CHAPTER 19

An Introduction to Decision Making

GOALS

When you have completed this chapter, you will be able to:

- Define the terms *state of nature*, *event*, *decision alternatives*, *payoff*, and *utility*
- Organize information in a payoff table or a decision tree
- Compute opportunity loss and utility function
- Find an optimal decision alternative based on a given decision criterion
- Assess the expected value of additional information.

Statistical decision theory was developed by Abraham Wald (1902–1950). This can also be considered as a special case of a much broader theory called *game theory*. The first recorded non-trivial result on game theory is generally accepted as the one by Baron Waldegrave of Chewton (1684–1741) in 1713. However, credit for systematic development of game theory goes to John von Neumann. It was only after the publication of his book *Theory of Games and Economic Behaviour,* with Oskar Morgenstern, in 1944, that the topic received widespread attention among mathematicians, economists, and social scientists. A very important concept in decision theory is that of utility, introduced by Gabriel Cramer (1704–1752). Von Neumann and Morgenstern developed an axiomatic theory of utility and applied it to issues in economics and social sciences. This particularly had a profound impact on research developments in the field of economics.

John von Neumann was born in Budapest, Hungary, the eldest of three children of Miksa Neumann and Margit Kann. His childhood name was János, but after migrating to the United States in 1930, he changed it to John.

Von Neumann was a child prodigy. He could memorize pages by reading them just a few times. At the age of six, he could divide two eight-digit numbers in his head and exchange jokes with his father in classical Greek. By eight, he had mastered calculus, and by twelve, he was at the graduate level in mathematics. At the age of eight, he developed an interest in history and read all 44 volumes of the universal history in his family library. He published his first research paper in mathematics at the age of seventeen, while still a high school student.

Although mathematics was always von Neumann's main area of interest, at the insistence of his father, he enrolled in an undergraduate program in chemical engineering at the University of Berlin in 1921. In the summer of 1923, he enrolled at the Budapest University for a PhD program in mathematics and in 1926, he received, simultaneously, an undergraduate degree in chemical engineering and a PhD degree in mathematics.

By the age of twenty-six, he had already achieved a reputation as a young genius. In 1930, von Neumann joined Princeton University as a visiting lecturer and he was appointed professor there in 1931. In 1933, he became one of the original six mathematics professors at the prestigious Institute for Advanced Study (IAS) at Princeton University. He kept this position till the end of his life.

Modern mathematics is so vast that it seems impossible for any single person to know more than a tiny fraction of it. Once when asked how much of mathematics he himself knew, von Neumann thought for some time and replied "twenty-eight percent." However, von Neumann made profound contributions to almost all branches of mathematics and science.

Von Neumann's contributions, which have most profoundly touched our lives, are: first, his pioneering work in the field of electronic computing. He built upon the work of Mauchly and Eckert to devise a computer architecture that allows both instructions and data to be stored in computer memory. This architecture is known as *von Neumann architecture* and is used (in modified form) even in present day computers. Second, his contributions to both the Manhattan project and the development of the hydrogen bomb. His valuable contribution here was the implosion method for exploding the bomb. Also, it is said that Von Neumann's computer hastened the hydrogen bomb explosion on November 1, 1952. Finally, the development of *game theory*, which had profound impact of the field of economics, among others.

Besides his obsession for his work, parties and nightlife held a special appeal for von Neumann. He was almost always dressed in a business suit, even while riding a mule up a Colorado mountain. One of his colleagues once suggested that he "buy an old jacket, sprinkle chalk on it, and look more like us." He was also reputed for his driving (in)ability.

Von Neumann won innumerable prestigious awards and honours, including two presidential awards. One of the rare honours is the naming of a lunar crater after him as *Crater von Neumann*.

For a more detailed biography of John von Neumann, see *John von Neumann,* by Norman Macrae, Pantheon books, New York, 1992.

INTRODUCTION

In Chapters 9 to 16, we considered classical statistical techniques for estimation of population parameters and conducting tests of hypothesis regarding population distribution(s). Classical statistics does not address the financial consequences of the conclusions. For example, in the classical statistical theory of hypothesis testing, the decision rule is selected so as to restrict the probability, α, of a Type I error to a pre-specified value, and minimize the probability, β, of a Type II error. No consideration is given here to the costs associated with the two types of errors.

Statistical decision theory was developed by Abraham Wald in 1947 as an alternative approach to hypothesis testing, in which a decision rule is designed so as to minimize the expected value of the cost of error. The theory has found applications in diverse real-world decision making scenarios and has grown rapidly since then. The term **Bayesian statistics** is also used to indicate this branch of statistics.

As the name implies, the focus of the statistical decision theory is on the process of making decisions that explicitly includes the payoffs that may result. Here, we are concerned with determining which decision, from a given set of possible alternatives, is optimal for a particular set of conditions. Consider the following examples of decision-theory problems.

- Nortel is considering introducing a new wireless telecommunications device into the market. They are considering three alternatives: (i) build a new full scale plant for manufacturing the new product; (ii) build a medium size plant; and (iii) do not market the product. If they decide to market the product, the annual profit will depend on the market response to the product. Suppose preliminary market analysis indicates that the market response to the product may be highly favourable, moderately favourable, or unfavourable. What decision should they make?

- Ford Motor Company must decide whether to purchase assembled door locks for the new model Ford F-150 or to manufacture and assemble the parts at their plant. If sales of the Escort continue to increase, it will be more profitable to manufacture and assemble the parts. If sales level off or decline, it will be more profitable to purchase the door locks assembled. Which decision should be made?

- Banana Republic developed a new line of jackets that are very popular in the cold-weather regions of the country. They would like to purchase commercial television time during the upcoming NCAA basketball final. If both teams who play in the game are from warmer parts of the country, they estimate that only a small proportion of the viewers will be interested in the jackets. However, a match-up between two teams from colder climates would reach a large proportion of viewers who wear jackets. What should be their decision?

In each of these cases the decision is characterized by several alternative courses of action and several factors not under the control of the decision maker. For example, Banana Republic has no control over which teams will reach the final. These cases characterize the nature of decision making. Possible decision alternatives can be listed, possible future events determined, and even probabilities established, but *the decisions are made in the face of uncertainty*.

19.1 ELEMENTS OF DECISION THEORY

There are three components to any decision theory problem: (i) the decision alternatives or the choices available to the decision maker; (ii) the states of nature, which are not under the control of the decision maker; and (iii) the payoffs. These concepts will be explained in the following paragraphs.

> *i* The **decision alternatives** are the choices available to the decision maker.

For example, Nortel can decide to build a full scale plant, a medium size plant or not to market the product at all. To simplify our presentation, we assume that the decision maker has only a finite number of choices.

> *i* The **states of nature** are the future events that are not under the control of the decision maker.

Nortel does not know whether the market response to its product will be highly favourable, moderately favourable, or unfavourable. Banana Republic cannot determine whether warm-weather or cold-weather teams will play in the NCAA basketball final.

> *i* A **payoff** is the numerical gain to the decision maker corresponding to each combination of a decision alternative and a state of nature.

Nortel may estimate that if they build a full scale plant and the market response is highly favourable, the payoff will be $400 million. However, if they build a full scale plant and the market condition is unfavourable, then the payoff will be $−800 million (that is, they will take a loss of $800 million).

A CASE OF DECISION MAKING UNDER UNCERTAINTY

Let us continue with the Nortel problem. Nortel has to make one of the following three decisions regarding marketing of one of their new products: (i) build a full size manufacturing plant (D_1); (ii) build a medium size plant (D_2); or (iii) do not market the product (D_3). A preliminary market analysis shows that the market response to the product may be very favourable (S_1), moderately favourable (S_2), or unfavourable (S_3). The estimated profits for each combination of a decision alternative and the market responses is shown in Table 19-1. We call this table the **payoff table**.

TABLE 19-1: Payoff Table for the Nortel Problem (values in millions of dollars)

Decision Alternatives \ Market Response	Very Favourable (S_1)	Moderately Favourable (S_2)	Unfavourable (S_3)
Build full-scale plant (D_1)	400	20	−800
Build medium-size plant (D_2)	80	60	−50
Do not market the product (D_3)	0	0	0

What should be Nortel's decision?

For each decision alternative, the payoff value is not fixed but depends on which of the three states of nature, S_1, S_2, and S_3, will occur. The approach we shall use is:

- associate with each decision alternative an appropriate single value; and
- choose the alternative for which the associated value is *maximum*.

19.2 NON-PROBABILISTIC CRITERIA

Here, we do not use any information about the probabilities of occurrence of the three states of nature, except that each of the probabilities is non-zero (positive).

MAXIMIN CRITERION

This is also called a *pessimistic* criterion and is likely to be used by an individual with a pessimistic outlook. Such a person will always be concerned about the worst case implication of every decision alternative. He or she will therefore associate with every decision alternative the minimum payoff value corresponding to the alternative and will choose an alternative for which the minimum value is maximum possible.

Suppose Nortel decides to take decision D_1. Then in the worst case, its payoff will be \$−800 million. (This is the minimum of the three payoffs corresponding to decision D_1.) Similarly, the minimum payoffs corresponding to decisions D_2 and D_3 are, respectively, \$−50 million and \$0. The maximum of the three minimum values $\{-800, -50, 0\}$ is 0, which corresponds to alternative D_3. Hence, if the decision maker is pessimistic, the right choice will be decision alternative D_3.
Thus,

> *i* In **maximin criterion**, we note the minimum payoff value corresponding to each decision alternative and select the decision alternative for which the minimum payoff value is maximum.

MAXIMAX CRITERION

This criterion is likely to be used by an optimistic individual. Such an individual will only be interested in the maximum possible payoff corresponding to each decision alternative and will associate with every decision alternative the maximum payoff value corresponding to the alternative. The choice of decision alternative will be the one for which the maximum payoff is maximum.

In the case of the Nortel problem, the maximum payoffs corresponding to decisions D_1, D_2, and D_3 are, respectively, (in millions) \$400, \$80, and \$0. The maximum of these three maximum values is \$400 million, which corresponds to alternative D_1. Hence, the choice of an optimistic decision maker is alternative D_1.

> *i* In **maximax criterion**, we note the maximum payoff value corresponding to each decision alternative and select the decision alternative for which the maximum payoff value is maximum.

THE PESSIMISTIC-OPTIMISTIC INDEX CRITERION OF HURWICZ

The maximin criterion is a totally pessimistic criterion that associates with every alternative the minimum payoff value corresponding to it. The maximax criterion, on the other hand, is extremely optimistic, which associates with every alternative the maximum payoff corresponding to it.

A criterion suggested by Hurwicz balances between these two extremes. It chooses a number α between 0 and 1, called the pessimistic-optimistic index. The value associated with each decision alternative is then α (minimum payoff corresponding to the alternative) + $(1 - \alpha)$(maximum payoff corresponding to the alternative).

In the Nortel problem, suppose we choose $\alpha = 0.4$.

The value associated with alternatives D_1 is $(0.4)(-800) + (0.6)(400) = \$-80$ million;

the value associated with D_2 is $(0.4)(-50) + (0.6)(80) = \$28$ million;

the value associated with D_3 is $(0.4)(0) + (0.6)(0) = \$0$ million.

The maximum of $\{-80, 28, 0\}$ is 28, which corresponds to D_2. Hence, optimal choice of alternative, as per this criterion, is D_2.

SELF-REVIEW 19-1

Bob Hill, a small investor, has \$1100 to invest. He has narrowed his choices to three common stocks, namely McGraw-Hill Ryerson, Petro-Canada, and Kayser Chemicals, and he intends to invest all his money in precisely one of these three stocks. His predictions regarding the value of his \$1100 investment for the three stocks for a bull market and for a bear market are shown in Table 19-2.

TABLE 19-2: Payoff Table for the Three Common Stocks under The Two Market Conditions

Purchase	Bull Market (in \$) (S_1)	Bear Market (in \$) (S_2)
McGraw-Hill Ryerson (D_1)	2400	1000
Petro-Canada (D_2)	2200	1100
Kayser Chemicals (D_3)	1900	1150

What should be his decision, if he decides to use

(a) the maximin criterion?

(b) the maximax criterion?

(c) the pessimistic-optimistic index criterion of Hurwicz with $\alpha = 0.7$?

19.3 PROBABILISTIC CRITERIA

Let us now assume that we have some prior information about the probabilities of occurrence of the states of nature. These probabilities are usually based on historical data or subjective estimates. For example, in the Nortel problem, suppose a preliminary market analysis gives estimates 0.4, 0.5, and 0.1 of the probabilities of occurrence of the three states of nature, S_1, S_2, and S_3, respectively. A commonly used criterion in this case is the expected monetary value criterion.

EXPECTED MONETARY VALUE CRITERION

One way to incorporate the information about the probability distribution of occurrence of states of nature is to associate with each decision alternative a value equal to the corresponding expected monetary value (EMV) and choose the alternative with the highest expected value.

Thus, if alternative D_1 is chosen, the payoff could be \$400 million, \$20 million, or \$−800 million, with probabilities 0.4, 0.5, and 0.1, respectively. The expected monetary value (EMV) corresponding to decision alternative D_1 is then

$$\text{EMV}(D_1) = (0.4)(400) + (0.5)(20) + (0.1)(-800) = \$90 \text{ million.}$$

Similarly, the expected monetary values corresponding to decision alternatives D_2 and D_3 are,

$$\text{EMV}(D_2) = (0.4)(80) + (0.5)(60) + (0.1)(-50) = \$57 \text{ million.}$$

$$\text{EMV}(D_3) = (0.4)(0) + (0.5)(0) + (0.1)(0) = \$0 \text{ million.}$$

Expected Monetary Value $$\text{EMV}(D_i) = \sum [P(S_j) \times V(D_i, S_j)] \quad \textbf{19-1}$$

where

- $\text{EMV}(D_i)$ refers to the expected monetary value of decision alternative D_i.
- $P(S_j)$ refers to the probability of the state of nature S_j.
- $V(D_i, S_j)$ refers to the value of the payoff corresponding to decision alternative D_i and state of nature S_j.

The decision alternative D_1 would yield the greatest expected profit (\$90 million). Hence, if management decides to use this criterion, the best choice will be decision alternative D_1.

SELF-REVIEW 19-2

Consider the decision problem in Self-Review 19-1. The past 10 years' record indicates that the stock market prices increased during six of the years and declined during four of the years. Thus, an estimate of the probability of a market rise is 0.60 and that of the probability of a market decline is 0.40. What should be Bob's decision, if he decides to use the expected monetary value criterion?

19.4 CRITERIA BASED ON OPPORTUNITY LOSS (REGRET)

L.J. Savage suggested alternative criteria based on a table of what are called **opportunity losses** (or **regrets**) rather than a payoff table. As an example, suppose in the Nortel problem management decides to build a medium-size plant (decision alternative D_2). Now if the market condition happens to be very favourable (S_1), then Nortel will expect to make a profit of \$80 million (as indicated in Table 19-1). In this case, had management known beforehand that the market condition would be S_1, they would have chosen the alternative D_1, and the expected value of profit would be \$400 million. Thus, management missed an opportunity for making an extra profit of \$(400 − 80) = \$320 million. To put it another way, the \$320 million represents the opportunity loss or regret if management chooses the alternative D_2 and the correct state of nature happens to be S_1. Similarly, if the market condition happens to be moderately favourable (S_2), then Nortel will expect to make a profit of \$60 million (from Table 19-1).

In this case, had management known beforehand that the market condition would be S_2, they would have still chosen alternative D_2. Thus, the opportunity loss or regret if management chooses D_2 and the correct state of nature happens to be S_2 is zero. Table 19-3(a) gives the opportunity losses or regrets for each combination of decision alternative and state of nature. Table 19-3(b) gives the negatives of the regrets. It may be noted that the table of negative regret values is obtained from the payoff table by subtracting from entries in each column, the maximum value in that column. Savage suggests basing our decision criterion on the negative regrets.

TABLE 19-3: Regret Values (a) and Negative Regret Values (b) for the Nortel Problem

(a) Market Response Decision	S_1	S_2	S_3	(b) Market Response Decision	S_1	S_2	S_3
D_1	0	40	800	D_1	0	−40	−800
D_2	320	0	50	D_2	−320	0	−50
D_3	400	60	0	D_3	−400	−60	0

PESSIMISTIC CRITERION

Suppose Nortel decides to choose decision alternative D_1. Then in the worst case, the opportunity loss will be $800 million. (This is the maximum value in the row corresponding to D_1 in Table 19-3(a).) We shall associate with D_1 the value (−800). (This is the minimum value in the row corresponding to D_1 in Table 19-3(b).) Similarly, we shall associate with decision alternatives D_2 and D_3 the minimum values corresponding to rows D_2 and D_3 in Table 19-3(b). These are, respectively, −320 and −400. The maximum of {−800, −320, −400} is −320, which corresponds to alternative D_2. Hence our choice will be D_2.

EXPECTED VALUE CRITERION

We can calculate the expected value of the negative regrets corresponding to each decision alternative and choose the alternative for which the expected value is maximum. However, this will always lead to the same decision as the one based on expected monetary value criterion. (Here is a challenge for you. Find out why.)

SELF-REVIEW 19-3

Refer to the problem in Self-Review 19-2. Suppose Bob decides to base his decision on the opportunity loss function. What should be his decision if he uses the pessimistic criterion?

19.5 CRITERION BASED ON UTILITY FUNCTION

Problems of decision making under uncertainty were considered by early probabilists in the seventeenth century and the criterion of expected monetary value was popularly accepted as a reasonable one to evaluate the value of a decision alternative when probabilities of occurrence of the possible states of nature are known. However, a serious doubt was cast on the suitability of this criterion by the famous problem called the St. Petersburg Paradox, posed in 1713 by Nicholas Bernoulli. (See the Statistics in Action box on this page.) This led Gabriel Cramer to introduce the concept of utility of an outcome. He did not, however, provide proper justification for his choice of the utility function and the use of expected value of utilities of individual outcomes to define the utility of the entire game.

In 1947, von Neumann and Morgenstern developed an axiomatic theory of utility. They showed that if a decision maker's preference between any pair of decision alternatives satisfies a certain rational set of rules, then (i) his or her utility function (we shall call it the VNM-utility) can be measured on an interval scale (they gave a scheme for measuring the utility function), and (ii) his or her VNM-utility of the outcome of a decision alternative equals the expected value of the VNM-utilities of the outcomes (of the decision alternative) corresponding to the different states of nature.

STATISTICS IN ACTION

The following problem, called the *St. Petersburg Paradox,* was posed by Nicholas Bernoulli in 1713: A fair coin is tossed until a head appears. If the game lasts for n tosses, the player receives 2^n ducats. (Ducats are gold coins formerly used in most European countries for trading.) Thus, he receives 2 ducats if heads appears at the very first toss, 4 ducats if it does at the second toss, then 8, 16, 32 ducats, etc. What is a decent price to pay for the privilege of playing this game? The expected monetary value is infinite: $(1/2)(2)+(1/4)(4)+(1/8)(8)+(1/16)(16)+\cdots=1+1+1+1+\cdots$ However, it is clear that nobody would ever pay more than a few ducats for a shot at this type of gamble.... Why? Gabriel Cramer (1704–1752), settled the paradox by introducing the concept of *moral value of goods,* which is now termed *utility*. He argued that individuals base their decisions on utility of the outcome and not the monetary value. He assigned to each monetary value w a utility equal to $\sqrt{w}$, and calculated the utility value of the entire game as the expected value of the utilities of the outcomes. The final monetary value of the game was then defined as the inverse (square) of the utility of the game, which equals $(3+2\sqrt{2})$ ducats, a reasonable number.

SCHEME FOR MEASURING VNM-UTILITY OF A DECISION MAKER

To illustrate, let us consider the Nortel problem. Suppose the decision maker's preferences satisfy the von Neumann/Morgenstern set of rules.

The minimum payoff value in Table 19-1 is $\$-800$ million, and the maximum payoff value is \$400 million. We shall therefore assign a utility of 0 to $\$-800$ million and a utility of 1 to \$400 million.

The middle of the seven numbers in Table 19-1 is \$20 million. We shall measure the utility of \$20 million. We shall offer the decision maker two alternatives: (i) a sure amount of \$20 million, and (ii) a lottery such that if he accepts it, he will win \$400 million with some probability (p) and he will lose \$800 million with probability $(1-p)$. We shall ask him for what value of p the two alternatives will be equally preferable to him. Suppose his answer is $p=0.86$. Then we shall assign to \$20 million, a utility value of 0.86.

Now between $\$-800$ million and \$20 million we have two values, $\$-50$ million and \$0. We shall choose one of them, say \$0. We shall offer the decision maker two alternatives: (i) A sure amount of \$0, and (ii) a lottery such that if he accepts the lottery, he will win \$20 million with probability p and he will loose \$800 million with probability $(1-p)$. We shall ask him for what value of p the two alternatives will be equally preferable to him. Suppose his answer is $p=0.95$. Then we shall assign to \$0 a utility value of (0.95)(utility of \$20 million) + (0.05)(utility of $\$-800$ million) = (0.95)(0.86) + (0.05)(0) = 0.817.

We shall continue the process, until we obtain the utilities of all the values in Table 19-1.

DECISION MAKING BASED ON THE VNM-UTILITIES

Suppose the measured values of VNM-utilities of the decision maker for all the payoff values in Table 19-1 are as tabulated below:

Monetary Value (in millions of dollars)	−800	−50	0	20	60	80	400
VNM-utility, $u(.)$	0	0.694	0.817	0.86	0.885	0.902	1.0

Now, the utility for the decision maker of the outcomes of the decision alternative D_1 will be

$$u(D_1) = (0.4)(\text{utility of } \$400 \text{ million}) + (0.5)(\text{utility of } \$20 \text{ million}) + (0.1)(\text{utility of } \$-800 \text{ million})$$
$$= (0.4)(1.0) + (0.5)(0.86) + (0.1)(0) = 0.83.$$

Similarly, his or her utilities for the outcomes of the decision alternatives D_2 and D_3 will be:

$$u(D_2) = (0.4)(0.902) + (0.5)(0.885) + (0.1)(0.694) = 0.8727$$
$$u(D_3) = (0.4)(0.817) + (0.5)(0.817) + (0.1)(0.817) = 0.817$$

Utility of the Outcome of Alternative D_i

$$u(D_i) = \sum(P(S_j) \times u(V(D_i, S_j))) \qquad \textbf{19-2}$$

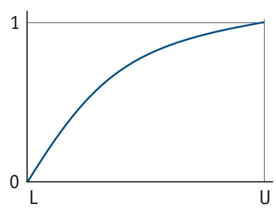
where
- $u(D_i)$ refers to the utility for the outcomes of the decision alternative i.
- $P(S_j)$ refers to the probability of the state of nature S_j.
- $u(V(D_i, S_j))$ refers to the utility of the value of the payoff corresponding to decision D_i and the state of nature S_j.

The alternative D_2 has the highest utility (= 0.8727). Hence, if this criterion is used, the best choice will be decision alternative D_2.

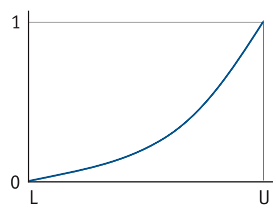
The shape of the utility function reflects the decision maker's attitude towards risk. Charts 19-1a, b, and c show the shapes of the utility functions of a risk averse person, a risk taker and a risk neutral person. Here, we have denoted by L and U the minimum and maximum values in the decision making scenarios to which we have assigned utilities of 0 and 1 respectively. (Note that VNM-utility is measured on an interval scale.)

For a risk averse person, the VNM-utility of a sure amount of $(0.5L + 0.5U)$ is more than 0.5. That is, he/she prefers this sure amount to a lottery that guarantees him amount U with probability 0.5 and amount L with probability 0.5. Though the two amounts have the same expected monetary value, the risk averse person is likely to avoid the uncertain situation because of the possibility of a low payoff of L. In the case of the risk taker, the situation is reversed. For a risk neutral person, the utility of a lottery is the same as that of its expected monetary value.

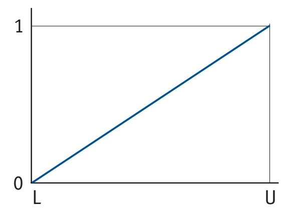
CHART 19-1: Utility Functions for Different Attitudes Towards Risk



(a) Risk Avoider's Utility



(b) Risk Taker's Utility



(c) Risk Neutral's Utility

SELF-REVIEW 19-4

Refer to the problem in Self-Review 19-2. Suppose Bob's VNM-utilities are as follows:

Monetary value (in millions of dollars)	1000	1100	1150	1900	2200	2400
VNM-utility, $u(.)$	0	0.4	0.5	0.85	0.97	1.0

What should be his decision based on the utility criterion?

EXERCISES 19-1 TO 19-6

19-1. Consider the following payoff table corresponding to a decision problem:

Alternative	State of Nature	
	S_1	S_2
D_1	\$50	\$70
D_2	\$90	\$40
D_3	\$70	\$60

(a) What is the optimal decision based on the maximin criterion and payoff values?
(b) What is the optimal decision based on the payoff values and the maximax criterion?
(c) What is the optimal decision based on the regret values and the pessimistic criterion?

19-2. The following is the payoff table corresponding to a decision problem:

Alternative	State of Nature		
	S_1	S_2	S_3
D_1	\$−20	\$50	\$90
D_2	\$50	\$20	\$70
D_3	\$90	\$10	\$80

(a) What is the optimal decision based on the maximin criterion and payoff values?
(b) What is the optimal decision based on the payoff values and the maximax criterion?
(c) What is the optimal decision based on the regret values and the pessimistic criterion?

19-3. Refer to Exercise 19-1. Let $P(S_1) = 0.40$ and $P(S_2) = 0.60$.
(a) Find the optimal decision based on expected monetary value criterion.
(b) Find the optimal decision based on the expected value criterion and the regret values. Verify that it is the same as the one in (a).
(c) Let the VNM-utility function of the decision maker be as follows.

Monetary value (in millions of dollars)	40	50	60	70	90
VNM-utility, $u(.)$	0	0.5	0.7	0.85	1.0

Find the optimal decision based on the utility criterion.

19-4. Refer to Exercise 19-2. Let $P(S_1) = 0.20$, $P(S_2) = 0.50$, and $P(S_3) = 0.30$.
(a) What is the optimal decision based on expected monetary value criterion?
(b) Let the VNM-utility function of the decision maker be as follows:

Monetary value (in millions of dollars)	−20	10	20	50	70	80	90
VNM-utility, $u(.)$	0	0.5	0.65	0.75	0.92	0.98	1.0

Find the optimal decision based on the utility criterion.

19-5. The Wilhelms' Cola Company plans to market a new pineapple-flavoured cola this summer. The decision is whether to package the cola in returnable or in non-returnable bottles. Currently, the provincial legislature is considering eliminating non-returnable bottles. The payoff table below shows the estimated monthly profits (in thousands of dollars) if the pineapple cola is bottled in returnable versus non-returnable bottles. Of course, if the law is passed and the decision is to bottle the cola in non-returnable bottles, all profits would be from out-of-province sales.

Alternative	**Law Is Passed (S_1)**	**Law Is Not Passed (S_2)**
Returnable bottle	80	40
Non-returnable bottle	25	60

(a) What would be the optimal decision based on the maximin criterion and payoff values?
(b) What would be the optimal decision based on the maximax criterion and payoff values?
(c) What would be the optimal decision based on the pessimistic criterion and regret values?

19-6. Refer to Exercise 19-5 involving the Wilhelms' Cola Company. Tybo Wilhelms, president of Wilhelms' Cola Company, has discussed the problem with his provincial representative and established the probability to be 0.7 that non-returnable bottles will be eliminated.
(a) What would be the optimal decision based on expected monetary value criterion?
(b) Let the VNM-utility function of Tybo Wilhelms' decision be as follows:

Monetary value (in millions of dollars)	25	40	60	80
VNM-utility, $u(.)$	0	0.6	0.9	1.0

What will be the optimal decision based on the utility criterion?

19.6 DECISION TREE

We introduced in Chapter 5 the concept of a tree diagram to analyze the outcomes of a complex random experiment that can be decomposed into a sequence of simpler random experiments. We shall now introduce a similar tool called a decision tree that helps in systematic analysis of a complex decision making scenario.

Basically, a decision tree is a pictorial representation of the sequence of all the possible decision alternatives, occurrences of states of nature, and the consequent possible outcomes.

> A **decision tree** is a pictorial representation of the sequence of all the possible decision alternatives, occurrences of states of nature, and the consequent possible outcomes.

A decision tree consists of two types of nodes: (i) a decision node, represented by a square, and (ii) a chance node, represented by a circle. A decision node (a square) is used to indicate the point at which a decision must be made, and the branches going out from the node indicate the set of decision alternatives from which the decision maker must choose one. A chance node (a circle) is used to indicate the point at which one of a set of states of nature occurs. Each branch going out from the node represents a possible state of nature.

STATISTICS IN ACTION

An estimated 120 000 total hip arthroplasty (THA) procedures are performed in North America every year, most of them on elderly people suffering from hip osteoarthritis. This high technology procedure is much more expensive in the short term than simple medical management.

G. Hazen et al[1] used a modification of a decision tree called **factored stochastic tree** to analyze the long-term cost effectiveness of this procedure. Their study showed that, when cost effectiveness is measured in terms of dollars per quality adjusted life year (QALY), this procedure can be among the most cost-effective comparable to well-accepted procedures such as cardiac bypass or renal dialysis.

CHART 19-2: A Decision Tree Representation of the Nortel Decision Problem

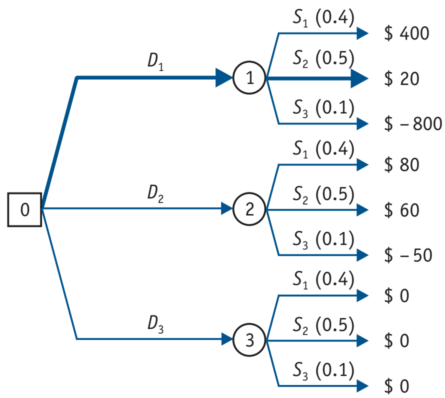


Chart 19-2 is a decision tree representation of the Nortel decision problem. On the left is the decision node (square node), labelled "0," with three branches starting from it, representing the three possible decision alternatives, D_1, D_2, and D_3, of management.

Each of the three chance nodes (circular nodes), labelled "1," "2," and "3," represents chance occurrence of one of the three states of nature. The three branches going out to the right from each of the chance nodes represent chance occurrence of the three states of nature, S_1, S_2, and S_3, and their corresponding probabilities are indicated in parentheses. At the extreme right of the branches we note the estimated payoff values corresponding to each choice of a decision alternative and a state of nature. (We could also use here negative regret values or utilities.) Thus, for example, if management chooses decision D_1 and the state of nature S_2 occurs, the estimated value of the payoff (from Table 19-1) is $20 million. (This scenario is indicated by bold lines in Chart 19-2.)

After the decision tree has been constructed, the best decision strategy can be found by a method called "pruning the tree."

PRUNING THE TREE

We choose a node of the tree that does not have any other node to the right of it and we evaluate this node as follows:

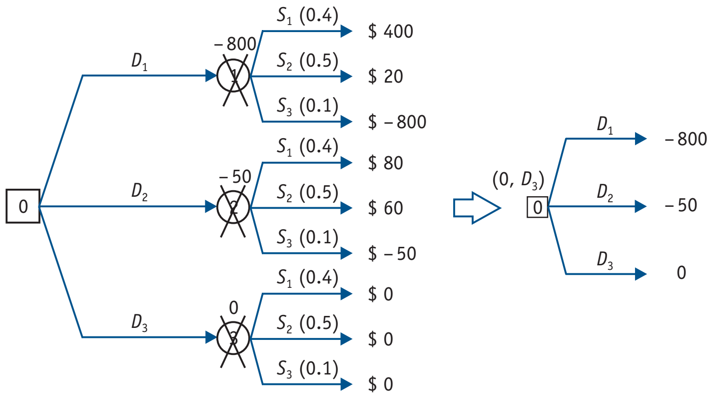
> *i* **Evaluation of a chance node** We calculate the value of the node based on the decision criterion selected, record this value on the top of the node and delete the node and all the branches going to the right from that node.

For example, consider the decision tree corresponding to the Nortel problem, given in Chart 19-2.

Suppose we decide to use the maximin criterion. Then we shall choose node "1," associate with it the value equal to minimum of {400, 20, −800}, which equals −800.

We shall then delete this node and all the branches emanating from it. Similarly we shall associate with nodes "2" and "3" values of −50 and 0, respectively, and we shall delete these nodes and all the branches emanating from them. These are shown in Chart 19-3(a).

CHART 19-3(a): Decision Tree Evaluation Using the Maximin Criterion



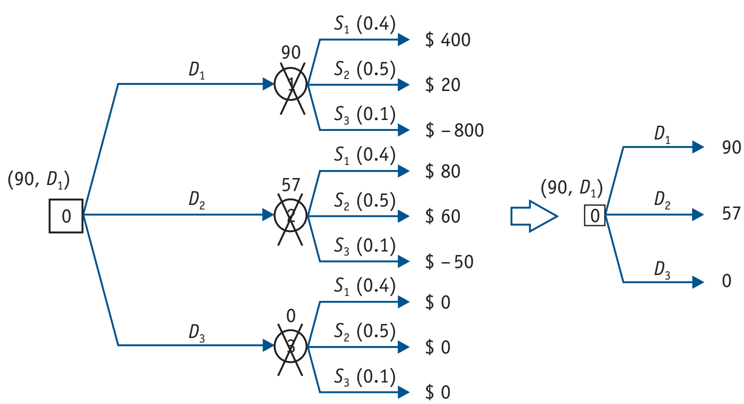
Suppose we decide to use the expected monetary value criterion. Then the value associated with node "1" will be (0.4)(400) + (0.5)(20) + (0.1)(−800) = $90 million. Similarly, the values associated with nodes "2" and "3" will be:

$$(0.4)(80) + (0.5)(60) + (0.1)(-50) = \$57 \text{ million, and}$$

$$(0.4)(0) + (0.5)(0) + (0.1)(0) = 0.$$

We record these values and delete nodes "1," "2," and "3," and all the branches emanating from them. These are shown in Chart 19-3(b).

CHART 19-3(b): Decision Tree Evaluation Using the Expected Monetary Value Criterion



Evaluation of a decision node: We compute the maximum of the values at the end of the branches emanating from the node, and we record this value as well as the branches (decision alternatives) corresponding to this value, on the top of the node. We then delete the node and all the branches emanating from it.

Let us continue with the Nortel example with maximin criterion. We already discussed above how to evaluate the nodes "1," "2," and "3." After deleting nodes "1," "2," and "3," we choose node "0" for evaluation. This is a decision node. The values recorded at the end of the three branches emanating from node "0" are −800, −50, and 0. The maximum of these is 0 and corresponds to decision alternative D_3. Hence we record (0, D_3) on the top of node "0" and delete node "0" and all the branches emanating from it. Since all the nodes have been evaluated, we are done. Our optimal decision is to choose the alternative D_3.

In Chart 19-3(b) we evaluated nodes "1," "2," and "3" of the Nortel decision tree using the expected monetary value criterion. The next node is "0." The values at the end of the three branches emanating from node "0" are 90, 57, and 0. The maximum of these is 90 and corresponds to decision alternative D_1. Hence we record (90, D_1) on the top of node "0" and delete node "0" and all the branches emanating from it. Since all the nodes have been evaluated, we are done. Our optimal decision is to choose the alternative D_1.

19.7 VALUE OF ADDITIONAL INFORMATION

Let us continue with the Nortel problem. Since the decision, whether to market the new product, involves hundreds of millions of dollars, the management would like to conduct a more thorough market analysis to obtain better estimates of the probabilities of occurrence of the three states of nature. Suppose the market analysis technique that will be used will only indicate whether the market response will be favourable (*RF*) or whether it will be unfavourable (*RUF*). Suppose the reliability of the technique is known to be as follows:

$$P(RF \mid S_1) = 0.9; \qquad P(RUF \mid S_1) = 0.1;$$
$$P(RF \mid S_2) = 0.6; \qquad P(RUF \mid S_2) = 0.4;$$
$$P(RF \mid S_3) = 0.2; \qquad P(RUF \mid S_3) = 0.8;$$

Let us find out how much the company should be willing to pay for the market analysis.

In the decision tree in Chart 19-2, there were only three decision alternatives, D_1, D_2, and D_3, which were represented by three branches emanating from the decision node "0." But now, with the additional option of conducting a more thorough market analysis, we have to modify the tree by adding a fourth branch starting from the node "0." To save space, we shall not draw the whole tree again, but we show in Chart 19-4, only this additional branch.

Node "4" represents the outcome of market analysis, which can be either *RF* or *RUF*. Based on the outcome of the market analysis, the management will have to choose one of the three decision alternatives, D_1, D_2, and D_3. Hence, we have three branches, corresponding to these three alternatives, emanating from each of the nodes "5" and "6." The reliability of the market analysis is not one hundred percent. Even if the outcome of the market analysis is *RF*, in reality the state of nature may be any one of S_1, or S_2, or S_3. The probabilities $P(RF)$, $P(RUF)$ can be computed using

STATISTICS IN ACTION

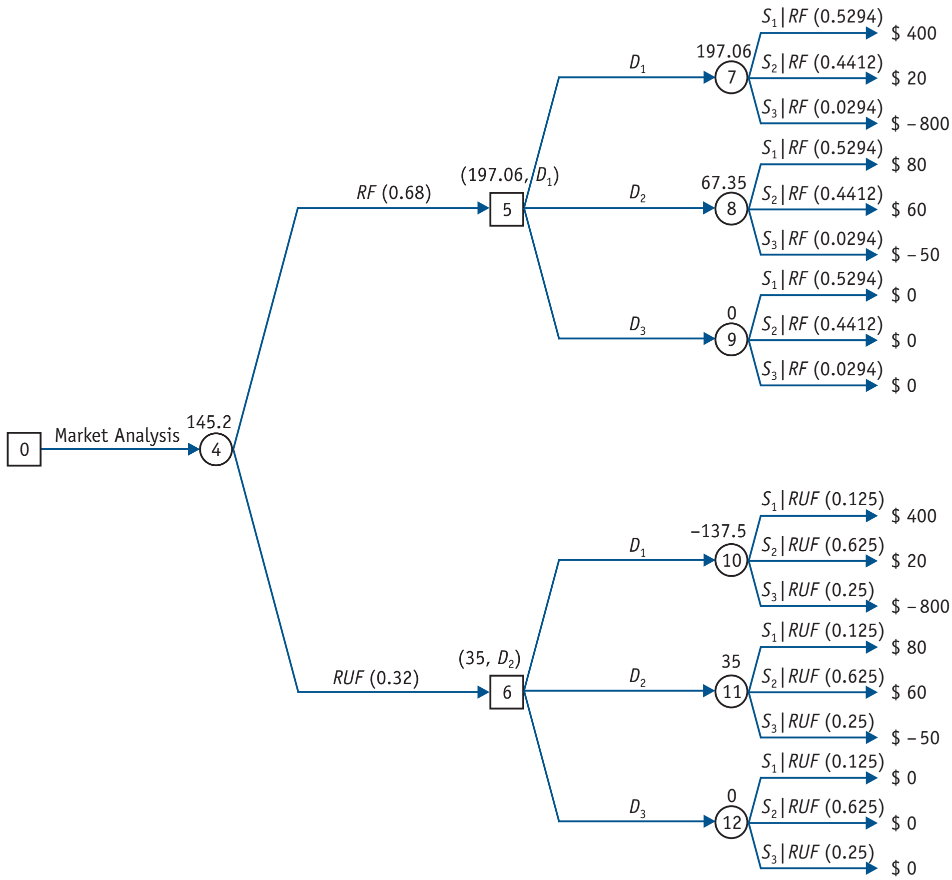
Decision analysis techniques are used to analyze a wide range of real world problems.

For example, they have been used by the electric utility industry to evaluate the health, environmental, and economic risks associated with their operations.[2]

They are used by banks to decide upon the choice of a contingency-planning program for its operations' services.[3]

Recent collapses of major commercial fisheries in Canada have led to significant research on the use of utility theory and other decision analysis tools in developing an effective fisheries' management system.[4]

CHART 19-4: Market Analysis Decision Branch of the Nortel Problem



Formula 5-5 on page 195 and the general law of multiplication (Formula 5-11 on page 201) in Chapter 5 of the textbook, as:

$$\begin{aligned} P(RF) &= P(RF \text{ and } S_1) + P(RF \text{ and } S_2) + P(RF \text{ and } S_3) \\ &= P(RF \mid S_1)P(S_1) + P(RF \mid S_2)P(S_2) + P(RF \mid S_3)P(S_3) \\ &= (0.9)(0.4) + (0.6)(0.5) + (0.2)(0.1) = 0.68 \end{aligned}$$

Similarly,

$$\begin{aligned} P(RUF) &= P(RUF \mid S_1)P(S_1) + P(RUF \mid S_2)P(S_2) + P(RUF \mid S_3)P(S_3) \\ &= (0.1)(0.4) + (0.4)(0.5) + (0.8)(0.1) = 0.32 \end{aligned}$$

The conditional probabilities of S_1, S_2, and S_3, given the outcome of the market analysis (which will be one of RF and RUF), can be obtained using Bayes' Theorem (Formula 5-14 on page 208) in Chapter 5 of the textbook, as

$$\begin{aligned} P(S_1 \mid RF) &= \frac{P(RF \mid S_1)P(S_1)}{P(RF \mid S_1)P(S_1) + P(RF \mid S_2)P(S_2) + P(RF \mid S_3)P(S_3)} \\ &= \frac{(0.9)(0.4)}{(0.9)(0.4) + (0.6)(0.5) + (0.2)(0.1)} = \frac{0.36}{0.68} = 0.5294 \end{aligned}$$

$$P(S_2|RF) = \frac{P(RF|S_2)P(S_2)}{P(RF|S_1)P(S_1) + P(RF|S_2)P(S_2) + P(RF|S_3)P(S_3)}$$
$$= \frac{(0.6)(0.5)}{0.68} = 0.4412$$

$$P(S_3|RF) = \frac{P(RF|S_3)P(S_3)}{P(RF|S_1)P(S_1) + P(RF|S_2)P(S_2) + P(RF|S_3)P(S_3)} = 0.0294$$

$$P(S_1|RUF) = \frac{P(RUF|S_1)P(S_1)}{P(RUF|S_1)P(S_1) + P(RUF|S_2)P(S_2) + P(RUF|S_3)P(S_3)}$$
$$= \frac{(0.1)(0.4)}{(0.1)(0.4) + (0.4)(0.5) + (0.8)(0.1)} = \frac{0.04}{0.32} = 0.125$$

$$P(S_2|RUF) = \frac{P(RUF|S_2)P(S_2)}{P(RUF|S_1)P(S_1) + P(RUF|S_2)P(S_2) + P(RUF|S_3)P(S_3)}$$
$$= \frac{(0.4)(0.5)}{0.32} = 0.625$$

$$P(S_3|RUF) = \frac{P(RUF|S_3)P(S_3)}{P(RUF|S_1)P(S_1) + P(RUF|S_2)P(S_2) + P(RUF|S_3)P(S_3)}$$
$$= 0.25$$

EVALUATION OF MARKET ANALYSIS BRANCH

Let us now evaluate this tree branch using the process of "pruning". Let us use the expected monetary value criterion.

The expected monetary values associated with chance nodes labeled "7," "8," "9," "10," "11," and "12" are:

Node	Expected Monetary Value
7	$(0.5294)(400) + (0.4412)(20) + (0.0294)(-800) = 197.06$
8	$(0.5294)(80) + (0.4412)(60) + (0.0294)(-50) = 67.35$
9	$(0.5294)(0) + (0.4412)(0) + (0.0294)(0) = 0$
10	$(0.125)(400) + (0.625)(20) + (0.25)(-800) = -137.5$
11	$(0.125)(80) + (0.625)(60) + (0.25)(-50) = 35$
12	$(0.125)(0) + (0.625)(0) + (0.25)(0) = 0$

We next move to nodes labelled "5" and "6."

The value corresponding to the decision node labelled "5" is the maximum of 197.06, 67.35, and 0. It is 197.06, corresponding to decision alternative D_1. The value corresponding to the decision node labelled "6" is the maximum of −137.5, 35, and 0. It is 35, corresponding to alternative D_2.

Next we evaluate the chance node labelled "4." The expected monetary value associated with the node "4" is $(0.68)(197.06) + (0.32)(35) = 145.2$. Now let us finally evaluate the decision node "0." There are four branches coming out of node "0." The three branches corresponding to decision alternatives D_1, D_2, D_3, shown in

Chart 19-3(b) in addition to the market analysis branch in Chart 19-4. The maximum of the values at the end of the branches corresponding to D_1, D_2, and D_3 is 90. (See Chart 19-3(b).)

Now suppose market analysis costs $\$\theta$. Then the expected value corresponding to this branch will be $\$(145.2 - \theta)$.

The decision to conduct market analysis will be optimal if $(145.2 - \theta) \geq 90$; that is, if $\theta \leq 145.2 - 90 = \$55.2$ million. So, if the company pays less than \$55.2 million to conduct the market analysis, its expected profit will increase. In this case, the optimal decision will be to conduct the market analysis: if the result of market analysis is "favourable (*RF*)," then choose decision alternative D_1; if the result is "unfavourable (*RUF*)," then choose decision alternative D_2.

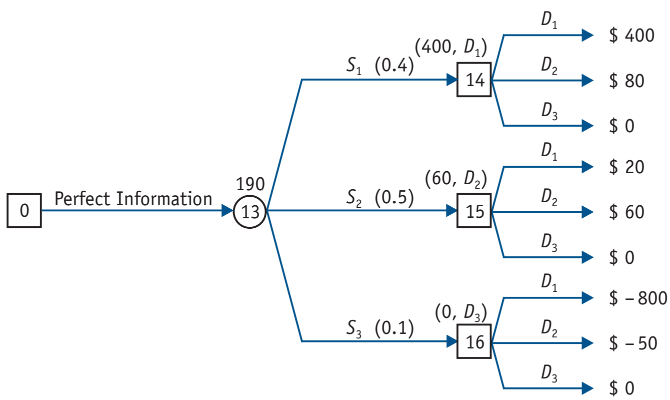
We call the amount \$55.2 million the **expected value of the market analysis**.

EXPECTED VALUE OF "PERFECT INFORMATION"

In the above example regarding market analysis, we assumed that the market analysis technique is imperfect (that is, it may sometimes give wrong results) and that its reliability is known. Now let's assume that a technique of analysis is available that is perfect and determines for sure which of the three states of nature, S_1, S_2, and S_3, will occur. Let us find out how much the management should be willing to pay for such "*perfect*" *information*.

Our decision tree in Chart 19-2 will now have an additional branch starting from node "0", corresponding to the decision alternative of seeking perfect information. To save space, we shall not draw the whole tree again but we will show in Chart 19-5 only this additional branch corresponding to the "perfect" technique.

CHART 19-5: Perfect Information Decision Branch of the Nortel Problem



The chance node labelled "13" corresponds to the outcome of the perfect market analysis. The result of the analysis will be S_1, or S_2, or S_3. Since the technique is perfect, the probabilities corresponding to these three branches are the same as the probabilities of occurrence of the three states of nature, which are given to be 0.4, 0.5, and 0.1, respectively. If the outcome is S_1, then management will have to make one of the three decisions D_1, D_2, and D_3. This is represented by the three branches starting from node "14" and the corresponding payoffs are the profits corresponding to D_1, D_2, and D_3, when the state of nature is S_1. These are given by Column 1 in Table 19-1 on page 4. Similarly the numbers at the end of the branches coming out of nodes labelled "15" and "16" are the payoffs in Table 19-1 in columns corresponding to S_2 and S_3, respectively.

Let us evaluate this branch, using the expected monetary value criterion.

Nodes labelled "14," "15," and "16" are decision nodes. The maximum of the values at the end of the branches starting from node "14" is 400. Hence, we replace this node and the branches starting from it by (400, D_1). Similarly, we replace the nodes labelled "15" and "16" by corresponding maximum values at the end of branches starting from them, which are (60, D_2) and (0, D_3), respectively.

Node "13" is a chance node. Hence, we replace it and the branches starting from it by the expected value of the values at the end of the branches, which is: $(0.4)(400) + (0.5)(60) + (0.1)(0) = 190$.

Now, let us evaluate the decision node "0." The maximum of the values at the end of the branches corresponding to D_1, D_2, and D_3 is 90. (See Chart 19-3(b).)

Now suppose perfect information costs $\$\theta$. Then the value corresponding to this branch will be $\$(190 - \theta)$. The decision to seek perfect information will be optimal if $(190 - \theta) \geq 90$; that is, if $\theta \leq \$100$ million.

So, if the company can obtain perfect information for less than \$100 million, its expected profit will increase. In this case, the optimal decision will be: seek perfect information; if the state S_1 is going to occur, then choose D_1; if the state S_2 is going to occur, then choose D_2; and, if the state S_3 is going to occur, then choose D_3.

We call the amount \$100 million, the ***expected value of perfect information (EVPI).***

i *Expected Value of Perfect Information (EVPI)* is the difference between the expected payoff with perfect information and the maximum expected payoff without additional information.

SELF-REVIEW 19-5

Consider the problem in Self-Review 19-2. Suppose Bob wants to use the expected monetary value criterion. Draw a decision tree to answer the following.

(a) What is the expected value of perfect information?

(b) Bob is considering hiring the services of an economist to obtain better estimates of future market behaviour. The reliability of the economist's forecasts of market rise (FS_1) and market decline (FS_2) is as given below:

$$P(FS_1 | S_1) = 0.9; P(FS_2 | S_1) = 0.1; P(FS_1 | S_2) = 0.2; P(FS_2 | S_2) = 0.8.$$

How much should Bob be willing to pay for the economist's services?

EXERCISES 19-7 TO 19-10

Assume in Exercises 19-7 to 19-10 that the decision maker wants to use the expected monetary value criterion.

19-7. Refer to Exercise 19-3. Compute the expected value of perfect information.

19-8. Refer to Exercises 19-4 and 19-6. In each case, compute the expected value of perfect information.

19-9. Refer to Exercise 19-3. The decision maker wants to hire the services of an expert to obtain better estimates of the probabilities of occurrence of the two states of nature – S_1 and S_2. The reliability of this expert's forecasts is known to be as follows. (We denote the forecasts of S_1 and S_2 by FS_1 and FS_2).

$$P(FS_1 | S_1) = 0.9; P(FS_2 | S_1) = 0.1; P(FS_1 | S_2) = 0.1; P(FS_2 | S_2) = 0.9.$$

How much should the decision maker be willing to pay the expert?

19-10. Refer to Exercise 19-6. Mr. Wilhelms wants to hire the services of an expert to obtain a better estimate of the probability that the non-returnable bottles will be eliminated. The reliability of this expert's forecasts is known to be as follows. (We denote the forecasts of S_1 and S_2 by FS_1 and FS_2.)

$$P(FS_1 | S_1) = 0.8; P(FS_2 | S_1) = 0.2; P(FS_1 | S_2) = 0.1; P(FS_2 | S_2) = 0.9.$$

How much should Wilhelms be willing to pay for the services of the expert?

CHAPTER OUTLINE

I. Statistical decision theory is concerned with choosing an optimal decision alternative from a given set of alternatives.
   A. The various courses of action are called the acts or decision alternatives.
   B. The uncontrollable future events are called the states of nature. Probabilities are usually assigned to the states of nature.
   C. The consequence of a particular decision alternative and state of nature is called the payoff. This is normally a monetary value.
   D. All possible combinations of decision alternatives and states of nature result in a payoff table.

II. There are several criteria for selecting the best decision alternative.
   A. The maximin criterion is a pessimistic criterion, while the maximax criterion is an optimistic criterion.
   B. In the expected monetary value (EMV) criterion, the expected value for each decision alternative is computed and the one corresponding to the largest expected value is selected.
   C. The decision can be based on the opportunity loss function.
      1. An opportunity loss table is constructed by taking, for each state of nature, the differences between optimal payoff for this state of nature, and payoffs corresponding to different decisions and this state of nature. Each entry in this table denotes the opportunity loss or regret due to making a decision if the corresponding state of nature occurs.

III. Decision trees are useful for structuring the various alternatives. They present a picture of the various courses of action and the possible states of nature.

IV. The expected value of additional information is the amount of money the decision maker should be willing to pay for the extra information, when he uses the expected monetary value criterion. It is the difference between the best expected payoff using the additional information and the best expected payoff without the additional information.

V. The expected value of perfect information (EVPI) is the amount of money the decision maker should be willing to pay for perfect information, when he uses the expected monetary value criterion.

CHAPTER EXERCISES 19-11 TO 19-18

19-11. The Twenge Manufacturing Company is considering introducing two new products. The company can add to the current line of products, both of the new products, add neither, or just add one of the two. The success of these products depends on the general economy and on consumers' reactions to the products. These reactions can be summarized as "good," $P(S_1) = 0.30$; "fair," $P(S_2) = 0.50$; or "poor," $P(S_3) = 0.20$. The company's estimated profits, in thousands of dollars, are given in the following payoff table.

	State of Nature		
Decision	S_1	S_2	S_3
Neither	0	0	0
Product 1 only	125	65	−20
Product 2 only	105	60	−30
Both	220	110	−60

(a) What should be the company decision based on maximin criterion and the monetary value?
(b) What should be the decision based on the expected monetary value criterion?
(c) Develop an opportunity loss table. What should be the decision based on the pessimistic criterion and the opportunity loss values?
(d) Compute the expected value of perfect information.

19-12. A financial executive lives in Toronto but frequently must travel to Montreal. She can go to Montreal by car, train, or plane. A plane ticket from Toronto to Montreal costs \$170, and it is estimated that the trip takes 55 minutes in good weather and 75 minutes in bad weather. The cost for a train ticket is \$60, and the trip takes five hours in good weather and six hours in bad weather. The cost of driving her own car from Toronto to Montreal is \$30, and this trip takes six hours in good weather and eight hours in bad weather. The executive places a value of \$25 per hour on her time. The weather forecast is for a 60 percent chance of bad weather tomorrow. What decision would you recommend based on expected monetary value criterion? What is the expected value of perfect information?

19-13. The Aggarwal Manufacturing Company has \$100 000 available to invest. Doctor Aggarwal, the president and CEO of the company, would like to choose one of the following alternatives: (i) expand his production, (ii) invest the money in stocks, or (iii) purchase a certificate of deposit from the bank. Of course, the unknown is whether the economy will remain in recession or there will be a recovery. He estimates the likelihood of a recession continuing at 0.20. Whether there is a recession or not, the certificate of deposit will result in a gain of 6 percent. If there is a recession, he predicts a 10 percent loss if he expands his production and a 5 percent loss if he invests in stocks. If there is a recovery, an expansion of production will result in a 15 percent gain and stock investment will produce a 12 percent gain.

(a) What decision should he make if he uses the maximin strategy?
(b) What decision should he make if the maximax strategy is used?
(c) What should be his decision if he uses the expected monetary value criterion?
(d) What is the expected value of perfect information?

19-14. The quality-assurance department at Malcomb Products must either inspect each part in a lot or not inspect any of the parts. That is, there are two decision alternatives: inspect all the parts or inspect none of the parts. Each lot contains 100 items and is known, from historical data, to belong to one of the three categories—I, II, and III—with probabilities 0.9, 0.05, and 0.05 of being defective, respectively. Each item in categories I, II, and III has a probability $p_1 = 0.01$, $p_2 = 0.02$, and $p_3 = 0.2$, of being defective, respectively.

For the decision not to inspect any parts, the expected cost incurred is $(180)p_jN$, where N is the lot size and \$180 is the cost of selling a defective part. For inspecting all the items in the lot, the cost is \$400.

(a) Develop a payoff table.
(b) What decision should be made if the expected monetary value criterion is used?
(c) What is the expected value of perfect information?
(d) The company is considering the third option. Randomly choose one item from the lot and inspect it. Based on the outcome of this inspection, decide either to inspect each of the remaining parts in the lot or not to inspect any of the remaining parts. If the decision is made to inspect all the remaining parts, the total cost will still be \$400. Otherwise, the inspection of the randomly selected part will cost \$20 and the expected cost of not inspecting the remaining parts will be $(180)p_j(99)$. Is this new option profitable?

19-15. Dude Ranches Incorporated has bought a large farm in eastern Canada to build a summer resort. They have constructed a lake, a swimming pool, and other facilities. However, to build a number of family cottages would require a considerable investment. Hence, they have decided to enter into an agreement with Mihome Inc., to supply very attractive mobile homes. Mihome Inc., agrees to deliver mobile homes every Saturday for \$300 per week per home. For each week, Mihome Inc. must know at least two weeks in advance how many mobile homes Dude Ranches Incorporated wants for the week. An analysis of the various costs involved indicated that \$350 a week should be charged for a ranch home. The basic problem is how many mobile ranch homes to order from Mihome Inc. each week. Based on their experience with similar projects elsewhere, they expect the probabilities that the number of homes rented per week will be 11, 12, or 13, to be 0.4, 0.35, and 0.25, respectively.

(a) Construct a payoff table.
(b) What should be the decision based on the expected monetary value criterion?

(c) Dude Ranches Incorporated is considering hiring the services of a marketing research firm that will give for each week, a two-week advance estimate of the demand for the number of homes for the week. The reliability of the firm's estimates is given in the following table, where each number gives the probability of the estimated value of demand for the given actual value of the demand.

Actual Demand \ Estimated Value	11	12	13
11	0.8	0.2	0.0
12	0.1	0.8	0.1
13	0.1	0.1	0.8

How much should Dude Ranches be willing to pay the marketing research firm for their services?

19-16. A firm is considering three sites for the location of its new plant. The profits from the new plant will depend on whether a proposed new highway will be constructed. The table below gives the expected profits (in thousands of dollars) corresponding to different scenarios.

Decision Alternatives	Highway built (S_1)	Highway not built (S_2)
Location A (D_1)	400	50
Location B (D_2)	200	90
Location C (D_3)	150	100

Based on the current information, the best estimate of the probability that the highway will be built is 0.7.

(a) What decision should be made based on the expected monetary value criterion?

(b) What is the expected value of perfect information?

(c) The VNM-utility of the firm's owner was measured and is tabulated below.

Monetary value (in thousands of dollars)	50	90	100	150	200	400
VNM-utility $u(.)$	0	0.4	0.5	0.75	0.85	1.0

What should be the decision based on the utility criterion?

19-17. Tim Waltzer owns and operates Waltzer's Wrecks, a discount car rental agency. He rents a wreck for \$20 a day. He has an arrangement with Landrum Leasing to purchase used cars at \$6000 each. His cars receive only necessary maintenance and, as a result, are worth only \$3500 at the end of the year of operation. Tim has decided to sell all his wrecks every year and purchase a complete set of used cars from Landrum Leasing. His clerk-accountant provided him with a probability distribution of the number of cars rented per day.

Number of Cars Rented per day	20	21	22	23	24	25
Probability	0.1	0.2	0.2	0.1	0.2	0.2

Tim's car rental agency operates only 300 days a year. The clerk-accountant estimated that it cost \$1.50 per car rental per day for minimal maintenance and cleaning. How many cars should he purchase to maximize expected profit?

19-18. A television network earns on average \$200 000 from a hit show, \$20 000 from an average show and loses on average \$50 000 from a flop show. The past record shows that of all the shows selected by their in-house experts, 20 percent were hits (H), 50 percent were average (A), and 30 percent were flops (F). A new show has just been selected by the in-house experts. The management is considering two options: (i) air the show, or (ii) hire the services of a market research firm to evaluate the show. The reliability of the market research firm is known to be as follows. (Here, MH, MA, and MF denote that the conclusion of the market research firm will be that the show will be a hit, an average show, or a flop.)

$P(MH \mid H) = 0.8$; $P(MA \mid H) = 0.1$; $P(MF \mid H) = 0.1$; $P(MH \mid A) = 0.2$; $P(MA \mid A) = 0.7$; $P(MF \mid A) = 0.1$; $P(MH \mid F) = 0.1$; $P(MA \mid F) = 0.1$; $P(MF \mid F) = 0.8$.

(a) Based on the expected monetary value criterion, how much should the company be willing to pay for the services of the market research firm?

(b) What is the expected value of perfect information?

CHAPTER 19 ANSWERS TO SELF-REVIEW

19-1. (a) The minimum payoffs corresponding to decision alternatives D_1, D_2, and D_3 are, respectively, \$1000, \$1100, and \$1150. The maximum of {1000, 1100, 1150} is \$1150, corresponding to D_3. Hence the optimal decision corresponding to maximin criterion is D_3.

(b) The maximum payoffs corresponding to decision D_1, D_2, and D_3 are, respectively, \$2400, \$2200, and \$1900. The maximum of {2400, 2200, 1900} is \$2400, corresponding to D_1. Hence the optimal decision corresponding to maximax criterion is D_1.

(c) maximum of {(0.7)(1000) + (0.3)(2400) = 1420, (0.7)(1100) + (0.3)(2200) = 1430, (0.7)(1150) + (0.3)(1900)} = 1375} is 1430, corresponding to D_2. Hence the optimal decision is D_2.

19-2. EMV(D_1) = (0.6)(2400) + (0.4)(1000) = \$1840,
EMV(D_2) = (0.6)(2200) + (0.4)(1100) = \$1760,
EMV(D_3) = (0.6)(1900) + (0.4)(1150) = \$1600.
The maximum of the expected monetary values is \$1840, and it corresponds to decision D_1. Hence the optimal choice is decision alternative D_1.

19-3. Negative of the Regret Values

Purchase	**Bull Market, (S_1)**	**Bear Market, (S_2)**
McGraw-Hill Ryerson (D_1)	0	−150
Petro-Canada (D_2)	−200	−50
Kayser Chemicals (D_3)	−500	0

The minimum of the negative regret values corresponding to decisions D_1, D_2, and D_3 are, respectively, −150, −200, and −500.

The maximum of {−150, −200, −500} is \$−150, corresponding to D_1. Hence the optimal decision is D_1.

19-4. The VNM-utilities of the decision alternatives D_1, D_2, and D_3 are,
$u(D_1) = (0.6)(1.0) + (0.4)(0.0) = 0.6$;
$u(D_2) = (0.6)(0.97) + (0.4)(0.4) = 0.742$;
$u(D_3) = (0.6)(0.85) + (0.4)(0.5) = 0.71$.
The maximum of {0.6, 0.742, 0.71} is 0.742, corresponding to D_2. Hence, the optimal choice of decision alternative is D_2.

19-5. $P(FS_1) = P(FS_1 | S_1)P(S_1) + P(FS_1 | S_2)P(S_2)$
$= (0.9)(0.6) + (0.2)(0.4) = 0.62$
$P(FS_2) = P(FS_2 | S_1)P(S_1) + P(FS_2 | S_2)P(S_2)$
$= (0.1)(0.6) + (0.8)(0.4) = 0.38$

$$P(S_1 | FS_1) = \frac{P(FS_1 | S_1)P(S_1)}{P(FS_1 | S_1)P(S_1) + P(FS_1 | S_2)P(S_2)} = 0.871$$

$P(S_2 | FS_1) = 1 - P(S_1 | FS_1) = 1 - 0.871 = 0.129$

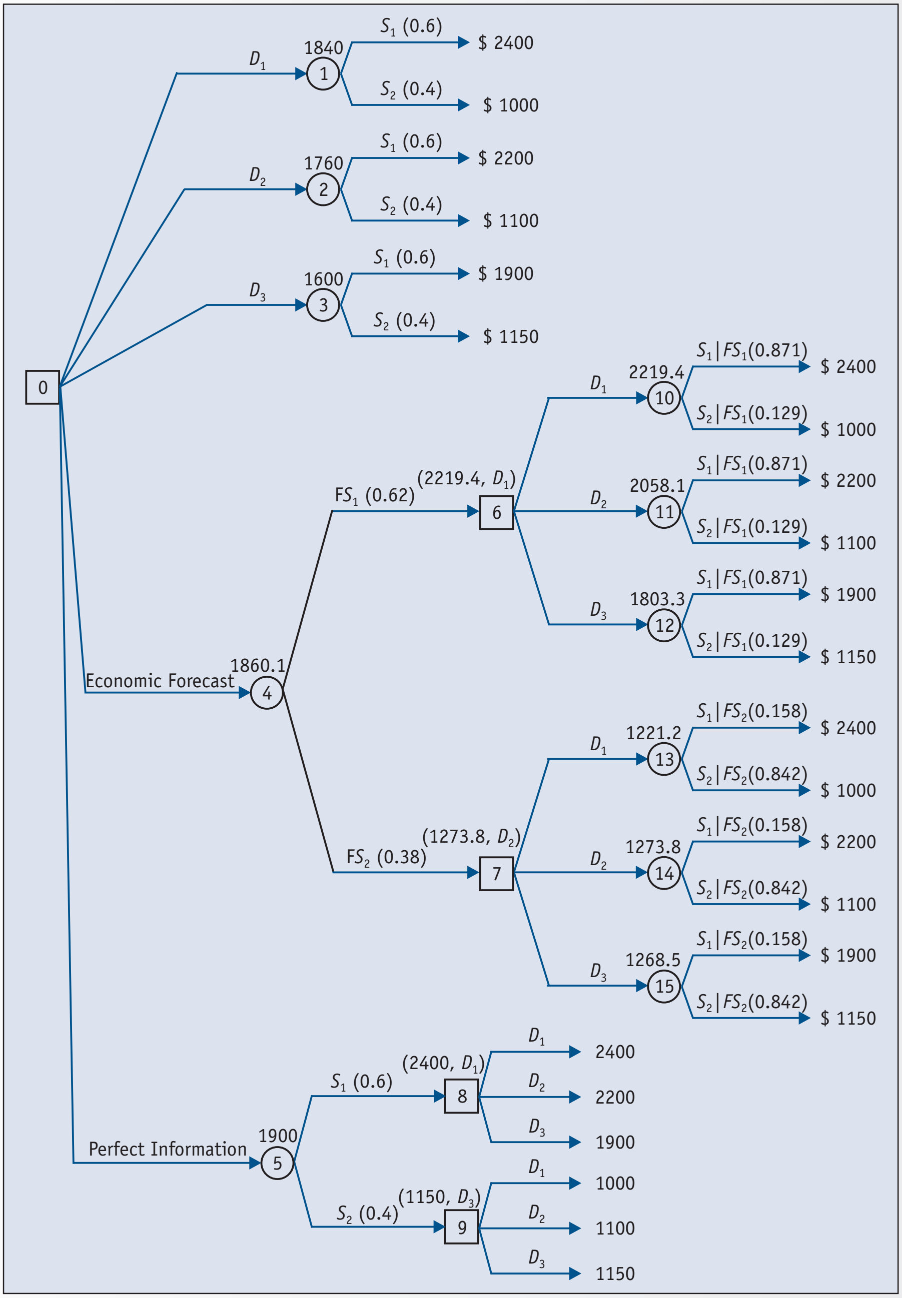
$$P(S_1 | FS_2) = \frac{P(FS_2 | S_1)P(S_1)}{P(FS_2 | S_1)P(S_1) + P(FS_2 | S_2)P(S_2)} = 0.158$$

$P(S_2 | FS_2) = 1 - P(S_1 | FS_2) = 1 - 0.158 = 0.842$

The decision tree for the problem is as given on the next page.

(a) Maximum of {1840, 1760, 1600} is 1840. The expected value of perfect information, θ, is such that: $1900 - \theta = 1840$. That is $\theta = (1900 - 1840) = \$60$.

(b) Let us denote the maximum amount that Bob should be willing to pay for the economist's services by θ. Then, $1860.1 - \theta \geq 1840$; that is, $\theta \leq (1860.1 - 1840) = \$20.1$.


0
D1
1840
1
S1 (0.6)
$ 2400
S2 (0.4)
$ 1000
D2
1760
2
S1 (0.6)
$ 2200
S2 (0.4)
$ 1100
D3
1600
3
S1 (0.6)
$ 1900
S2 (0.4)
$ 1150
Economic Forecast
1860.1
4
FS1 (0.62)
(2219.4, D1)
6
D1
2219.4
10
S1|FS1(0.871)
$ 2400
S2|FS1(0.129)
$ 1000
D2
2058.1
11
S1|FS1(0.871)
$ 2200
S2|FS1(0.129)
$ 1100
D3
1803.3
12
S1|FS1(0.871)
$ 1900
S2|FS1(0.129)
$ 1150
FS2 (0.38)
(1273.8, D2)
7
D1
1221.2
13
S1|FS2(0.158)
$ 2400
S2|FS2(0.842)
$ 1000
D2
1273.8
14
S1|FS2(0.158)
$ 2200
S2|FS2(0.842)
$ 1100
D3
1268.5
15
S1|FS2(0.158)
$ 1900
S2|FS2(0.842)
$ 1150
Perfect Information
1900
5
S1 (0.6)
(2400, D1)
8
D1
2400
D2
2200
D3
1900
S2 (0.4)
(1150, D3)
9
D1
1000
D2
1100
D3
1150

NOTES

Chapter 19

1. G. Hazen, J. Pellissier, and R. Chang, "Factored Stochastic Tree Modeling For Arthritic Joint Replacement Decisions," Decision Analysis Society, working paper no. WP980015, June 1998.
2. W. Balson, J. Welsh, and D. Wilson, "Using Decision Analysis and Risk Analysis to Manage Utility Environment Risk," *Interfaces*, 22, 6, pp. 126–39, 1992.
3. K. Engemann and H. Miller, "Operations Risk Management at a Major Bank," *Interfaces*, 22, 6, pp. 140–49, 1992.
4. D. Lane and R. Stephenson, "A Framework for Risk Analysis in Fisheries Decision Making," PRISM paper series, working paper no. 97–03.

ANSWERS

to Odd-Numbered Chapter Exercises

19-1. a. D_3.
b. D_2.
c. D_3.

19-3. a. D_3.
b. Optimal decision is D_3, same as in (a).
c. D_3.

19-5. a. Use returnable bottles.
b. Use returnable bottles.
c. Use returnable bottles.

19-7. \$(78 – 64) = \$14.

19-9. \$(74.6596 – 64) = \$10.6596.

19-11. a. Do not market any of the products.
b. Market both the products.
c. Market both the products.
d. \$12 000.

19-13. a. Purchase certificate of deposit.
b. Expand production.
c. Expand production.
d. 3.2 percent.

19-15. a. Payoff table

	Demand for 11 cottages	**Demand for 12 cottages**	**Demand for 13 cottages**
Rent 11 cottages	550	550	550
Rent 12 cottages	250	600	600
Rent 13 cottages	–50	300	650

b. Rent 11 cottages every week.
c. \$11.75/week.

19-17. Buy 23 cars.