


# VISUAL WALKTHROUGH

**3**

**STRAIN ENERGY AND THEORIES OF FAILURES**



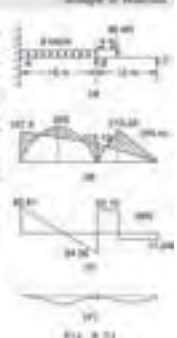
**3.1 INTRODUCTION**

When an elastic body is loaded within elastic limits, it deforms and stores work to show which is stored within the body in the form of internal energy. This stored energy in the deformed body is known as strain energy and is denoted by  $U$ . It is increasing strain due to stress as the load is increased from the body. However, if the elastic limit is exceeded, there is permanent set of deformation and the particles of the material of the body slide one over another. The work done in doing so is spent in increasing the extension of the particles and the energy spent appears as heat in the strained material of the body. The concept of strain energy is very important in strength of materials as it is associated with the deformation of the body. The deformation of a body depends upon the nature of application of the load, i.e. whether the applied load is

Introduction at the beginning of each chapter sums up the aim and contents of the chapter.

A variety of solved examples to reinforce the concepts.

**Example 8.22** A continuous beam ABC is built-in at A and simply supported at B and C. Span AB is 15 m long and carries a uniformly distributed load of 2 kN/m run and span BC is 12 m long and carries a point load of 80 kN at 4 m from support B. Draw the bending moment and shear force diagrams if the support B sink 12 mm relative to A and C.  $E = 200$  GPa and  $I = 800 \times 10^8 \text{ mm}^4$ .



**Solution:** Figure 8.11a shows the loaded beam. First assuming the continuous beam ABC to be made up of fixed spans AB and BC.

- For span AB: Fixing moments at A,  $M_A = -\frac{wL^2}{12} = -\frac{2 \times 15^2}{12} = -37.5 \text{ kNm}$
- Fixing moments at B,  $M_B = 37.5 \text{ kNm}$
- For span BC: Fixing moments at B,  $M_B = \frac{80 \times 4 \times 8}{12} = 213.33 \text{ kNm}$
- Fixing moments at C,  $M_C = \frac{80 \times 8 \times 4}{12} = 213.33 \text{ kNm}$


\* In span AB, moments at A and B due to sinking of support B by 12 mm:

$$\Delta l = -\frac{12 \times 15^3}{2^3}$$

(Example 8.22, on being correct electronic)

**3.6 REINFORCED CONCRETE BEAMS**

Concrete is a material which has compressive strength but is very weak in tension. As stress develops, cracks, thus reducing its tensile strength to zero. The compensation for this weakness of concrete, steel reinforcement is done on the tension side of concrete beams and to have the maximum advantage it is put at the maximum distance from the neutral axis of the beam (Fig. 8.17).



The following assumptions are made in the reinforced concrete beams:

1. Equal strains in the concrete on tension side.
2. Uniform stress in the steel.
3. Stress proportional to strain in the concrete.
4. Strain proportional to distance from neutral axis.

Assumption 1 is not true as concrete does not obey the Hooke's law. However, a stress value may be taken of the modulus over the range of stress yield. The law assumption is true for pure bending and it also implies that there is no relative slip between concrete and steel.

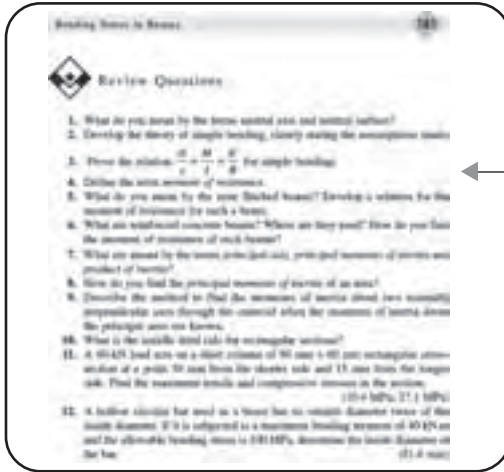
Consider the case of rectangular section as shown in Fig. 8.18.

Let:

- $d$  = depth of rectangular beam from compression edge.
- $x_c$  = maximum stress in the concrete.
- $x_s$  = maximum stress in the steel.
- $A_s$  = area of steel reinforcement.
- $m$  = modular ratio  $E_s/E_c$ .

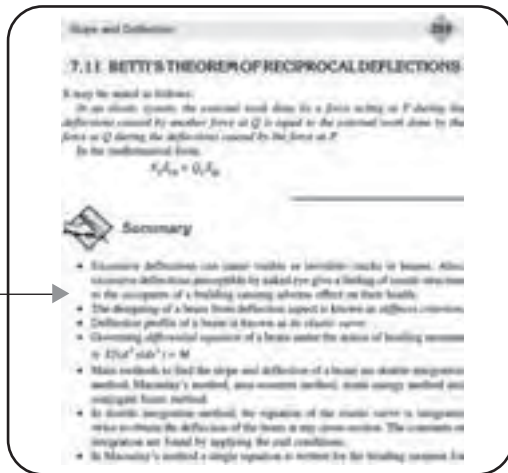
As stress are considered proportional to distance from neutral axis,

Concise and comprehensive treatment of topics with emphasis on fundamental concepts.



A number of theoretical questions and unsolved problems for practice to widen the horizon of comprehension of the topic.

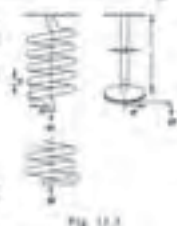
Summary at the end of each chapter recapitulates the inferences for quick revision.



International system of units (SI) throughout the book for universal approval.

**Stresses**

When they are acted upon by an axial load, there is axial extension and when there is an axial torque there is a change in the pitch of coarves of the spring coils. In the latter case, there is an angular extension of the free end and the action is known as wind up.



(i) Under Axial Load

Let  $W$  = Axial load  
 $D$  = Mean coil diameter  
 $P$  = Mean coil radius  
 $d$  = wire diameter  
 $\alpha$  = axial angle of twist along the wire  
 $\delta$  = deflection of  $W$  along the axis of the coil  
 $n$  = number of coils  
 $l$  = length of wire

As shown in FIG. 11.1, the action of load  $W$  on any cross-section is to twist it like a shaft with a pure torque  $W\alpha$ . Bending and shear effects may be neglected. Then  $\delta = \alpha l$  (approximately).

$$\delta = \frac{Wl}{GJ} = \frac{Wl}{G \cdot \frac{\pi}{32} d^4} = \frac{32Wl}{\pi G d^4} \quad (11.1)$$

$$\text{Since } \alpha = \frac{\delta}{l} \quad \therefore \alpha = \frac{32W}{\pi G d^4} \quad (11.2)$$


$$\text{Deflection of the spring, } \delta = \alpha l = \frac{32Wl^2}{\pi G d^4} \quad (11.3)$$

Simple diagrams for easy visualization of the explanations.

Appendix containing multiple choice questions to prepare for competitive examinations.

### Appendix I

#### OBJECTIVE TYPE QUESTIONS




#### I. DIRECT STRESS

- The ratio of stress to the strain is called:
  - strain
  - stress
  - modulus of elasticity
  - none of these
- The resistance to deformation of a body per unit area is known as:
  - strain
  - stress
  - modulus of elasticity
  - modulus of rigidity
- Stress is defined as deformation per unit:
  - area
  - length
  - force
  - volume
- Units of stress are:
  - kg/cm<sup>2</sup>
  - kg/cm<sup>3</sup>
  - kg/cm
  - kg/cm<sup>2</sup>

### Appendix II

#### IMPORTANT RELATIONS AND RESULTS



- Displacement of a bar:  $\delta = \frac{Wl}{AE}$
- Temperature stress in bar:  $\sigma = \alpha \cdot \Delta T \cdot E$
- Poisson's ratio:  $\nu = \frac{\epsilon_y}{\epsilon_x} = \frac{\epsilon_z}{\epsilon_x} = \frac{\epsilon_r}{\epsilon_x}$
- Relation between elastic constants:  $E = 2G(1 + \nu)$  and  $E = 3K(1 - 2\nu)$
- Strain energy on an inclined plane:  $U = \frac{1}{2} W \delta$
- Strain energy stored in a bar:  $U = \frac{1}{2} W \delta = \frac{1}{2} \int \sigma \epsilon \, dV$

Appendix containing important relations for ready reference.