CHAPTER 4

Valuing Bonds

Investment in a new plant and equipment requires money—often a lot of money. Sometimes firms may be able to save enough out of previous earnings to cover the cost of investments, but often they need to raise cash from investors. In broad terms, we can think of two ways to raise new money from investors: borrow the cash or sell additional shares of common stock.

If companies need the money for only a short while, they may borrow it from a bank; if they need it to make long-term investments, they generally issue bonds, which are simply long-term loans. When companies issue bonds, they promise to make a series of fixed interest payments and then to repay the debt. As long as the company generates sufficient cash, the payments on a bond are certain. In this case bond valuation involves straightforward time value of money computations. But there is some chance that even the most blue-chip company will fall on hard times and will not be able to repay its debts. Investors take this default risk into account when they price the bonds and demand a higher interest rate to compensate.

In the first part of this chapter we sidestep the issue of default risk and we focus on Government of Canada bonds. We show how bond prices are determined by market interest rates and how those prices respond to changes in rates. We also consider the yield to maturity and discuss why a bond's yield may vary with its time to maturity.

Later in the chapter we look at corporate bonds where there is also a possibility of default. We will see how bond ratings provide a guide to the default risk and how low-grade bonds offer higher promised yields.

In Chapter 13 we will look in more detail at the securities that companies issue, and we will see that there are many variations on bond design. But for now, we'll keep our focus on garden variety bonds and general principles of bond valuation.

After studying this chapter you should be able to

- Distinguish among the bond's coupon rate, current yield, and yield to maturity.
- Find the market price of a bond given its yield to maturity, find a bond's yield given its price, and demonstrate why prices and yields vary inversely.
- Show why bonds exhibit interest rate risk.
- Understand why investors pay attention to bond ratings and demand a higher interest rate for bonds with low ratings.

4.1

Bond Characteristics

bond Security that obligates the issuer to make specified payments to the bondholder.

coupon The interest payments paid to the bondholder.

face value Payment at the maturity of the bond. Also called *par value*, or *maturity value*, or *principal*.

coupon rate Annual interest payment as a percentage of face value.

Governments and corporations borrow money by selling **bonds** to investors. The money they collect when the bond is *issued,* or sold to the public, is the amount of the loan. In return, they agree to make specified payments to the bondholders, who are the lenders. When you own a bond, you generally receive a fixed interest payment each year until the bond matures. This payment is known as the **coupon** because most bonds used to have coupons that the investors clipped off and mailed to the bond issuer to claim the interest payment. At maturity, the debt is repaid: The borrower pays the bondholder the bond's **face value** (equivalently, its *par value*, *maturity value* or *principal*).

How do bonds work? Several years ago, the federal government raised money by selling 6.5 percent coupon, June 1, 2004 maturity, Government of Canada bonds. Each bond has a face value of $1,000. Because the **coupon rate** is 6.5 percent, the government makes coupon payments of 6.5 percent of $1,000, or $65 each year.[1] When the bond matures on June 1, 2004, the government must pay the face value of the bond, $1,000, in addition to the final coupon payment.

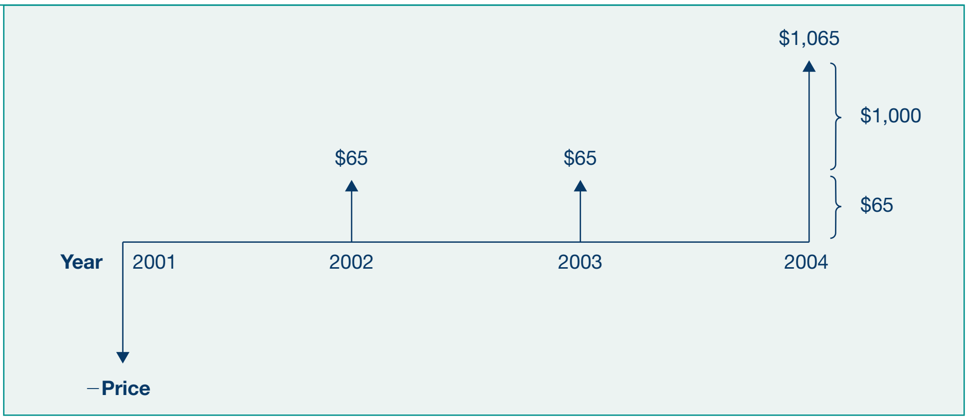
Suppose that in 2001 you decided to buy the 6.5s of 2004, that is, the 6.5 percent coupon bonds maturing in 2004. If you planned to hold the bond until maturity, you would then have looked forward to the cash flows depicted in Figure 4.1. The initial cash flow is negative and equal to the price you have to pay for the bond. Thereafter, the cash flows equal the annual coupon payment until the maturity date in 2004, when you receive the face value of the bond, $1,000, in addition to the final coupon payment.

READING THE FINANCIAL PAGES

Bond prices are reported in the financial press and are always for the previous day's trading activity. Figure 4.2 is an excerpt from the bond quotations for June 12, 2001, reported in the *Globe and Mail*, on June 13, 2001. The entry for the 6.5 percent Canada bonds maturing in June 2004 that we just looked at is highlighted. The prices reported in this newspaper were provided by RBC Dominion Securities, one of Canada's largest bond dealers (www.rbcds.com).

The prices quoted are the "final bid-side price as of 5 pm." The *bid* price is the price

FIGURE 4.1
Cash flows to an investor in the 6.5 percent coupon bond, maturing in the year 2004



[1] In Canada, these coupon payments typically would come in two semiannual installments of $32.50 each. To keep things simple for now, we will assume one coupon payment per year.

FIGURE 4.2

Government of Canada bond quotes from the Globe and Mail*, June 13, 2001*

CANADIAN BONDS

Provided by RBC Dominion Securities

Selected quotations, with changes since the previous day, on actively traded bond issues yesterday. Yields are calculated to full maturity. Price is the final bid-side price as of 5 pm yesterday.

Issuer	Coupon	Maturity	Price	Yield	Price $ Chg
GOVERNMENT OF CANADA					
Canada	5.500	Sep 01/02	101.15	4.51	–0.01
Canada	6.000	Dec 01/02	101.96	4.60	–0.01
Canada	11.250	Dec 15/02	109.49	4.64	–0.02
Canada	11.750	Feb 01/03	110.90	4.73	–0.02
Canada	7.250	Jun 01/03	104.58	4.78	–0.01
Canada	5.750	Jun 01/03	101.80	4.78	–0.01
Canada	5.250	Sep 01/03	100.74	4.89	0.00
Canada	7.500	Dec 01/03	105.81	4.97	0.00
Canada	10.250	Feb 01/04	112.62	5.06	0.02
Canada	6.500	Jun 01/04	103.77	5.11	0.03
Canada	5.000	Sep 01/04	99.51	5.17	0.05
Canada	9.000	Dec 01/04	111.82	5.22	0.05
Canada	12.000	Mar 01/05	122.28	5.30	0.07
Canada	6.000	Sep 01/05	102.53	5.32	0.08
Canada	12.250	Sep 01.05	125.83	5.32	0.08
Canada	8.750	Dec 01/05	113.28	5.36	0.08
Canada	5.750	Sep 01/06	101.59	5.39	0.10
Canada	14.000	Oct 01/06	138.78	5.46	0.11
Canada	7.000	Dec 01/06	107.29	5.44	0.11
Canada	7.250	Jun 01/07	108.87	5.48	0.12
Canada	12.750	Mar 01/08	139.49	5.60	0.23
Canada	10.000	Jun 01/08	125.44	5.54	0.22
Canada	6.000	Jun 01/08	102.54	5.55	0.20
Canada	5.500	Jun 01/09	99.36	5.60	0.26
Canada	9.500	Jun 01/10	127.03	5.62	0.34
Canada	5.500	Jun 01/10	99.10	5.63	0.30
Canada	9.000	Mar 01/11	124.39	5.69	0.37
Canada	6.000	Jun 01/11	102.61	5.65	0.34
Canada	10.250	Mar 15/14	140.04	5.77	0.48
Canada	11.250	Jun 01/15	151.93	5.78	0.54
Canada	9.750	Jun 01/21	144.05	5.95	0.48
Canada	9.250	Jun 01/22	138.75	5.98	0.48
Canada	8.000	Jun 01/23	125.28	5.93	0.46
Canada	9.000	Jun 01/25	138.16	5.98	0.51
Canada	8.000	Jun 01/27	127.14	5.94	0.45
Canada	5.750	Jun 01/29	98.70	5.84	0.35
Real Return	4.250	Dec 01/21	109.66	3.58	0.00
Real Return	4.250	Dec 01/26	111.11	3.58	0.00
Real Return	4.000	Dec 01/31	107.65	3.59	0.00

Source: The *Globe and Mail*, June 13, 2001.

you receive if you *sell* the bond to a bond dealer. Prices are quoted as percentage of face value. Thus for the 6.5 percent bond, the price of 103.77 means 103.77 percent of the $1,000 face value, or $1,037.7. If you want to *buy* the bond, you pay the *asked* price. Just as used car dealers earn their living by reselling cars at higher prices than they paid for them, so the bond dealer needs to charge a *spread* between the bid and asked prices.

The next column, Yield, shows the *yield to maturity*, the rate of return investors will receive if they buy the bond at the quoted bond price and hold it to maturity in 2004. You can see that the Canada bonds at 6.5 percent offer investors a return of 5.11 percent. We will explain shortly how this figure was calculated. The final column, Price $ Chg, shows the change in price from the previous day. Thus the Canada 6.5s of 2004 price on June 11 was 103.74.

SEE BOX P. 115

If you wanted to buy the Canada 6.5s of 2004, you would contact a bond dealer. Dealers hold inventories of bonds and are typically part of financial institutions such as banks and brokerage houses. This is an *over-the-counter market*, where securities are not traded in one central place. For example, TD Waterhouse offers bonds to retail investors through its online brokerage service (www.TDWaterhouse.com). See the nearby Finance in Action box for a discussion of important developments in online bond trading. At TD Waterhouse's website on June 12, 2001, you could sell the 6.5 percent bond for 103.289 (bid price) and buy it for 104.299 (ask price). The spread between the bid and ask prices is 104.299 – 103.289 = 1.01, or about 1 percent of the bond's value. The spread for a large investor, such as a pension fund, would be much smaller, closer to 0.1 percent of the bond's value.

☑ CHECK POINT 4.1

Find the 5.5 June 1, 2010, Canada bond in Figure 4.2.

a. If you already own this bond, at what price can you sell it?
b. If you want to buy the bond, will the price you pay be higher or lower than the price quoted in the paper? Why?
c. How much did the price change from the previous day?
d. What annual interest payment does the bond make?
e. What is the bond's yield to maturity?

FINANCE IN ACTION

Hope for Retail Bond Traders

The development by the banks and independent dealers of online bond trading systems is good news for retail investors. It should help bond trading become more visible and comparable to what online investors in stocks now have—eventually.

Why Is Visibility a Problem for Individual Bond Investors?

Most bonds are fixed-income investments—that is, the owner receives a fixed amount of dollars of interest at intervals of usually six months, over the fixed life of the bond.

At the end, the owner receives the full face value amount of the bond as well. This means the rate of return on your investment depends directly on the price you pay. The more you pay, the lower the interest you get on each dollar you invest.

How Do You Know What's a Reasonable Price?

In the stock market you can easily find a reasonable starting point. Investors are accustomed to being able to check the last price at which a stock traded from information that's available to everyone. But when did you last see a ticker tape showing bond prices?

This lack of reliable price information is because there is no central reporting system for actual bond transaction prices. The usual industry excuse is that it's because bonds are traded over a telecommunications network by bond dealers, scattered around the country, and buying and selling them as owners not agents.

But so are the well-known stocks that trade on the Nasdaq stock market in the United States. Recent transaction prices in that market are as widely available as the [stock trading] prices on the New York and Toronto stock exchanges. There are other problems with the Nasdaq market but that is not one of them.

A would-be investor in bonds has to go to a bond dealer and ask for a quote on available issues. Oddly, for stock investors who first encounter this, there will be no mention of commission. What's on the broker's mind is how much he or she can mark up the bond price to a buyer, over the so-called transfer price. This is an internal price set by the firm at which the bond is transferred to the broker from the firm's inventory. This may or may not be what the bond cost the firm.

If the client wants to sell the bond instead, the broker has to decide how much to mark down the price the client gets. The broker gets the mark up or the markdown instead of a commission.

What decides the size of the mark ups or markdowns? They vary from firm to firm, from broker to broker, and from one minute to the next. If you are a good client who provides a lot of business and are not a pest, your "commission" may be small or even zero. You may do similarly well if your broker is hungry.

But these hidden charges provide less scrupulous brokers an opportunity to levy an unconscionably high "commission" without the client being aware of what's happening. That's because, unlike stocks, no commission charge is indicated to the client on an order confirmation, and there is no widely available, reliable source of the current market price for bonds against which you can check what your broker quotes.

The arrival of more online bond trading firms will be welcome because their published quotes improve the visibility of bond prices.

But until the Canadian investment industry stops dragging its feet on a long delayed project to provide an online source of actual bond transaction prices, there will be nothing for bond investors comparable to the ubiquitous stock market ticker.

Source: Adapted from Hugh Anderson, "Light Appears at the End of the Online Bond Tunnel: Consortium of Industry Players Ready to Launch Site," *Financial Post* (*National Post*), F 24'01, C5.

4.2 Bond Prices and Yields

In Figure 4.1, we examined the cash flows that an investor in 6.5 percent Canada bonds would receive. How much would you be willing to pay for this stream of cash flows? To find out, you need to look at the interest rate that investors could earn on similar securities. In 2001, Canada bonds with 3-year maturities offered a return of about 5.1 percent. Therefore, to value the 6.5s of 2004, we need to discount the prospective stream of cash flows at 5.1 percent:

$$PV = \frac{\$65}{(1+r)} + \frac{\$65}{(1+r)^2} + \frac{\$1,065}{(1+r)^3}$$
$$= \frac{\$65}{(1.051)} + \frac{\$65}{(1.051)^2} + \frac{\$1,065}{(1.051)^3} = \$1,038.05$$

Bond prices are usually expressed as a percentage of their face value. Thus we can say that our 6.5 percent Canada bond is worth 103.805 percent of face value, and its price would usually be quoted as 103.805.

Did you notice that the coupon payments on the bond are an annuity? In other words, the holder of our 6.5 percent Canada bond receives a level stream of coupon payments of \$65 per year for each of the 3 years. At maturity the bondholder gets an additional payment of \$1,000. Therefore, you can use the annuity formula to value the coupon payments and then add on the present value of the final payment of face value:

$$\begin{aligned} \textbf{PV} &= \textbf{PV (coupons) + PV (face value)} \\ &= \textbf{(coupon} \times \textbf{annuity factor) + (face value} \times \textbf{discount factor)} \qquad (4.1) \\ &= \$65 \times \left[\frac{1}{.051} - \frac{1}{.051(1.051)^3}\right] + 1,000 \times \frac{1}{1.051^3} \\ &= \$176.68 + \$861.37 = \$1,038.05 \end{aligned}$$

If you need to value a bond with many years to run before maturity, it is usually easiest to value the coupon payments as an annuity and then add on the present value of the final payment.

CHECK POINT 4.2

Calculate the present value of a 6-year bond with a 9 percent coupon. The interest rate is 12 percent.

EXAMPLE 4.1 *Bond Prices and Semiannual Coupon Payments*

Thus far we've assumed that interest payments occur annually. This is the case for bonds in many European countries, but in Canada most bonds make coupon payments *semiannually.* So when you hear that a bond in Canada has a coupon rate of 6.5 percent, you can generally assume that the bond makes a payment of \$65/2 = \$32.50 every 6 months. Similarly, when investors in Canada refer to the bond's interest rate, they usually mean the semiannually compounded interest rate. Thus an interest rate quoted at 5.1 percent really means that the 6-month rate is 5.1/2 = 2.55 percent.[2] The actual cash flows on the Canada bond are illustrated in Figure 4.3. To value the bond a bit more precisely, we should have discounted the series of semiannual payments by the semiannual rate of interest as follows:

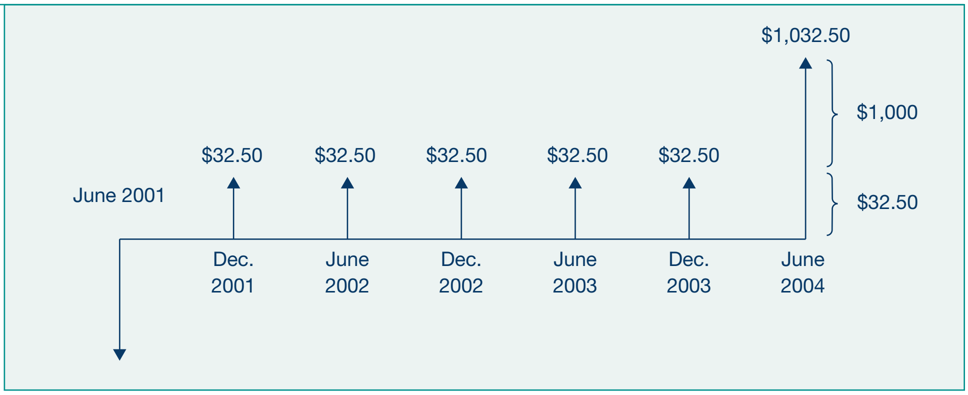
[2] You may have noticed that the interest rate compounded semiannually on the bond is also the bond's APR, although this term is generally not used by bond investors. To find the effective rate, we can use a formula that we used in Section 3.6:

$$\text{Effective annual rate} = \left(1 + \frac{\text{APR}}{m}\right)^m - 1$$

where m is the number of payments each year. In the case of our Canada bond,

$$\text{Effective annual rate} = \left(1 + \frac{.051}{2}\right)^2 - 1 = 1.0255^2 - 1 = .0517, \text{ or } 5.17\%$$

FIGURE 4.3
Cash flows to an investor in the 6.5 percent coupon bond, maturing in 2004. The bond pays semiannual coupons, so there are two payments of $32.50 each year.



$$\begin{aligned} PV &= \frac{\$32.5}{(1.0255)} + \frac{\$32.5}{(1.0255)^2} + \frac{\$32.5}{(1.0255)^3} + \frac{\$32.5}{(1.0255)^4} + \frac{\$32.5}{(1.0255)^5} + \frac{\$1{,}032.5}{(1.0255)^6} \\ &= \$1{,}038.49 \end{aligned}$$

which is slightly more than the value of $1,038.05 that we obtained when we treated the coupon payments as annual rather than semiannual.[3] Since semiannual coupon payments just add to the arithmetic, we will stick to our approximation for the rest of the chapter and assume annual interest payments.

HOW BOND PRICES VARY WITH INTEREST RATES

As interest rates change, so do bond prices. For example, suppose that investors demanded an interest rate of 6.5 percent on 3-year government bonds. What would be the price of the Canada 6.5s of 2004? Just repeat the last calculation with a discount rate of $r = .065$:

$$\text{PV at 6.5\%} = \frac{\$65}{(1.065)} + \frac{\$65}{(1.065)^2} + \frac{\$1{,}065}{(1.065)^3} = \$1{,}000.00$$

Thus when the interest rate is the same as the coupon rate (6.5 percent in our example), the bond sells for its face value.

We first valued the Canada bond with an interest rate of 5.1 percent, which is lower than the coupon rate. In that case the price of the bond was *higher* than its face value. We then valued it using an interest rate that is equal to the coupon rate and found that bond price equalled face value. You have probably already guessed that when the cash flows are discounted at a rate that is *higher* than the bond's coupon rate, the bond is worth *less* than its face value. The following example confirms that this is the case.

EXAMPLE 4.2 *Bond Prices and Interest Rates*

Investors will pay $1,000 for a 6.5 percent, 3-year Canada bond, when the interest rate is 6.5 percent. Suppose that the interest rate is higher than the coupon rate at, say, 15 percent.

[3] Why is the present value a bit higher in this case? Because now we recognize that half the annual coupon payment is received only 6 months into the year, rather than at year-end. Because part of the coupon income is received earlier, its present value is higher.

Now what is the value of the bond? Simple! We just repeat our initial calculation but with $r = .15$:

$$\text{PV at 15\%} = \frac{\$65}{(1.15)} + \frac{\$65}{(1.15)^2} + \frac{\$1{,}065}{(1.15)^3} = \$805.93$$

The bond sells for 80.59 percent of face value.

We conclude that when the market interest rate exceeds the coupon rate, bonds sell for less than face value. When the market interest rate is below the coupon rate, bonds sell for more than face value.

YIELD TO MATURITY VERSUS CURRENT YIELD

Suppose you are considering the purchase of a 3-year bond with a coupon rate of 10 percent. Your investment adviser quotes a price for the bond. How do you calculate the rate of return the bond offers?

For bonds priced at face value the answer is easy. The rate of return is the coupon rate. We can check this by setting out the cash flows on your investment:

	Cash Paid to You in Year			
You Pay	**1**	**2**	**3**	**Rate of Return**
$1,000	$100	$100	$1,100	10%

Notice that in each year you earn 10 percent on your money ($100/$1,000). In the final year you also get back your original investment of $1,000. Therefore, your total return is 10 percent, the same as the coupon rate.

Now suppose that the market price of the 3-year bond is $1,136.16. Your cash flows are as follows:

	Cash Paid to You in Year			
You Pay	**1**	**2**	**3**	**Rate of Return**
$1,136.16	$100	$100	$1,100	?

current yield Annual coupon payments divided by bond price.

What's the rate of return now? Notice that you are paying out $1,136.16 and receiving an annual income of $100. So your income as a proportion of the initial outlay is $100/$1,136.16 = .088, or 8.8 percent. This is sometimes called the bond's **current yield.**

However, total return depends on both interest income and any capital gains or losses. A current yield of 8.8 percent may sound attractive, only until you realize that the bond's price must fall. The price today is $1,136.16, but when the bond matures 3 years from now, the bond will sell for its face value, or $1,000. A price decline (i.e., a *capital loss*) of $136.16 is guaranteed, so the overall return over the next 3 years must be less than the 8.8 percent current yield.

Let us generalize. A bond that is priced above its face value is said to sell at a *premium.* Investors who buy a bond at a premium face a capital loss over the life of the bond, so the return on these bonds is always *less* than the bond's current yield. A bond priced below face value sells at a *discount.* Investors in discount bonds face a capital *gain* over the life of the bond; the return on these bonds is *greater* than the current yield:

Because it focuses only on current income and ignores prospective price increases or decreases, the current yield mismeasures the bond's total rate of return. It overstates the return of premium bonds and understates that of discount bonds.

yield to maturity
Interest rate for which the present value of the bond's payments equals the price.

We need a measure of return that takes account of both current yield and the change in a bond's value over its life. The standard measure is called **yield to maturity.** The yield to maturity is the answer to the following question: At what interest rate would the bond be correctly priced?

The yield to maturity is defined as the discount rate that makes the present value of the bond's payments equal to its price.

If you can buy the 3-year bond at face value, the yield to maturity is the coupon rate, 10 percent. We can confirm this by noting that when we discount the cash flows at 10 percent, the present value of the bond is equal to its $1,000 face value:

$$\text{PV at 10\%} = \frac{\$100}{(1.10)} + \frac{\$100}{(1.10)^2} + \frac{\$1,100}{(1.10)^3} = \$1,000.00$$

But if you have to buy the 3-year bond for $1,136.16, the yield to maturity is only 5 percent. At that discount rate, the bond's present value equals its actual market price, $1,136.16:

$$\text{PV at 5\%} = \frac{\$100}{(1.05)} + \frac{\$100}{(1.05)^2} + \frac{\$1,100}{(1.05)^3} = \$1,136.16$$

EXAMPLE 4.3 *Calculating Yield to Maturity for the Canada Bond*

We found the value of the 6.5 percent coupon Canada bond by discounting at a 5.1 percent interest rate. We could have phrased the question the other way around: If the price of the bond is $1,038.05, what return do investors expect? We need to find the yield to maturity, in other words, the discount rate r that solves the following equation:

$$\text{Price} = \frac{\$65}{(1+r)} + \frac{\$65}{(1+r)^2} + \frac{\$1,065}{(1+r)^3} = \$1,038.05$$

To find the yield to maturity, most people use a financial calculator. For our Government of Canada bond you would enter a PV of $1,038.05.[4] The bond provides a regular payment of $65, entered as PMT = 65. The bond has a future value of $1,000, so FV = 1,000. The bond life is 3 years, so $n = 3$. Now compute the interest rate, and you will find that the yield to maturity is 5.1 percent. The nearby Financial Calculator box reviews the use of the financial calculator in bond valuation problems.

SEE BOX P. 120

Example 4.3 illustrates that the yield to maturity depends on the coupon payments that you receive each year ($65), the price of the bond ($1,038.05), and the final repayment of face value ($1,000). Thus it is a measure of the total return on this bond, accounting for

[4] Actually on most calculators you would enter 1,038.05 as a negative number, (–1,038.05), because the purchase of the bond represents a cash *outflow.*

FINANCIAL CALCULATOR

Bond Valuation on a Financial Calculator

In Chapter 3 we saw that financial calculators can compute the present values of level annuities as well as the present values of one-time future cash flows. Coupon bonds present both of these characteristics: The coupon payments are level annuities and the final payment of par value is an additional one-time payment. Thus for the coupon bond we looked at in Example 4.3, you would treat the periodic payment as PMT = \$65, the final or future one-time payment as FV = \$1,000, the number of periods as $n = 3$ years, and the interest rate as the yield to maturity of the bond, $i = 5.1$ percent. You would thus compute the value of the bond using the following sequence of key strokes. By the way, the order in which the various inputs for the bond valuation problem are entered does not matter.

Hewlett-Packard HP-10B	Sharp EL-733A	Texas Instruments BA II Plus
65 PMT	65 PMT	65 PMT
1000 FV	1000 FV	1000 FV
3 N	3 n	3 N
5.1 I/YR	5.1 i	5.1 I/Y
PV	COMP PV	CPT PV

Your calculator should now display a value of –1,038.05. The minus sign reminds us that the initial cash flow is negative: You have to pay to buy the bond.

You can also use the calculator to find the yield to maturity of a bond. For example, if you buy this bond for \$1,038.05, you should find that its yield to maturity is 5.1 percent. Let's check that this is so. You enter the PV as –1,038.05 because you buy the bond for this price. Thus to solve for the interest rate, use the following key strokes:

Hewlett-Packard HP-10B	Sharp EL-733A	Texas Instruments BA II Plus
65 PMT	65 PMT	65 PMT
1000 FV	1000 FV	1000 FV
3 N	3 n	3 N
–1038.05 PV	–1038.05 PV	–1038.05 PV
I/YR	COMP i	CPT I/Y

Your calculator should now display 5.1 percent, the yield to maturity of the bond.

both coupon income and price change for someone who buys the bond today and holds it until maturity. Bond investors often refer loosely to a bond's "yield." It's a safe bet that they are talking about its yield to maturity rather than its current yield.

The only *general* procedure for calculating yield to maturity is trial and error. You guess at an interest rate and calculate the present value of the bond's payments. If the present value is greater than the actual price, your discount rate must have been too low, so try a higher interest rate (since a higher rate results in a lower PV). Conversely, if PV is less than price, you must reduce the interest rate. In fact, when you use a financial calculator to compute yield to maturity, you will notice that it takes the calculator a few moments to compute the interest rate. This is because it must perform a series of trial-and-error calculations.[5]

CHECK POINT 4.3

A 4-year maturity bond with a 14 percent coupon rate can be bought for \$1,200. What is the yield to maturity? You will need to use the trial-and-error method, or a financial calculator to answer this question.

5 If you don't have a financial calculator, estimate the yield to maturity with an approximation formula. Check the estimate and fine tune it using the trial-and-error method to be confident of its accuracy. The approximation formula is

$$\text{YTM} = \frac{\text{annual coupon payment} + (\text{par value} - \text{current price})/\text{years to maturity}}{(\text{par value} + \text{current price})/2}$$

Figure 4.4 is a graphical view of yield to maturity. It shows the present value of the 6.5 percent Canada bond for different interest rates. The actual bond price, \$1,038.05, is marked on the vertical axis. A line is drawn from this price over to the present value curve and then down to the interest rate, 5.1 percent. If we picked a higher or lower figure for the interest rate, then we would not obtain a bond price of \$1,038.05. Thus we know that the yield to maturity on the bond must be 5.1 percent.

Figure 4.4 also illustrates a fundamental relationship between interest rates and bond prices:

When the interest rate rises, the present value of the payments to be received by the bondholder falls, and bond prices fall. Conversely, declines in the interest rate increase the present value of those payments and result in higher prices.

A gentle warning! People sometimes confuse the *interest rate*—that is, the return that investors currently require—with the interest, or coupon, payment on the bond. Although interest rates change from day to day, the \$65 coupon payments on our Canada bonds are fixed when the bond is issued. Changes in interest rates affect the *present value* of the coupon payments but not the payments themselves.

RATE OF RETURN

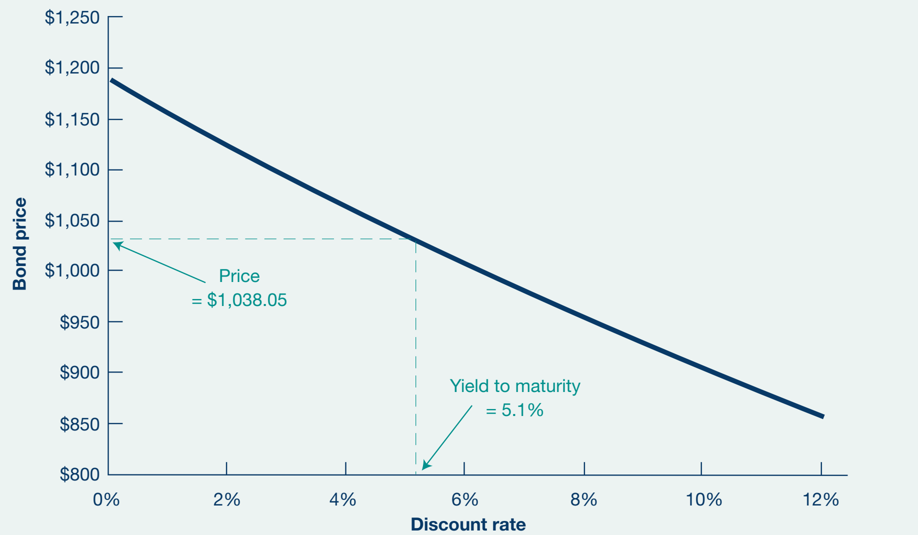
When you invest in a bond, you receive a regular coupon payment. As bond prices change, you may also make a capital gain or loss. For example, suppose you buy the 6.5 percent Canada bond today for a price of \$1,038.05 and sell it next year at a price of \$1,045. The return on your investment is the \$65 coupon payment plus the price change of (\$1,045 – \$1,038.05) = \$6.95. The **rate of return** on your investment of \$1,038.05 is

rate of return Total income per period per dollar invested.

$$\text{Rate of return} = \frac{\text{coupon income + price change}}{\text{investment}} \tag{4.2}$$

$$= \frac{\$65 + \$6.95}{\$1{,}038.05} = .0693, \text{ or } 6.93\%$$

FIGURE 4.4
The value of the 6.5 percent bond is lower at higher discount rates. The yield to maturity is the discount rate at which price equals present value of cash flows.



Because bond prices fall when market interest rates rise and rise when market rates fall, the rate of return that you earn on a bond will also fluctuate with market interest rates. This is why we say bonds are subject to interest rate risk.

Do not confuse the bond's rate of return over a particular investment period with its yield to maturity. The yield to maturity is defined as the discount rate that equates the bond's price to the present value of all its promised cash flows. It is a measure of the average rate of return you will earn over the bond's life if you hold it to maturity. In contrast, the rate of return can be calculated for any particular holding period and is based on the actual income and the capital gain or loss on the bond over that period. The difference between yield to maturity and rate of return for a particular period is emphasized in the following example.

EXAMPLE 4.4 *Rate of Return versus Yield to Maturity*

Our 6.5 percent coupon bond with maturity 2004 currently has 3 years left until maturity and sells today for $1,038.05. Its yield to maturity is 5.1 percent. Suppose that by the end of the year, interest rates have fallen and the bond's yield to maturity is now only 4 percent. What will be the bond's rate of return?

At the end of the year, the bond will have only 2 years to maturity. If investors then demand an interest rate of 4 percent, the value of the bond will be

$$\text{PV at 4\%} = \frac{\$65}{(1.04)} + \frac{\$1{,}065}{(1.04)^2} = \$1{,}047.15$$

You invested $1,038.05. At the end of the year you receive a coupon payment of $65 and have a bond worth $1,047.15. Your rate of return is therefore

$$\text{Rate of return} = \frac{\$65 + (\$1{,}047.15 - \$1{,}038.05)}{\$1{,}038.05} = .0714\text{, or } 7.14\%$$

The yield to maturity at the start of the year was 5.1 percent. However, because interest rates fell during the year, the bond price rose, and this increased the rate of return.

CHECK POINT 4.4

Suppose that the bond's yield to maturity had risen to 7 percent during the year. Show that its rate of return would have been *less* than the yield to maturity.

Is there *any* connection between yield to maturity and the rate of return during a particular period? Yes: If the bond's yield to maturity remains unchanged during an investment period, its rate of return will equal that yield. We can check this by assuming that the yield on 6.5 percent Canada bonds stays at 5.1 percent. If investors still demand an interest rate of 5.1 percent at the end of the year, the value of the bond will be

$$\text{PV} = \frac{\$65}{(1.051)} + \frac{\$1{,}065}{(1.051)^2} = \$1{,}025.99$$

At the end of the year you receive a coupon payment of $65 and have a bond worth $1,025.99, somewhat less than you paid for it. Your total profit is $65 + ($1,025.99 – $1,038.05) = $52.94. The return on your investment is therefore $52.94/$1,038.05 = .051, or 5.1 percent, just equal to the yield to maturity.

When interest rates do not change, the bond price changes with time so that the total return on the bond is equal to the yield to maturity. If the bond's yield to maturity increases, the rate of return during the period will be less than that yield. If the yield decreases, the rate of return will be greater than the yield.

Check Point 4.5

Suppose you buy the bond next year for $1,025.99, and hold it for yet another year, so that at the end of that time it has only 1 year to maturity. Show that if the bond's yield to maturity is still 5.1 percent, your rate of return also will be 5.1 percent and the bond price will be $1,013.32.

The solid curve in Figure 4.5 plots the price of a 30-year maturity, 6.5 percent Canada bond over time assuming that its yield to maturity remains at 5.1 percent. The price declines gradually until the maturity date, when it finally reaches face value. In each period, the price decline offsets the coupon income by just enough to reduce total return to 5.1 percent. The dotted curve in Figure 4.5 shows the corresponding price path for a 30-year maturity, 3 percent coupon Canada bond, also assuming its yield to maturity remains at 5.1 percent. This low-coupon bond currently sells at a discount to face value. The coupon income provides less than a competitive rate of return, so the bond sells below par. Its price gradually approaches face value, however, and the price gain each year brings its total return up to the market interest rate of 5.1 percent.

TAXES AND RATES OF RETURN

Taxes reduce the rate of return on an investment. Let's go back to Example 4.4. You bought a bond for $1,038.05 and sold it 1 year later for $1,047.15 and received one coupon payment of $65. The *before-tax* rate of return on your 1-year investment was 7.14 percent. However, as we discussed in Chapter 2, interest income is fully taxable, and 50 percent of

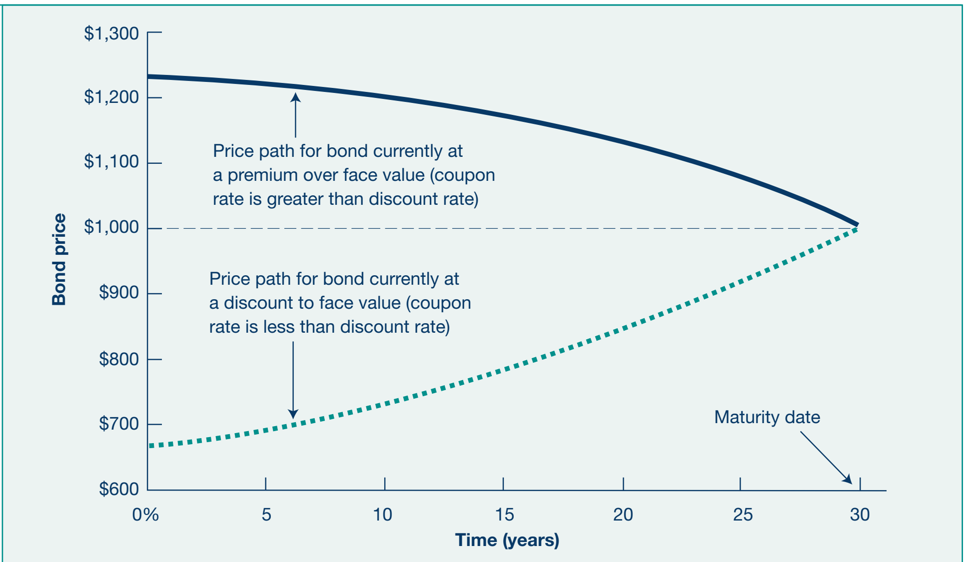


FIGURE 4.5
Bond prices over time, assuming an unchanged yield to maturity. Prices of both premium and discount bonds approach face value as their maturity date approaches.

capital gains are taxable. To figure out the *after-tax* rate of return on the investment, convert the cash flows to their after-tax values by subtracting the relevant taxes. If your personal tax rate is 35 percent, the tax on the coupon payment of $65 is

$$\begin{aligned}\text{Tax on coupon income} &= \text{personal tax rate} \times \text{coupon income}\\ &= .35 \times \$65 = \$22.75\end{aligned}$$

After taxes, the coupon income is

$$\begin{aligned}\text{After-tax coupon income} &= \text{coupon income} - \text{tax on coupon income}\\ &= \$65 - \$22.75 = \$42.25\end{aligned}$$

Likewise, the tax on the capital gain is

$$\begin{aligned}\text{Tax on capital gain} &= \text{personal tax rate} \times .5 \times \text{capital gain}\\ &= .35 \times .5 \times (\$1{,}047.15 - \$1{,}038.05) = \$1.5925\end{aligned}$$

and the after-tax capital gain is

$$\begin{aligned}\text{After-tax capital gain} &= \text{capital gain} - \text{tax on capital gain}\\ &= (\$1{,}047.15 - \$1{,}038.05) - \$1.5925 = \$7.5075\end{aligned}$$

Your after-tax rate of return is therefore

$$\begin{aligned}\text{After-tax rate of return} &= \frac{\text{after-tax coupon income} + \text{after-tax capital gain}}{\text{investment}}\\ &= \frac{\$42.25 + \$7.5075}{\$1{,}038.05} = .0479, \text{ or } 4.79\%\end{aligned}$$

As you can see, taxes have a material effect on the rate of return on your investment! Here, the 7.14 percent before-tax rate of return is only 4.79 percent once you consider the taxes you must pay on your investment income.

CHECK POINT 4.6

Suppose you bought an 8 percent coupon bond for $1,200 and sold it 1 year later for $1,150. Calculate the before-tax and after-tax rate of return on your investment, if your personal tax rate is 40 percent and you can use the capital loss to reduce your taxes.

In our examples we have only considered 1-year investments. How do you calculate the rate of return if the investment lasts longer than 1 year? Suppose you buy the 6.5 percent coupon bond for $1,038.05 and sell it in 2 years for $1,015.25. You receive cash flows at two different points in time: a $65 coupon payment after one year, and then another $65 coupon plus the cash from selling the bond after 2 years. If you ignore the fact that you received the first $65 early, you can add up all the coupon payments and calculate the rate of return like we did above. This method understates your rate of return—you ignored the value of investing the first coupon during the time of the bond investment. The standard approach to calculating the rate of return is to assume that the first coupon is reinvested for the remaining life of the investment. In other words, calculate the future value of that first coupon payment at the end of the second year.

Suppose when you received the first coupon payment you immediately invested at 4 percent for 1 year. That coupon payment will be worth $65 × 1.04, or $67.6, 1 year later. At the end of the 2 years, the total value of coupon income received is $67.6 + $65, or $132.6. The price change on the bond is a capital loss: $1,015.25 – $1,038.05, or –$22.8. The rate of return on the investment is

$$\text{Rate of return} = \frac{\$132.6 - \$22.8}{\$1{,}038.05} = .10578, \text{ or } 10.58\%$$

Did you notice that this is a *two-year* rate of return? The effective annual equivalent is $(1.1058)^{1/2} - 1$, or 5.2 percent.

How do you know the rate at which the intermediate coupons payments are invested? You can use the actual rates available at the time you received the coupons. Another approach is to use a variation on the yield-to-maturity calculation. Using this method, your rate of return is the discount rate, that equates the purchase price to the present value of the coupons and the price you receive when you sell the bond. This assumes that all of the coupons are invested at that discount rate for the remaining time you own the bond. You can use this approach to calculate the after-tax rate of return too—just use the after-tax cash flows. We will see this approach to calculating rates of return again in Chapter 6, but there we will call it the *internal rate of return*.

EXAMPLE 4.5 *Calculating the Rate of Return on a Two-Year Bond Investment*

You buy a 6.5 percent bond for $1,038.05 and sell it for $1,015.25 2 years later. What is the rate of return on your investment if you use the yield-to-maturity approach? Using a calculator, enter PV of –$1,038.05, PMT of $65, FV of $1,015.25, and $n = 2$. Now compute the interest rate, which is your rate of return. You should get 5.19 percent.

INTEREST RATE RISK

interest rate risk The risk in bond prices due to fluctuations in interest rates.

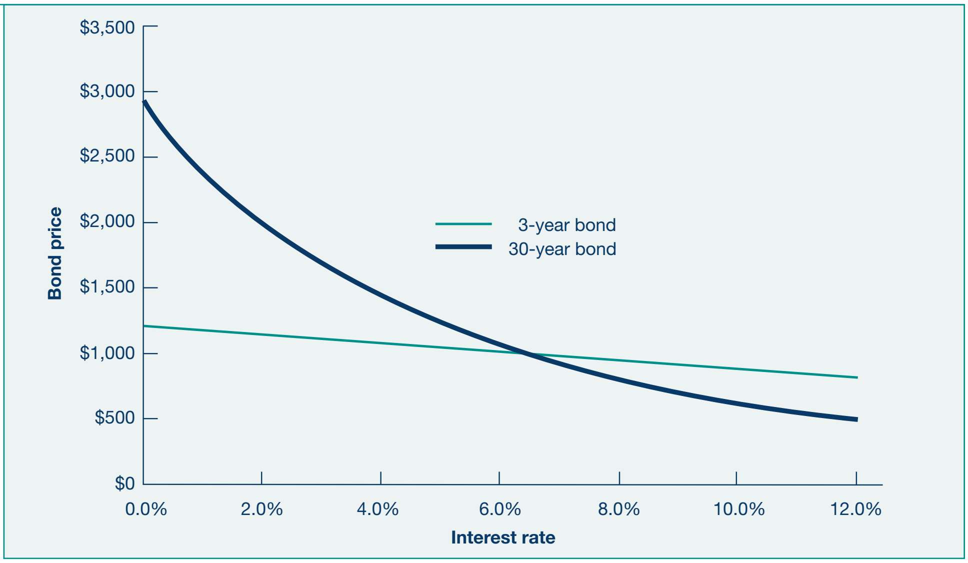
We have seen that bond prices fluctuate as interest rates change. In other words, bonds exhibit **interest rate risk.** Bond investors cross their fingers that market interest rates will fall, so that the price of their bond will rise. If they are unlucky and the market interest rate rises, the value of their investment falls.

But all bonds are not equally affected by changing interest rates. Compare the two curves in Figure 4.6. The green line shows how the value of the 3-year, 6.5 percent coupon bond varies with the level of the interest rate. The blue line shows how the price of a 30-year, 6.5 percent bond varies with the level of interest rates. You can see that the 30-year bond is more sensitive to interest rate fluctuations than the 3-year bond. This should not surprise you. If you buy a 3-year bond when the interest rate is 5.1 percent, and rates then rise, you will be stuck with a bad deal—you have just loaned your money at a lower interest rate than if you had waited. However, think how much worse it would be if the loan had been for 30 years, rather than 3 years. The longer the period of the loan, the more income you have lost by accepting what turns out to be a low coupon rate. This shows that the price of the longer-term bond had a greater decline. Of course, there is a flip side to this effect, which you can also see from Figure 4.6. When interest rates fall, the longer-term bond responds with a greater increase in price.

CHECK POINT 4.7

Suppose that the interest rate rises overnight from 5.1 percent to 10 percent. Calculate the present values of the 6.5 percent, 3-year bond and of the 6.5 percent, 30-year bond both before and after this change in interest rates. Confirm that your answers correspond with Figure 4.6. Use your financial calculator.

FIGURE 4.6
Plots of bond prices as a function of the interest rate. Long-term bond prices are more sensitive to the interest rate than prices of short-term bonds.



THE YIELD CURVE

Look back for a moment to Figure 4.2. The Canada bonds are arranged in order of their maturity. Notice that the longer the maturity, the higher the yield. This is usually the case, though sometimes long-term bonds offer *lower* yields.

yield curve Graph of the relationship between time to maturity and yield to maturity.

In addition to showing the yields on individual bonds, the *Globe and Mail* also shows a plot of the relationship between bond yields and maturity. This is known as the **yield curve.** You can see from the yield curve in Figure 4.7 that bonds with 3 months to maturity offered a yield of about 4.25 percent; those with more than 10 years of maturity offered a yield of almost 6 percent.

Why didn't everyone buy long-maturity bonds and earn the extra 1.75 percentage points? Who were those investors who put their money into short-term Canada bonds at only 4.25 percent?

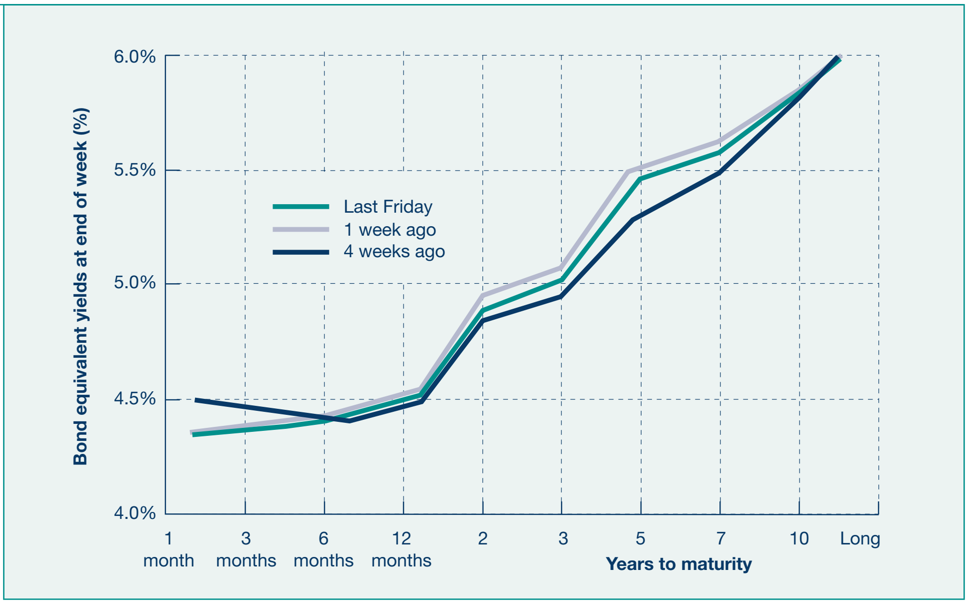
Even when the yield curve is upward sloping, investors might rationally stay away from long-term bonds for two reasons. First, the prices of long-term bonds fluctuate much more than prices of short-term bonds. Figure 4.6 illustrates that long-term bond prices are more sensitive to shifting interest rates. A sharp increase in interest rates could easily knock 20 or 30 percent off long-term bond prices. If investors don't like price fluctuations, they will invest their funds in short-term bonds unless they receive a higher yield to maturity on long-term bonds.

Second, short-term investors can profit if interest rates rise. Suppose you hold a 1-year bond. A year from now when the bond matures you can reinvest the proceeds and enjoy whatever rates the bond market offers then. Rates may be high enough to offset the first year's relatively low yield on the 1-year bond. Thus you often see an upward-sloping yield curve when future interest rates are expected to rise. We return to this issue in Chapter 12.

NOMINAL AND REAL RATES OF INTEREST

In Chapter 3 we drew a distinction between nominal and real rates of interest. The cash flows on the 6.5 percent Canada bonds are fixed in nominal terms. Investors are sure to receive an interest payment of $65 each year, but they do not know what that money will buy them. The *real* interest rate on the bonds depends on the rate of inflation. For example, if

FIGURE 4.7
The yield curve. A plot of yield to maturity as a function of time to maturity for Government of Canada bonds, reported June 1, 2001.



Source: RBC Capital Market, Fixed Income.

the nominal rate of interest is 5.1 percent and the inflation rate is 3 percent, then the real interest rate is calculated as follows:

$$(1 + \text{real interest rate}) = \frac{1 + \text{nominal interest rate}}{1 + \text{inflation rate}} = \frac{1.051}{1.03} = 1.0204$$

$$\text{Real interest rate} = .0204, \text{ or } 2.04\%$$

Since the inflation rate is uncertain, so is the real rate of interest on the bonds.

real return bonds
Bonds with a nominal coupon payment, determined by a fixed real coupon payment and the inflation rate.

You *can* nail down a real rate of interest by buying an indexed or **real return bond**, whose payments are linked to inflation. The Government of Canada began issuing inflation-indexed or real return bonds, RRBs, in 1991. The real cash flows are fixed, but the nominal cash flows (coupon payments and principal) are increased as the consumer price index increases. For example, the 4.25 percent RRB due December 1, 2021, pays annual real coupons of $42.50. In Figure 4.2, prices and yields of the three issues of Government of Canada real return bonds are seen at the bottom of the third column.

To see how the nominal coupon is calculated, suppose the Government of Canada issues a 3 percent, 2-year real return bond. The real cash flows are fixed but the nominal cash flows will depend on the increase in the consumer price index. Suppose inflation turns out to be 5 percent in Year 1 and a further 4 percent in Year 2. The real and nominal cash flows of the bonds would be

	Year 1	Year 2
Real cash flows	$30	$1030
Nominal cash flows	$30 × 1.05 = $31.50	$1030 × 1.05 × 1.04 = $1,124.76

For the 4.25 percent RRB, the nominal value of each coupon is calculated when the coupon payment is due and reflects the inflation that has occurred since the issue of the bond. We won't know the nominal value of the principal until just before the bond matures in 2021.

Currently, all real return bond issues of the Government of Canada have maturities of at least 20 years. As we write this in mid-2001, the yield to maturity on Canada RRBs is 3.58 percent. This yield is a real interest rate. It measures the amount of extra goods your investment will allow you to buy. In contrast, the yield on nominal Canada bonds with similar term to maturity is 5.95 percent. An estimate of the expected annual inflation rate used by market participants when discounting future cash flows can be found by rearranging the formula for the real interest rate:

$$(1 + \text{inflation rate}) = \frac{1 + \text{nominal interest rate}}{1 + \text{real interest rate}} = \frac{1.0595}{1.0358} = 1.0229$$

$$\text{Inflation rate} = .0229, \text{ or } 2.29\%$$

If the annual inflation rate proves to be higher than 2.29 percent, you will earn a higher return by holding RRBs, if the inflation rate is lower than 2.29 percent, the reverse will be true.

Inflation-indexed bonds have been issued by other governments and corporations. The United Kingdom has issued indexed bonds since 1982. The United States Treasury began to issue Treasury Inflation-Protected Securities, or TIPs, in 1997 and structured them similarly to the Government of Canada Real Return Bonds. In 2000, 407 International Inc., owner of largest electronic toll highway in Canada, just north of Toronto, sold real return bonds with 5.29 percent real coupon rate, maturing in 2039.

Real interest rates depend on the supply of savings and the demand for new investment. As this supply-demand balance changes, real interest rates change. But they do so gradually. The green line in Figure 4.8 shows that the real interest rate on the Government of Canada Real Return Bonds has fluctuated within a relatively narrow range.

Suppose that investors upwardly revise their forecast of inflation by 1 percent. How will this affect interest rates? If investors are concerned about the purchasing power of their money, the changed forecast should not affect the real rate of interest. The *nominal* interest rate must therefore rise by 1 percent to compensate investors for the higher inflation prospects.

The blue line in Figure 4.8 shows the nominal rate of interest in Canada since 1991. You can see that the nominal rate is much more variable than the real rate. In 1991, the nominal interest rate was almost 5 percent above the real rate. You can clearly see the impact of steady decline in inflation through most of the 1990s, causing nominal yields to fall.

FIGURE 4.8

Real and nominal yields to maturity on the Government of Canada's real return and nominal bonds

DEFAULT RISK

Our focus so far has been on Government of Canada bonds. But the federal government is not the only issuer of bonds. Provincial and municipal governments borrow by selling bonds. So do corporations. Canadian governments and corporations also borrow in the United States and in other countries. The bonds may be denominated in Canadian dollars, U.S. dollars and sometimes in another currency, such as the British pound or Japanese yen.

There is an important distinction between bonds issued by corporations and those issued by the Government of Canada. National governments don't go bankrupt—they just print more money.[5] So investors do not worry that the Canadian government will *default* on its bonds. However, there is some chance that corporations may get into financial difficulties and may default on their bonds. Thus the payments promised to corporate bondholders represent a best case scenario: The firm will never pay more than the promised cash flows, but in hard times it may pay less.

default (or credit) risk The risk that a bond issuer may default on its bonds.

default premium or credit spread The additional yield on a bond investors require for bearing credit risk.

The risk that a bond issuer may default on its obligations is called **default risk** (or **credit risk**). It should be no surprise to find that to compensate for this default risk companies need to promise a higher rate of interest than the Canadian government when borrowing money. The difference between the promised yield on a corporate bond and the yield on a Canada bond with the same coupon and maturity is called the **default premium**, or **credit spread**. The greater the chance that the company will get into trouble, the higher the default premium demanded by investors.

investment grade Bonds rated Baa or above by Moody's or BBB, or above by Standard & Poor's or DBRS.

junk bond Bond with a rating below Baa or BBB.

The safety of most corporate bonds can be judged from bond ratings provided by the Dominion Bond Rating Service (DBRS), Moody's, Standard & Poor's, or other bond-rating firms. Table 4.1 lists the possible bond ratings in declining order of quality. For example, the bonds that receive the highest rating are known as AAA, or triple A bonds. Then come AA, or double A, A bonds, BBB bonds, and so on. Bonds rated BBB and above are called **investment grade,** while those with a rating of BB or below are referred to as *speculative grade, high-yield,* or **junk bonds.**

It is rare for highly rated bonds to default. For example, since 1971 fewer than one in a thousand triple A bonds have defaulted within 10 years of issue. On the other hand, almost half of the bonds that were rated CCC by Standard & Poor's at issue have defaulted within 10 years. Of course, bonds rarely fall suddenly from grace. As time passes and the

TABLE 4.1
Key to DBRS, S&P, and Moody's bond ratings. (Moody's ratings in brackets.) The highest quality bonds are rated triple A, then come double A bonds, and so on.

Bond Rating	Safety
AAA, (Aaa)	The strongest rating; ability to repay interest and principal is very strong.
AA, (Aa)	Very strong likelihood that interest and principal will be repaid.
A, (A)	Strong ability to repay, but some vulnerability to change in circumstances.
BBB, (Baa)	Adequate capacity to repay; more vulnerability to changes in economic circumstances.
BB, (Ba)	Considerable uncertainty about ability to repay.
B, (B)	Likelihood of interest and principal payments over sustained periods is questionable.
CCC, (Caa) CC, (Ca)	Bonds in the CCC and CC classes may already be in default or in danger of imminent default.
C, (C)	Little prospect for interest or principal on the debt ever to be repaid.

[5] But they can't print money of other countries. Therefore when a government borrows in a foreign currency, investors worry that in some future crisis the government may not be able to come up with enough of the foreign currency to repay the debt. This worry shows up in the yield that investors demand on such debt. For example, during the Asian financial crisis in 1998, yields on the U.S. dollar bonds issued by the Indonesian government rose to 18 percentage points above the yields on comparable U.S. Treasury issues.

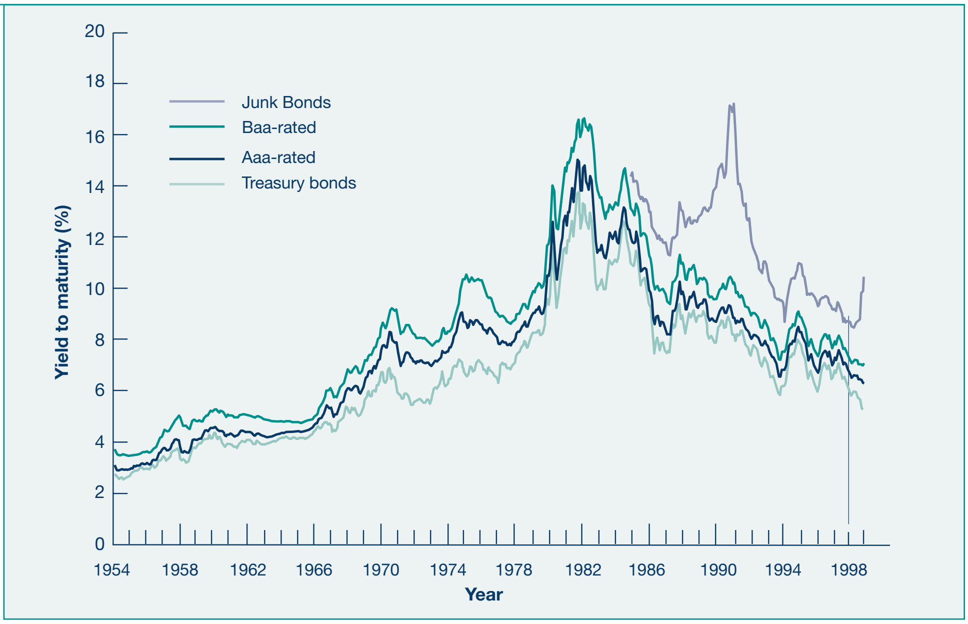


FIGURE 4.9
Yields on U.S. long-term bonds. Bonds with greater credit risk promise higher yields to maturity.

company becomes progressively more shaky, the agencies revise the bond's rating downward to reflect the increasing probability of default.

As you would expect, the yield on corporate bonds varies with the bond rating. Figure 4.9 presents the yields on default-free long-term U.S. Treasury (government) bonds, AAA-rated corporate bonds, and BBB-rated bonds since 1954. It also shows junk bond yields starting in November 1984. You can see that yields on the four groups of bonds track each other closely. However, promised yields go up as safety falls off. Although these data are for U.S. bonds, the picture would look much the same for Canadian bonds. One notable difference would be the much later start for Canadian junk bond yield data as this market developed much later than in the U.S.

EXAMPLE 4.6 *Promised versus Expected Yield to Maturity*

Bad Bet Inc. issued bonds several years ago with a coupon rate (paid annually) of 10 percent and face value of $1,000. The bonds are due to mature in 6 years. However, the firm is currently in bankruptcy proceedings, the firm has ceased to pay interest, and the bonds sell for only $200. Based on *promised* cash flow, the yield to maturity on the bond is 63.9 percent. (On your calculator, set PV = –200, FV = 1,000, PMT = 100, n = 6, and compute i.) But this calculation is based on the very unlikely possibility that the firm will resume paying interest and come out of bankruptcy. Suppose that the most likely outcome is that after 3 years of litigation, during which no interest will be paid, debtholders will receive $.27 on the dollar—that is, they will receive $270 for each bond with $1,000 face value. In this case the expected return on the bond is 10.5 percent. (On your calculator, set PV = –200, FV = 270, PMT = 0, n = 3, and compute i.) When default is a real possibility, the promised yield can depart considerably from the expected return. In this example, the default premium is greater than 50 percent.

VARIATIONS IN CORPORATE BONDS

Most corporate bonds are similar to the 6.5 percent Canada bonds that we examined earlier in the chapter. In other words, they promise to make a fixed nominal coupon payment for each year until maturity, at which point they also promise to repay the face value. However, you will find that there is greater variety in the design of corporate bonds. We will return to this issue in Chapter 13, but here are a few types of corporate bonds that you may encounter.

Zero-Coupon Bonds. Corporations sometimes issue zero-coupon bonds. In this case, investors receive $1,000 face value at the maturity date but do not receive a regular coupon payment. In other words, the bond has a coupon rate of zero. You learned how to value such bonds in Chapter 3. These bonds are issued at prices considerably below face value, and the investor's return comes from the difference between the purchase price and the payment of face value at maturity.

Floating-Rate Bonds. Sometimes the coupon rate can change over time. For example, floating-rate bonds make coupon payments that are tied to some measure of current market rates. The rate might be reset once a year to the current Treasury bill rate plus 2 percent. So if the Treasury bill rate at the start of the year is 6 percent, the bond's coupon rate over the next year would set at 8 percent. This arrangement means that the bond's coupon rate always approximates current market interest rates.

Convertible Bonds. If you buy a convertible bond, you can choose later to exchange it for a specified number of shares of common stock. For example, a convertible bond that is issued at par value of $1,000 may be convertible into 50 shares of the firm's stock. Because convertible bonds offer the opportunity to participate in any price appreciation of the company's stock, investors will accept lower interest rates on convertible bonds.

4.3 Summary

1. **What are the differences between the bond's coupon rate, current yield, and yield to maturity?**

   A bond is a long-term debt of a government or corporation. When you own a bond, you receive a fixed interest payment each year until the bond matures. This payment is known as the coupon. The **coupon rate** is the annual coupon payment expressed as a fraction of the bond's **face value.** At maturity the bond's face value is repaid. In Canada most bonds have a face value of $1,000. The **current yield** is the annual coupon payment expressed as a fraction of the bond's price. The **yield to maturity** measures the average rate of return to an investor who purchases the bond and holds it until maturity, accounting for coupon income as well as the difference between purchase price and face value.

2. **How can one find the market price of a bond given its yield to maturity and find a bond's yield given its price? Why do prices and yields vary inversely?**

   Bonds are valued by discounting the coupon payments and the final repayment by the yield to maturity on comparable bonds. The bond payments discounted at the bond's yield to maturity equal the bond price. You may also start with the bond price and ask what interest rate the bond offers. This interest rate that equates the present value of bond payments to the bond price is the yield to maturity. Because present values are lower when discount rates are higher, price and yield to maturity vary inversely.

3. **Why do bonds exhibit interest rate risk?**

   Bond prices are subject to interest rate risk, rising when market interest rates fall and falling when market rates rise. Long-term bonds exhibit greater **interest rate risk** than short-term bonds.

4. **Why do investors pay attention to bond ratings and demand a higher interest rate for bonds with low ratings?**

   Investors demand higher promised yields if there is a high probability that the borrower will run into trouble and default. **Credit risk** implies that the promised yield to maturity on the bond is higher than the expected yield. The additional yield investors require for bearing credit risk is called the **default premium.** Bond ratings measure the bond's credit risk.

Related Web Links

www.finpipe.com The Financial Pipeline is an Internet site dedicated to financial education; see the page on bonds
www.investinginbonds.com All about bond pricing
www.bloomberg.com/markets/C13.html A look at the U.S. yield curve, updated daily
www.bondmarkets.com/publications/IGCORP/what.htm A guide to corporate bonds
www.moodys.com The website of the bond rating agency
www.standardandpoors.com/ratings Standard & Poor's provides information on how it rates securities
www.rbcds.com RBC Capital Markets, one of Canada's largest bond dealers
www.tdwaterhouse.com Online brokerage service
www.bankofcanada.ca For yield-to-maturity data
www.bondsonline.com/asp/corp/spreadbank.html Yields to maturity for corporate bonds
www.ebond.ca Current Canadian federal and provincial bond prices

Key Terms

bond	113	current yield	118	real return bonds	127
coupon	113	yield to maturity	119	default (or credit) risk	129
face value, par value, maturity value, principal	113	rate of return	121	default premium	129
		interest rate risk	125	investment grade	129
coupon rate	113	yield curve	126	junk bond	129

Questions and Problems

*Answers in Appendix B

Basic

*1. **Bond Yields.** A 30-year Canada bond is issued with par value of $1,000, paying interest of $80 per year. If market yields increase shortly after the bond is issued, what happens to the bond's
   a. coupon rate
   b. price
   c. yield to maturity
   d. current yield

2. **Bond Yields.** If a bond with par value of $1,000 and a coupon rate of 8 percent is selling at a price of $970, is the bond's yield to maturity more or less than 8 percent? What about the current yield?

*3. **Bond Yields.** A bond with par value $1,000 has a current yield of 7.5 percent and a coupon rate of 8 percent. What is the bond's price?

*4. **Bond Pricing.** A 6-year Circular File bond pays interest of $80 annually and sells for $950. What is its coupon rate, current yield, and yield to maturity?

5. **Bond Pricing.** If Circular File (see question 4) wants to issue a new 6-year bond at face value, what coupon rate must the bond offer?

6. **Bond Yields.** A BCE bond has 10 years until maturity, a coupon rate of 8 percent, and sells for $1,050.
   a. What is the current yield on the bond?
   b. What is the yield to maturity?

7. **Coupon Rate.** General Matter's outstanding bond issue has a coupon rate of 10 percent and a current yield of 9.6 percent, and it sells at a yield to maturity of 9.25 percent. The firm wishes to issue additional bonds to the public at par value. What coupon rate must the new bonds offer in order to sell at par?

8. **Financial Pages.** Turn back to Figure 4.2. What is the current yield of the 8.75 percent, December 1, 2005 maturity? What was the closing bid price of the bond on the previous day?

*9. **Rate of Return.** You bought a 10-year, 5 percent coupon bond for $1,000 and sold it 1 year later for $1,100. What is the rate of return on your investment if the bond pays interest annually?

*10. **After-Tax Rate of Return.** Refer to problem 9. If your marginal tax rate is 30 percent, what is the after-tax rate of return on your bond investment?

PRACTICE

*11. **Bond Prices and Returns.** One bond has a coupon rate of 8 percent, another a coupon rate of 12 percent. Both bonds have 10-year maturities and sell at a yield to maturity of 10 percent. If their yields to maturity next year are still 10 percent, what is the rate of return on each bond? Does the higher coupon bond give a higher rate of return?

*12. **Bond Returns.**

a. If the BCE bond in problem 6 has a yield to maturity of 8 percent 1 year from now, what will its price be?

b. What will be your rate of return if you buy it today and sell it in 1 year?

c. If the inflation rate during the year is 3 percent, what is the real rate of return on the bond?

*13. **Bond Pricing.** A Stelco bond carries a coupon rate of 8 percent, has 9 years until maturity, and sells at a yield to maturity of 9 percent.

a. What interest payments do bondholders receive each year?

b. At what price does the bond sell? (Assume annual interest payments.)

c. What will happen to the bond price if the yield to maturity falls to 7 percent?

*14. **Bond Pricing.** A 30-year maturity bond with $1,000 face value makes annual coupon payments and has a coupon rate of 8 percent. What is the bond's yield to maturity if the bond is selling for

a. $900

b. $1,000

c. $1,100

15. **Bond Pricing.** Repeat the previous problem if the bond makes semiannual coupon payments.

16. **Bond Pricing.** Fill in the table below for the following zero-coupon bonds. The face value of each bond is $1,000.

Price	Maturity (Years)	Yield to Maturity
$300	30	__
$300	__	8%
__	10	10%

17. **Consol Bonds.** Perpetual Life Corp. has issued consol bonds with coupon payments of $80. (Consols pay interest forever and never mature. They are perpetuities.) If the required rate of return on these bonds at the time they were issued was 8 percent, at what price were they sold to the public? If the required return today is 12 percent, at what price do the consols sell?

*18. **Bond Pricing.** Sure Tea Co. has issued 9 percent annual coupon bonds, which are now selling at a yield to maturity of 10 percent and current yield of 9.8375 percent. What is the remaining maturity of these bonds?

19. **Bond Pricing.** Large Industries bonds sell for $1,065.15. The bond life is 9 years, and the yield to maturity is 7 percent. What must be the coupon rate on the bonds?

*20. **Bond Prices and Yields.**

a. Several years ago, Castles in the Sand, Inc., issued bonds at face value at a yield to maturity of 8 percent. Now, with 8 years left until the maturity of the bonds, the company has run into hard times, and the yield to maturity on the bonds has increased to 14 percent. What has happened to the price of the bond?

b. Suppose that investors believe that Castles can make good on the promised coupon payments, but that the company will go bankrupt when the bond matures and the principal comes due. The expectation is that investors will receive only 80 percent of face value at maturity. If they buy the bond today, what yield to maturity do they expect to receive?

*21. **Bond Returns.** You buy an 8 percent coupon, 10-year maturity bond for $980. A year later, the bond price is $1,050.

a. What is the new yield to maturity on the bond?
b. What is your rate of return over the year?

22. **Bond Returns.** You buy an 8 percent coupon, 10-year maturity bond when its yield to maturity is 9 percent. A year later, the yield to maturity is 10 percent. What is your rate of return over the year?

23. **Interest Rate Risk.** Consider three bonds with 8 percent coupon rates, all selling at face value. The short-term bond has a maturity of 4 years, the intermediate-term bond has maturity of 8 years, and the long-term bond has maturity of 30 years.

a. What will happen to the price of each bond if their yields increase to 9 percent?
b. What will happen to the price of each bond if their yields decrease to 7 percent?
c. What do you conclude about the relationship between time to maturity and the sensitivity of bond prices to interest rates?

*24. **Rate of Return.** A 2-year maturity bond with $1,000 face value makes annual coupon payments of $80 and is selling at face value. What will be the rate of return on the bond if its yield to maturity at the end of the year is

a. 6 percent
b. 8 percent
c. 10 percent

25. **Rate of Return.** A bond that pays coupons annually is issued with a coupon rate of 4 percent, maturity of 30 years, and a yield to maturity of 8 percent. What rate of return will be earned by an investor who purchases the bond and holds it for 1 year if the bond's yield to maturity at the end of the year is 9 percent?

*26. **Rate of Return.** Five years ago you purchased an 8 percent coupon bond for $975. Today you sold the bond for $1,000. What is your rate of return on the bond in each of the following situations:

a. All coupons were immediately spent when received.
b. All coupons were reinvested in your bank account, which pays 1 percent interest until the bond is sold.
c. All coupons were reinvested at 8.64 percent until the bond is sold.

*27. **Rate of Return.** Looking back at the previous question, use the yield-to-maturity method to compute the rate of return on your bond investment.

*28. **Bond Risk.** A bond's credit rating provides a guide to its risk. Long-term bonds rated AA currently offer yields to maturity of 8.5 percent EAR. A-rated bonds sell at yields of 8.8 percent EAR. If a 10-year bond with a coupon rate of 8 percent, paid semiannually, is downgraded by DBRS from AA to A rating, what is the likely effect on the bond price?

*29. **Real Returns.** Suppose that you buy a 1-year maturity bond for $1,000 that will pay you $1,000 plus a coupon payment of $60 at the end of the year. What real rate of return will you earn if the inflation rate is

a. 2 percent
b. 4 percent
c. 6 percent
d. 8 percent

30. **Real Returns.** Now suppose that the bond in the previous problem is a real return bond with a coupon rate of 4 percent. What will the cash flow provided by the bond be for each of the four inflation rates? What will be the real and nominal rates of return on the bond in each scenario?

31. **Real Returns.** Now suppose the real return bond in the previous problem is a 2-year maturity bond. What will be the bondholder's cash flows in each year in each of the inflation scenarios?

CHALLENGE

32. **Interest Rate Risk.** Suppose interest rates increase from 8 percent to 9 percent. Which bond will suffer the greater percentage decline in price: a 30-year bond paying annual coupons of 8 percent, or a 30-year zero coupon bond? Can you explain intuitively why the zero exhibits greater interest rate risk even though it has the same maturity as the coupon bond?

*33. **After-Tax Rate of Return.** Using the information in problem 26, calculate your after-tax rate of return on your bond investment assuming that your marginal tax rate is 35 percent. You pay tax on the interest when it is received.

34. **Bond Prices and Yields.** Big Time Company is planning to raise $15 million by selling 10-year bonds. The bond rating agency has advised the company that the bonds will have an A rating. Currently, the difference between the yield to maturity of A-rated corporate bonds over similar maturity Government of Canada bonds is 150 basis points (which is called the credit spread). If 10-year Canada bonds are currently priced to yield 5 percent, how many bonds will Big Time have to sell to raise the needed funds? Note that the convention is to set the coupon rate on the corporate bond issue so that the new bonds will sell at par value.

INTERNET PROBLEMS

1. Use historical yield-to-maturity data from the Bank of Canada website at www.bankofcanada.ca to look at bonds of different types. From the main page, click on "Bonds and Securities" on the left side of the main page and then select "Historic Bond Yields" (www.bankofcanada.ca/en/bond-look.htm). Follow the instructions and download 60 months of yield-to-maturity data for long-term corporate bonds (series B14048), long-term provincial bonds (B14047) and long-term Canada bonds (series B14013) and put the data into a spreadsheet. Calculate the average spreads of the corporate and provincial bonds over the Canada bonds. Graph the yields to maturity over time. What do you see? Does it make sense?

2. It is difficult to get online yield-to-maturity and credit spread data for Canadian corporate bonds with different debt ratings. Currently, www.ebond.ca is not providing bond data but may do so in the future. However, U.S. data is available. Go to www.bondsonline.com/asp/corp/spreadbank.html. You will see a table showing current extra yields to maturity (credit spreads) of corporate bonds of different risk over comparable term U.S. government bonds. Spreads are provided for different industries. Select "Industrials" (at the bottom of the page). Using the data in the table, estimate the required rate of return on a 10-year debt issue by a U.S. company with A1 or A-rated debt. What if its debt had a B1 or B rating? The current yield to maturity on U.S. government bonds is at www.bondsonline.com/asp/news/composites.html. To learn more about bonds and credit spreads, check out www.finpipe.com then click on "Bonds." Select "Bond Spreads" and read about the factors affecting spreads.

3. From either the *Globe and Mail* or the *National Post*, find five different corporate bonds. Go to one of the bond rating agencies such as www.dbrs.com or www.standardpoor.com/RatingsActions/RatingsLists/CanadianIssuers/index.html and look up the bonds' ratings. Compare the bonds' yields to maturity to comparable term Government of Canada bonds. Do the yields make sense relative to their bond ratings?

Solutions to Check Points

4.1 a. The bid price, your selling price, is 99.1 percent of face value, or $991.

b. To buy the bond, you pay a higher price. The dealer sets a spread to cover her costs of holding inventory of bonds. She buys the bond from you at a lower price than she is willing to sell to you.

c. The bond price increased by .03 percent of face value, or $0.3.

d. The annual coupon is 5.5 percent of face value, or $55, paid in two semiannual installments.

e. The yield to maturity, based on the bid price, is 5.63 percent.

4.2 The coupon is 9 percent of $1,000, or $90, a year. First value the 6-year annuity of coupons:

$$\begin{aligned} PV &= \$90 \times (\text{6-year annuity factor}) \\ &= \$90 \times \left[\frac{1}{.12} - \frac{1}{.12(1.12)^6}\right] \\ &= \$90 \times 4.11 = \$370.03 \end{aligned}$$

Then value the final payment and add up:

$$PV = \frac{\$1{,}000}{(1.12)^6} = \$506.63$$

$$\text{PV of bond} = \$370.03 + \$506.63 = \$876.66$$

4.3 The yield to maturity is about 8 percent because the present value of the bond's cash returns is $1,199 when discounted at 8 percent:

$$\begin{aligned} PV &= PV\ (\text{coupons}) + PV\ (\text{final payment}) \\ &= (\text{coupon} \times \text{annuity factor}) + (\text{face value} \times \text{discount factor}) \\ &= \$140 \times \left[\frac{1}{.08} - \frac{1}{.08(1.08)^4}\right] + \$1{,}000 \times \frac{1}{1.08^4} \\ &= \$463.70 + \$735.03 = \$1{,}199 \end{aligned}$$

4.4 The 6.5 percent coupon bond with maturity 2004 starts with 3 years left until maturity and sells for $1,038.05. At the end of the year, the bond has only 2 years to maturity and investors demand an interest rate of 7 percent. Therefore, the value of the bond becomes

$$\text{PV at 7\%} = \frac{\$65}{(1.07)} + \frac{\$1{,}065}{(1.07)^2} = \$990.06$$

You invested $1,038.05. At the end of the year you receive a coupon payment of $65 and have a bond worth $990.96. Your rate of return is therefore

$$\text{Rate of return} = \frac{\$65 + (\$990.96 - \$1{,}038.05)}{\$1{,}038.05} = .0173, \text{ or } 1.73\%$$

The yield to maturity at the start of the year was 7.14 percent. However, because interest rates rose during the year, the bond price fell and the rate of return was below the yield to maturity.

4.5 By the end of this year, the bond will have only 1 year left until maturity. It will make only one more payment of coupon plus face value, so its price will be $1,065/1.051 = $1,013.32. The rate of return is therefore

$$\frac{\$65 + (\$1{,}013.32 - \$1{,}025.99)}{\$1{,}025.99} = .051, \text{ or } 5.1\%$$

4.6 The coupon payment is .08 x $1,000, or $80, before tax. The tax on the coupon interest is .4 x $80, or $32. A capital loss of $1,150 – $1,200 = $50 is made. The capital loss can only be used to reduce taxable capital gain. You must have had a capital gain of at least $50 to use the loss to reduce your capital gains tax. The tax savings from the loss is .5 × .4 × $50, or $10. The before tax rate of return is

$$\frac{\$80 + (\$1{,}150 - \$1{,}200)}{\$1{,}200} = .025, \text{ or } 2.5\%$$

The after-tax rate of return is

$$\frac{\$80 - \$32 + (\$1{,}150 - \$1{,}200) + \$10}{\$1{,}200} = .0067, \text{ or } 0.67\%$$

If you had no capital gains to use up the loss, your after-tax return would be worse: – .167%!

4.7 At an interest rate of 5.1 percent, the 3-year bond sells for $1,038.05. If the interest rate jumps to 10 percent, the bond price falls to $912.96, a decline of 12.1 percent. The 30-year bond sells for $1,212.78 when the interest rate is 5.1 percent, but its price falls to $670.06 at an interest rate of 10 percent, a much larger percentage decline of 44.8 percent.