{ km/h}$$

The distinction arises because the average velocity is the average of a vector quantity whereas the average speed is the average of a scalar.

Instantaneous Velocity

The speedometer of a car does not indicate the average speed, but shows how fast the car is going at any instant in time—the *instantaneous speed.* When a speedometer reads 55 mi/h, it does *not* necessarily mean that the car will travel 55 miles in the next hour; the car could change its speed or stop during that hour. The speedometer reading indicates how far the car will travel during a very short time interval—short enough that the speed does not change appreciably. For instance, at 55 mi/h (= 25 m/s), we can calculate that in 0.010 s the car moves 25 m/s × 0.010 s = 0.25 m—assuming the speed does not change significantly during that 0.010 s interval.

The **instantaneous velocity** is a *vector* quantity whose magnitude is the instantaneous speed and whose direction is the direction of motion. The direction of an object's instantaneous velocity at any point is tangent to the path of the object at that point. Repeating the word *instantaneous* can get cumbersome. When we refer simply to the *velocity*, we mean the *instantaneous* velocity.

The mathematical definition of instantaneous velocity starts with the average velocity during a short time interval. Then we consider shorter and shorter time intervals Δt; the corresponding displacements $\Delta\vec{\mathbf{r}}$ that take place during Δt get smaller and smaller. We let the time interval approach—but never reach—zero (Fig. 3.6). (This mathematical process is called finding the *limit* and is written $\lim\limits_{\Delta t \to 0}$.) As Δt approaches zero, the average velocity during the increasingly short time interval approaches the instantaneous velocity.

The *rate of change* of a quantity Q (whether scalar or vector) is

$$\text{rate of change of } Q = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t}$$

Velocity is the rate of change of position.

Definition of instantaneous velocity

$$\vec{\mathbf{v}} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\mathbf{r}}}{\Delta t} \qquad \text{(3-4)}$$

Both numerator and denominator in Eq. (3-4) approach zero, but $\Delta\vec{\mathbf{r}}/\Delta t$ approaches a limiting value that is neither undefined nor infinite; it can be but is not necessarily zero. Suppose Δt is small enough that the velocity is approximately constant during the interval. If we now cut the time interval Δt in half, the displacement during the time interval is also cut in half (or nearly so), so $\Delta\vec{\mathbf{r}}/\Delta t$ (the average velocity during the interval) changes very little.

For motion along the x-axis, we can rewrite the definition of velocity in terms of x-components:

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \qquad \text{(3-5)}$$

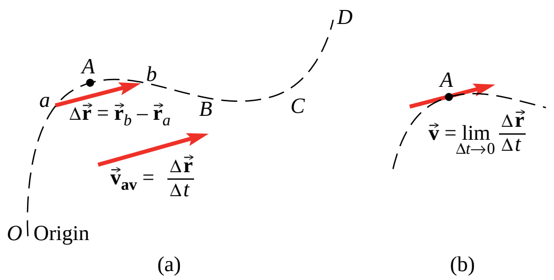


Figure 3.6 (a) A small displacement $\Delta\vec{\mathbf{r}}$ of a butterfly from point a to point b. If the time to travel from point a to point b is Δt, the average velocity during that time interval is $\vec{\mathbf{v}}_{\text{av}} = \dfrac{\Delta\vec{\mathbf{r}}}{\Delta t}$. (b) As points a and b get closer and closer to point A, the displacement $\Delta\vec{\mathbf{r}}$ becomes tangent to the path of travel at point A. The average velocity during the displacement from a to b approaches the instantaneous velocity at point A.

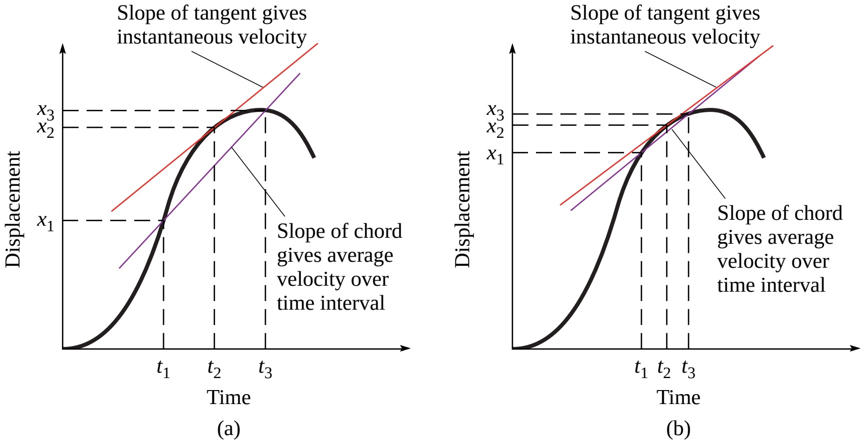


Figure 3.7 Instantaneous velocity at time t_2 is the slope of the tangent to the curve of displacement versus time at that time: (a) average velocity measured over a longer time interval and (b) average velocity measured over a shorter time interval

Table 3.1

Velocities Determined from the Graph of Fig. 3.4

t (min)	14	16	19	21	23	28	32	38	47	56
v_x (km/min)	0	0.5	1.8	0.7	0	0	–0.5	–1.3	–2.0	–1.3
v_x (m/s)	0	8.3	30	11	0	0	–8.3	–22	–33	–22

Graphical Relationship between Position and Velocity

How can we determine the instantaneous velocities from a graph of position versus time? Again, we look at the average velocity during a short time interval. For motion along the x-axis, the displacement (in component form) is Δx. Then the average velocity can be represented on the graph of x versus t as the slope of a line connecting two points (called a "chord"). In Fig. 3.7a, the displacement $\Delta x = x_3 - x_1$ is the "rise" of the graph and the time interval $\Delta t = t_3 - t_1$ is the "run" of the graph. The slope of the graph is the "rise over run," which is equal to the average velocity $\Delta x/\Delta t = v_{\text{av},x}$ for that time interval.

To find the *instantaneous* velocity at some time $t = t_2$, we draw lines showing the average velocity for smaller and smaller time intervals. As the time interval is reduced (Fig. 3.7b), the average velocity changes. As $\Delta t \to 0$, the chord approaches a tangent to the graph. Therefore, v_x is the *slope of the line tangent to the graph of x versus t* at the chosen time.

v_x is the slope of the graph of x versus t

Average and instantaneous velocities for the train can be found from the graph of x versus t (Fig. 3.4). Instantaneous velocities are found from the slopes of the tangents to the curve at various points. Table 3.1 shows some instantaneous velocities of the train in km/min along with equivalent velocities in m/s.

What about the other way around? Given a graph of the *velocity* as a function of time (v_x versus t), how can we determine displacements? For motion along a straight line it is most convenient to work in terms of the x-components of velocity and displacement (v_x and Δx). If v_x is constant during a time interval, then the average velocity is equal to the instantaneous velocity:

$$v_x = v_{\text{av},x} = \frac{\Delta x}{\Delta t}$$

and therefore

$$\Delta x = v_x\,\Delta t \text{ (for constant } v_x\text{)}$$

The graph of Fig. 3.8 shows v_x versus t for an object moving along the x-axis with constant speed v_1 from time t_1 to t_2. The displacement Δx during the time interval $\Delta t = t_2 - t_1$ is $v_1\Delta t$. The shaded rectangle has "height" v_1 and "width" Δt. Since the area of a rectangle is the product of the height and the width, the displacement Δx is represented by the area of the rectangle between the graph of $v_x(t)$ and the time axis for the time interval considered.

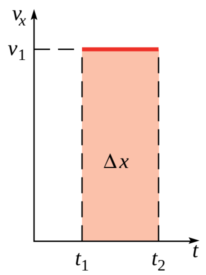


Figure 3.8 Displacement Δx between t_1 and t_2 is represented by the shaded area under the $v_x(t)$ graph.

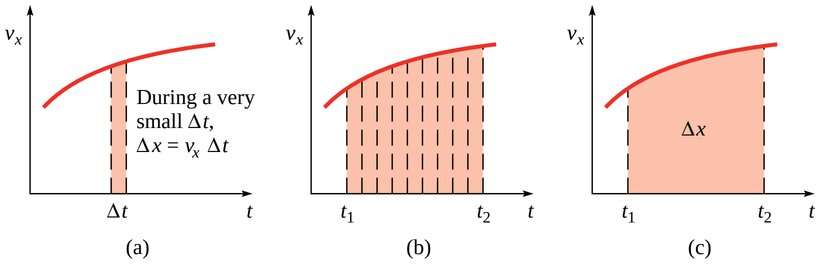


Figure 3.9 Displacement Δx is the area under the v_x versus time graph for the time interval considered.

When we speak of the area under a graph, we are not talking about the literal number of square centimeters of paper or computer screen. The figurative area under a graph usually does not have dimensions of an ordinary area ($[L]^2$). In a graph of v_x versus t, v_x has dimensions [L]/[T] and time has dimensions [T]; areas on such a graph have dimensions $\frac{[L]}{[T]} \times [T] = [L]$, which is correct for a displacement. The units of Δx are determined by the units used on the axes of the graph. If v_x is in meters per second and t is in seconds, then the displacement is in meters.

What if the velocity is not constant? The displacement Δx during a *very small* time interval Δt can be found in the same way as for constant velocity since, during a short enough time interval, the velocity does not change appreciably. Then v_x and Δt are the height and width of a narrow rectangle (Fig. 3.9a) and the displacement during that short time interval is the area of the rectangle. To find the total displacement during any time interval, the areas of all the narrow rectangles are added together.

Δx is the area under the graph of v_x versus t.
The area is negative when the graph is beneath the time axis ($v_x < 0$).

In Fig. 3.9b the time from t_1 to t_2 is subdivided into many short time intervals so that many narrow rectangles of varying heights are formed. The width Δt is allowed to approach zero and the areas of the rectangles are added together. The total displacement Δx between t_1 and t_2 is the sum of the areas of the rectangles. Thus, *the displacement* Δx *during any time interval equals the area under the graph of* $v_x(t)$ (Fig. 3.9c). If v_x is negative, x is decreasing and the displacement is negative, so we must count the area as negative when it is below the time axis.

The magnitude of the train's displacement from time $t = 14$ min to time $t = 23$ min is the area under the v_x versus t graph during that time interval. In Fig. 3.10 the area

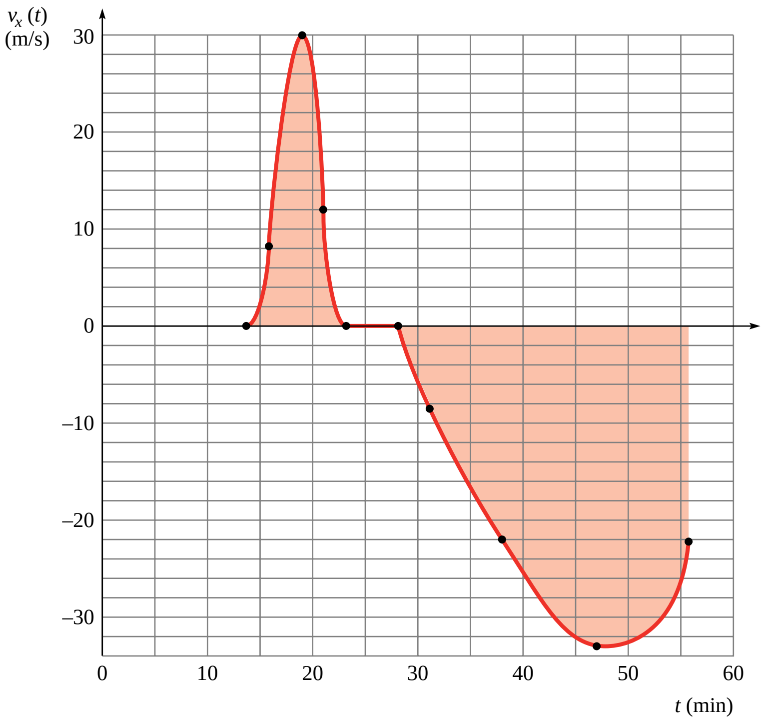


Figure 3.10 Train velocity versus time from $t = 14$ to 56 min using values from Table 3.1.

under the graph for that time interval is shaded. One way to estimate the area is to count the number of grid boxes under the curve. Each box is 2 m/s in height and 5 min (300 s) in width, so each box represents an “area” (displacement) of 2 m/s × 300 s = 600 m. When counting the number of boxes under the curve, we make our best estimate for the fraction of the boxes that are only partly below the curve.

The total number of shaded boxes for this time interval is about 12, so the displacement magnitude is 12×0.60 km ≈ 7 km, as expected—during this time interval the train went from +3 km to +10 km. To get more exact measurements of areas under a curve, we could divide the area into a finer grid.

The shaded area for the time interval $t = 28$ min to $t = 56$ min is below the x-axis, indicating a negative “area” or negative displacement. During this interval the train is headed west (in the $-x$-direction). The number of shaded grid boxes in this interval is approximately 60. The magnitude of the displacement is the product of the number of boxes times “area” of a single box and is negative since v_x is negative; $\Delta x \approx -(60) \times (0.60\text{ km}) \approx -36$ km. By adding this value to the displacement during the first 14 min, we have the total displacement from $t = 0$ to $t = 56$ min:

$$\Delta x = +7\text{ km} + (-36\text{ km}) = -29\text{ km}$$

which agrees with Fig. 3.1.

3.3 ACCELERATION

One goal of this chapter is to quantify changes in velocity, so we can predict the effect on an object’s motion of a nonzero net force. Just as we defined velocity in terms of displacements and time intervals, now we define **acceleration** in terms of changes in velocity and time intervals. The **average acceleration** during a time interval Δt is defined to be

$$\vec{\mathbf{a}}_{\text{av}} = \frac{\vec{\mathbf{v}}_{\text{f}} - \vec{\mathbf{v}}_0}{t_{\text{f}} - t_0} = \frac{\Delta\vec{\mathbf{v}}}{\Delta t} \tag{3-6}$$

The velocity vector can change magnitude, direction, or both. Acceleration is a vector quantity because it is the product of a scalar1/Δt and a vector $\Delta\vec{\mathbf{v}}$; *the direction of the average acceleration is the direction of the vector* $\Delta\vec{\mathbf{v}}$. The direction of the *change* in velocity $\Delta\vec{\mathbf{v}}$ is not necessarily the same as either the initial or the final velocity direction.

Acceleration is the rate of change of velocity.

Just as instantaneous velocity is the limit of average velocity as the time interval approaches zero, the **instantaneous acceleration** is defined as the limit of the average acceleration as the time interval approaches zero.

$$\vec{\mathbf{a}} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\mathbf{v}}}{\Delta t} \tag{3-7}$$

The instantaneous acceleration is the rate of change of the velocity.

The SI units of acceleration are m/s^2, read as “meters per second squared.” Just as with instantaneous velocity, the word *instantaneous* is not always repeated; acceleration without the adjective means *instantaneous* acceleration.

In physics, the word acceleration does not necessarily mean speeding up. A velocity can also change by decreasing speed or by changing direction. A car going around a curve at constant speed has a nonzero acceleration because its velocity is changing; the change is in the *direction* of the velocity rather than in the magnitude. Of course, both the magnitude and direction of the velocity can be changing simultaneously, as when a skateboarder goes up a curved ramp.

Conceptual Example 3.4

Direction of Acceleration While Slowing Down

When Damon approaches a stop sign on his motor scooter, he "decelerates" before coming to a full stop. If he moves in the negative x-direction while he slows down, is the scooter's acceleration component, a_x, positive or negative?

Strategy and Solution The term "decelerate" is not a scientific term. In common usage it means the scooter is slowing: the scooter's velocity is decreasing in magnitude. The acceleration vector $\vec{\mathbf{a}}$ points in the direction of the *change* in the velocity vector $\Delta\vec{\mathbf{v}}$ during a short time interval. Since the velocity is decreasing in magnitude, the direction of $\Delta\vec{\mathbf{v}}$ is opposite to $\vec{\mathbf{v}}$ (Fig. 3.11). The acceleration vector is in the same direction as $\Delta\vec{\mathbf{v}}$—the positive x-direction. Thus a_x is positive (see Fig. 3.12).

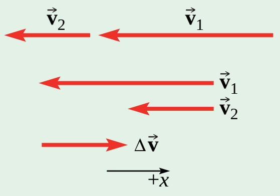


Figure 3.11
The velocity vectors at two different times as the scooter moves to the left with decreasing speed. The change in velocity $\Delta\vec{\mathbf{v}} = \vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1$ is to the right.

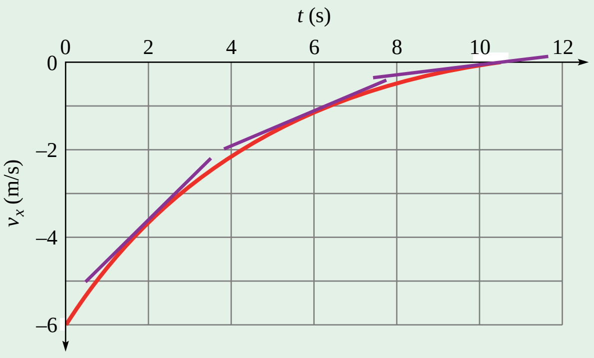


Figure 3.12
In this graph of v_x versus t, as Damon is stopping, v_x is negative, while a_x (the slope) is positive. The value of v_x is increasing, but—since it is less than zero to begin with and is getting closer to zero as time goes on—the speed is *decreasing*.

Practice Problem 3.4 Continuing on his way

As Damon pulls away from the stop sign, continuing in the $-x$-direction, his speed gradually increases. What is the sign of a_x?

a_x is the slope of a $v_x(t)$ graph.
Δv_x is the area under an $a_x(t)$ graph.

Both velocity and acceleration measure rates of change: velocity is the rate of change of position and acceleration is the rate of change of velocity. Therefore the graphical relationship of acceleration to velocity is the same as the graphical relationship of velocity to position: a_x is the slope of a $v_x(t)$ graph and Δv_x is the area under an $a_x(t)$ graph. On a graph of *any* quantity Q as a function of time, the slope of the graph represents the instantaneous rate of change of Q. On a graph of the *rate of change of* Q as a function of time, the area under the graph represents ΔQ.

Example 3.5

Acceleration of Sports Car

A sports car can accelerate from 0 to 30.0 m/s in 4.7 s according to the advertisements. Figure 3.13 shows data for the speed of the car as a function of time as the sports car starts from rest and accelerates to 60.0 m/s while traveling in a straight line headed east.

(a) What is the average acceleration of the sports car from 0 to 30.0 m/s?
(b) What is the maximum acceleration of the car?
(c) How far has the car traveled when it reaches 60.0 m/s?
(d) What is the car's average velocity during the entire 19.2 s interval?

Strategy The velocity of the car is always in the same direction. If we choose that direction as the +x-axis, the x-component of the car's velocity is always positive. Therefore, v_x is equal to the car's speed (which is the *magnitude* of the velocity). We can reinterpret the graph as a graph of v_x versus t.

(a) To find the average acceleration, the change in velocity for the time interval is divided by the time interval. (b) The instantaneous acceleration is the slope of the velocity graph, so it is maximum where the graph is steepest. At that point, the velocity is changing at a high rate. We expect the maximum acceleration to take place early on; the magnitude of acceleration must decrease as the velocity gets higher and higher—there is a maximum velocity for the car, after all. (c) The displacement is the area under the $v_x(t)$ graph. The graph is not a simple shape such as a triangle or rectangle, so an estimate of the area is made. (d) Once we have a value for the displacement, we can apply the definition of average velocity.

continued on next page

Example 3.5 *continued*

Given: Graph of $v(t)$ in Fig. 3.13

To find: (a) $a_{av,x}$ for $v_x = 0$ to 30.0 m/s; (b) a_{max}; (c) Δx from $v_x = 0$ to 60.0 m/s; (d) $v_{av,x}$ from $t = 0$ to 19.2 s

Solution (a) During the first 4.7 s, the car is moving in a straight line to the east, so the average component of acceleration in the east direction is, by definition,

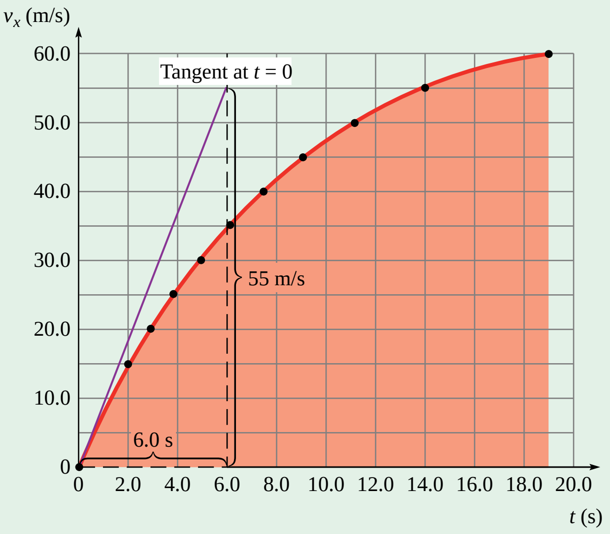
$$a_{av,x} = \frac{\Delta v_x}{\Delta t} = \frac{30.0\ \text{m/s}}{4.7\ \text{s}} = 6.4\ \text{m/s}^2$$

The positive sign indicates the acceleration is directed to the east. On average, then, the car's velocity increases 6.4 m/s each second during the first 4.7 s.

(b) The acceleration, at any instant of time, is the slope of the tangent line to the $v_x(t)$ graph, at that time. To find the maximum acceleration, notice where the graph is steepest. In this case, the largest slope occurs near $t = 0$, just as the car is starting out. In Fig. 3.13, a tangent line to the $v_x(t)$ graph at $t = 0$ passes through $t = 0$. Values for the change in velocity and change in time are read from the graph; the tangent line passes through the two points ($t = 0$, $v_x = 0$ and $t = 6.0$ s, $v_x = 55$ m/s) on the graph so that the change in velocity is 55 m/s for a change in time of 6.0 s. The slope of this line is

$$a_{max} = \frac{\text{rise}}{\text{run}} = \frac{55\ \text{m/s}}{6.0\ \text{s}} = +9.2\ \text{m/s}^2$$

Since the slope is positive, the direction of the acceleration is east.



v_x (m/s)	0	15.0	20.0	25.0	30.0	35.0	40.0	45.0	50.0	55.0	60.0
t (s)	0	2.0	2.9	3.8	4.9	6.2	7.6	9.1	11.2	14.0	19.1

Figure 3.13
Speed as a function of time for a sports car.

(c) Displacement is the area under the $v_x(t)$ graph shown shaded in Fig. 3.13. The area can be estimated by counting the number of grid boxes under the curve. Each box is 5.0 m/s in height and 2.0 s in width, so each represents an "area" (displacement) of 10 m. When counting the number of boxes under the curve, a best estimate is made for the fraction of the boxes that are only partly below the curve. Approximately 75 boxes lie below the curve, so the displacement magnitude is $(75 \times 10\ \text{m}) = 750$ m.

(d) The average velocity for the 19.2-s interval is

$$v_{av,x} = \frac{\Delta x}{\Delta t}$$

$$= \frac{750\ \text{m}}{19.2\ \text{s}} = 39\ \text{m/s}$$

This result is reasonable; if the acceleration were constant, the average velocity would be $\frac{1}{2}(0 + 60.0\ \text{m/s}) = 30.0$ m/s. The actual average velocity is somewhat higher because the acceleration is greater at the start so less of the time interval is spent going (relatively) slowly and more is spent going fast. The speed is less than 30.0 m/s for only 4.7 s, but is greater than 30.0 m/s for 14.5 s.

Discussion The graph of velocity as a function of time is often the most helpful graph to have when solving a problem. If that graph is not given in the problem, it is useful to sketch one. The $v(t)$ graph shows all three kinematic quantities at once: the velocity is given by the points or the curve graphed, the displacement is the area under the curve, and the acceleration is the slope of the curve.

Practice Problem 3.5 Braking a car

An automobile is traveling along a straight road heading to the southeast at 24 m/s when the driver sees a deer begin to cross the road ahead of her. She steps on the brake and brings the car to a complete stop in an elapsed time of 8.0 s. A data recording device, triggered by the sudden braking action, records the following velocities and times as the car slows (Table 3.2). Let the positive x-axis be directed to the southeast. Plot a graph of v_x versus t and find (a) the average acceleration as the car comes to a stop and (b) the instantaneous acceleration at a time of 2.0 s after braking begins.

Table 3.2

Velocities and Times as Car Slows

v_x (m/s)	24	17.3	12.0	8.7	6.0	3.5	2.0	0.75	0
t (s)	0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0

When the acceleration and the velocity of an object are both pointing in the same direction, the object is speeding up. In terms of components, a_x and v_x must either both be positive or both be negative when speed is increasing. When they are both positive,

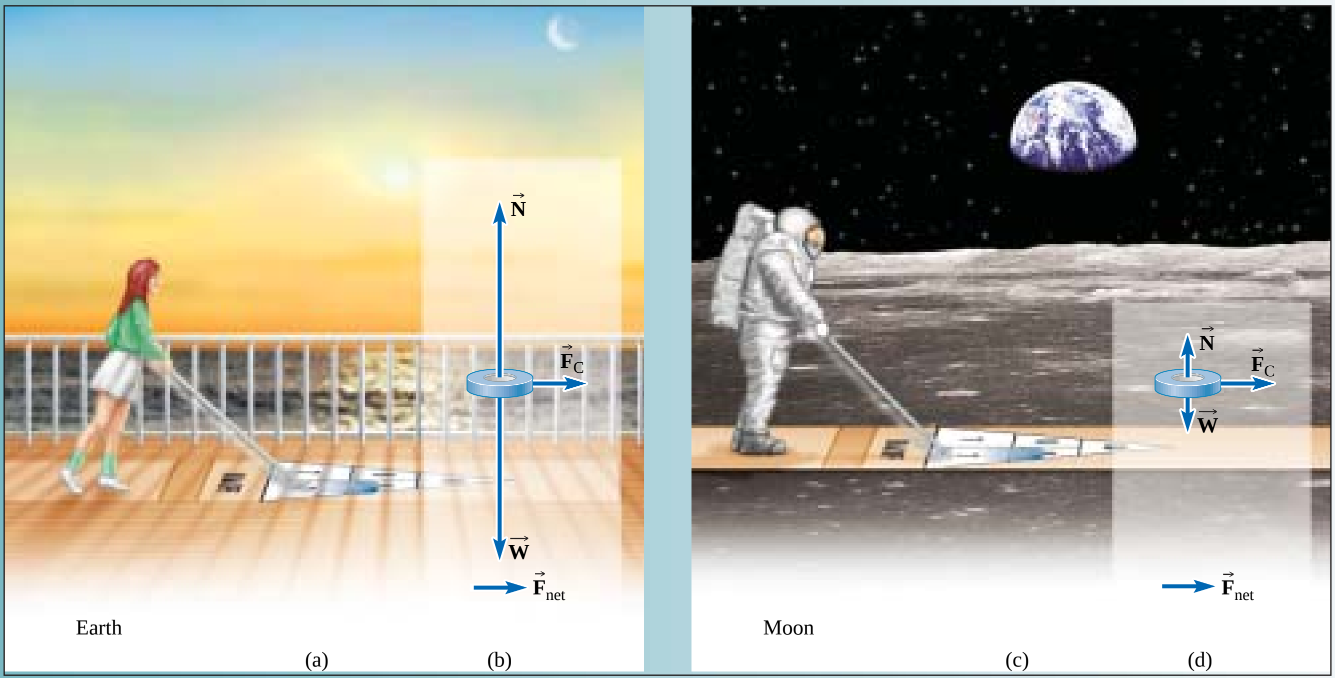


Figure 3.14 An astronaut playing shuffleboard (a) on Earth and (c) on the Moon. Free-body diagrams for a puck being given the same acceleration on a *frictionless* court on (b) Earth and (d) on the Moon. The contact force on the puck due to the *pushing stick* ($\vec{F}_C$) must be the same since the mass of the puck is the same.

the object is moving in the +x-direction and is speeding up. If they are both negative, the object is moving in the –x-direction and is speeding up.

When the velocity and acceleration point in opposite directions, so that their components have opposite signs, the object is slowing down. When v_x is positive and a_x is negative, the body is moving in the positive x-direction, but it is slowing down. When v_x is negative and a_x is positive, the body is moving in the negative x-direction and is slowing down (its speed is decreasing).

The acceleration does not contain any information about the initial velocity. Acceleration is the *rate of change* of velocity. Can an object have a velocity of zero and a nonzero acceleration at the same time? Yes, but the velocity does not *remain* zero. A ball thrown straight up into the air has a velocity of zero at its highest point. Its acceleration is *not* zero at that point; if it were, the ball would not fall back down. On the way up, the ball's velocity is upward and decreasing in magnitude. At the highest point, the velocity is zero but then the ball starts moving downward. On the way down, the velocity is downward and increasing in magnitude. For the entire flight of the ball, its acceleration is downward.

We can define the upward direction as positive or negative as we like; the choice is arbitrary, but we must remain consistent throughout a particular problem. For vertical motion, upward is usually defined as the positive y-direction. For a ball thrown into the air, v_y is positive on the way up and negative on the way down; a_y is negative and constant throughout the motion—on the way up, at the highest point, and on the way down. Since v_y and a_y have opposite signs on the way up, the ball is slowing down (the velocity is decreasing in magnitude). On the downward trip, v_y and a_y have the same sign so the ball's speed is increasing.

3.4 NEWTON'S SECOND LAW: FORCE AND ACCELERATION

According to Newton's second law, the acceleration of an object is proportional to the net force on it and is in the same direction. The larger the net force, the larger the acceleration.

If the net force is zero, the acceleration is zero and the object moves with constant velocity—possibly zero velocity, but not necessarily. Newton's second law also says that the acceleration is inversely proportional to the object's mass. The same net force acting on two different objects causes a smaller acceleration on the object with greater mass. Mass is a measure of an object's inertia—the amount of resistance to *changes in velocity.*

Mass and weight measure different physical properties. The mass of a body is a measure of its inertia, while weight is a measure of the gravitational force acting on it. Imagine taking a shuffleboard puck to the Moon. Since the Moon's gravitational field is weaker than the Earth's, the puck's weight $\vec{\mathbf{W}}$ would be smaller. A smaller normal force $\vec{\mathbf{N}}$ would be required to hold it up. On the other hand, the puck's mass, an intrinsic property, is the same. Neglecting the effects of friction, an astronaut playing shuffleboard on the Moon would have to exert the same horizontal force on the puck as on Earth to give it the same acceleration (Fig. 3.14).

Newton's law relating net force and acceleration is

Newton's Second Law

$$\vec{\mathbf{F}}_{\text{net}} = m\vec{\mathbf{a}} \qquad (3\text{-}8)$$

or

$$\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}}$$

where Σ, the Greek capital letter sigma, stands for *the sum of.* $\Sigma\vec{\mathbf{F}}$ means the sum of all the forces acting on a system. The order of the symbols in Eq. (3-8) does not reflect a cause-and-effect relationship; the net force causes the acceleration, not the other way around. The SI unit of force, the newton, is defined in terms of SI base units so that a 1-N net force acting on a 1-kg mass produces an acceleration of 1 m/s^2; therefore,

$$1\ \text{N} = 1\ \text{kg}\cdot\text{m/s}^2$$

When calculating the net force on a system, only *external* forces need be considered. According to Newton's third law, internal forces always add to zero.

Example 3.6

Coupling Force on First and Last Freight Cars

A train engine pulls out of a station along a straight track with five identical freight cars behind it, each of which weigh 90.0 kN. The train reaches a speed of 15.0 m/s within 5.00 min of starting out. Assuming the acceleration is constant, with what magnitude of force must the coupling between cars pull forward on the first and last of the freight cars? Ignore friction and air resistance. Assume g = 9.80 N/kg.

Strategy A sketch of the situation is shown in Fig. 3.15a. We can calculate the acceleration of the train from the initial and final velocities and the elapsed time. Then we can relate the acceleration to the net force using Newton's second law.

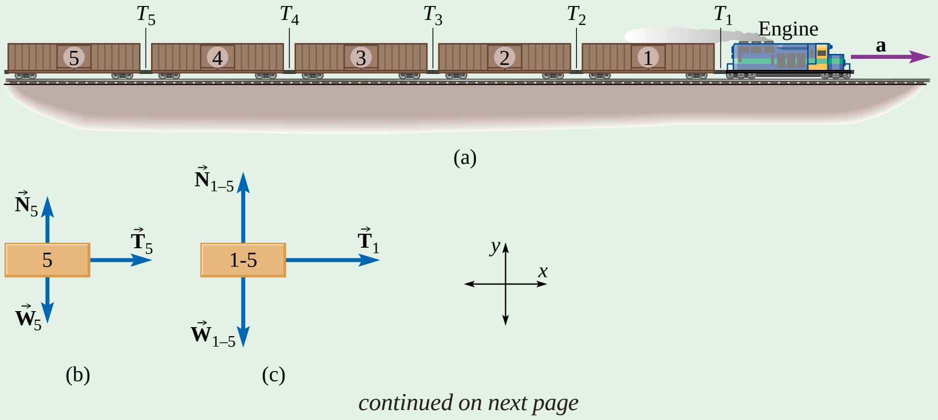


Figure 3.15 (a) An engine pulling five identical freight cars. The entire train has a constant acceleration $\vec{\mathbf{a}}$ to the right. (b) FBD for car 5. (c) FBD for cars 1–5.

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Example 3.6 *continued*

To find the force of the first coupling, we can consider all five cars to be one system so that we do not have to worry about the force exerted on the first car by the second car. Once we identify a system, we draw a free-body diagram before applying Newton's second law.

Given: W = weight of each freight car = 90.0 kN = 9.00×10^4N; v_x = 15.0 m/s at t = 5.00 min = 300 s; v_{0x} = 0 since the train starts from rest; a_x = constant

To find: tensions T_1 and T_5

Solution The acceleration of the train is

$$a_x = \frac{\Delta v_x}{\Delta t} = \frac{15.0\text{ m/s}}{300\text{ s}} = 0.0500\text{ m/s}^2$$

First consider the last freight car (car 5). If we ignore friction and air resistance, the only forces acting are the force $\vec{\mathbf{T}}_5$ due to the tension in the coupling, the normal force $\vec{\mathbf{N}}_5$, and the car's weight $\vec{\mathbf{W}}_5$; an FBD is shown in Fig. 3.15b. The normal force and the weight are vertical and act in opposite directions. They must be equal in magnitude; the vertical component of the net force is zero since the vertical component of the acceleration is zero. Then the net force is equal to the tension in the coupling. The mass of the car is $m = W/g$, where g = 9.80 N/kg = 9.80 m/s^2. Then, from Newton's second law,

$$T_5 = \Sigma F_x = ma_x = \frac{W}{g}a_x$$

$$T_5 = 9.00 \times 10^4\text{ N} \times \frac{0.0500\text{ m/s}^2}{9.80\text{ m/s}^2} = 459\text{ N}$$

For the tension in the first coupling, consider the five cars as *one system.* Fig. 3.15c shows an FBD in which cars 1–5 are treated as a single object. Again, the vertical forces on the system add to zero. The only external horizontal force is the force $\vec{\mathbf{T}}_1$ due to the tension in the first coupling. The mass of the system is five times the mass of one car. Therefore,

$$T_1 = \Sigma F_x = ma_x = (5 \times 9.00 \times 10^4\text{ N}) \times \frac{0.0500\text{ m/s}^2}{9.80\text{ m/s}^2} = 2.30\text{ kN}$$

Discussion The solution to this problem is much simpler when Newton's second law is applied to a system comprised of all five cars, rather than to each car individually. Although the problem can be solved by looking at individual cars, to find the tension in the first coupler you would have to draw five free-body diagrams (one for each car) and apply Newton's second law five times. That's because each car, except the fifth, is acted on by the unequal tensions in the couplers on either side. You'd have to first find the tension in the fifth coupler, then in the fourth, then the third, and so on.

Practice Problem 3.6 Coupling force between first and second freight cars

With what force does the coupling between the first and second cars pull forward on the second car? [*Hint:* Try two methods. One of them is to draw an FBD for the first car and apply Newton's *third* law as well as the second.]

Example 3.7

Two Blocks Hanging on a Pulley

In Fig. 3.16a, two blocks are connected by a massless, flexible cord that does not stretch; the cord passes over a massless, frictionless pulley. If the masses are m_1 = 26.0 kg and m_2 = 42.0 kg, what are the accelerations of each block and the tension in the cord? The gravitational field strength is 9.80 N/kg.

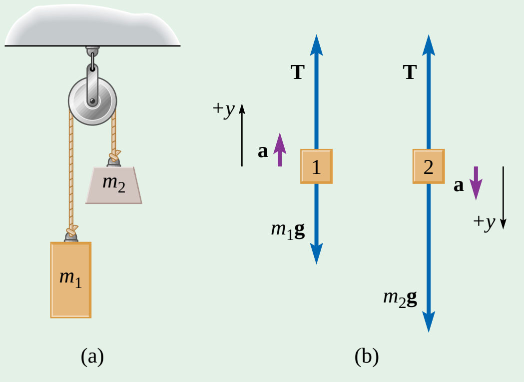


Figure 3.16
(a) Two hanging blocks connected on either side of a frictionless pulley by a massless, flexible cord that does not stretch. (b) Free-body diagrams for the hanging blocks.

Strategy Since m_2 is greater than m_1, the downward pull of gravity is stronger on the right side than on the left. We expect m_2 to accelerate downward and m_1 to accelerate upward.

Since the cord does not stretch, the accelerations of the two blocks are equal in magnitude. If the accelerations had different magnitudes, then soon the two blocks would be moving with different speeds. That could only happen if the cord either stretches or contracts. The fixed length of the cord constrains the blocks to move with equal speeds (in opposite directions) at all times, so the magnitudes of their accelerations must be equal.

The tension in the cord must be the same everywhere along the cord since the masses of the cord and pulley are negligible and the pulley turns without friction.

We treat each block as a separate system, draw free-body diagrams for each, and then apply Newton's second law to each. It is convenient to choose the positive y-direction differently for the two blocks. For each, we choose the +y-axis in the direction of the acceleration. Doing so means that a_y has the same magnitude *and sign* for the two.

continued on next page

Example 3.7 continued

Given: $m_1 = 26.0$ kg and $m_2 = 42.0$ kg
To find: a_y; T

Solution Figure 3.16b shows free-body diagrams for the two blocks. Two forces act on each: gravity and the pull of the cord. The acceleration vectors are drawn next to the free-body diagrams. Thus we know the direction of the net force: it is always the same as the direction of the acceleration. Then we know that the tension must be greater than m_1g to give block 1 an upward acceleration and less than m_2g to give block 2 a downward acceleration. The +y-axes are drawn for each block to be in the direction of the acceleration.

From the free-body diagram of block 1, the pull of the cord is in the +y-direction and the gravitational force is in the –y-direction. Then

$$\Sigma F_y = T - m_1g = m_1a_y$$

For block 2, the pull of the cord is in the –y-direction and the gravitational force is in the +y-direction. Therefore,

$$\Sigma F_y = m_2g - T = m_2a_y$$

Both T and a_y are identical in these two equations. We then have a system of two equations with two unknowns. Adding the equations, we obtain

$$m_2g - m_1g = m_2a_y + m_1a_y$$

Solving for a_y we find

$$a_y = \frac{(m_2 - m_1)g}{m_2 + m_1}$$

Substituting numerical values,

$$a_y = \frac{(42.0\text{ kg} - 26.0\text{ kg}) \times 9.80\text{ N/kg}}{42.0\text{ kg} + 26.0\text{ kg}} = \frac{16.0\text{ kg}}{68.0\text{ kg}} \times 9.80\text{ N/kg}$$

$$a_y = 2.31\text{ m/s}^2$$

since

$$1\,\frac{\text{N}}{\text{kg}} = 1\,\frac{\text{kg}\cdot\text{m/s}^2}{\text{kg}} = 1\text{ m/s}^2$$

The blocks have the same magnitude acceleration. For block 1 the acceleration points upward and for block 2 it points downward.

To find T we can substitute the expression for a_y into either of the two original equations. Using the first equation,

$$T - m_1g = m_1\frac{(m_2 - m_1)g}{m_2 + m_1}$$

Solving for T yields

$$T = \frac{2m_1m_2}{m_1 + m_2}g$$

Substituting,

$$T = \frac{2 \times 26.0\text{ kg} \times 42.0\text{ kg}}{68.0\text{ kg}} \times 9.80\text{ N/kg} = 315\text{ N}$$

Discussion A few quick checks:

- a_y is positive, which means that the accelerations are in the directions we expect.
- The tension (315 N) is between m_1g (255 N) and m_2g (412 N) as expected.
- The units and dimensions are correct for all equations.

It is also instructive to examine what happens to the expressions for a_y and T for special cases of hanging blocks with: equal masses, masses just slightly unequal, or one mass much greater than the other. We often have intuition about what should happen in such special cases. See Practice Problem 3.7 for some examples.

Note that we did *not* find out which way the blocks move. We found the directions of their *accelerations*. If the blocks start out at rest, then the block of mass m_2 moves downward and the block of mass m_1 moves upward. However, if initially m_2 is moving up and m_1 down, they continue to move in those directions, slowing down since their accelerations are opposite to their velocities. Eventually, they come to rest and then reverse directions.

Practice Problem 3.7 Equal and slightly unequal masses

Suppose that the blocks attached to the pulley are of equal mass ($m_1 = m_2$). What do the expressions for a_y and T yield in that case? Can you explain why that must be correct? What if the blocks are only slightly unequal so that their masses $m_2 - m_1 \ll m_2 \approx m_1$? What is the tension in that case? What about the magnitude of the acceleration?

These steps are helpful in most problems that involve Newton's second law.

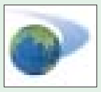
Problem-Solving Strategies for Newton's Second Law

- Decide what objects will have Newton's second law applied to them.
- Identify all the interactions affecting that object.
- Draw a free-body diagram to show all the forces acting on the object.
- Find the net force by adding the forces as vectors.
- Use Newton's second law to relate the net force to the acceleration.

Forces acting on an object determine the motion of the object. The sum of the forces is the net force, which causes an acceleration. The acceleration is a measure of the *rate of change* of the object's velocity. To describe the motion, one other piece of information is required: the object's initial velocity. From the initial velocity and the changes in velocity caused by the forces acting, the velocity at any later time can be calculated. The velocity is the *rate of change* of the position. This strategy for finding the velocity is a triumph of Newton's method of analyzing and then predicting the motion based on the initial conditions and the forces acting on the object.

Example 3.8

Forward Acceleration of a Grocery Cart



Alfredo is shopping at the supermarket (Fig. 3.17a). Alfredo's mass is 72 kg and the mass of his shopping cart plus groceries is 46 kg. At some particular instant, Alfredo's foot pushes backward on the floor with a force of 147 N. What is the acceleration of the cart at that instant? The forces that oppose the forward motion of the cart—friction in the rotation of the cart wheels, air resistance, and so on—add to 5 N.

Strategy To simplify this problem we may choose a system composed of Alfredo and the cart; they move together with the same acceleration. Two horizontal external forces act on the system: the forces opposing the motion of the cart, $\vec{\mathbf{f}}$, and a force of the floor pushing forward on Alfredo's foot. There is no motion in the vertical direction so the net vertical force is zero; the weight of the system is equal to the normal force with which the floor pushes up on the system. We apply both Newton's third law and Newton's second law to solve this problem.

Given: Alfredo's mass: $m_1 = 72$ kg
cart + groceries mass: $m_2 = 46$ kg
force on floor by Alfredo: $\vec{\mathbf{F}}_{fA} = 147$ N to the left
force opposing motion of cart: $\vec{\mathbf{f}} = 5$ N to the left

To find: $\vec{\mathbf{a}}$

Solution From Newton's third law, we know that the floor pushes forward on the system with the same force that Alfredo pushes backward on the floor.

$$\vec{\mathbf{F}}_{Af} = -\vec{\mathbf{F}}_{fA}$$

Since $\vec{\mathbf{F}}_{fA}$ is to the left, $\vec{\mathbf{F}}_{Af}$ is to the right and of magnitude 147 N. We draw a free-body diagram (Fig. 3.17b), showing the external forces acting on the system. Since the vertical acceleration component is zero, the free-body diagram indicates that the weight of the system is balanced by an equal normal force. There may be a nonzero horizontal acceleration component. The $+x$-axis direction is chosen to the right, in the forward direction of the cart.

From Newton's second law for the vertical direction,

$$\Sigma F_y = N - (m_1 + m_2)g = ma_y = 0$$

and for the horizontal direction,

$$\Sigma F_x = F_{Af} - f = ma_x$$

Solving for the acceleration,

$$a_x = \frac{F_{Af} - f}{m_1 + m_2}$$

Substituting the given values,

$$a_x = \frac{147\text{ N} - 5\text{ N}}{72\text{ kg} + 46\text{ kg}} = \frac{142\text{ N}}{118\text{ kg}} = 1.20\text{ m/s}^2$$

The direction of the acceleration is to the right.

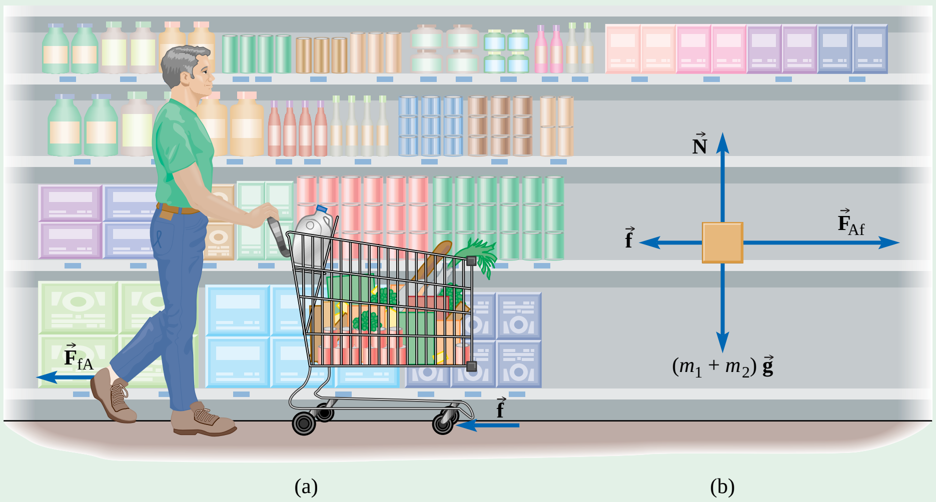


Figure 3.17
(a) Alfredo pushes backward on the floor with force $\vec{\mathbf{F}}_{fA}$ and a force $\vec{\mathbf{f}}$ opposes the forward motion of the cart; (b) a free-body diagram for the system of Alfredo and cart + groceries.

continued on next page

Example 3.8 *continued*

Discussion By choosing the system to be composed of Alfredo and the cart and groceries, we did not have to worry about the forces internal to the system. Alfredo pushes on the cart handle while the cart handle pushes back on Alfredo's hands—these are two internal forces within the system that cancel each other. The only forces that are of concern for the motion of the system as a whole are the external forces, shown in Fig. 3.17b, that act on the system.

Practice Problem 3.8 Alfredo's pushing force

What is the force with which Alfredo's hands push on the grocery cart? [*Hint:* Choose a new system so that Alfredo is external to the system and consider forces acting external to the chosen system.]

3.5 MOTION WITH CONSTANT ACCELERATION

If the net force acting on an object is constant, then the acceleration of the object is also constant, both in magnitude and direction. *Uniform acceleration* is a synonym for constant acceleration.

Three essential relationships between position, velocity, and acceleration can be used to solve *any* constant acceleration problem. We write these relationships using these conventions:

- For motion along the x-axis, write position, velocity, and acceleration in terms of their x-components (x, v_x, a_x).
- At a time $t = 0$, we designate the initial position and velocity as x_0 and v_{0x}.
- At a later time $t > 0$, the position and velocity are x and v_x.

Just as the origin of a coordinate system can be chosen at any convenient point in space, the time at which $t = 0$ can be freely chosen to be any instant in time—whatever makes the problem easiest to solve.

The three essential relationships are: First, since the acceleration is constant, the change in velocity over a given time interval Δt is just the acceleration—the rate of change of velocity—times the elapsed time:

$$\Delta v_x = v_x - v_{0x} = a_x \Delta t \quad \text{for } a_x \text{ constant} \tag{3-9}$$

Second, the displacement is the average velocity times the time interval:

$$\Delta x = x - x_0 = v_{\text{av},x} \Delta t \tag{3-10}$$

Equation (3-10) is true whether the acceleration is constant or not; it comes directly from the definition of average velocity.

Third, since the velocity changes linearly with time, the average velocity is given by

$$v_{\text{av},x} = \frac{v_{0x} + v_x}{2} \quad \text{for } a_x \text{ constant} \tag{3-11}$$

Equation (3-11) is *not* true in *general*, but it is true for constant acceleration. To see why, refer to the velocity versus time graph in Fig. 3.18a. The graph is linear because the acceleration is constant. The displacement during any time interval is represented by the area under the $v(t)$ graph. The average velocity is found by forming a rectangle with an area equal to the area under the curve in Fig. 3.18a, because the average velocity should give the same displacement in the same time interval. Fig. 3.18b shows that, to make the area of one colored triangle equal to the area of the other, the average velocity must be exactly halfway between the initial and final velocities.

If the acceleration is *not* constant, there is no reason why the average velocity would have to be exactly halfway between the initial and the final velocity. As an illustration, imagine a trip where you drive along a straight highway at 80 km/h for 50 min and then at 60 km/h for 30 min. Your acceleration is zero for the entire trip *except* during the few seconds while you slowed from 80 km/h to 60 km/h. The magnitude of your average velocity would *not* be 70 km/h. You spent more time going 80 km/h than you did going

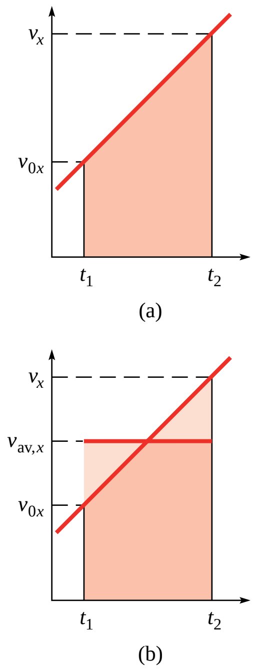


Figure 3.18 Finding average velocity with the aid of a graph

60 km/h, so the magnitude of your average velocity would be greater than 70 km/h (see Problem 16).

To summarize:

If a_x is constant during the entire time interval from $t = 0$ until a later time t, when the time interval is from $t = 0$ until a later time t, $\Delta t = t - 0 = t$

$$\Delta v_x = v_x - v_{0x} = a_x \Delta t$$

$$\Delta x = x - x_0 = v_{av,x} \Delta t$$

$$v_{av,x} = \frac{v_{0x} + v_x}{2}$$

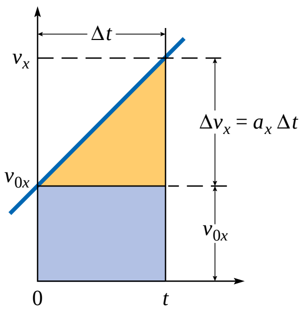


Figure 3.19 Graphical interpretation of Eq. (3-12)

Other relationships can be formed between the various quantities (displacement, velocity, acceleration, and time interval), but it is usually better to start with the three basic relationships and work from there. Any constant acceleration problem can be solved using just these three, and they are not difficult to remember—two of them are really definitions.

Two such relationships that are often useful shortcuts can be derived from Eqs. (3-9) through (3-11) (see Problems 37 and 38). They are

$$\Delta x = x - x_0 = v_{0x}t + \tfrac{1}{2}a_x t^2 \quad \textbf{(3-12)}$$

$$v_x^2 - v_{0x}^2 = 2a_x \Delta x \quad \textbf{(3-13)}$$

Note that Δt is replaced by t because the time interval begins at $t = 0$ and $\Delta t = t$. Equation (3-12) is useful when the final velocity is not known, while Eq. (3-13) is useful when the elapsed time is not known.

We can interpret Eq. (3-12) graphically. Figure 3.19 shows a $v_x(t)$ graph for constant-acceleration motion. The displacement that occurs between $t = 0$ and a later time t is the area under the graph for that time interval. Partition this area into a rectangle plus a triangle. The area of the rectangle is

$$\text{height} \times \text{base} = v_{0x}t$$

The height of the triangle is the change in velocity, which is equal to $a_x t$. The area of the triangle is

$$\tfrac{1}{2} \times \text{height} \times \text{base} = \tfrac{1}{2} \times a_x t \times t = \tfrac{1}{2}\, a_x t^2$$

Adding these "areas" gives Eq. (3-12).

Equations (3-9) through (3-13) are called *kinematic* equations for motion with constant acceleration, since they deal with the relationships between position, velocity, acceleration, and time, but not with forces.

Example 3.9

Displacement of Motorboat

A motorboat accelerates from rest at a dock with a constant acceleration of magnitude 2.8 m/s^2. After traveling directly to the east for 140 m the motor is throttled down so that the boat slows down at 1.2 m/s^2 while still moving east until its speed is 16 m/s. Just as the boat attains the velocity of 16 m/s, it passes a buoy due east of the dock. What is the total displacement of the motorboat from the dock at that time?

Strategy This problem involves two different values of acceleration, so it must be divided into two subproblems. The kinematic equations for constant acceleration cannot be applied to a time interval during which the acceleration changes. But for each of two time intervals, the acceleration of the boat is constant. The two subproblems are connected by the position and velocity of the boat at the instant that the acceleration changes.

For the first subproblem, the boat speeds up with a constant acceleration of 2.8 m/s^2 to the east. We know the acceler-

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Example 3.9 continued

ation, the displacement (140 m east), and the initial velocity: the boat starts from rest, so the initial velocity is zero. We need to calculate the final velocity, which then becomes the initial velocity for the second subproblem. The boat is always headed to the east, so we let east be the positive x-direction.

First subproblem: $v_{0x} = 0$; $a_x = +2.8\ \text{m/s}^2$
$\Delta x = 140$ m; find v_x

For the second subproblem, we know acceleration, final velocity, and we have just found initial velocity from the first subproblem. Since the boat is slowing down, its acceleration is in the direction opposite its velocity; therefore a_x is negative. From these three quantities we can find the displacement of the boat during the second time interval.

Second subproblem: v_{0x} comes from first subproblem
$a_x = -1.2\ \text{m/s}^2$; $v_x = +16$ m/s; find Δx

Adding the displacements for the two time intervals gives the total displacement.

Solution (1) To find v_x without knowing the time, the most convenient kinematic equation is

$$v_x^2 - v_{0x}^2 = 2a_x \Delta x$$

Solving for v_x,

$$v_x = \sqrt{v_{0x}^2 + 2a_x \Delta x}$$

$$v_x = \sqrt{0 + 2 \times 2.8\ \text{m/s}^2 \times 140\ \text{m}} = +28\ \text{m/s}$$

(2) For the second interval the *initial* velocity is the final velocity for the first interval: $v_{0x} = +28$ m/s. Then

$$v_x^2 - v_{0x}^2 = 2a_x \Delta x$$

This time we solve for the displacement.

$$\Delta x = \frac{v_x^2 - v_{0x}^2}{2a_x}$$

$$\Delta x = \frac{(16\ \text{m/s})^2 - (28\ \text{m/s})^2}{2 \times (-1.2\ \text{m/s}^2)} = +220\ \text{m}$$

And the total displacement is

$$\Delta x = \Delta x_1 + \Delta x_2 = 140\ \text{m} + 220\ \text{m} = +360\ \text{m}$$

The boat is 360 m east of the dock.

Discussion This problem is solved by applying the same kinematic equation twice, once to find a velocity and once to find a displacement. The natural division of the problem into two parts occurs because the boat has two different constant accelerations during two different time periods.

In problems that can be subdivided in this way, the final velocity found in the first part becomes the initial velocity for the second part. The same is true for position.

Practice Problem 3.9 Time to reach buoy and average velocities

(a) What is the time required by the boat in the previous example to reach the buoy? (b) Find the average velocity for the entire trip from the dock to the buoy.

Visualizing Motion with Constant Acceleration

In Fig. 3.20 three carts move in the same direction with three different values of constant acceleration. The position of each cart is depicted as it would appear in a stroboscopic photograph with one picture taken every second.

The yellow cart has zero acceleration and therefore constant velocity. During each 1.0-s time interval its displacement is the same: 1.0 m/s × 1.0 s = 1.0 m to the right.

The red cart has a constant acceleration of 0.2 m/s^2 to the right. Although m/s^2 is normally read “meters per second squared,” it can be useful to think of it as “m/s per second”: the cart’s velocity changes by 0.2 m/s during each 1.0-s time interval. In this case,

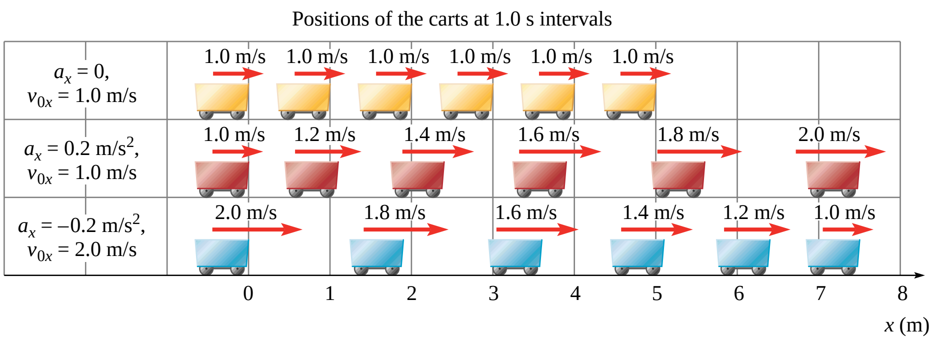

Figure 3.20 Each cart is shown as if stroboscopic photographs were taken with time intervals of 1.0 s between flashes. The arrows above each cart indicate velocity vectors as the strobe flashes occur.

acceleration is in the same direction as the velocity, so the velocity increases (Fig. 3.21a). The displacement of the cart during successive 1.0-s time intervals gets larger and larger.

The blue cart experiences a constant acceleration of 0.2 m/s^2 in the $-x$-direction—the direction *opposite* to the velocity. The magnitude of the velocity then decreases (Fig. 3.21b); during each one-second interval the speed decreases by 0.2 m/s. Now the displacements during one-second intervals get smaller and smaller.

Figure 3.22 shows graphs of $x(t)$, $v_x(t)$, and $a_x(t)$ for each of the carts. The acceleration graphs are horizontal since each of the carts has a constant acceleration. All three v_x graphs are straight lines. Since a_x is the rate of change of v_x, the slope of the v_x graph at any value of t is a_x at that value of t. With constant acceleration, the slope is the same everywhere and the graph is linear. Note that a positive a_x does mean that v_x is increasing, but not necessarily that the *speed* is increasing; if v_x is negative then a positive a_x indicates a *decreasing* speed. (See Conceptual Example 3.4 and Fig. 3.12.) Speed is increasing when the acceleration and velocity are in the same direction (a_x and v_x both positive *or* both negative). Speed is decreasing when acceleration and velocity are in opposite directions—when a_x and v_x have opposite signs.

The position graph is linear for the yellow cart because it has constant velocity. For the red cart the slope of the $x(t)$ graph increases, showing that v_x is increasing; for the blue cart the slope of the $x(t)$ graph decreases, showing that v_x is decreasing.

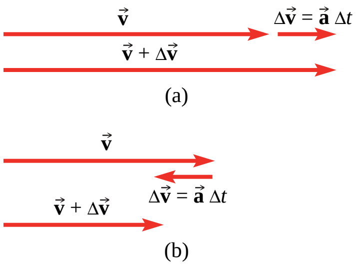

Figure 3.21 (a) If the acceleration is parallel to the velocity, then the change in velocity ($\Delta\vec{v} = \vec{a}\,\Delta t$) is also parallel to the velocity. The result is an increase in the magnitude of the velocity: the object speeds up. (b) If the acceleration is antiparallel (parallel but pointed in opposite direction) to the velocity, then the change in velocity ($\Delta\vec{v} = \vec{a}\,\Delta t$) is also antiparallel to the velocity. The result is a decrease in the magnitude of the velocity: the object slows down.

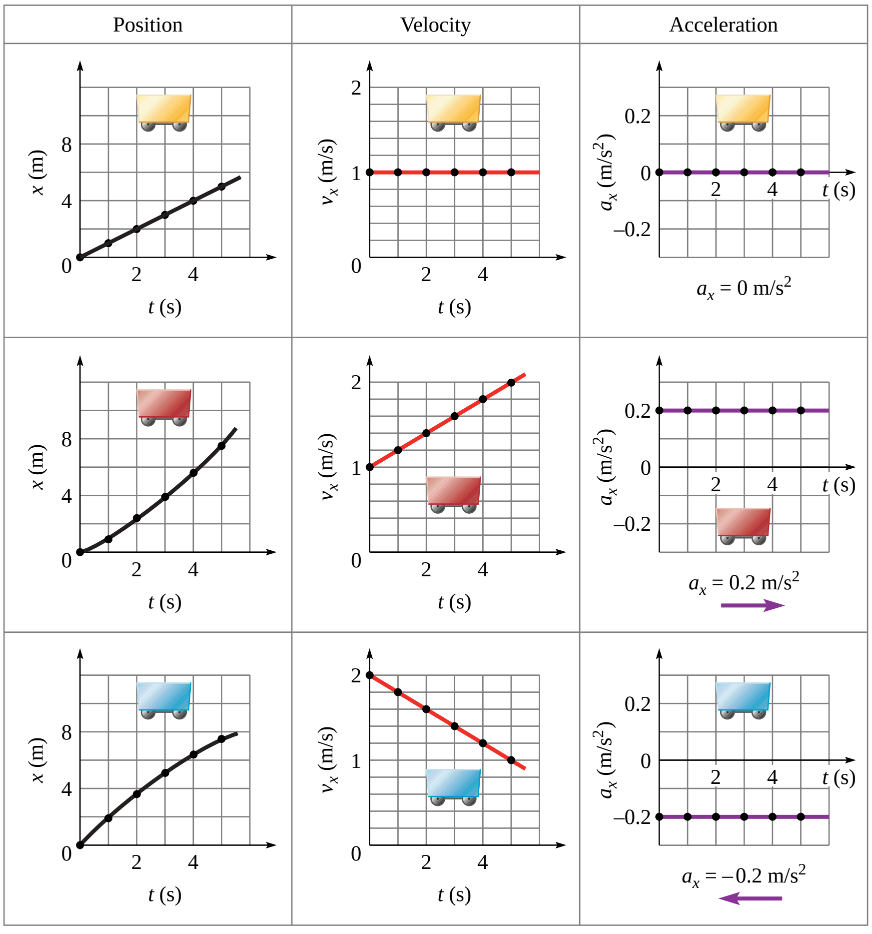

Figure 3.22 Plots of position, velocity, and acceleration along the x-axis for the carts of Fig. 3.20.

Example 3.10

Drag-Racing Spaceships

Two spaceships are moving from the same starting point in the $+x$-direction with constant accelerations. In component form, the silver spaceship starts with an initial velocity of +2.00 km/s and has an acceleration of +0.400 km/s^2. The black spaceship starts with a velocity of +6.00 km/s and has an acceleration of –0.400 km/s^2. Find the time at which the silver spaceship just overtakes the black spaceship.

Strategy We can find the positions of the spaceships at later times from the initial velocities and the accelerations. At first, the black spaceship is moving faster, so it pulls out ahead. Later, the silver ship overtakes the black ship when their *positions are equal*.

Solution The position of either spaceship at a later time is given by

$$x = x_0 + v_{0x}t + \tfrac{1}{2}at^2$$

We set the positions of the spaceships equal to each other ($x_{\text{silver}} = x_{\text{black}}$) and solve algebraically for the time at which this occurs. The initial positions are the same: $x_{0s} = x_{0b} = x_0$.

$$x_0 + v_{0sx}t + \tfrac{1}{2}a_{sx}t^2 = x_0 + v_{0bx}t + \tfrac{1}{2}a_{bx}t^2$$

Subtracting x_0 from each side, moving all terms to one side, and factoring out one power of t yields

$$t(v_{0sx} + \tfrac{1}{2}a_{sx}t - v_{0bx} - \tfrac{1}{2}a_{bx}t) = 0$$

One solution of the equation is $t = 0$ (the two spaceships at the same *initial* position). That is not the solution we seek, so the expression in the parentheses must be equal to zero. Solving for t,

$$t = \frac{2(v_{0sx} - v_{0bx})}{a_{bx} - a_{sx}} = \frac{2(2.00\text{ km/s} - 6.00\text{ km/s}}{-0.400\text{ km/s}^2 - 0.400\text{ km/s}^2} = 10.0\text{ s}$$

The silver spaceship overtakes the black spaceship at $t = 10.0$ s.

Discussion Quick check: the two ships must have the same displacement at $t = 10.0$ s.

$$\Delta x_s = v_{0sx}t + \tfrac{1}{2}a_{sx}t^2$$

$$= 2.00\text{ km/s} \times 10.0\text{ s} + \tfrac{1}{2} \times 0.400\text{ km/s}^2 \times (10.0\text{ s})^2 = 40.0\text{ km}$$

$$\Delta x_b = v_{0bx}t + \tfrac{1}{2}a_{bx}t^2$$

$$= 6.00\text{ km/s} \times 10.0\text{ s} + \tfrac{1}{2} \times (-0.400\text{ km/s}^2) \times (10.0\text{ s})^2$$

$$= 40.0\text{ km}$$

Practice Problem 3.10 Time to reach same velocity

Find the time at which the two spaceships have the same *velocity*.

Example 3.11

Two Blocks, One Sliding and One Hanging

A block of mass $m_1 = 3.0$ kg rests on a frictionless horizontal surface. A second block of mass $m_2 = 2.0$ kg hangs from a flexible cord of negligible mass that runs over an ideal pulley and then is connected to the first block (Fig. 3.23a). The blocks are released from rest. (a) Find the accelerations of the two blocks after they are released. (b) Find the tension in the cord connecting the blocks. (c) What is the velocity of the first block 1.2 s after the release of the blocks, assuming the first block does not run out of room on the table and the second block does not land on the floor? (d) What is the minimum distance from the pulley for block 1 to be located so that it can attain the velocity found in part (c)?

Strategy We consider each block as a separate system and draw a free-body diagram for each. The tension in the cord is the same at both ends of the cord since the cord and pulley are ideal. We choose the $+x$-axis to the right and the $+y$-axis up. To find the accelerations of the blocks (which are equal in magnitude if the cord length is fixed) and the tension in the cord, we apply Newton's second law. Then we use kinematic equations for constant acceleration to answer (c) and (d).

Given: $m_1 = 3.0$ kg; $m_2 = 2.0$ kg; $\vec{\mathbf{v}}_0 = 0$ for both; $\Delta t = 1.2$ s

Solution (a) Figure 3.23b shows free-body diagrams for the two blocks. Block 1 slides along the table surface, so the vertical component of acceleration is zero; the normal force must

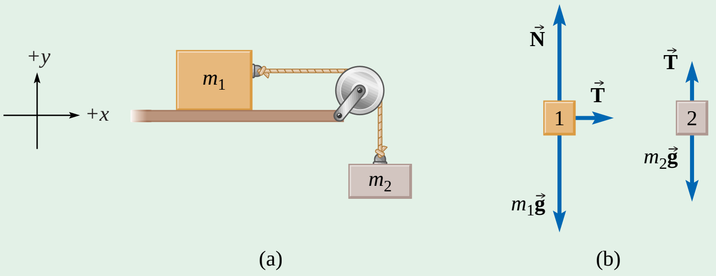

Figure 3.23
(a) Two blocks connected by a cord, one supported by a frictionless table and one hanging freely and
(b) free-body diagrams for blocks 1 and 2 with force magnitudes labeled.

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Example 3.11 *continued*

be equal in magnitude to the weight. With the two vertical forces canceling, the only remaining force is horizontal: the pull of the cord.

$$\Sigma F_{1x} = T = m_1 a_{1x}$$

Only vertical forces act on block 2. From the free-body diagram for block 2, we write Newton's second law:

$$\Sigma F_{2y} = T - m_2 g = m_2 a_{2y}$$

The two accelerations have the same magnitude. The acceleration of block 1 is in the $+x$-direction while that of block 2 is in the $-y$-direction. Therefore we substitute

$$a_{1x} = a \quad \text{and} \quad a_{2y} = -a$$

where a is the magnitude of the acceleration.

Substituting $T = m_1 a$ and $a_{2y} = -a$ into the second equation,

$$m_1 a - m_2 g = -m_2 a$$

Now we solve for a:

$$a = \frac{m_2}{m_1 + m_2} g$$

Substituting the known quantities

$$a = \frac{2.0\text{ kg}}{3.0\text{ kg} + 2.0\text{ kg}} \times 9.8\text{ m/s}^2 = 3.9\text{ m/s}^2$$

The acceleration of block 1 is 3.9 m/s^2 to the right and that of block 2 is 3.9 m/s^2 downward.

(b) The tension is now found from the acceleration:

$$T = m_1 a = 3.0\text{ kg} \times 3.9\text{ m/s}^2 = 12\text{ N}$$

(c) Next we find the velocity of block 1 after 1.2 s. The problem gives the initial velocity, $v_{0x} = 0$ at $t = 0$, and the elapsed time, $\Delta t = t = 1.2$ s.

$$v_x = v_{0x} + a_x t$$

$$v_x = 0 + 3.9\text{ m/s}^2 \times 1.2\text{ s} = 4.7\text{ m/s}$$

The positive sign indicates that block 1 moves to the right

(d) To find the minimum distance from the pulley, we find the distance traveled during 1.2 s after starting from rest

$$\Delta x = v_{0x} t + \tfrac{1}{2} a_x t^2$$

$$\Delta x = 0 + \tfrac{1}{2}\left[3.9\text{ m/s}^2 \times (1.2\text{s})^2\right] = 2.8\text{ m}$$

Discussion An algebraic expression for the tension is

$$T = m_1 a = \frac{m_1 m_2}{m_1 + m_2} g$$

An algebraic expression can lead to insights that are lost when a numerical answer is calculated (see Practice Problem 3.11).

Practice Problem 3.11 A quick check

Check the expressions for acceleration and tension in the special case $m_1 \gg m_2$. [*Hint:* What is the sum $m_1 + m_2$ approximately equal to if $m_1 \gg m_2$?]

Example 3.12

Towing a Glider

A small plane of mass 760 kg requires 120 m of runway to take off by itself (120 m is the horizontal displacement of the plane just before it lifts off the runway, not the entire length of the runway). (a) When the plane is towing a 330-kg glider, how much runway does it need? (b) If the final speed of the plane just before it lifts off the runway is 28 m/s, what is the tension in the tow cable while the plane and glider are moving along the runway?

Strategy The plane's engines produce thrust—the forward force on the plane due to the air. There is also a backward force on the plane due to the air: drag. The drag force increases as the plane's speed increases, but it is much less than the thrust. A small backward force is exerted on the rolling tires by the runway. As a simplified model we assume that the sum of the horizontal forces on the plane (thrust plus drag plus runway) is constant.

As the glider is towed along the runway, the tension in the cable pulls forward on the sailplane and backward on the plane. Ignoring the small mass of the cable, the tension is the same at both ends. Drag on the glider is negligible—it is designed to have very little drag.

Since the plane and glider are moving along the runway, the vertical component of acceleration is zero. We need not be concerned with gravity and with lift (the upward force on the aircraft's wings due to the air) since they add to zero to produce zero vertical acceleration.

Solution (a) When the plane takes off by itself, we assume a constant horizontal net force. From Newton's second law,

$$\Sigma F_x = m_1 a_{1x}$$

where m_1 is the plane's mass and a_{1x} is its horizontal acceleration component.

When the glider is towed, we can consider the plane, glider, and cable to be a single system. The net horizontal force on the system is the same as ΣF_x above. The tension in the cable is an internal force; the glider produces no thrust; and we ignore drag on the glider. Therefore

$$m_1 a_{1x} = (m_1 + m_2) a_{2x}$$

where m_2 is the glider's mass and a_{2x} is the horizontal acceleration component of the plane and glider system. The same net force applied to a larger mass produces a smaller acceleration:

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Example 3.12 *continued*

$a_{2x} < a_{1x}$. Rearranging the last equation shows that the acceleration is inversely proportional to the total mass:

$$\frac{a_{2x}}{a_{1x}} = \frac{m_1}{m_1 + m_2}$$

How is the acceleration related to the runway distance? The plane must get to the same final speed in order to lift off the runway. From

$$v_x^2 - v_{0x}^2 = 2a_x \Delta x$$

with the same values of v_x and v_{0x} in both cases, we see that Δx is inversely proportional to a_x; a smaller acceleration means a longer runway distance is required. Since the acceleration is inversely proportional to the total mass, and the runway distance is inversely proportional to the acceleration, the runway distance is directly proportional to the total mass:

$$\frac{\Delta x_2}{\Delta x_1} = \frac{a_{1x}}{a_{2x}} = \frac{m_1 + m_2}{m_1} = \frac{1090 \text{ kg}}{760 \text{ kg}} = 1.43$$

$$\Delta x_2 = 1.43 \times 120 \text{ m} = 172 \text{ m} \approx 170 \text{ m}$$

The plane uses 170 m of runway when towing the glider.

(b) The final speed given enables us to find the acceleration:

$$v_x^2 - v_{0x}^2 = 2a_x \Delta x$$

With $v_x = 28$ m/s, $v_{0x} = 0$, and $\Delta x = 172$ m,

$$a_x = \frac{v_x^2}{2\,\Delta x} = \frac{(28 \text{ m/s})^2}{2 \times 172 \text{ m}} = 2.28 \text{ m/s}^2$$

The tension in the cable is the only horizontal force acting on the glider. Therefore

$$\Sigma F_x = T = m_2\, a_x = 330 \text{ kg} \times 2.28 \text{ m/s}^2 = 752 \text{ N}$$

The tension in the cable is approximately 750 N.

Discussion This solution is based on a simplified model, so we can only regard the answers as approximate. Nevertheless, it illustrates Newton's second law. The same net force produces an acceleration inversely proportional to the mass of the object on which it acts. Here we have the same net force acting on two different objects: first the plane alone, then the plane and glider together.

Alternatively, we can look at forces acting only on the plane. When towing the glider, the cable pulls backward on the plane. The net force *on the plane* is smaller, so its acceleration is smaller. The smaller acceleration means that it takes more time to reach takeoff speed and travels a longer distance before lifting off the runway.

Practice Problem 3.12 Engine thrust

Neglecting air resistance, what is the thrust provided by the airplane's engines in the preceding example?

Example 3.13

Hauling a Crate Up To a Third-Floor Window

A student is moving into a dorm room on the third floor and he decides to use a block and tackle arrangement (Fig. 3.24) to move a crate of mass 91 kg from the ground up to his window. If the breaking strength of the available cable is 550 N, what is the minimum time required to haul the crate to the level of the window, 30.0 m above the ground?

Strategy The tension in the cable is T and is the same at both ends or anywhere along the cable, assuming the cable and pulleys are ideal. Two cable strands support the crate, each pulling upward with a force of magnitude T. The weight of the crate acts downward. We draw a free-body diagram and set the tension equal to the breaking force of the rope to find the maximum possible acceleration of the crate. Then from the maximum acceleration, we use kinematic relationships to find the minimum time to move the required distance to the third-floor window with that acceleration.

Given: $m = 91$ kg; $\Delta y = 30.0$ m; $g = 9.8$ m/s^2; $T_{max} = 550$ N;
$v_{0y} = 0$

To find: Δt, the minimum time to raise the crate 30.0 m

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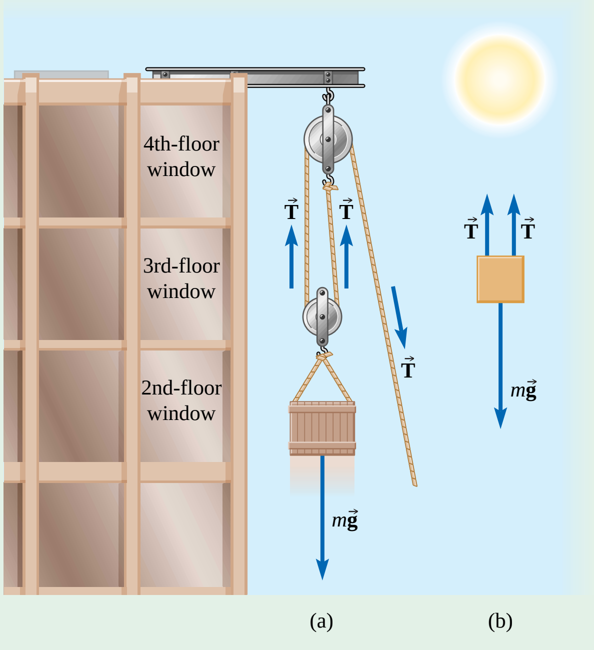

Figure 3.24
(a) Block and tackle setup and (b) free-body diagram for the crate

Example 3.13 *continued*

Solution From the free-body diagram (Fig. 3.24b), if the forces acting up are greater than the force acting down, the net force is upward and the crate's acceleration is upward. In terms of components, with the $+y$-direction chosen to be upward,

$$\Sigma F_y = T + T - mg = ma_y$$

Solving for the acceleration,

$$a_y = \frac{T + T - mg}{m}$$

Setting T = 550 N, the maximum possible value before the cable breaks, and substituting the other known values,

$$a_y = \frac{550\text{ N} + 550\text{ N} - 91\text{ kg} \times 9.8\text{ m/s}^2}{91\text{ kg}}$$

$$= 2.3\text{ m/s}^2$$

The minimum time to move the crate up a distance Δy starting from rest can be found from

$$\Delta y = v_{0y}t + \tfrac{1}{2}a_y t^2$$

Setting $v_{0y} = 0$ and solving for t, we find

$$t = \pm\sqrt{\frac{2\,\Delta y}{a_y}}$$

Our equation applies only for $t \geq 0$ (the crate reaches the window *after* it leaves the ground). Taking the positive root and substituting numerical values,

$$t = \sqrt{\frac{2 \times 30.0\text{ m}}{2.3\text{ m/s}^2}} = 5.1\text{ s}$$

This is the minimum time if a_y is the maximum acceleration.

Discussion If the crate is accelerated upward, the tension of the cable is greater than if the crate is moved at a constant velocity. For the crate to accelerate, there must be an upward net force. For motion with a constant velocity, the tension would be equal to half the weight of the crate, 450 N.

Practice Problem 3.13 Hauling the crate with a single pulley

If only a single pulley, attached to the pole above the fourth-floor, were available and if the student had a few friends to help him pull on the cable, could they haul the crate up to the third-floor window? If so, what is the minimum time required to do so?

3.6 FALLING OBJECTS

Suppose you are standing on a bridge over a deep gorge. If you drop a stone into the gorge, how fast does it fall? You know from experience that it does not fall at a constant velocity; the longer it falls, the faster it goes. A better question is: what is the stone's acceleration?

First, let us simplify the problem. If the stone were moving very fast, an appreciable force of air resistance would oppose its motion. When it is not falling so fast, air resistance is negligibly small. If air resistance is negligible, the only appreciable force is that of gravity. **Free fall** is a situation in which no forces act on an object other than the gravitational force that makes the object fall. On Earth, free fall is an idealization since there is always *some* air resistance.

What is the acceleration of an object in free fall? More massive objects are harder to accelerate: the acceleration of an object subjected to a given force is inversely proportional to its mass. However, the stronger gravitational force on a more massive object compensates for its greater inertia, giving it the same acceleration as a less massive object. The gravitational force on an object is

$$\vec{\mathbf{W}} = m\vec{\mathbf{g}}$$

From Newton's second law,

$$\vec{\mathbf{F}}_{\text{net}} = m\vec{\mathbf{g}} = m\vec{\mathbf{a}}$$

Dividing by the mass yields

$$\vec{\mathbf{a}} = \vec{\mathbf{g}} \tag{3-14a}$$

The acceleration of an object in free fall is $\vec{\mathbf{g}}$, regardless of the object's mass. Since 1 N = 1 kg·m/s^2, an object in free fall near the Earth's surface has an acceleration of magnitude

$$a = g = 9.8\,\frac{\text{N}}{\text{kg}} = 9.8\,\frac{\cancel{\text{N}}}{\cancel{\text{kg}}} \times 1\,\frac{\cancel{\text{kg}}\cdot\text{m/s}^2}{\cancel{\text{N}}} = 9.8\text{ m/s}^2$$

Thus any object in free fall near the Earth's surface has a constant downward acceleration of magnitude 9.8 m/s^2. For this reason, $\vec{\mathbf{g}}$ is sometimes called *the freefall acceleration*—the acceleration of an object near the surface of the Earth when the *only* force acting is gravity.

When dealing with vertical motion, the y-axis is usually chosen to be positive pointing upward. In two-dimensional motion, the x-axis is often used for the horizontal direction and the y-axis for the vertical direction. The direction of the acceleration is down, so in free fall

$$a_y = -g \tag{3-14b}$$

In free fall near the Earth's surface, $a_y = -g$ (if the y-axis points up).

The same techniques and equations used for other constant acceleration situations are used with free fall. The only change is that the constant acceleration in free fall is always directed toward the center of the Earth and has a known constant magnitude of approximately 9.8 m/s^2 for objects near the surface of the Earth.

Earth's gravity always pulls downward, so the acceleration of an object in free fall is always downward, regardless of whether the object is moving up, down, or is at rest. If the object is moving downward, the downward acceleration makes it speed up; if it is moving upward, the downward acceleration makes it slow down; and if it is at rest, the downward acceleration makes it start moving downward.

If an object is thrown straight up, its velocity is zero at the highest point of its flight. Why? On the way up, the y-component of its velocity v_y is positive if the positive y-axis is pointing up. On the way down, v_y is negative. Since v_y changes continuously, at a rate of 9.8 m/s^2, it must pass through zero to change sign. At the one instant of time at its highest point, the object is neither moving up nor down. The object's acceleration is *not* zero at the top of flight. If the acceleration were to suddenly become zero at the top of flight, the velocity would no longer change; the object would get *stuck* at the top rather than fall back down! The velocity is zero at the top but it does not stay zero; it is still changing at the same rate.

Example 3.14

Throwing Stones

Standing on a bridge, you throw a stone straight upward. The stone hits a stream, 44.1 m below the point at which you release it, 4.00 s later. Assume g = 9.81 m/s^2. (a) What is the speed of the stone just after it leaves your hand? (b) What is the speed of the stone just before it hits the water?

Strategy Ignoring air resistance, the acceleration is constant. Choose the positive y-axis pointing up. Let the stone be thrown at $t = 0$ and hit the stream at a later time t.

Known: $a_y = -9.81$ m/s^2; $\Delta y = -44.1$ m at $t = 4.00$ s

To find: $|v_{0y}|$ (speed at $t = 0$) and $|v_y|$ (speed at $t = 4.00$ s)

Solution (a) Equation (3-12) can be used to solve for v_{0y}, since all the other quantities in it (Δy, t, and a_y) are known.

$$\Delta y = v_{0y}t + \tfrac{1}{2}a_y t^2$$

Solving for v_{0y},

$$v_{0y} = \frac{\Delta y}{t} - \frac{1}{2}a_y t = \frac{-44.1\text{ m}}{4.00\text{ s}} - \frac{1}{2}(-9.81\text{ m/s}^2 \times 4.00\text{ s}) \tag{1}$$

$$= -11.0\text{ m/s} + 19.6\text{ m/s} = 8.6\text{ m/s}$$

The initial speed is 8.6 m/s.

(b) The change in v_y is $a_y t$ from Eq. (3-9):

$$v_y = v_{0y} + a_y t$$

Substituting the expression for v_{0y} found in Eq. (1),

$$v_y = \left(\frac{\Delta y}{t} - \frac{1}{2}a_y t\right) + a_y t \tag{2}$$

$$= \frac{\Delta y}{t} + \frac{1}{2}a_y t$$

$$= \frac{-44.1\text{ m}}{4.00\text{ s}} + \frac{1}{2}(-9.81\text{ m/s}^2 \times 4.00\text{ s})$$

$$v_y = -11.0\text{ m/s} - 19.6\text{ m/s} = -30.6\text{ m/s}$$

The final speed is 30.6 m/s.

Discussion The final speed is greater than the initial speed, as expected.

Equations (1) and (2) have a direct interpretation, which is a good check on their validity. The first term, $\Delta y/t$, is the average velocity of the stone during the 4.00 s of free fall. The second term, $\frac{1}{2}a_y t$, is *half* the change in v_y since $(v_y - v_{0y}) = a_y t$. Because the acceleration is constant, the average velocity is halfway between the initial and final velocities. Therefore, the initial velocity is the average velocity minus half of the change, while the final velocity is the average velocity plus half of the change.

Practice Problem 3.14 Height attained by stone

(a) How high above the bridge does the stone go? [*Hint:* What is v_y at the highest point?] (b) If you dropped the stone instead of throwing it, how long would it take to hit the water?

Figure 3.25 A stroboscopic photograph shows two objects falling through the air with very different terminal speeds. The exposures are taken at fixed time intervals of 1/15 s.

Air Resistance—Falling with Varying Acceleration

A skydiver relies on a parachute to provide a large drag force of air resistance. Even with the parachute closed, drag is not negligible when the skydiver is falling rapidly. The drag force exerted on a body falling through air increases dramatically with speed; it is proportional to the *square* of the speed:

$$F_\mathrm{d} = bv^2$$

where b is a constant that depends on the size and shape of the object. The direction of the drag force is opposite to the direction of motion.

Since the drag force increases as the speed increases, a falling object may eventually reach equilibrium when the drag force is equal in magnitude to the weight. The speed at which the drag force is equal in magnitude to the weight is called the object's **terminal speed**. As the speed gets near the terminal speed, the acceleration gets smaller and smaller. The acceleration is zero when the object falls at its terminal speed.

In Fig. 3.25, a baseball and a coffee filter are released from rest and fall through the air. The strobe photograph shows the positions of the two at equal time intervals. The baseball has a terminal velocity of about 40 m/s, so air resistance is negligible for the speeds shown in the photo. The displacement of the baseball in equal time intervals increases linearly, showing that its acceleration is constant. The coffee filter has a very large surface area for its small mass. As a result, its terminal speed is much smaller—about 1 m/s. The displacement of the filter barely changes from one strobe flash to the next, showing that it is falling at a small, nearly constant velocity.

At terminal speed v_t, the drag force is equal in magnitude to the weight. Therefore, $F_\mathrm{d} = mg = bv_\mathrm{t}^2$ and

$$b = \frac{mg}{v_\mathrm{t}^2}$$

Therefore, at any speed v,

$$F_\mathrm{d} = mg\,\frac{v^2}{v_\mathrm{t}^2} \tag{3-15}$$

The terminal speed of an object depends on its size, shape, and mass (see Table 3.3). A skydiver with the parachute closed will reach a terminal speed of about 50 m/s (≈110 mi/h) in the spread-eagle position or as much as 100 m/s (≈220 mi/h) in a dive. When the parachute is opened, the drag force increases dramatically—the larger surface area of the parachute means that more air has to be pushed out of the way. The terminal speed with the parachute open is typically about 9 m/s (20 mi/h). When the parachute is opened, the skydiver is initially moving *faster* than the new terminal speed. For $v > v_t$, the drag force is larger in magnitude than the weight and the acceleration is *upward.* The skydiver slows down, approaching the new terminal speed. Note that the terminal speed is not the maximum possible speed; it is the speed that the falling object approaches, regardless of initial conditions, when the only forces acting are drag and gravity.

Table 3.3

Some Typical Terminal Speeds

Object	Terminal Speed (m/s)
Feather	0.5
Snowflake	1
Raindrop	7
Skydiver (open parachute)	5–9
Basketball	20
Baseball	40
Skydiver (spread-eagle)	50–60
Skydiver (dive)	100
Bullet	100

Example 3.15

Skydivers Falling Freely

Two skydivers have identical parachutes. Their masses (including parachutes) are 62.0 kg and 82.0 kg. Which of the skydivers has the larger terminal speed? What is the ratio of their terminal speeds?

Strategy With identical parachutes, we expect the same amount of drag at a given speed. The more massive skydiver must fall faster in order for the drag force to equal his weight, so the 82.0-kg skydiver should have a larger terminal speed. For the ratio of the terminal speeds, we first find how the terminal speed depends on mass, all other things being equal. Then we work by proportions.

Solution At terminal speed v_t, the drag force must be equal in magnitude to the weight.

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Example 3.15 *continued*

$$mg = F_d = bv_t^2$$

Since the parachutes are identical, we expect the constant b to be the same for the two divers. Therefore,

$$v_t \propto \sqrt{m}$$

The more massive skydiver has a larger terminal speed—he must move faster in order for the drag force to equal his larger weight. The ratio of the terminal speeds is

$$\frac{v_{t2}}{v_{t1}} = \sqrt{\frac{m_2}{m_1}} = \sqrt{\frac{82.0}{62.0}} = 1.15$$

The terminal speed of the 82-kg diver is 1.15 times that of the less massive skydiver, or 15% faster.

Discussion The 82.0-kg skydiver is 32% more massive:

$$\frac{82.0 \text{ kg}}{62.0 \text{ kg}} = 1.32$$

but his terminal speed is only 15% greater. That is because the drag force is proportional to the *square* of the speed. It only takes a 15% greater speed to make the drag force 32% greater:

$$(1.15)^2 = 1.32$$

Practice Problem 3.15 Air resistance at terminal speed

A pilot has bailed out of her airplane at a height of 2000 m above the surface of the Earth. The mass of pilot plus parachute is 112 kg. What is the force of air resistance when the pilot reaches terminal speed?

Example 3.16

Dropping the Ball

A basketball is dropped off a tall building. (a) What is the initial acceleration of the ball, just after it is released? (b) What is the acceleration of the ball when it is falling at its terminal speed? (c) What is the acceleration of the ball when falling at half its terminal speed?

Strategy We choose the positive y-axis to point upward as usual. The ball is dropped from rest so *initially* the only force acting is gravity—the drag force is zero when the velocity is zero. Once the ball is moving, air drag contributes to the net force on the basketball.

Solution (a) The initial acceleration is the free-fall acceleration ($\vec{\mathbf{a}} = \vec{\mathbf{g}}$) since the drag force is zero.

(b) Once the ball reaches terminal speed, the drag force is equal in magnitude to the weight of the ball but acts in the opposite direction. The net force on the ball is zero, so the acceleration is zero. At terminal speed, $\vec{\mathbf{a}} = \vec{\mathbf{0}}$.

(c) When the ball is falling at half its terminal speed, the drag force is significant but it is smaller than the weight. The net force is down and therefore the acceleration is still downward, though with a smaller magnitude. The drag force at any speed is given by

$$F_d = mg\frac{v^2}{v_t^2}$$

and this drag force acts in the opposite direction to the weight; it acts upward.

The net vertical force is

$$\Sigma F_y = F_d - mg = mg\frac{v^2}{v_t^2} - mg = mg\left(\frac{v^2}{v_t^2} - 1\right)$$

Now we apply Newton's second law.

$$\Sigma F_y = ma_y$$

Solving for the acceleration yields

$$a_y = g\left(\frac{v^2}{v_t^2} - 1\right)$$

continued on next page

Example 3.16 *continued*

At a time when the velocity is at half the terminal speed,

$$v = \frac{1}{2}v_t \quad \text{and} \quad \frac{v^2}{v_t^2} = \frac{1}{4}$$

$$a_y = g\left(\tfrac{1}{4} - 1\right) = -\tfrac{3}{4}g$$

so that the acceleration of the ball is

$$\vec{\mathbf{a}} = \tfrac{3}{4}\vec{\mathbf{g}}$$

where $\vec{\mathbf{a}}$ and $\vec{\mathbf{g}}$ both point downward.

Discussion How do we know when air resistance is negligible? If we know the approximate terminal speed of an object, then air resistance is negligible as long as its speed moving through the air is small compared to the terminal speed.

Practice Problem 3.16 Acceleration graph sketch

Sketch a qualitative graph of $v_y(t)$ for the basketball using a y-axis that is positive pointing upward. [*Hint:* At first air resistance is negligible. After a long time the basketball is in equilibrium. Figure out what the beginning and end of the graph look like and then connect them smoothly.]

Physics at Home

Go to a balcony or climb up a ladder and drop a basket-style paper coffee filter (or a cupcake paper) and a penny simultaneously. Air resistance on the penny is negligible unless it is dropped from a very high balcony. At the other extreme, the effect of air resistance on the coffee filter is very noticeable; it reaches its terminal speed almost immediately. Stack several (two to four) coffee filters together and drop them simultaneously with a single coffee filter. Why is the terminal speed higher for the stack? Crumple a coffee filter into a ball and drop it simultaneously with the penny. Air resistance on the coffee filter is now reduced, but still noticeable.

3.7 APPARENT WEIGHT

Imagine being in an elevator when the cable snaps. Assume that some safety mechanism brings you to rest after you have been in free fall for a while. While you are in free fall, you *seem* to be "weightless," but your weight has not changed; the Earth still pulls downward with the same gravitational force. In free fall, gravity makes the elevator and everything in it accelerate downward at 9.8 m/s^2. The floor of the elevator stops pushing up on you, as it does when the elevator is at rest. If you jump up from the elevator floor, you seem to "float" up to the ceiling of the elevator. Your *weight* hasn't changed, but your **apparent weight** is zero while you are in free fall.

You don't need a disastrous elevator mishap to notice an apparent weight that differs from your true weight. A normally operating elevator will do quite nicely. Step in the elevator and push a button for a higher floor. When the elevator accelerates upward, you can feel your apparent weight increase. When the elevator slows down to stop, the elevator's acceleration is downward and your apparent weight is less than your true weight.

What is happening in the body while the elevator accelerates? Blood tends to collect in the lower extremities during acceleration upward and in the upper body during acceleration downward. The internal organs shift position within the body cavity resulting in a funny feeling in the gut as the elevator starts and stops. To avoid this problem, high-speed express elevators in skyscrapers keep the acceleration relatively small, but maintain that acceleration long enough to reach high speeds. That way, the elevator can travel quickly to the upper floors without making the passengers feel too uncomfortable.

Imagine an object resting on a bathroom scale. The scale measures the object's *apparent* weight W', which is equal to the true weight only if the object has zero acceleration. Newton's second law requires that

$$\vec{\mathbf{F}}_{net} = \vec{\mathbf{N}} = m\vec{\mathbf{g}} = m\vec{\mathbf{a}}$$

where $\vec{\mathbf{N}}$ is the normal force of the scale pushing up. The apparent weight is the reading of the scale—the magnitude of $\vec{\mathbf{N}}$:

$$W' = |\vec{\mathbf{N}}| = N$$

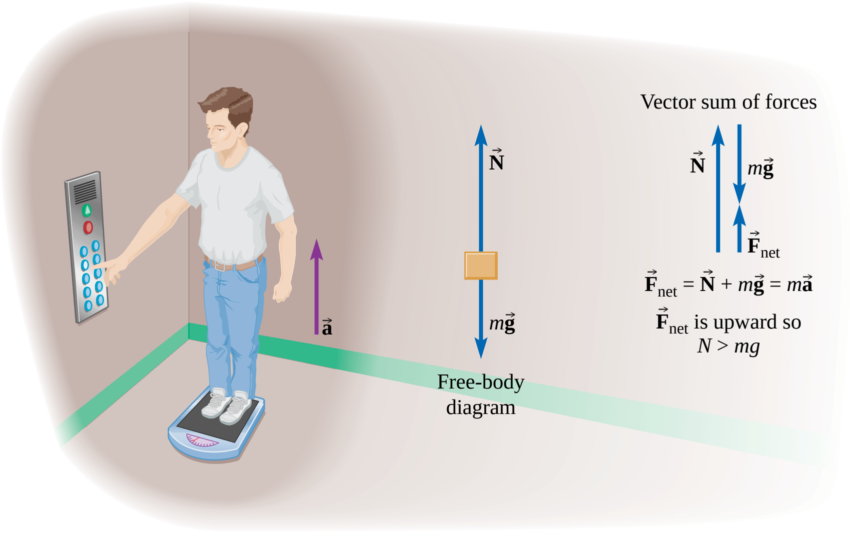


Figure 3.26 (a) Apparent weight in an elevator with acceleration upward (b) Free-body diagram for the passenger (c) Normal force must be greater than the weight to have an upward net force.

In Fig. 3.26a, the acceleration of the elevator is upward. The normal force $\vec{N}$ must be larger than the weight $m\vec{g}$ in order for the net force to be upward (Fig. 3.26c). Writing the forces in component form where the $+y$-direction is upward,

$$\Sigma F_y = N - mg = ma_y$$

or

$$N = mg + ma_y$$

Therefore,

$$W' = N = m(g + a_y) \tag{3-16}$$

Since the elevator accelerates upward, $a_y > 0$; the apparent weight is greater than the true weight (Fig. 3.26c).

In Fig. 3.27a, the acceleration is downward. Then the net force must also point downward. The normal force is still upward, but it must be smaller than the weight in order to produce a downward net force (Fig. 3.27c). It is still true that

$$W' = m(g + a_y)$$

but now the acceleration is downward ($a_y < 0$). The apparent weight is less than the true weight.

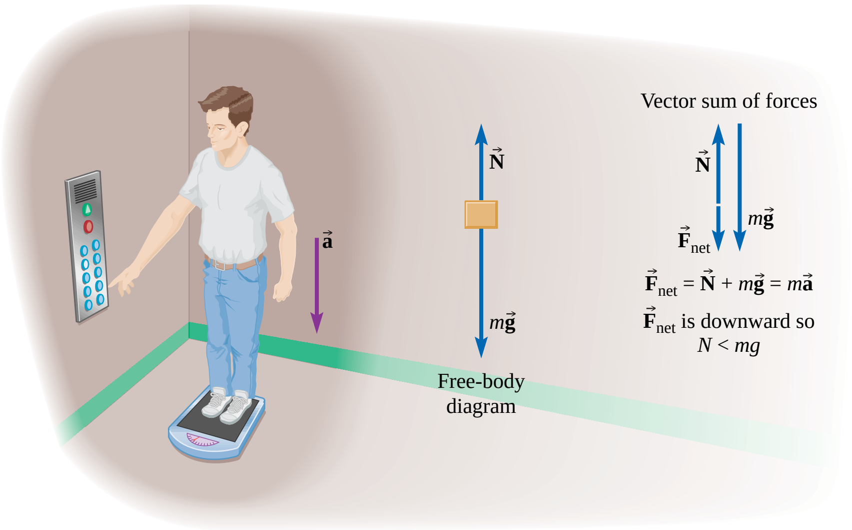


Figure 3.27 (a) Apparent weight in an elevator with acceleration down (b) Free-body diagram for the passenger (c) Normal force must be less than the weight to have a downward net force.

In both these cases—and in general—the apparent weight is given by

$$W' = m|\vec{\mathbf{g}} - \vec{\mathbf{a}}| \qquad (3\text{-}17)$$

where $\vec{\mathbf{g}}$ and $\vec{\mathbf{a}}$ are both vector quantities. When $\vec{\mathbf{g}}$ and $\vec{\mathbf{a}}$ point in opposite directions, the magnitude $|\vec{\mathbf{g}} - \vec{\mathbf{a}}|$ is greater than it is when they point in the same direction. If the acceleration of an elevator is upward, then $\vec{\mathbf{g}}$ and $\vec{\mathbf{a}}$ have opposite directions and the apparent weight is greater than the true weight. In an elevator with a downward acceleration, $\vec{\mathbf{g}}$ and $\vec{\mathbf{a}}$ have the same direction and the apparent weight is less than the true weight. If an elevator is in free fall, then $\vec{\mathbf{g}}$ and $\vec{\mathbf{a}}$ are in the same direction and are equal in magnitude; in free fall the apparent weight is zero. Draw the vectors and perform the vector subtraction to satisfy yourself that Eq. (3-17) is correct.

Physics at Home (or in the Garden):

Take an empty half-gallon paper milk carton or a plastic milk jug and poke one hole in the bottom and another in the side of the carton with a pencil. Fill the carton with water while sealing the holes with your fingers so the water does not pour out. Throw the carton straight up into the air and watch what happens at the holes. Does water start to pour out of the unsealed holes as the carton ascends? What about when the carton is falling downward? Can you explain your observations using the ideas of free fall and apparent weight?

Example 3.17

Apparent Weight in an Elevator

A passenger weighing 598 N rides in an elevator. The gravitational field strength is 9.80 N/kg. What is the apparent weight of the passenger in each of the following situations? In each case, the magnitude of the elevator's acceleration is 0.500 m/s^2. (a) The passenger is on the 1st floor and has pushed the button for the 15th floor; the elevator is beginning to move upward. (b) The elevator is slowing down as it nears the 15th floor.

Strategy Let the $+y$-axis be upward. The apparent weight is equal to the magnitude of the normal force exerted by the floor on the passenger. Newton's second law lets us find the normal force from the weight and the acceleration.

Given: $W = 598$ N; magnitude of the acceleration is $a = 0.500$ m/s^2

Find: W'

Solution (a) When the elevator starts up from the first floor it accelerates in the upward direction as its speed increases. Since the elevator accelerates upward, $a_y > 0$ (as in Fig. 3.26). We expect the apparent weight $W' = N$ to be greater than the true weight—the floor must push up with a force greater than W to cause an upward acceleration.

$$\Sigma F_y = N - W = ma_y$$

Since $m = W/g$,

$$W' = N = W + ma_y = W + \frac{W}{g}a_y = W\left(1 + \frac{a_y}{g}\right)$$

$$= 598\ \text{N} \times \left(1 + \frac{0.500\ \text{m/s}^2}{9.80\ \text{m/s}^2}\right) = 629\ \text{N}$$

(b) When the elevator approaches the 15th floor, it slows down while still moving upward; its acceleration is downward ($a_y < 0$) as in Fig. 3.27. The apparent weight is less than the true weight. Again, $\sum F_y = N - W = ma_y$, but this time $a_y = -0.500$ m/s^2.

$$N = W\left(1 + \frac{a_y}{g}\right)$$

$$= 598\ \text{N} \times \left(1 + \frac{-0.500\ \text{m/s}^2}{9.80\ \text{m/s}^2}\right) = 567\ \text{N}$$

Discussion The apparent weight is greater when the direction of the elevator's acceleration is upward. That can happen in two cases: either the elevator is moving up with increasing speed, or it is moving down with decreasing speed.

Practice Problem 3.17 Elevator descending

What is the apparent weight of a passenger of mass 42.0 kg traveling in an elevator in each of the following situations? The gravitational field strength is 9.80 N/kg. In each case, the magnitude of the elevator's acceleration is 0.460 m/s^2. (a) The passenger is on the 15th floor and has pushed the button for the 1st floor; the elevator is beginning to move downward. (b) The elevator is slowing down as it nears the 1st floor.

MASTER THE CONCEPTS

Summary

- Position (symbol $\vec{\mathbf{r}}$) is a vector from the origin to an object's location. Its magnitude is the distance from the origin and its direction points from the origin to the object.
- Displacement is the change in position: $\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_f - \vec{\mathbf{r}}_0$. The displacement depends only on the starting and ending positions, not on the path taken. The magnitude of the displacement vector is not necessarily equal to the total distance traveled; it is the straight line distance from the initial position to the final position.
- Vector components: If $\vec{\mathbf{A}}$ points along the $+x$-axis, then $A_x = +A$; if $\vec{\mathbf{A}}$ points in the opposite direction—along the negative x-axis—then $A_x = -A$. When adding or subtracting vectors, we can add or subtract their components.
- The average velocity states at what constant speed and in what direction to travel to cause that same displacement in the same amount of time.

$$\vec{\mathbf{v}}_{av} = \frac{\Delta\vec{\mathbf{r}}}{\Delta t}$$

- Velocity is a vector that states how fast and in what direction something moves. Its direction is the direction of the object's motion and its magnitude is the instantaneous speed.

$$\vec{\mathbf{v}} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\mathbf{r}}}{\Delta t} \tag{3-4}$$

- Average acceleration is the constant acceleration that would give the same velocity change in the same amount of time. In terms of changes in velocity and time,

$$\vec{\mathbf{a}}_{av} = \frac{\Delta\vec{\mathbf{v}}}{\Delta t} \tag{3-6}$$

- Acceleration is the instantaneous rate of change of velocity:

$$\vec{\mathbf{a}} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\mathbf{v}}}{\Delta t} \tag{3-7}$$

  Acceleration does not necessarily mean speeding up. A velocity can also change by decreasing speed or by changing directions.
- Interpreting graphs: On a graph of $x(t)$, the slope at any point is v_x. On a graph of $v_x(t)$, the slope at any point is a_x and the area under the graph during any time interval is the displacement during that time interval. If v_x is negative, the displacement is also negative, so we must count the area as negative when it is below the time axis. On a graph of $a_x(t)$, the area under the curve is the change in v_x during that time interval.
- Newton's second law:

$$\vec{\mathbf{F}}_{net} = m\vec{\mathbf{a}} \tag{3-8}$$

- The SI unit of force is the newton; 1 N = 1 kg·m/s^2. One newton is the magnitude of net force that gives a 1-kg object an acceleration of magnitude 1 m/s^2.
- Essential relationships for solving any constant acceleration problem: if a_x is constant during the entire time interval from $t = 0$ until a later time t,

$$\Delta v_x = v_x - v_{0x} = a_x t \tag{3-9}$$

$$\Delta x = x - x_0 = v_{av,x} t \tag{3-10}$$

$$v_{av,x} = \frac{v_{0x} + v_x}{2} \tag{3-11}$$

$$\Delta x = x - x_0 = v_{0x}t + \tfrac{1}{2}a_x t^2 \tag{3-12}$$

$$v_x^2 - v_{0x}^2 = 2a_x \Delta x \tag{3-13}$$

Highlighted Figures and Tables

F3.1 Displacement is the change in position: $\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_f - \vec{\mathbf{r}}_0$ (p. 64)

F3.5 Displacement and average velocity (p. 67)

F3.6 As Δt approaches zero, the average velocity during the increasingly short time interval approaches the instantaneous velocity (p. 68)

F3.7 Graphical relationship between position and velocity (p. 69)

F3.9 Displacement Δx is the area under the v_x versus time graph for the time interval considered (p. 70)

F3.14 Mass versus weight (p. 76)

F3.18 Finding average velocity with the aid of a graph (p. 79)

F3.19 Graphical interpretation of Eq. 3-12 (p. 80)

F3.20 Visualizing motion with constant acceleration (p. 81)

T3.3 Some typical terminal speeds (p. 88)

CONCEPTUAL QUESTIONS

1. Explain the difference between distance traveled, displacement, and displacement magnitude.
2. Explain the difference between speed and velocity.
3. On a graph of v_x versus time, what quantity does the area under the graph represent?
4. On a graph of v_x versus time, what quantity does the slope of the graph represent?

5. On a graph of a_x versus time, what quantity does the area under the graph represent?
6. On a graph of x versus time, what quantity does the slope of the graph represent?
7. What is the relationship between average velocity and instantaneous velocity? An object can have different instantaneous velocites at different times. Can the same object have different average velocities? Explain.
8. If an object is traveling at a constant velocity, is it necessarily traveling in a straight line? Explain.
9. Can the average speed and the magnitude of the average velocity ever be equal? If so, under what circumstances?
10. If a feather and a lead brick are dropped simultaneously from the top of a ladder, the lead brick hits the ground first. What would happen if the experiment is repeated on the surface of the moon?
11. Name a situation where the speed of an object is constant while the velocity is not.
12. Can the velocity of an object be zero and the acceleration be nonzero at the same time? Explain.
13. Why does a 1-kg sandbag fall with the same acceleration as a 5-kg sandbag? Explain in terms of Newton's second law and his law of gravitation.
14. If an object is acted on by a single constant force, is it possible for the object to remain at rest? Is it possible for the object to move with constant velocity? Is it possible for the object's speed to be decreasing? Is it possible for it to change direction?
15. If an object is acted on by two constant forces is it possible for the object to move at constant velocity? If so, what must be true about the two forces?
16. An object is placed on a scale. Under what conditions does the scale read something other than the object's weight, even though the scale is functioning properly and is calibrated correctly? Explain.
17. What is meant by the terminal speed of a falling object? Can an object ever move through air faster than the object's terminal speed? If so, give an example.
18. What force(s) act on a parachutist descending to Earth with a constant velocity? What is the acceleration of the parachutist?
19. A baseball is tossed straight up. Taking into consideration the force of air resistance, is the magnitude of the baseball's acceleration zero, less than g, equal to g, or greater than g on the way up? At the top of the flight? On the way down? Explain.
20. What is the acceleration of an object thrown straight up into the air at the highest point of its motion? Does the answer depend on whether air resistance is negligible or not? Explain.
21. You are bicycling along a straight north-south road. Let the x-axis point north. Describe your motion in each of the following cases. Example: $a_x > 0$ and $v_x > 0$ means you are moving north and speeding up. (a) $a_x > 0$ and $v_x < 0$. (b) $a_x = 0$ and $v_x < 0$. (c) $a_x < 0$ and $v_x = 0$. (d) $a_x < 0$ and $v_x < 0$. (e) Based on your answers, explain why it is not a good idea to use the expression "negative acceleration" to mean slowing down.

MULTIPLE CHOICE QUESTIONS

1. A go-kart travels around a circular track at a constant speed. Which of these is a true statement?
   (a) The go-kart has a constant velocity.
   (b) The go-kart has zero acceleration.
   (c) Both (a) and (b) are true.
   (d) Neither (a) nor (b) is true.
2. A ball is thrown straight up into the air. Neglect air resistance. While the ball is in the air its acceleration
   (a) increases (b) is zero (c) remains constant
   (d) decreases on the way up and increases on the way down
   (e) changes direction
3. A stone is thrown upward and reaches a height Δy. After an elapsed time Δt, measured from the time the stone was first thrown, the stone has fallen back down to the ground. The magnitude of the average velocity of the stone during this time is
   (a) zero (b) $2\dfrac{\Delta y}{\Delta t}$ (c) $\dfrac{\Delta y}{\Delta t}$ (d) $\dfrac{1}{2}\dfrac{\Delta y}{\Delta t}$
4. A stone is thrown upward and reaches a height Δy. After an elapsed time Δt, measured from the time the stone was first thrown, the stone has fallen back down to the ground. The average speed of the stone during this time is
   (a) zero (b) $2\dfrac{\Delta y}{\Delta t}$ (c) $\dfrac{\Delta y}{\Delta t}$ (d) $\dfrac{1}{2}\dfrac{\Delta y}{\Delta t}$
5. A ball is thrown straight up. At the top of its trajectory the ball is
   (a) instantaneously at rest.
   (b) instantaneously in equilibrium.
   (c) Both (a) and (b) are true.
   (d) Neither (a) nor (b) is true.

Multiple Choice Questions 6–15 refer to Fig. 3.28.

6. ◆ What distance does the jogger travel during the first 10.0 min ($t = 0$ to 10 min)?
   (a) 8.5 m (b) 510 m (c) 900 m (d) 1020 m
7. What is the displacement of the jogger from $t = 18.0$ min to $t = 24.0$ min?
   (a) 720 m, south (b) 720 m, north
   (c) 2160 m, south (c) 3600 m, north
8. What is the displacement of the jogger for the entire 30.0 min?
   (a) 3120 m, south (b) 2400 m, north
   (c) 2400 m, south (d) 3840 m, north
9. What is the total distance traveled by the jogger in 30.0 min?
   (a) 3840 m (b) 2340 m (c) 2400 m (d) 3600 m
10. What is the average velocity of the jogger during the 30.0 min?
    (a) 1.3 m/s, north (b) 1.7 m/s, north
    (c) 2.1 m/s, north (d) 2.9 m/s, north

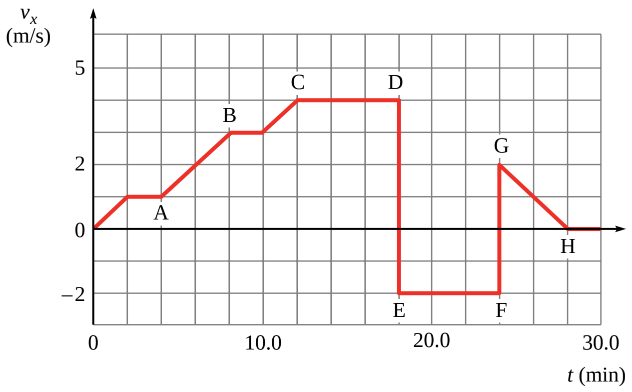


Figure 3.28 Multiple Choice Questions 6–15. A jogger is exercising along a long, straight road that runs north-south. She starts out heading north.

11. What is the average speed of the jogger for the 30 min?
    (a) 1.4 m/s (b) 1.7 m/s (c) 2.1 m/s (d) 2.9 m/s
12. In what direction is she running at time $t = 20$ min?
    (a) south (b) north (c) not enough information
13. In which region of the graph is a_x positive?
    (a) A to B (b) C to D (c) E to F (d) G to H
14. In which region is a_x negative?
    (a) A to B (b) C to D (c) E to F (d) G to H
15. In which region is the velocity directed to the south?
    (a) A to B (b) C to D (c) E to F (d) G to H

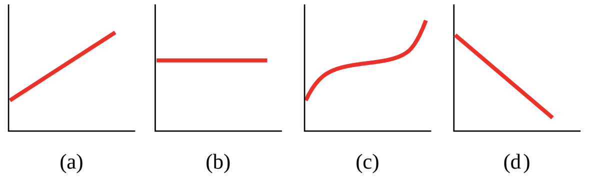


Figure 3.29 Multiple Choice Questions 16–20

Multiple Choice Questions 16–20 refer to Fig. 3.29.

16. If Fig. 3.29 shows four graphs of x versus time, which graph shows a constant, positive, nonzero velocity?
17. If Fig. 3.29 shows four graphs of v_x versus time, which graph shows a constant velocity?
18. If Fig. 3.29 shows four graphs of v_x versus time, which graph shows a_x constant and positive?
19. If Fig. 3.29 shows four graphs of v_x versus time, which graph shows a_x constant and negative?
20. If Fig. 3.29 shows four graphs of v_x versus time, which graph shows a changing a_x that is always positive?
21. Two blocks are connected by a light string passing over a pulley [see Fig. 3.23a]. The block with mass m_1 slides on the frictionless horizontal surface, while the block with mass m_2 hangs vertically. ($m_1 > m_2$.) The tension in the string is:
    (a) zero (b) less than m_2g
    (c) equal to m_2g (d) greater than m_2g, but less than m_1g
    (e) equal to m_1g (f) greater than m_1g

PROBLEMS

Note: Ⓒ indicates a combination conceptual/quantitative problem. Gold diamonds ◆, ◆◆ are used to indicate the increasing level of difficulty of each problem. Problem numbers appearing in blue, 9., denote problems that have a detailed solution available in the Student Solutions Manual. Some problems are *paired* by concept; their numbers are connected by a ruled box.

3.1 Position and Displacement; 3.2 Velocity

1. Two cars, a Porsche and a Honda, are traveling in the same direction, although the Porsche is 186 m behind the Honda. The speed of the Porsche is 24.4 m/s and the speed of the Honda is 18.6 m/s. How much time does it take for the Porsche to catch the Honda? [*Hint:* What must be true about the displacements of the two cars when they meet?]
2. To get to a concert in time, a harpsichordist has to drive 121 mi in 2.00 h. If he drove at an average speed of 55 mi/h for the first 1.20 h, what must be his average speed for the remaining 0.80 h?
3. Figure 3.30 shows the vertical velocity component of an elevator versus time. How high is the elevator above the starting point ($t = 0$ s) after 20 s have elapsed?
4. Figure 3.30 shows the vertical velocity component of an elevator versus time. If the elevator starts to move at $t = 0$ s, when is the elevator at its highest location above the starting point?

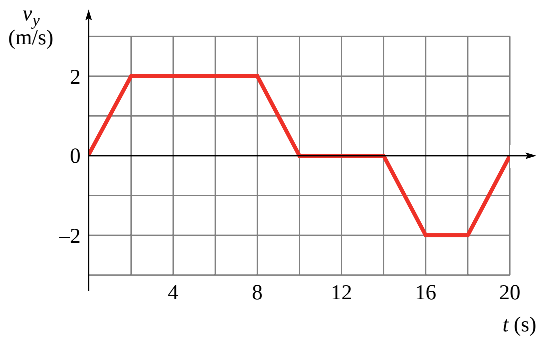


Figure 3.30 Problems 3 and 4

5. Figure 3.31 shows a graph of speedometer readings obtained as a car comes to a stop along a straight-line path. How far does the car move between $t = 0$ and $t = 16$ s?
6. Figure 3.32 shows a graph of speedometer readings, in meters per second (on the vertical axis), obtained as a car travels along a straight-line path. How far does the car move between $t = 3.00$ s and $t = 8.00$ s?

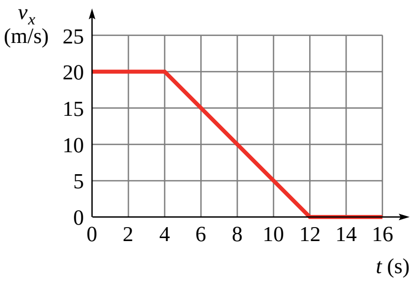


Figure 3.31 Problems 5, 17, and 79

Figure 3.32 Problems 6, 7, and 8.

7. Figure 3.32 shows a graph of $x(t)$ in meters, on the vertical axis, for an object traveling in a straight line. (a) What is $v_{av,x}$ for the interval from $t = 0$ to $t = 4.0$ s? (b) from $t = 0$ to $t = 5.0$ s?
8. Figure 3.32 shows a graph of $x(t)$ in meters for an object traveling in a straight line. What is v_x at $t = 2.0$ s?
9. A rabbit, nervously trying to cross a road, first moves 80 cm to the right, then 30 cm to the left, then 90 cm to the right, and then 310 cm to the left. (a) What is the rabbit's total displacement? (b) If the elapsed time was 18 s, what was the rabbit's average speed? (c) What was its average velocity?
10. An object is moving along a straight line. The graph in Fig. 3.33 shows its position from the starting point as a function of time. (a) In which section(s) of the graph does the object have the highest speed? (b) At which time(s) does the object reverse its direction of motion? (c) How far does the object move from $t = 0$ to $t = 3$ s?
11. A motor scooter travels east at a speed of 12 m/s. The driver then reverses direction and heads west at 15 m/s. What was the change in velocity of the scooter? Give magnitude and direction.

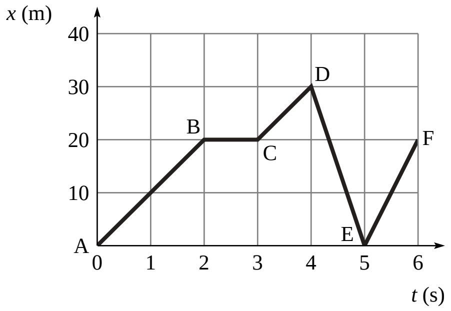


Figure 3.33 Problem 10

3.3 Acceleration

12. If a car traveling at 28 m/s is brought to a full stop in 4.0 s after the brakes are applied, find the average acceleration during braking.
13. If a pronghorn antelope accelerates from rest in a straight line with a constant acceleration of 1.7 m/s^2, how long does it take for the antelope to reach a speed of 22 m/s?
14. An airtrack glider, 8.0 cm long, blocks light as it goes through a photocell gate (Fig. 3.34). The glider is released from rest on a frictionless inclined track and the gate is positioned so that the glider has traveled 96 cm when it is in the middle of the gate. The timer gives a reading of 333 ms for the glider to pass through this gate. Friction is negligible. What is the acceleration (assumed constant) of the glider along the track?
15. Figure 3.35 shows a graph of v_x versus t for a body moving along a straight line. (a) What is a_x at $t = 11$ s? (b) What is a_x at $t = 3$ s? (c) How far does the body travel from $t = 12$ to $t = 14$ s?
16. Figure 3.36 shows a plot of $v_x(t)$ for a car. (a) What is $a_{av,x}$ between $t = 6$ s and $t = 11$ s? (b) What is $v_{av,x}$ for the same time interval? (c) What is $v_{av,x}$ for the interval $t = 0$ to $t = 20$ s? (d) What is the increase in the car's speed between 10 and 15 s? (e) How far does the car travel from time $t = 10$ s to time $t = 15$ s?
17. Figure 3.31 shows a graph of speedometer readings as a motorcycle comes to a stop. What is the magnitude of the acceleration at $t = 7.0$ s?
18. At 3:00 P.M., a bank robber is spotted driving north on I-15 at milepost 126. His speed is 112.0 mi/h. At 3:37 P.M. he is spotted at milepost 185 doing 105.0 mi/h. During this time interval, what are the bank robber's displacement, average velocity, and average acceleration? (Assume a straight highway.)

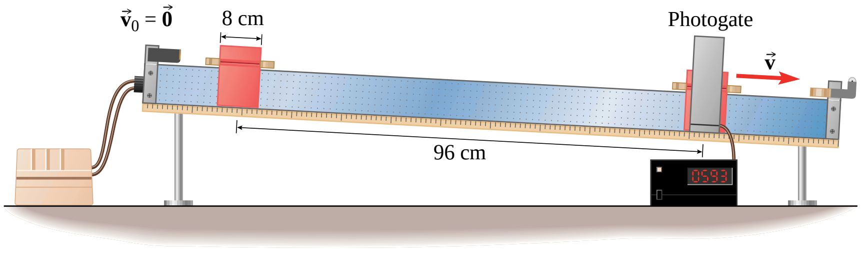


Figure 3.34 Problem 14

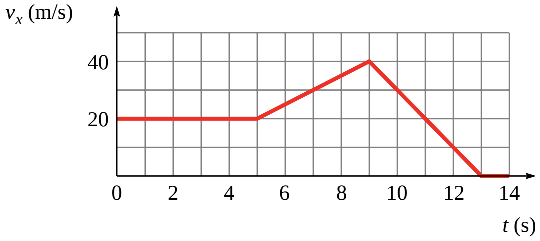


Figure 3.35 Problems 15, 33, and 34

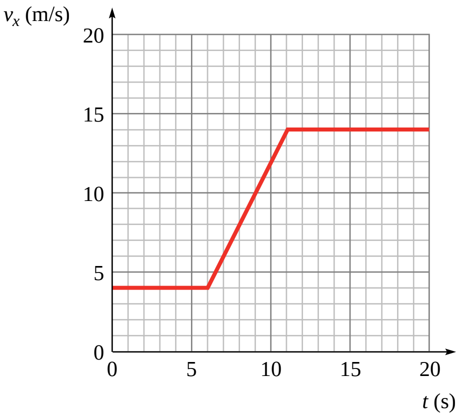


Figure 3.36 Problem 16

3.4 Newton's Second Law: Force and Acceleration

19. An engine accelerates a train of 20 freight cars, each having a mass of 5.0×10^4 kg, from rest to a speed of 4.0 m/s in 20.0 s on a straight track. Neglecting friction and assuming constant acceleration, what is the force with which the 10th car pulls the 11th one (at the middle of the train)?
20. In Fig. 3.16a, two blocks are connected by a lightweight, flexible cord that passes over a frictionless pulley. (a) If m_1 = 3.0 kg and m_2 = 5.0 kg, what are the accelerations of each block? (b) What is the tension in the cord?
21. A 2.0-kg toy locomotive is pulling a 1.0-kg caboose. The frictional force of the track on the caboose is 0.50 N backward along the track. If the train is accelerating forward at 3.0 m/s^2, what is the magnitude of the force exerted by the locomotive on the caboose?
22. A 2010-kg elevator accelerates upward at 1.5 m/s^2. What is the tension in the cable that supports the elevator?
23. A 2010-kg elevator accelerates downward at 1.5 m/s^2. What is the tension in the cable that supports the elevator?
24. A model sailboat is slowly sailing west across a pond. A gust of wind gives the sailboat a constant acceleration of magnitude 0.30 m/s^2 during a time interval of 2.0 s. If the net force on the sailboat during the 2.0-s interval has magnitude 0.375 N, what is the sailboat's mass?
25. The vertical component of the acceleration of a sailplane is zero when the air pushes up against its wings with a force of 3.0 kN. (a) Assuming that the only forces on the glider are that due to gravity and that due to the air pushing against its wings, what is the gravitational force on the Earth due to the glider? (b) If the wing stalls and the upward force decreases to 2.0 kN, what is the acceleration of the glider?
26. A man lifts a 2.0-kg stone vertically with his hand at a constant velocity of 1.5 m/s. What is the force exerted by his hand on the stone?
27. A man lifts a 2.0-kg stone vertically with his hand at a constant upward *acceleration* of 1.5 m/s^2. What is the magnitude of the total force of the stone on the man's hand?
28. ✦ A crate of oranges weighing 180 N rests on a flatbed truck 2.0 m from the back of the truck. The coefficients of friction between the crate and the bed are μ_s = 0.30 and μ_k = 0.20. The truck drives on a straight, level highway at a constant 8.0 m/s. (a) What is the force of friction acting on the crate? (b) If the truck speeds up with an acceleration of 1.0 m/s^2, what is the force of friction on the crate? (c) What is the maximum acceleration the truck can have without the crate starting to slide?

3.5 Motion with Constant Acceleration

29. A trolley car in New Orleans starts from rest at the St. Charles Street stop and accelerates uniformly at 1.20 m/s^2 for 12.0 s. (a) How far has the train traveled at the end of the 12.0 s? (b) What is the speed of the train at the end of the 12.0 s?
30. A train, traveling at a constant speed of 22 m/s, comes to an incline with a constant slope. While going up the incline, the train slows down with a constant acceleration of magnitude 1.4 m/s^2. (a) What is the speed of the train after 8.0 s on the incline? (b) How far has the train traveled up the incline after 8.0 s?
31. A car is speeding up and has an instantaneous speed of 10.0 m/s when a stopwatch reads 10.0 s. It has a constant acceleration of 2.0 m/s^2. (a) What change in speed occurs between t = 10.0 s and t = 12.0 s? (b) What is the speed when the stopwatch reads 12.0 s?
32. A train is traveling south at 24.0 m/s when the brakes are applied. It slows down at a constant rate to a speed of 6.00 m/s in a time of 9.00 s. (a) What is the acceleration of the train during the 9.00 s interval? (b) How far does the train travel during the 9.00 s?
33. Figure 3.35 shows the graph of v_x versus time for a body moving along the x-axis. How far does the body go between t = 9.0 s and t = 13.0 s? Solve using two methods: a graphical analysis and an algebraic solution using kinematic equations.
34. Figure 3.35 shows the graph of v_x versus time for a body moving along the x-axis. What is the average acceleration between t = 5.0 s and t = 9.0 s? Solve using two methods: a graphical analysis and an algebraic solution using kinematic equations.
35. An airplane lands and starts down the runway at a southwest velocity of 55 m/s. What constant acceleration allows it to come to a stop in 1.0 km?
36. ✦ The minimum stopping distance of a car moving at 30.0 mi/h is 12 m. Under the same conditions (so that the maximum braking force is the same), what is the minimum stopping distance for 60.0 mi/h? Work by proportions to avoid converting units.

37. Derive Eq. (3-12) using these steps. (a) Start with $\Delta x = v_{av,x}t$. Rewrite this equation with the average velocity expressed in terms of the initial and final velocities. (b) Replace the final velocity with the initial velocity plus the change in velocity ($v_{0x} + \Delta v_x$). (c) Find the change in velocity in terms of a_x and t and substitute. Rearrange to form Eq. (3-12).

38. Derive Eq. (3-13) using these steps. (a) Start with $\Delta x = v_{av,x}t$. Rewrite this equation with the average velocity expressed in terms of the initial and final velocities. (b) Use $v_x - v_{0x} = a_x t$ to eliminate t. (c) Rearrange algebraically to form Eq. (3-13). [*Hint:* $A^2 - B^2 = (A + B)(A - B)$.]

39. A train of mass 55,200 kg is traveling along a straight, level track at 26.8 m/s (60.0 mi/h). Suddenly the engineer sees a truck stalled on the tracks 184 m ahead. If the maximum possible braking force has magnitude 84.0 kN, can the train be stopped in time?

40. In a television tube, electrons are accelerated from rest by a constant electric force of magnitude 6.4×10^{-17} N during the first 2.0 cm of the tube's length; then they move at essentially constant velocity another 45 cm before hitting the screen. (a) Find the speed of the electrons when they hit the screen. (b) How long does it take them to travel the length of the tube?

3.6 Falling Objects

In the problems, please assume g = 9.8 m/s^2 unless a more precise value is given in the problem.

41. A penny is dropped from the observation deck of the Empire State building (369 m above ground). With what velocity does it strike the ground? Ignore air resistance.

42. (a) How long does it take for a golf ball to fall from rest for a distance of 12.0 m? (b) How far would the ball fall in twice that time?

43. Grant Hill jumps 1.3 m straight up into the air to slam-dunk a basketball into the net. With what speed did he leave the floor?

44. A student, looking toward his fourth-floor dormitory window, sees a flowerpot with nasturtiums (originally on a window sill above) pass his 2.0-m-high window in 0.093 s. The distance between floors in the dormitory is 4.0 m. From a window on which floor did the flowerpot fall?

45. A balloonist, riding in the basket of a hot air balloon that is rising vertically with a constant velocity of 10.0 m/s, releases a sandbag when the balloon is 40.8 m above the ground. Neglecting air resistance, what is the bag's speed when it hits the ground? Assume $g = 9.80$ m/s^2.

46. A 55-kg lead ball is dropped from the leaning tower of Pisa. The tower is 55 m high. (a) How far does the ball fall in the first 3.0 s of flight? (b) What is the speed of the ball after it has traveled 2.5 m downward? (c) What is the speed of the ball 3.0 s after it is released?

47. During a walk on the Moon, an astronaut accidentally drops his camera over a 20.0-m cliff. It leaves his hands with zero speed, and after 2.0 s it has attained a velocity of 3.3 m/s downward. How far has the camera fallen after 4.0 s?

48. A stone is launched straight up by a slingshot. Its initial speed is 19.6 m/s and the stone is 1.50 m above the ground when launched. Assume $g = 9.80$ m/s^2. (a) How high above the ground does the stone rise? (b) How much time elapses before the stone hits the ground?

49. How far must something fall for its speed to get close to its terminal speed? Use dimensional analysis to come up with an estimate based on the object's terminal speed v_t and the gravitational field g. Estimate this distance for (a) a raindrop ($v_t \approx 7$ m/s) and (b) a skydiver in a dive ($v_t \approx 100$ m/s).

50. A paratrooper with a fully loaded pack has a mass of 120 kg. The force due to air resistance on him when falling with an unopened parachute has magnitude $F_D = bv^2$, where $b = 0.14\ \dfrac{\text{N}\cdot\text{s}^2}{\text{m}^2}$. (a) If he is falling with an unopened parachute at 64 m/s, what is the force of air resistance acting on him? (b) What is his acceleration? (c) What is his terminal speed?

51. A bobcat weighing 72 N jumps out of a tree. (a) What is the drag force on the bobcat when it falls at its terminal velocity? (b) What is the drag force on the bobcat when it falls at 75% of its terminal velocity? (c) What is the acceleration of the bobcat when it falls at its terminal velocity? (d) What is its acceleration when it falls at 75% of its terminal velocity?

52. In free fall, we assume the acceleration to be constant. Not only is air resistance neglected, but the gravitational field strength is assumed to be constant. (a) From what height can an object fall to the Earth's surface such that the gravitational field strength changes less than 1.00% during the fall? (b) In most cases, which do we have to worry about first: air resistance becoming significant or g changing?

3.7 Apparent Weight

53. Refer to Example 3.17. What is the apparent weight of the same passenger (weighing 598 N) in the following situations? In each case, the magnitude of the elevator's acceleration is 0.50 m/s^2. (a) After having stopped at the 15th floor, the passenger pushes the 8th floor button; the elevator is beginning to move downward. (b) Elevator is slowing down as it nears the 8th floor.

54. You are standing on a bathroom scale inside an elevator. Your weight is 140 lb, but the reading of the scale is 120 lb. (a) What is the magnitude and direction of the acceleration of the elevator? (b) Can you tell whether the elevator is speeding up or slowing down?

55. Felipe is going for a physical before joining the swim team. He is concerned about his weight, so he carries his scale into the elevator to check his weight while heading to the doctor's office on the 21st floor of the building. If his scale reads 750 N while the elevator is accelerating upward at 2.0 m/s^2, what does the nurse measure his weight to be?

56. Luke stands on a scale in an elevator that has a constant acceleration upward. The scale reads 0.960 kN. When Luke picks up a box of mass 20.0 kg, the scale reads 1.200 kN. (The acceleration remains the same.) (a) Find the acceleration of the elevator. (b) Find Luke's weight.

COMPREHENSIVE PROBLEMS

In the problems, please assume $g = 9.8\ \mathrm{m/s^2}$ unless a more precise value is given in the problem.

57. The conduction electrons in a copper wire have instantaneous speeds of around 10^6 m/s. The electrons keep colliding with copper atoms and reversing direction, so that even when a current is flowing in the wire, they only drift along with average velocities on the order of centimeters per hour. If the drift velocity in a particular wire has magnitude 10 cm/h, calculate the distance an electron travels in order to move 1 m down the wire.

58. Imagine a trip where you drive along a straight east-west highway at 80.0 km/h for 50.0 min and then at 60.0 km/h for 30.0 min. What is your average velocity for the trip (a) if you drive east for both parts of the trip? (b) if you drive east for the first 50.0 min and then west for the last 30.0 min?

59. A rocket is launched from rest. After 8.0 min, it is 160 km above the Earth's surface and is moving at a speed of 7.6 km/s. Assuming the rocket moves up in a straight line, what are its (a) average velocity and (b) average acceleration?

60. Based on the information given in Problem 59, is it possible that the rocket moves with constant acceleration? Explain.

61. An unmarked police car starts from rest just as a speeding car passes at a speed of v_0. If the police car accelerates at a constant value of a, what is the speed of the police car when it catches up to the speeder, who does not realize she is being pursued and does not vary her speed?

62. In Fig. 3.23a, the block of mass m_1 slides to the right with coefficient of kinetic friction μ_k on a horizontal surface. The block is connected to a hanging block of mass m_2 by a light cord that passes over a light, frictionless pulley. (a) Find the acceleration of each of the blocks and the tension in the cord. (b) Check your answers in the special cases $m_1 \ll m_2$, $m_1 \gg m_2$, and $m_1 = m_2$. (c) For what value of m_2 (if any) do the two blocks slide at constant velocity? What is the tension in the cord in that case?

63. While passing a slower car on the highway, you accelerate uniformly from 39 mi/h to 61 mi/h in a time of 10.0 s. (a) How far do you travel during this time? (b) What is your acceleration in (mi/h)/s and $\mathrm{m/s^2}$?

64. A cheetah can accelerate from rest to 24 m/s in 2.0 s. Assuming the acceleration is uniform, (a) what is the acceleration of the cheetah? (b) What is the distance traveled by the cheetah in these 2.0 s? (c) A runner can accelerate from rest to 6.0 m/s in the same time, 2.0 s. What is the acceleration of the runner? By what factor is the cheetah's average acceleration magnitude greater than that of the runner?

65. Neglecting air resistance, (a) from what height must a hockey puck drop if it is to attain a speed of 30.0 m/s (approximately 67 mi/h) before striking the ground? (b) If the puck comes to a full stop in a time of 1.00 s from initial impact, what acceleration (assumed constant) is experienced by the puck during the impact with the ground?

66. Locusts can jump to heights of 0.30 m. (a) Assuming the locust jumps straight up, and ignoring air resistance, what is the takeoff speed of the locust? (b) The locust actually jumps at an angle of about 55° to the horizontal, and air resistance is not negligible. The result is that the takeoff speed is about 40% higher than the value you calculated in part (a). If the mass of the locust is 2.0 g and its body moves 4.0 cm in a straight line while accelerating from rest to the takeoff speed, calculate the acceleration of the locust (assumed constant). (c) Ignore the locust's weight and estimate the force exerted on the hind legs by the ground. Compare this force to the locust's weight. Was it OK to ignore the locust's weight?

67. In Fig. 3.16a, two blocks are connected by a lightweight, flexible cord that passes over a frictionless pulley. If m_1 is 3.6 kg and m_2 is 9.2 kg, and block 2 is initially at rest 140 cm above the floor, how long does it take block 2 to reach the floor?

68. A locomotive pulls a train of 10 identical cars with a force of 2.0×10^6 N directed east. What is the force with which the *last* car pulls on the rest of the train?

69. A woman of mass 51 kg is standing in an elevator. If the elevator floor pushes up on her feet with a force of 408 N, what is the acceleration of the elevator?

70. A drag racer crosses the finish line of a 400.0-m track with a final speed of 104 m/s. (a) Assuming constant acceleration during the race, find the racer's time and the minimum coefficient of static friction between the tires and the road. (b) If, because of bad tires or wet pavement, the acceleration were 30.0% smaller, how long would it take to finish the race?

71. The graph in Fig. 3.37 shows the position x of a switch engine in a rail yard as a function of time t. At which of the labeled times t_0 to t_7 is (a) $a_x < 0$, (b) $a_x = 0$, (c) $a_x > 0$, (d) $v_x = 0$, (e) the speed decreasing?

72. An elevator starts at rest on the ninth floor. At $t = 0$, a passenger pushes a button to go to another floor. The graph in Fig. 3.38 shows the acceleration a_y of the elevator as a function of time. Let the y-axis point upward. (a) Has the passenger gone to a higher or lower floor? (b) Sketch a graph of the velocity

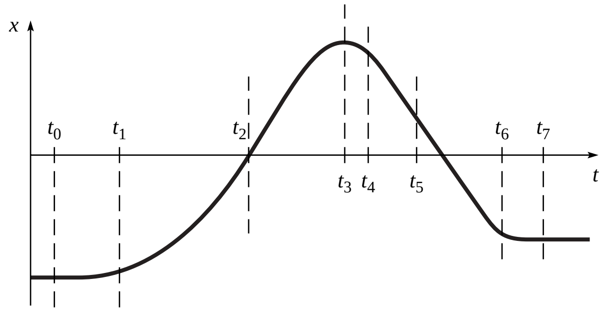


Figure 3.37 Problem 71

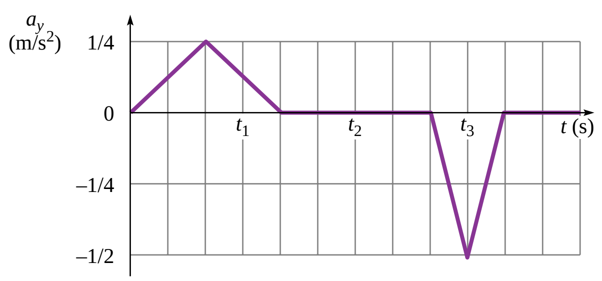


Figure 3.38 Problem 72

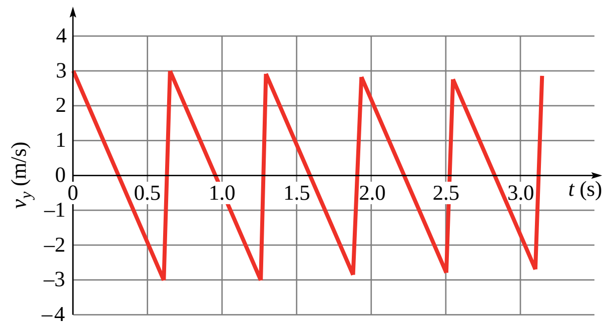


Figure 3.39 Problem 74

v_y of the elevator versus time. (c) Sketch a graph of the position y of the elevator versus time. (d) If a 1.40×10^2 lb person stands on a scale in the elevator, what does the scale read at times t_1, t_2, and t_3?

73. A streetcar named Desire travels between two stations 0.60 km apart. Leaving the first station, it accelerates for 10.0 s at 1.0 m/s^2 and then travels at a constant speed until it is near the second station, when it brakes at 2.0 m/s^2 in order to stop at the station. How long did this trip take? [*Hint:* What's the average velocity?]

C ✦ 74. The graph in Fig. 3.39 is the vertical velocity component v_y of a bouncing ball as a function of time. The y-axis points up. Answer these questions based on the data in the graph. (a) At what time does the ball reach its maximum height? (b) For how long is the ball in contact with the floor? (c) What is the maximum height of the ball? (d) What is the acceleration of the ball while in the air? (e) What is the average acceleration of the ball while in contact with the floor?

✦ 75. A rocket engine can accelerate a rocket launched from rest vertically up with an acceleration of 20.0 m/s^2. However, after 50.0 s of flight the engine fails. Neglect air resistance. (a) What is the rocket's altitude when the engine fails? (b) When does it reach its maximum height? (c) What is the maximum height reached? [*Hint:* a graphical solution may be easiest.] (d) What is the velocity of the rocket just before it hits the ground?

76. Two blocks lie side by side on a frictionless table. The block on the left is of mass m; the one on the right is of mass 2 m. The block on the right is pushed to the left with a force of magnitude F, pushing the other block in turn. What force does the block on the left exert on the block to its right?

✦ 77. A helicopter of mass M is lowering a truck of mass m onto the deck of a ship. (a) At first, the helicopter and the truck move downward together (the length of the cable doesn't change). If their downward speed is decreasing at a rate of 0.10g, what is the tension in the cable? (b) As the truck gets close to the deck, the helicopter stops moving downward. While it hovers, it lets out the cable so that the truck is still moving downward. If the truck's downward speed is decreasing at a rate of 0.10g, while the helicopter is at rest, what is the tension in the cable?

C ✦ 78. Fish don't move as fast as you might think. A small trout has a top swimming speed of only about 2 m/s, which is about the speed of a brisk walk (for a human, not a fish!). It may seem to move faster because it is capable of large *accelerations*—it can dart about, changing its speed or direction very quickly. (a) If a trout starts from rest and accelerates to 2 m/s in 0.05 s, what is the trout's average acceleration? (b) During this acceleration, what is the average net force on the trout? Express your answer as a multiple of the trout's weight. (c) Explain how the trout gets the water to push it forward.

79. Figure 3.31 shows a graph of speedometer readings obtained as an automobile came to a stop. What is the displacement along a straight line path in the first 16 s?

✦ 80. In Fig. 3.16a, two blocks are connected by a lightweight, flexible cord that passes over a frictionless pulley. If $m_1 \gg m_2$, find (a) the acceleration of each block and (b) the tension in the cord.

✦✦ 81. Find the point of no return for an airport runway of 1.50 mi in length if a jet plane can accelerate at 10.0 ft/s^2 and decelerate at 7.00 ft/s^2. The point of no return occurs when the pilot can no longer abort the takeoff without running out of runway. What length of time is available from the start of the motion in which to decide on a course of action?

82. Two blocks, masses m_1 and m_2, are connected by a massless cord (Fig. 3.40). If the two blocks are accelerated uniformly on a frictionless surface by applying a force of magnitude T_2 to a second cord connected to m_2, what is the ratio of the tensions in the two cords in terms of the masses? $T_1/T_2 = ?$

83. In the human nervous system, signals are transmitted along neurons as *action potentials* that travel at speeds of up to 100 m/s. (An action potential is a traveling influx of sodium ions through the membrane of a neuron.) The signal is passed from one neuron to another by the release of neurotransmitters in the synapse. Suppose someone steps on your toe. The pain signal travels along a 1.0-m-long sensory neuron to the spinal column, across a synapse to a second 1.0-m-long neuron, and across a second synapse to the brain. Suppose that the synapses are each 100 nm wide, that it takes 0.10 ms for the signal to cross each synapse, and that the action potentials travel at 100 m/s. (a) At what average speed does the signal cross a synapse? (b) How long does it take the signal to reach the brain? (c) What is the average speed of propagation of the signal?

84. A car traveling at 65 mi/h runs into a bridge abutment after the driver falls asleep at the wheel. (a) If the driver is wearing a seat belt and comes to rest within a 1.0-m distance, what is his acceleration (assumed constant)? (b) A passenger who isn't wearing a seat belt is thrown into the windshield and comes to a stop in a distance of 10.0 cm. What is the acceleration of the passenger? (c) Express these accelerations in terms of the free fall acceleration, g = 9.8 m/s^2, and find "how many g's" are felt in each case. (Test pilots can black out at accelerations of 4g or greater; pressure suits are designed to help force blood to the brain and prevent such a loss of consciousness.)

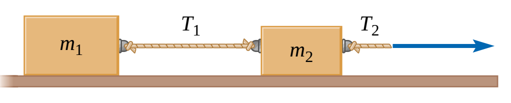


Figure 3.40 Problem 82

ANSWERS TO PRACTICE PROBLEMS

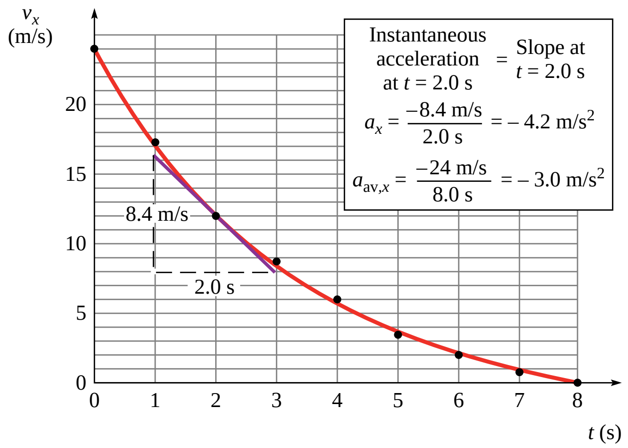
3.1 His displacement is zero since he ends up at the same place from which he started (home plate).

3.2 $\Delta x = -2.9$ km, so the displacement is 2.9 km to the west.

3.3 1.05 m/s to the north

3.4 The velocity is increasing in magnitude, so $\Delta\vec{\mathbf{v}}$ and $\vec{\mathbf{a}}$ are in the same direction as the velocity (the $-x$-direction). Thus a_x is negative.

3.5 (a) $a_{av,x} = -3.0$ m/s^2, where the negative sign means the acceleration is directed to the northwest; (b) $a_x = -4.2$ m/s^2 (northwest)



3.6 1.84 kN

3.7 For equal masses, $a_y = 0$ and $T = mg$. The pull of gravity is equal on the two sides. For slightly unequal masses, $m_1 \approx m_2$, so again $T \approx mg$; the acceleration is very small since there is only a slight excess pull of gravity on the heavier side but plenty of inertia.

3.8 60 N

3.9 (a) 20 s; (b) 18 m/s east

3.10 5.00 s

3.11 If $m_1 \gg m_2$, a is very small and $T \approx m_2 g$. Block 1 is so massive that it barely accelerates; block 2 is essentially just hanging there, being supported by the cord.

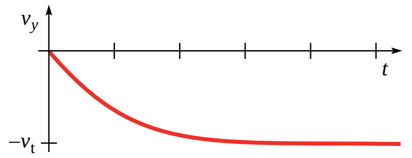
3.12 2500 N

3.13 It is impossible to pull the crate up with a single pulley. The entire weight of the crate would be supported by a single strand of cable and that weight exceeds the breaking strength of the cable.

3.14 (a) 3.8 m: (b) 3.00 s

3.15 1.1 kN

3.16 At first, constant acceleration: straight line with slope $-g$. After a long time, constant velocity.



3.17 (a) 392 N; (b) 431 N