CHAPTER 1

Simple Interest and Simple Discount

Learning Objectives

Money is invested or borrowed in thousands of transactions every day. When an investment is cashed in or when borrowed money is repaid, there is a fee that is collected or charged. This fee is called interest.

In this chapter, you will learn how to calculate interest using simple interest. Although most financial transactions use compound interest (introduced in chapter 2), simple interest is still used in many short-term transactions. Many of the concepts introduced in this chapter will be used throughout the rest of this book and are applicable to compound interest.

The chapter starts off with some fundamental relationships for calculating interest and how to determine the future, or accumulated, value of a single sum of money invested today. You will also learn how to calculate the time between dates. Section 1.2 introduces the concept of discounting a future sum of money to determine its value today. This "present value of a future cash flow" is one of the fundamental calculations underlying the mathematics of finance.

Section 1.3 introduces the concept of the time value of money. This section also introduces equations of value, which allow you to accumulate or discount a series of financial transactions and are used to solve many problems in financial mathematics. Section 1.4 introduces two methods that are used to pay off a loan through a series of partial payments. The chapter ends with section 1.5 in which a less common form of simple interest — simple discount — is introduced along with the concept of discounted loans.

Section 1.1 Simple Interest

In any financial transaction, there are two parties involved: an investor, who is lending money to someone, and a debtor, who is borrowing money from the investor. The debtor must pay back the money originally borrowed, and also the fee charged for the use of the money, called **interest**. From the investor's point of view, interest is income from invested capital. The capital originally invested in an interest transaction is called **the principal**. The sum of the principal and interest due is called the **amount** or **accumulated value**. Any interest transaction can be described by the **rate of interest**, which is the ratio of the interest earned in one time unit on the principal.

In early times, the principal lent and the interest paid might be tangible goods (e.g., grain). Now, they are most commonly in the form of money. The practice of charging interest is as old as the earliest written records of humanity. Four thousand years ago, the laws of Babylon referred to interest payments on debts.

At **simple interest**, the interest is computed on the original principal during the whole time, or term of the loan, at the stated annual rate of interest.

We shall use the following notation:

P = the principal, or the present value of S, or the discounted value of S, or the proceeds.
I = simple interest.
S = the amount, or the accumulated value of P, or the future value of P, or the maturity value of P.
r = annual rate of simple interest.
t = time in years.

Simple interest is calculated by means of the formula

$$\boxed{\boldsymbol{I = Prt}} \tag{1}$$

From the definition of the amount S we have

$$S = P + I$$

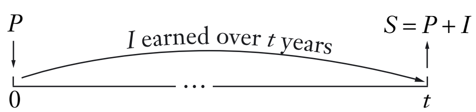
By substituting for $I = Prt$, we obtain S in terms of P, r, and t:

$$S = P + Prt$$

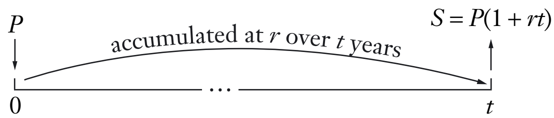
$$\boxed{\boldsymbol{S = P(1 + rt)}} \tag{2}$$

> The factor $(1 + rt)$ in formula **(2)** is called an **accumulation factor at a simple interest rate *r*** and the process of calculating S from P by formula **(2)** is called **accumulation at a simple interest rate *r***.

We can display the relationship between S and P on a time diagram.



Alternatively,



The time t must be in years. When the time is given in months, then

$$t = \frac{\text{number of months}}{12}$$

When the time is given in days, there are two different varieties of simple interest in use:

1. **Exact interest**, where $t = \dfrac{\text{number of days}}{365}$

   i.e., the year is taken as 365 days (leap year or not).

2. **Ordinary interest**, where $t = \dfrac{\text{number of days}}{360}$

   i.e., the year is taken as 360 days.

CALCULATION TIP:

The general practice in Canada is to use exact interest, whereas the general practice in the United States and in international business transactions is to use ordinary interest (also referred to as the Banker's Rule). In this textbook, exact interest is used all the time unless specified otherwise. When the time is given by two dates we calculate the exact number of days between the two dates from a table listing the serial number of each day of the year (see the table on the inside back cover). The exact time is obtained as the difference between serial numbers of the given dates. In leap years (years divisible by 4) an extra day is added to February.

OBSERVATION:

When calculating the number of days between two dates, the most common practice in Canada is to count the starting date, but not the ending date. The reason is that financial institutions calculate interest each day on the closing balance of a loan or savings account. On the day a loan is taken out or a deposit is made, there is a non-zero balance at the end of that day, whereas on the day a loan is paid off or the deposit is fully withdrawn, there is a zero balance at the end of that day.

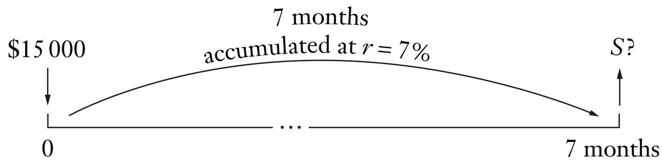
However, it is easier to assume the opposite when using the table on the inside back cover. That is, unless otherwise stated, you should assume that interest is not calculated on the starting date, but is calculated on the ending date. That way, in order to determine the number of days between two dates, all you have to do is subtract the two values you find from the table on the inside back cover. Example 1 will illustrate this.

EXAMPLE 1 A loan of \$15 000 is taken out. If the interest rate on the loan is 7%, how much interest is due and what is the amount repaid if

a) The loan is due in seven months;

b) The loan was taken out on April 7 and is due in seven months?

Solution a We have $P = 15\,000$, $r = 0.07$ and since the actual date the loan was taken out is not given, we use $t = \frac{7}{12}$.



Interest due, $I = Prt = \$15\,000 \times 0.07 \times \frac{7}{12} = \$612.50$

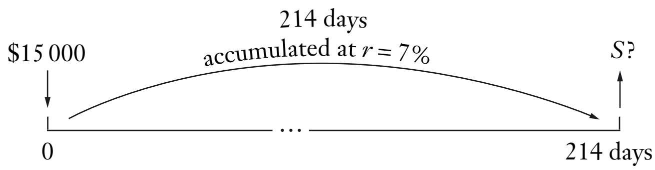
Amount repaid = Future or accumulated value,

$$S = P + I = \$15\,000 + \$612.50 = \$15\,612.50$$

Alternatively, we can obtain the above answer in one calculation:

$$S = P(1 + rt) = \$15\,000[1 + 0.07(\tfrac{7}{12})] = \$15\,612.50$$

Solution b Since a date is given when the loan was actually taken out, we must use days. Seven months after April 7 is November 7. Using the table on the inside back cover, we find that April 7 is day 97 and November 7 is day 311. The exact number of days between the two dates is $311 - 97 = 214$. Thus, $t = \frac{214}{365}$.



Interest due, $I = Prt = \$15\,000 \times 0.07 \times \frac{214}{365} = \$615.52$

Future value, $S = P + I = \$15\,000 + \$615.52 = \$15\,615.52$

Alternatively,

$$S = P(1 + rt) = \$15\,000[1 + 0.07(\tfrac{214}{365})] = \$15\,615.52$$

EXAMPLE 2 Determine the exact and ordinary simple interest on a 90-day loan of \$8000 at $8\frac{1}{2}\%$.

Solution We have $P = 8000$, $r = 0.085$, numerator of $t = 90$ days.

Exact interest, $I = Prt = \$8000 \times 0.085 \times \frac{90}{365} = \$167.67$

Ordinary Interest, $I = Prt = \$8000 \times 0.085 \times \frac{90}{360} = \$170.00$

OBSERVATION:

Notice that ordinary interest is always greater than the exact interest and thus it brings increased revenue to the lender.

EXAMPLE 3 A loan shark made a loan of \$100 to be repaid with \$120 at the end of one month. What was the annual interest rate?

Solution We have $P = 100$, $I = 20$, $t = \frac{1}{12}$, and

$$r = \frac{I}{Pt} = \frac{20}{100 \times \frac{1}{12}} = 240\%$$

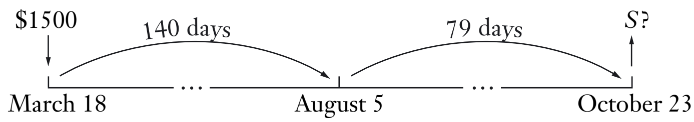
EXAMPLE 4 How long will it take \$3000 to earn \$60 interest at 6%?

Solution We have $P = 3000$, $I = 60$, $r = 0.06$, and

$$t = \frac{I}{Pr} = \frac{60}{3000 \times 0.06} = \frac{1}{3} = 4 \text{ months}$$

EXAMPLE 5 A deposit of \$1500 is made into a fund on March 18. The fund earns simple interest at 5%. On August 5, the interest rate changes to 4.5%. How much is in the fund on October 23?

Solution Using the table on the inside back cover, we determine the number of days between March 18 and August 5 = 217 − 77 = 140 and the number of days between August 5 and October 23 = 296 − 217 = 79.



$$\begin{aligned} S &= \text{original deposit} + \text{interest for 140 days} + \text{interest for 79 days} \\ &= \$1500 + (\$1500 \times 0.05 \times \tfrac{140}{365}) + (\$1500 \times 0.045 \times \tfrac{79}{365}) \\ &= \$(1500 + 28.77 + 14.61) \\ &= \$1543.38 \end{aligned}$$

Demand Loans

On a **demand loan**, the lender may demand full or partial payment of the loan at any time and the borrower may repay all of the loan or any part at any time without notice and without interest penalty. Interest on demand loans is based on the unpaid balance and is usually payable monthly. The interest rate on demand loans is not usually fixed but fluctuates with market conditions.

EXAMPLE 6 Jessica borrowed \$1500 from her credit union on a demand loan on August 16. Interest on the loan, calculated on the unpaid balance, is charged to her account on the 1st of each month. Jessica made a payment of \$300 on

September 17, a payment of \$500 on October 7, a payment of \$400 on November 12 and repaid the balance on December 15. The rate of interest on the loan on August 16 was 12% per annum. The rate was changed to 11.5% on September 25 and 12.5% on November 20. Calculate the interest payments required and the total interest paid.

Solution

Interest period	# of days	Balance	Rate	Interest
Aug. 16–Sep. 1	16	\$1500	12%	$1500(0.12)(\frac{16}{365}) = 7.89$
				Sep. 1 interest = \$ 7.89
Sep. 1–Sep. 17	16	\$1500	12%	$1500(0.12)(\frac{16}{365}) = 7.89$
Sep. 17–Sep. 25	8	\$1200	12%	$1200(0.12)(\frac{8}{365}) = 3.16$
Sep. 25–Oct. 1	6	\$1200	11.5%	$1200(0.115)(\frac{6}{365}) = 2.27$
				Oct. 1 interest = \$13.32
Oct. 1–Oct. 7	6	\$1200	11.5%	$1200(0.115)(\frac{6}{365}) = 2.27$
Oct. 7–Nov. 1	25	\$ 700	11.5%	$700(0.115)(\frac{25}{365}) = 5.51$
				Nov. 1 interest = \$ 7.78
Nov. 1–Nov. 12	11	\$ 700	11.5%	$700(0.115)(\frac{11}{365}) = 2.43$
Nov. 12–Nov. 20	8	\$ 300	11.5%	$300(0.115)(\frac{8}{365}) = 0.76$
Nov. 20–Dec. 1	11	\$ 300	12.5%	$300(0.125)(\frac{11}{365}) = 1.13$
				Dec. 1 interest = \$ 4.32
Dec. 1–Dec. 15	14	\$ 300	12.5%	$300(0.125)(\frac{14}{365}) = 1.44$
				Dec. 15 interest = \$ 1.44

Total interest paid = \$(7.89 + 13.32 + 7.78 + 4.32 + 1.44) = \$34.75

Invoice Cash Discounts

To encourage prompt payments of invoices many manufacturers and wholesalers offer cash discounts for payments in advance of the final due date. The following typical credit terms may be printed on sales invoices:

2/10, *n*/30 — Goods billed on this basis are subject to a cash discount of 2% if paid within ten days. Otherwise, the full amount must be paid not later than thirty days from the date of the invoice.

A merchant may consider borrowing the money to pay the invoice in time to receive the cash discount. Assuming that the loan would be repaid on the day the invoice is due, the interest the merchant should be willing to pay on the loan should not exceed the cash discount.

EXAMPLE 7 A merchant receives an invoice for a motor boat for \$20 000 with terms 4/30, *n*/100. What is the highest simple interest rate at which he can afford to borrow money in order to take advantage of the discount?

Solution Suppose the merchant will take advantage of the cash discount of 4% of \$20 000 = \$800 by paying the bill within 30 days from the date of invoice. He needs to borrow \$20 000 = \$800 = \$19 200. He would borrow this money on day 30 and repay it on day 100 (the day the original invoice is due) resulting in a 70-day loan. The interest he should be willing to pay on borrowed money should not exceed the cash discount \$800.

We have $P = 19\,200$, $I = 800$, $t = \frac{70}{365}$, and we calculate

$$r = \frac{I}{Pt} = \frac{800}{19\,200 \times \frac{70}{365}} = 21.73\%$$

The highest simple interest rate at which the merchant can afford to borrow money is 21.73%. This is a break-even rate. If he can borrow money, say at a rate of 15%, he should do so. He would borrow \$19 200 for 70 days at 15%. Maturity value of the loan is $\$19\,200\,[1 + (0.15)(\frac{70}{365})] = \$19\,752.33$. Thus his savings would be \$20 000 – \$19 752.33 = \$247.67.

Exercise 1.1

1. Determine the maturity value of
 a) a \$2500 loan for 18 months at 12% simple interest,
 b) a \$1200 loan for 120 days at 8.5% ordinary simple interest, and
 c) a \$10 000 loan for 64 days at 7% exact simple interest.

2. At what rate of simple interest will
 a) \$1000 accumulate to \$1420 in $2\frac{1}{2}$ years,
 b) money double itself in 7 years, and
 c) \$500 accumulate \$10 interest in 2 months?

3. How many days will it take \$1000 to accumulate to at least \$1200 at 5.5% simple interest?

4. Determine the ordinary and exact simple interest on \$5000 for 90 days at $10\frac{1}{2}\%$.

5. A student lends his friend \$10 for one month. At the end of the month he asks for repayment of the \$10 plus purchase of a chocolate bar worth 50¢. What simple interest rate is implied?

6. What principal will accumulate to \$5100 in 6 months if the simple interest rate is 9%?

7. What principal will accumulate to \$580 in 120 days at 18% simple interest?

8. Determine the accumulated value of \$1000 over 65 days at $6\frac{1}{2}\%$ using both ordinary and exact simple interest.

9. A man borrows \$1000 on February 16 at 7.25% simple interest. What amount must he repay in 7 months?

10. A sum of \$2000 is invested from May 18, 2006, to April 8, 2007, at 4.5% simple interest. Determine the amount of interest earned.

11. On May 13, 2006, Jacob invested \$4000. On February 1, 2007, he intends to pay Fred \$4300 for a used car. The bank assured Jacob that his investment would be adequate to cover the purchase. Determine the minimum simple interest rate that Jacob's money must be earning.

12. On January 1, Mustafa borrows \$1000 on a demand loan from his bank. Interest is paid at the end of each quarter (March 31, June 30, September 30, December 31) and at the time of the last payment. Interest is calculated at the rate of 12% on the balance of the loan outstanding. Mustafa repaid the loan with the following payments:

March 1	\$ 100
April 17	\$ 300
July 12	\$ 200
August 20	\$ 100
October 18	\$ 300
	\$1000

Calculate the interest payments required and the total interest paid.

13. On February 3, a company borrowed \$50 000 on a demand loan from a bank. Interest on the loan is charged to the company's current account on the 11th of each month. The company repaid the loan with the following payments:

February 20	\$10 000
March 20	\$10 000
April 20	\$15 000
May 20	\$15 000
	\$50 000

The rate of interest on the loan was originally 6%. The rate was changed to 7% on April 1, and to 6.5% on May 1. Calculate the interest payments required and the total interest paid.

14. A cash discount of 2% is given if a bill is paid 20 days in advance of its due date. At what interest rate could you afford to borrow money to take advantage of this discount?

15. A merchant receives an invoice for \$2000 with terms 2/20, *n*/60. What is the highest simple interest rate at which he can afford to borrow money in order to take advantage of the discount?

16. The ABC general store receives an invoice for goods totalling \$500. The terms were 3/10, *n*/30. If the store were to borrow the money to pay the bill in 10 days, what is the highest interest rate at which the store could afford to borrow?

17. I.C.U. Optical receives an invoice for \$2500 with terms 2/5, *n*/50.

a) If the company is to take advantage of the discount, what is the highest simple interest rate at which it can afford to borrow money?

b) If money can be borrowed at 10% simple interest, how much does I.C.U. Optical actually save by using the cash discount?

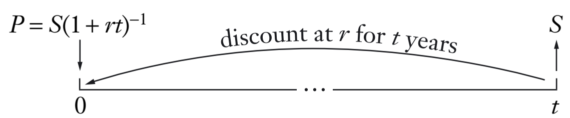
Section 1.2 Discounted Value at Simple Interest

From formula **(2)** we can express P in terms of S, r, and t and obtain

$$\boldsymbol{P = \frac{S}{1+rt} = S(1+rt)^{-1}} \qquad \textbf{(3)}$$

The factor $(1+rt)^{-1}$ in formula **(3)** is called a **discount factor at a simple interest rate *r*** and the process of calculating P from S is called **discounting at a simple interest rate *r***, or simple discount at an interest rate r.

We can display the relationship between P and S on a time diagram.



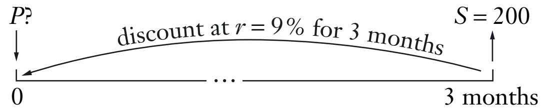
When we calculate P from S, we call P the present value of S or the discounted value of S. The difference $D = S - P$ is called the simple discount on S at an interest rate.

For a given interest rate r, the difference $S - P$ has two interpretations.

1. The interest I on P which when added to P gives S.
2. The discount D on S which when subtracted from S gives P.

EXAMPLE 1 Three months after borrowing money, a person pays back exactly \$200. How much was borrowed if the \$200 payment includes the principal and simple interest at 9%?

Solution We have $S = 200$, $r = 0.09$, and $t = \frac{3}{12}$. We can display this on a time diagram.

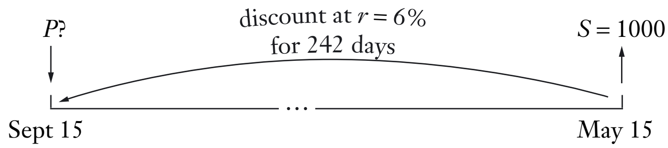


$$P = S(1 + rt)^{-1} = \frac{S}{(1 + rt)} = \frac{\$200}{1 + 0.09(\frac{3}{12})} = \$195.60$$

EXAMPLE 2 Jennifer wishes to have \$1000 in eight months time. If she can earn 6%, how much does she need to invest on September 15? What is the simple discount?

Solution Eight months after September 15 (day 258) is May 15 (day 135). The number of days between these two dates = (365 − 258) + 135 = 242.

We have $S = 1000$, $r = 0.06$, $t = \frac{242}{365}$



$$P = S(1 + rt)^{-1} = \frac{S}{(1 + rt)} = \frac{\$1000}{1 + 0.06(\frac{242}{365})} = \$961.74$$

The simple discount is

$$D = S - P = \$(1000 - 961.74) = \$38.26$$

Promissory Notes

A **promissory note** is a written promise by a debtor, called the **maker** of the note, to pay to, or to the order of, the creditor, called the **payee** of the note, a sum of money, with or without interest, on a specified date.

The following is an example of an interest-bearing note.

> \$2000.00 Toronto, September 1, 2006
> Sixty days after date, I promise to pay to the order of Mr. A
> Two thousand and $\frac{00}{100}$ dollars
> for value received with interest at 8% per annum.
> (Signed) Mr. B

The **face value** of the note is $2000. The **term** of the note is 60 days. The **due date** is 60 days after September 1, 2006, that is October 31, 2006.

In Canada, *three days of grace** are added to the 60 days to arrive at the **legal due date**, or the **"maturity date,"** that is November 3, 2006. By a **maturity value** of a note we shall understand the value of the note at the maturity date. In our example, the maturity value of the note is the accumulated value of $2000 for 63 days at 8%, i.e., $\$2000[1 + (0.08)(\frac{63}{365})] = \$2027.62$.

If Mr. B chooses to pay on October 31, he will pay interest for 60 days, not 63 days. However, no legal action can be taken against him until the expiry of the three days of grace. Thus, Mr. B would be within his legal rights to repay as late as November 3, and in that case he would pay interest for 63 days.

A promissory note is similar to a short-term bank loan, but it differs in two important ways:

1. The lender does not have to be a financial institution.
2. A promissory note may be sold one or several times before its maturity. To determine the price a buyer will pay for the note, he/she will take the amount to be received in the future (the maturity value), and discount it to the date of the sale at a rate of interest the buyer wishes to earn on his/her investment. The resulting value is called the proceeds and is paid to the seller. The proceeds are determined by formula **(3)**.

The procedure for discounting of promissory notes can be summarized in 2 steps:

Step 1 Calculate the maturity value, S, of the note.

The maturity value of a noninterest-bearing note is the face value of the note. The maturity value of an interest-bearing note is the accumulated value of the face value of the note at the maturity date.

Step 2 Calculate the proceeds, P, by discounting the maturity value, S, at a specified simple interest rate from the maturity date back to the date of sale.

**Three days of grace*. In Canada, "three days of grace" are allowed for the payment of a promissory note. This means that the payment becomes due on the third day after the due date of the note.

If interest is being charged, the three days of grace must be added to the time stated in the note. If the third day of grace falls on a holiday, payment will become due the next day that is itself not a holiday.

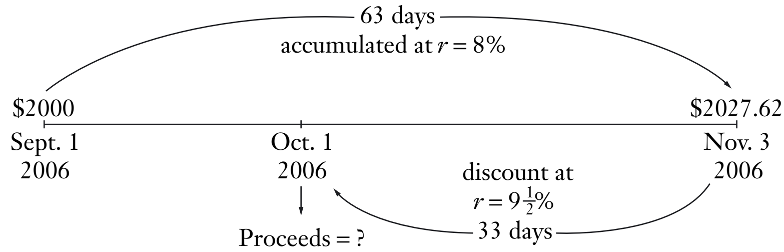
If the time is stated in months, these must be calendar months and not months of 30 days. For example, 2 months after July 5 is September 5. Thus the legal due date is September 8, and the number of days to be used in calculating the interest (if any) will be 65 days.

In the month in which the payment falls due, there may be no corresponding date to that from which the time is computed. In such a case, the last day of the month is taken as the corresponding date. For example, two months after December 31 is February 28 (or February 29 in leap years) and March 3 would be the legal due date.

EXAMPLE 3 The promissory note described in the beginning of this section is sold by Mr. A on October 1, 2006 to a bank that discounts notes at a $9\frac{1}{2}\%$ simple interest rate.

a) How much money would Mr. A receive for the note?
b) What rate of interest will the bank realize on its investment, if it holds the note till maturity?
c) What rate of interest will Mr. A realize on his investment, when he sells the note on October 1, 2006?

Solution a We arrange the dated values on a time diagram below.



Maturity value of the note is $S = \$2000[1 + (0.08)(\frac{63}{365})] = \$2027.62$.
Proceeds on October 1, 2006, are $P = \$2027.62[1 + (0.095)(\frac{33}{365})]^{-1} = \$2010.35$.
Thus Mr. A would receive $2010.35 for the note.

Solution b The bank will realize a profit of $\$2027.62 - \$2010.35 = \$17.27$ for 33 days. Thus we have $P = \$2010.35$, $I = \$17.27$, and $t = \frac{33}{365}$ and calculate

$$r = \frac{I}{Pt} = \frac{\$17.27}{\$2010.35 \times \frac{33}{365}} = 9.50\%$$

Solution c Mr. A will realize a profit of $\$2010.35 - \$2000 = \$10.35$ on his investment of $2000 for the 30 days he held the note. The rate of interest Mr. A will realize is

$$r = \frac{I}{Pt} = \frac{10.35}{2000 \times \frac{30}{365}} = 6.30\%$$

EXAMPLE 4 On April 21, a retailer buys goods amounting to $5000. If he pays cash he will get a 4% cash discount. To take advantage of this cash discount, he signs a 90-day noninterest-bearing note at his bank that discounts notes at an interest rate of 9%. What should be the face value of this note to give him the exact amount needed to pay cash for the goods?

Solution The cash discount 4% of $5000 is $200. The retailer needs $4800 in cash. He will sign a noninterest-bearing note with maturity value S calculated by formula **(2)**, given $P = 4800$, $r = 9\%$, and $t = \frac{93}{365}$. Thus

$$S = P(1 + rt) = \$4800[1 + (0.09)(\tfrac{93}{365})] = \$4910.07$$

The face value of the noninterest-bearing note should be \$4910.07.

Note: The retailer could also sign a 90-day 9% interest-bearing note with a face value of \$4800 and a maturity value of \$4910.07 in 93 days. He would receive cash of \$4800 immediately as the proceeds $P = \$4910.07[1 + (0.09)(\frac{93}{365})]^{-1} = \$4800$.

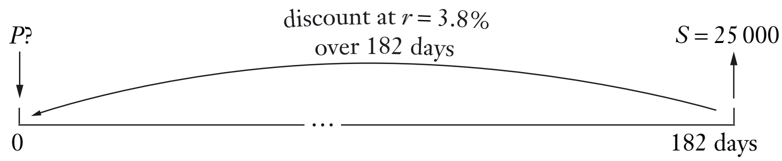
Treasury Bills (or T-bills)

Treasury bills are popular short-term and low-risk securities issued by the Federal Government of Canada every other Tuesday with maturities of 13, 26, or 52 weeks (91, 182, or 364 days). They are basically promissory notes issued by the government to meet their short-term financing needs.

Treasury bills are issued in denominations, or face values, of \$1000, \$5000, \$25 000, \$100 000 and \$1 000 000. The face value of a T-bill is the amount the government guarantees it will pay on the maturity date. There is no interest rate stated on a T-bill. Instead, to determine the purchase price of a T-bill, you need to discount the face value to the date of sale at an interest rate that is determined by market conditions. Since all T-bills have a term of less than one year, you would use simple interest formulas.

EXAMPLE 5 A 182-day T-bill with a face value of \$25 000 is purchased by an investor who wishes to yield 3.80%. What price is paid?

Solution We have $S = 25\,000$, $r = 0.038$, $t = \frac{182}{365}$.



$$P = S[1 + rt]^{-1} = \$25\,000\,[1 + 0.038(\tfrac{182}{365})]^{-1}$$
$$= \$24\,535.11$$

The investor will pay \$24 535.11 for the T-bill and receive \$25 000 in 182 days' time.

EXAMPLE 6 A 91-day T-bill with a face value of \$100 000 is purchased for \$97 250. What rate of interest is assumed?

Solution We have $S = 100\,000$, $P = 97\,250$, $t = \frac{91}{365}$ and we calculate

$$r = \frac{I}{Pt} = \frac{2750}{97\,250(\frac{91}{365})} = 0.113421283 = 11.34\%$$

Exercise 1.2

For each promissory note in problems 1 to 4 determine the maturity date, maturity value, discount period, and the proceeds:

No.	Date of Note	Face Value	Interest Rate on Note	Term	Date of Discount	Interest Rate for Discounting
1.	Nov. 3	$3000	None	3 months	Dec. 1	10.25%
2.	Sept. 1	$1200	5.5%	60 days	Oct. 7	4.75%
3.	Dec. 14	$ 500	7.0%	192 days	Dec. 24	7.00%
4.	Feb. 4	$4000	None	2 months	Mar. 1	8.50%

5. A 90-day note promises to pay Ms. Chiu $2000 plus simple interest at 13%. After 51 days it is sold to a bank that discounts notes at a 12% simple interest rate.
a) How much money does Ms. Chiu receive?
b) What rate of interest does Ms. Chiu realize on her investment?
c) What rate of interest will the bank realize if the note is paid in full in exactly 90 days?

6. An investor lends $5000 and receives a promissory note promising repayment of the loan in 90 days with 8.5% simple interest. This note is immediately sold to a bank that charges 8% simple interest. How much does the bank pay for the note? What is the investor's profit? What is the bank's profit on this investment when the note matures?

7. A 90-day note for $800 bears interest at 6% and is sold 60 days before maturity to a bank that uses a 8% simple interest rate. What are the proceeds?

8. Jacob owes Kieran $1000. Kieran agrees to accept as payment a noninterest-bearing note for 90 days that can be discounted immediately at a local bank that charges a simple interest rate of 10%. What should be the face value of the note so that Kieran will receive $1000 as proceeds?

9. Justin has a note for $5000 dated October 17, 2006. The note is due in 120 days with interest at 7.5%. If Justin discounts the note on January 15, 2007, at a bank charging a simple interest rate of 7%, what will be the proceeds?

10. A merchant buys goods worth $2000 and signs a 90-day noninterest-bearing promissory note. Determine the proceeds if the supplier sells the note to a bank that uses a 13% simple interest rate. How much profit did the supplier make if the goods cost $1500?

11. On August 16 a retailer buys goods worth $2000. If he pays cash he will get a 3% cash discount. To take advantage of this he signs a 60-day noninterest-bearing note at a bank that discounts notes at a 6% simple interest rate. What should be the face value of this note to give the retailer the exact amount needed to pay cash for the goods?

12. Monique has a note for $1500 dated June 8, 2006. The note is due in 120 days with interest at 6.5%.
a) If Monique discounts the note on August 1, 2006, at a bank charging an interest rate of 8.5%, what will the proceeds be?
b) What rate of interest will Monique realize on her investment?
c) What rate of interest will the bank realize if the note is paid off in full in exactly 120 days?

13. Yufeng needs to have $3500 in 10 months time. If she can earn 6% simple interest,
a) how much does she need to invest today?
b) how much does she need to invest today if the $3500 is needed on July 31? What is the simple discount?

14. An investor bought a 91-day treasury bill to yield 3.45%.
a) What was the price paid by the investor if the face value was $5000?
b) The investor sold the T-bill 40 days later to another investor who wishes to yield 3.10%. What price did the T-bill sell for?
c) What rate of return did the original investor earn on his investment?

15. An investment dealer bought a $25 000 364-day treasury bill for $23 892.06.
a) What yield rate is implied?
b) The treasury bill was purchased on September 7, 2006. The dealer sold it to another investor on January 25, 2007 for $24 102.25. What yield rate did the other investor wish to earn? What rate of return did the dealer end up earning?

Section 1.3

Equations of Value

All financial decisions must take into account the basic idea that **money has time value**. In other words, receiving $100 today is not the same as receiving $100 one year ago, nor receiving $100 one year from now if there is a positive interest rate. In a financial transaction involving money due on different dates, every sum of money should have an attached date, the date on which it falls due. That is, the mathematics of finance deals with **dated values**. This is one of the most important facts in the mathematics of finance.

Illustration: At a simple interest rate of 8%, $100 due in 1 year is considered to be equivalent to $108 in 2 years since $100 would accumulate to $108 in 1 year. In the same way

$$\$100(1 + 0.08)^{-1} = \$92.59$$

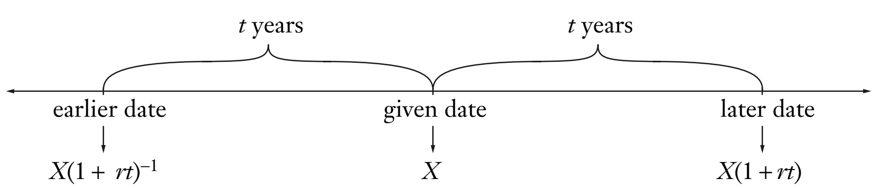
would be considered an equivalent sum today.

Another way to look at this is to suppose you were offered $92.59 today, or $100 one year from now, or $108 two years from now. Which one would you choose? Most people would probably take the $92.59 today because it is money in their hands. However, from a financial point of view, all three values are the same, or are equivalent, since you could take the $92.59 and invest it for one year at 8%, after which you would have $100. This $100 could then be invested for one more year at 8% after which it would have accumulated to $108. Note that the three dated values are not equivalent at some other rate of interest.

In general, we compare dated values by the following **definition of equivalence:**

> $\$X$ due on a given date is equivalent at a given simple interest rate r to $\$Y$ due t years later if
>
> $$Y = X(1 + rt) \quad \text{or} \quad X = \frac{Y}{1 + rt} = Y(1 + rt)^{-1}$$

The following time diagram illustrates dated values equivalent to a given dated value X.

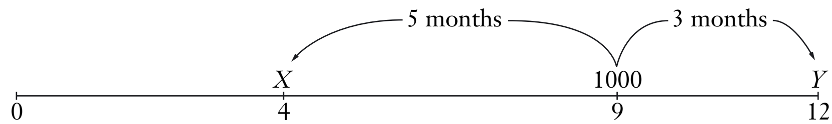


Based on the time diagram above we can state the following simple rules:

1. When we move money forward in time, we accumulate, i.e., multiply the sum by an accumulation factor $(1 + rt)$.
2. When we move money backward in time, we discount, i.e., multiply the sum by a discount factor $(1 + rt)^{-1}$.

EXAMPLE 1 A debt of \$1000 is due at the end of 9 months. Determine an equivalent debt at a simple interest rate of 9% at the end of 4 months and at the end of 1 year.

Solution Let us arrange the data on a time diagram below.



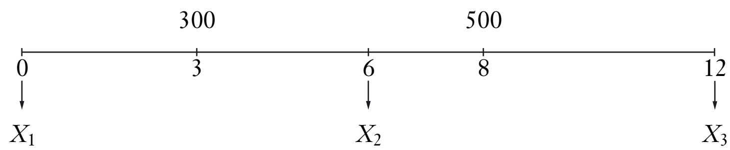
According to the definition of equivalence

$$X = 1000[1 + rt]^{-1} = \$1000[1 + (0.09)(\tfrac{5}{12})]^{-1} = \$963.86$$
$$Y = 1000[1 + rt] = \$1000[1 + (0.09)(\tfrac{3}{12})] = \$1022.50$$

The sum of a set of dated values, due on different dates, has no meaning. We must take into account the time value of money, which means we have to replace all the dated values by **equivalent dated values**, due on the same date. The sum of the equivalent values is called the **dated value of the set**.

EXAMPLE 2 A person owes \$300 due in 3 months and \$500 due in 8 months. The lender agrees to allow the person to pay off these two debts with a single payment. What single payment a) now; b) in 6 months; c) in 1 year, will liquidate these obligations if money is worth 8%?

Solution



We calculate equivalent dated values of both obligations at the three different times and arrange them in the table below.

Obligations	Now	In 6 Months	In 1 Year
First	$300[1 + (0.08)(\frac{3}{12})]^{-1} = 294.12$	$300[1 + (0.08)(\frac{3}{12})] = 306.00$	$300[1 + (0.08)(\frac{9}{12})] = 318.00$
Second	$500[1 + (0.08)(\frac{8}{12})]^{-1} = 474.68$	$500[1 + (0.08)(\frac{2}{12})]^{-1} = 493.42$	$500[1 + (0.08)(\frac{4}{12})] = 513.33$
Sum	$X_1 = 768.80$	$X_2 = 799.42$	$X_3 = 831.33$

Equation of Value

One of the most important problems in the mathematics of finance is the replacing of a given set of payments by an equivalent set.

We say that two sets of payments are equivalent at a given simple interest rate if the dated values of the sets, on any common date, are equal. An equation stating that the dated values, on a common date, of two sets of payments are equal is called an **equation of value** or an **equation of equivalence**. The date used is called the **focal date** or the **comparison date** or the **valuation date**.

A very effective way to solve many problems in mathematics of finance is to use the equation of value. The procedure is carried out in the following steps:

Step 1 Make a good time diagram showing the dated values of original debts on one side of the time line and the dated values of replacement payments on the other side. A good time diagram is of great help in the analysis and solution of problems.

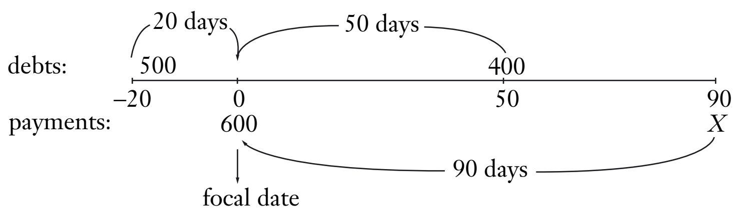
Step 2 Select a focal date and bring all the dated values to the focal date using the specified interest rate.

Step 3 Set up an equation of value at the focal date.
dated value of payments = dated value of debts

Step 4 Solve the equation of value using methods of algebra.

EXAMPLE 3 Debts of \$500 due 20 days ago and \$400 due in 50 days are to be settled by a payment of \$600 now and a final payment 90 days from now. Determine the value of the final payment at a simple interest rate of 11% with a focal date at the present.

Solution We arrange the dated values on a time diagram.



Equation of value at the present time:

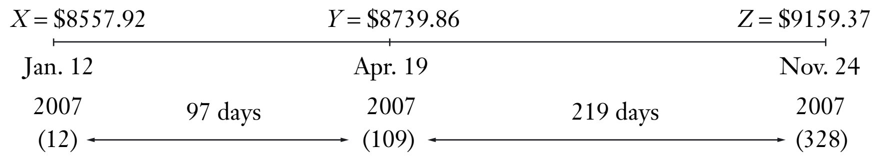
dated value of payments = dated value of debts

$$\begin{aligned} X[1 + (0.11)(\tfrac{90}{365})]^{-1} + 600 &= 500[1 + (0.11)(\tfrac{20}{365})] + 400[1 + (0.11)(\tfrac{50}{365})]^{-1} \\ 0.973592958X + 600 &= 503.01 + 394.06 \\ 0.973592958X &= 297.07 \\ X &\doteq 305.13 \end{aligned}$$

The final payment to be made in 90 days is \$305.13.

EXAMPLE 4 Dated payments X, Y, and Z are as follows: X is \$8557.92 on January 12, 2007; Y is \$8739.86 on April 19, 2007; and Z is \$9159.37 on November 24, 2007. If the annual rate of simple interest is 8%, show that X is equivalent to Y and Y is equivalent to Z but that X is not equivalent to Z.

Solution We arrange the dated values X, Y, and Z on a time diagram, listing for each date the day number from the table on the inside back cover.



At a simple rate of interest $r = 0.08$

X is equivalent to Y since $Y = \$8557.92[1 + (0.08)(\frac{97}{365})] = \$8739.86$

and Y is equivalent to Z since $Z = \$8739.86[1 + (0.08)(\frac{219}{365})] = \$9159.37$

but X **is not** equivalent to Z since $Z \neq \$8557.92[1 + (0.08)(\frac{97+219}{365})] = \$9150.64$

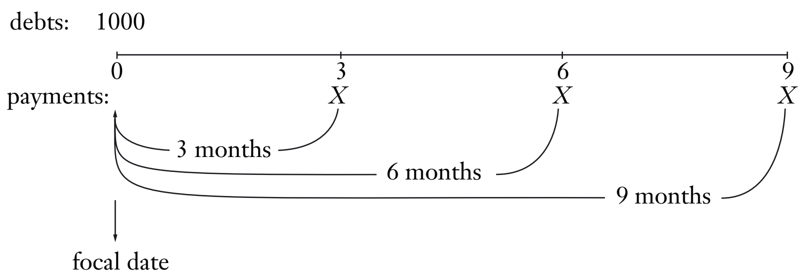
In mathematics, an equivalence relationship must satisfy the so-called **property of transitivity,** that is, if X is equivalent to Y and Y is equivalent to Z, then X is equivalent to Z.

OBSERVATION:

Example 4 illustrates that the definition of equivalence of dated values at simple interest r does not satisfy the property of transitivity. As a result, the solutions to the problems of equations of value at simple interest do depend on the selection of the focal date. The following Example 5 illustrates that in problems involving equations of value at simple interest the answer will vary slightly with the location of the focal date. It is therefore important that the parties involved in the financial transaction agree on the location of the focal date.

EXAMPLE 5 A person borrows \$1000 at 6% simple interest. She is to repay the debt with 3 equal payments, the first at the end of 3 months, the second at the end of 6 months, and the third at the end of 9 months. Determine the size of the payments. Put the focal date a) at the present time; b) at the end of 9 months.

Solution a Arrange all the dated values on a time diagram.



Equation of value at the present time:

$$\begin{aligned} X[1 + (0.06)(\tfrac{3}{12})]^{-1} + X[1 + (0.06)(\tfrac{6}{12})]^{-1} + X[1 + (0.06)(\tfrac{9}{12})]^{-1} &= 1000.00 \\ 0.98522167X + 0.97087379X + 0.95693780X &= 1000.00 \\ 2.91303326X &= 1000.00 \\ X &\doteq 343.28 \end{aligned}$$

Solution b Equation of value at the end of 9 months:

$$\begin{aligned} X[1+(0.06)(\tfrac{6}{12})] + X[1+(0.06)(\tfrac{3}{12})] + X &= 1000[1+(0.06)(\tfrac{9}{12})] \\ 1.03X + 1.015X + X &= 1045.00 \\ 3.045X &= 1045.00 \\ X &= 343.19 \end{aligned}$$

Notice the slight difference in the answer for the different focal dates.

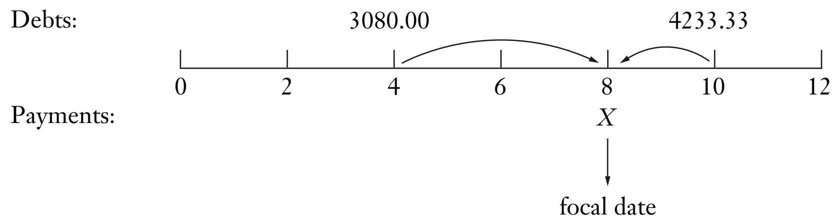
EXAMPLE 6 A woman owes \$3000 in 4 months with interest at 8% and another \$4000 is due in 10 months with interest at 7%. These two debts are to be replaced with a single payment due in 8 months. Determine the value of the single payment if money is worth 6.5%, using the end of 8 months as the focal date.

Solution Since the two original debts are due with interest, we first must determine the maturity value of each of them.

First debt: $S_1 = P_1[1 + r_1t_1] = 3000[1 + 0.08(\tfrac{4}{12})] = 3080.00$

Second debt: $S_2 = P_2[1 + r_2t_2] = 4000[1 + 0.07(\tfrac{10}{12})] = 4233.33$

Now arrange all the dated values on a time diagram.



Equation of value at end of 8 months, using $r_3 = 6.5\%$:

$$\begin{aligned} X &= 3080.00[1 + 0.065(\tfrac{4}{12})] + 4233.33[1 + 0.065(\tfrac{2}{12})]^{-1} \\ &= 3146.7333 \qquad\qquad + 4187.9604 \\ &\doteq 7334.69 \end{aligned}$$

The size of the single payment needed in 8 months is \$7334.69.

Exercise 1.3

For problems 1 to 6 make use of the following table. In each case determine the equivalent payments for each of the debts.

No.	Debts	Equivalent Payments	Focal Date	Rate
1.	\$1200 due in 80 days	In full	Today	12%
2.	\$100 due in 6 months, \$150 due in 1 year	In full	Today	6%
3.	\$200 due in 3 months, \$800 due in 9 months	In full	6 months hence	8%
4.	\$600 due 2 months ago, \$400 due in 3 months	\$500 today and the balance in 6 months	Today	13%
5.	\$800 due today	Two equal payments due in 4 and 7 months	7 months hence	5%
6.	\$2000 due 300 days ago	Three equal payments due today, in 60 days and in 120 days	Today	11%

7. Therèse owes \$500 due in 4 months and \$700 due in 9 months. If money is worth 7%, what single payment a) now; b) in 6 months; c) in 1 year, will liquidate these obligations?

8. Andrew owes Nicola \$500 in 3 months and \$200 in 6 months both due with interest at 6%. If Nicola accepts \$300 now, how much will Andrew be required to repay at the end of 1 year, provided they agree to use an interest rate of 8% and a focal date at the end of 1 year?

9. A person borrows \$1000 to be repaid with two equal instalments, one in six months, the other at the end of 1 year. What will be the size of these payments if the interest rate is 8% and the focal date is 1 year hence? What if the focal date is today?

10. Frank borrows \$5000 on January 8, 2007. He pays \$2000 on April 30, 2007, and \$2000 on August 31, 2007. The last payment is to be on January 2, 2008. If interest is at 6% and the focal date is January 2, 2008, determine the size of the final payment.

11. Mrs. Adams has two options available in repaying a loan. She can pay \$200 at the end of 5 months and \$300 at the end of 10 months, or she can pay \$$X$ at the end of 3 months and \$$2X$ at the end of 6 months. Determine X if interest is at 12% and the focal date is 6 months hence and the options are equivalent. What is the answer if the focal date is 3 months hence and the options are equivalent?

12. Mr. A will pay Mr. B \$2000 at the end of 5 years and \$8000 at the end of 10 years if Mr. B will give him \$3000 today plus an additional sum of money \$$X$ at the end of 2 years. Determine X if interest is at 8% and the comparison date is today. Determine X if the comparison date is 2 years hence (i.e., at the time \$$X$ is paid).

13. Mr. Malczyk borrows \$2000 at 14%. He is to repay the debt with 4 equal payments, one at the end of each 3-month period for 1 year. Determine the size of the payments given a focal date a) at the present time; b) at the end of 1 year.

14. A person borrows \$800 at 10%. He agrees to pay off the debt with payments of size \$$X$, \$$2X$, and \$$4X$ in 3 months, 6 months, and 9 months respectively. Determine X using all four transaction dates as possible focal dates.

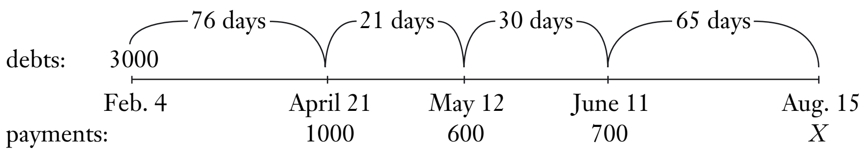
Section 1.4 Partial Payments

Financial obligations are sometimes liquidated by a series of partial payments during the term of obligation. Then it is necessary to determine the balance due on the final due date. There are two common ways to allow interest credit on short-term transactions.

Method 1 (also known as **Declining Balance Method**) This is the method most commonly used in practice. The interest on the unpaid balance of the debt is computed each time a partial payment is made. If the payment is greater than the interest due, the difference is used to reduce the debt. If the payment is less than the interest due, it is held without interest until other partial payments are made whose sum exceeds the interest due at the time of the last of these partial payments. (This point is illustrated in Example 3.) The balance due on the final date is the outstanding balance after the last partial payment carried to the final due date.

EXAMPLE 1 On February 4, David borrowed $3000 at a simple interest rate of 11%. He paid $1000 on April 21, $600 on May 12, and $700 on June 11. What is the balance due on August 15 using the declining balance method?

Solution We arrange all dated values on a time diagram.



Instead of having one comparison date we have 4 comparison dates. Each time a payment is made, the preceding balance is accumulated at a simple interest rate of 11% to this point and a new balance is obtained.

The calculations are given below.

Original debt	$3000.00
Interest for 76 days	68.71
Amount due on April 21	3068.71
First partial payment	1000.00
Balance due on April 21	2068.71
Interest for 21 days	13.09
Amount due on May 12	2081.80
Second partial payment	600.00
Balance due on May 12	1481.80
Interest for 30 days	13.40
Amount due on June 11	1495.20
Third partial payment	700.00
Balance due on June 11	795.20
Interest for 65 days	15.58
Balance due on August 15	$ 810.78

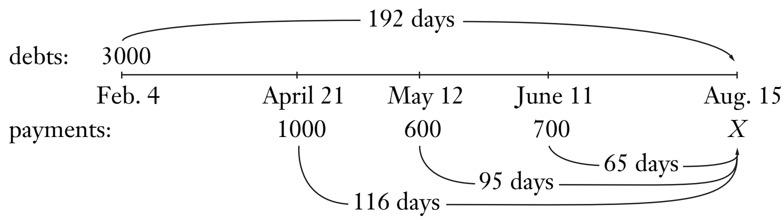
The above calculations also may be carried out in a shorter form, as shown below.

Balance due on April 21	$\$3000[1 + (0.11)(\frac{76}{365})] - \$1000 = \$2068.71$
Balance due on May 12	$\$2068.71[1 + (0.11)(\frac{21}{365})] - \$600 = \$1481.80$
Balance due on June 11	$\$1481.80[1 + (0.11)(\frac{30}{365})] - \$700 = \$795.20$
Balance due on August 15	$\$795.20[1 + (0.11)(\frac{65}{365})] = \$810.78$

Method 2 (also known as **Merchant's Rule**) The entire debt and each partial payment earn interest to the final settlement date. The balance due on the final due date is simply the difference between the accumulated value of the debt and the accumulated value of the partial payments.

EXAMPLE 2 Solve Example 1 using the Merchant's Rule.

Solution We arrange all dated values on a time diagram.



Simple interest is computed at 11% on the original debt of \$3000 for 192 days, on the first partial payment of \$1000 for 116 days, on the second partial payment of \$600 for 95 days, and on the third partial payment of \$700 for 65 days.

Calculations are given below:

Original debt	\$3000.00
Interest for 192 days	173.59
Accumulated value of the debt	\$3173.59

1st partial payment	\$1000.00
Interest for 116 days	34.96
2nd partial payment	600.00
Interest for 95 days	17.18
3rd partial payment	700.00
Interest for 65 days	13.71
Accumulated value of the partial payments	\$2365.85

Balance due on August 15: \$(3173.59 − 2365.85) = \$807.74

Alternative Solution We can write an equation of value with August 15 as the focal date.

On August 15: dated value of payments = dated value of debts

$$X + 1000[1 + (0.11)(\tfrac{116}{365})] + 600[1 + (0.11)(\tfrac{95}{365})] + 700[1 + (0.11)(\tfrac{65}{365})] = 3000[1 + (0.11)(\tfrac{192}{365})]$$

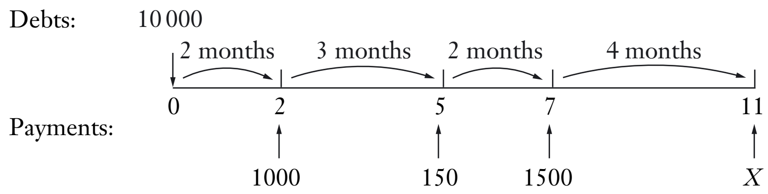
$$X + 1034.96 + 617.18 + 713.71 \doteq 3173.59$$

$$X = 807.74$$

Note: The methods result in two different concluding payments. It is important that the two parties to a business transaction agree on the method to be used. Common business practice is the Declining Balance Method.

EXAMPLE 3 Suzanne borrowed \$10 000 at 8%. She pays \$1000 in 2 months, \$150 in 5 months, and \$1500 in 7 months. What is the balance due in 11 months under the a) Declining Balance Method, b) Merchant's Rule?

Solution a We arrange all dated values on a time diagram.



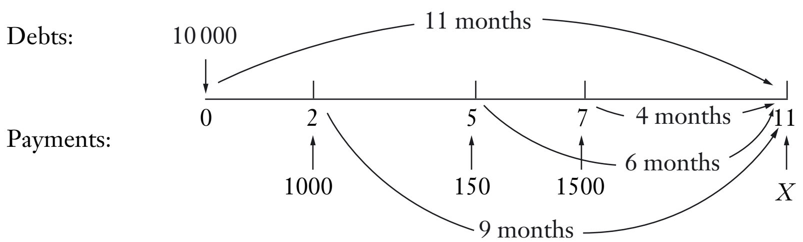
$$\text{Balance due in two months} = \$10\,000[1 + 0.08(\tfrac{2}{12})] - \$1000 = \$9133.33$$

$$\text{Interest due in three months} = Prt = \$9133.33(0.08)(\tfrac{3}{12}) = \$182.67 > \$150$$

Thus, the \$150 payment is not applied towards reducing the loan, but instead is carried forward, without interest, to the next payment date.

$$\begin{aligned}\text{Balance due in five months} &= \$9133.33[1 + 0.08(\tfrac{3+2}{12})] - \$(150 + 1500)\\ &= \$7787.77\\ \text{Balance due in four months, } X &= \$7787.77[1 + 0.08(\tfrac{4}{12})] = \$7995.44\end{aligned}$$

Solution b We arrange all dated values on a time diagram.



Setting up an equation of value, with the end of 11 months as the focal date,

$$\begin{aligned}1000[1 + 0.08(\tfrac{9}{12})] + 150[1 + 0.08(\tfrac{6}{12})] + 1500[1 + 0.08(\tfrac{4}{12})] + X &= 10\,000[1 + 0.08(\tfrac{11}{12})]\\ 1060.00 + 156.00 + 1540.00 + X &= 10\,733.33\\ X &= 7977.33\end{aligned}$$

Again, note that the declining balance method leads to a higher final balance.

Exercise 1.4

1. Jean-Luc borrowed $1000, repayable in one year, with interest at 9%. He pays $200 in 3 months and $400 in 7 months. Determine the balance in one year using the Declining Balance Method and the Merchant's Rule.
2. On June 1, 2006, Sheila borrows $2000 at 7%. She pays $800 on August 17, 2006; $400 on November 20, 2006; and $500 on February 2, 2007. What is the balance due on April 18, 2007, by the Declining Balance Method? By the Merchant's Rule?
3. A debt of $5000 is due in six months with interest at 5%. Partial payments of $3000 and $1000 are made in 2 and 4 months respectively. What is the balance due on the final statement date by the Declining Balance Method? By the Merchant's Rule?
4. Alexandra borrows $1000 on January 8 at 16%. She pays $350 on April 12, $20 on August 10, and $400 on October 3. What is the balance due on December 15 using the Declining Balance Method? The Merchant's Rule?
5. A loan of $1400 is due in one year with simple interest at 12%. Partial payments of $400 in 2 months, $30 in 6 months, and $600 in 8 months are made. Determine the balance due in one year using the Declining Balance Method.
6. A loan of $2500 is taken out on May 12. It is to be paid back within a one-year period with partial payments of $1000 on August 26, $40 on November 19, and $350 on January 9. If the rate of interest is 8%, what is the final balance due on March 20 by the Declining Balance Method? By the Merchant's Rule?

Section 1.5 Simple Discount at a Discount Rate

The calculation of present (or discounted) value over durations of less than one year is sometimes based on a simple discount rate. The **annual simple discount rate** d is the ratio of the discount D for the year to the amount S on which the discount is given. The simple discount D on an amount S for t years at the discount rate d is calculated by means of the formula:

$$\boxed{D = Sdt} \tag{4}$$

and the discounted value P of S, or the proceeds P, is given by

$$P = S - D$$

By substituting for $D = Sdt$ we obtain P in terms of S, d, and t

$$P = S - Sdt$$

$$\boxed{P = S(1 - dt)} \tag{5}$$

The factor $(1 - dt)$ in formula **(5)** is called **a discount factor at a simple discount rate *d*.**

Simple discount is not very common. However, it should not be ignored. Some financial institutions offer what are referred to as **discounted loans.** For these types of loans, the interest charge is based on the final amount, S, rather than on the present value. The lender calculates the interest (which is called discount) D, using formula **(4)** and deducts this amount from S. The difference, P, is the amount the borrower actually receives, even though the actual loan amount is considered to be S. The borrower pays back S on the due date. For this reason the interest charge on discounted loans is sometimes referred to as **interest in advance**, as the interest is paid up front, at the time of the loan, instead of the more common practice of paying interest on the final due date.

In theory, we can also accumulate a sum of money using simple discount, although it is not commonly done in practice. From formula **(5)**, we can express S in terms of P, d, and t and obtain

$$\boldsymbol{S = \frac{P}{1 - dt} = P(1 - dt)^{-1}} \tag{6}$$

The factor $(1 - dt)^{-1}$ in formula **(6)** is called **an accumulation factor at a simple discount rate *d*.**

Formula **(6)** can be used to calculate the maturity value of a loan for a specified proceeds.

EXAMPLE 1 A person takes out a discounted loan with a face value of \$500 for 6 months from a lender who charges a $9\frac{1}{2}\%$ discount rate. a) What is the discount, and how much money does the borrower receive? b) What size loan should the borrower ask for if he wants to receive \$500 cash?

Solution a We have $S = 500$, $d = 9\frac{1}{2}\%$, $t = \frac{1}{2}$ and calculate

$$\text{discount } D = Sdt = \$500 \times 0.095 \times \tfrac{1}{2} = \$23.75$$
$$\text{proceeds } P = S - D = \$(500 - 23.75) = \$476.25$$

Alternatively, we could calculate the proceeds by formula **(5)**

$$P = S(1 - dt) = \$500[1 - (0.095)(\tfrac{1}{2})] = \$476.25$$

What is happening in this situation is that the borrower receives a loan of \$500, but he must pay \$23.75 in interest charges today. He ends up with a net of \$476.25 in his pocket. Six months later, he repays the original loan amount, \$500, but does not pay any interest at that time, as he has already paid the interest up front.

Solution b We have $P = 500$, $d = 9\frac{1}{2}\%$, $t = \frac{1}{2}$ and calculate the maturity value of the loan

$$S = \frac{P}{1 - dt} = \frac{\$500}{1 - (0.095)(\frac{1}{2})} = \$524.93$$

The borrower should ask for a loan of \$524.93 to receive proceeds of \$500.

EXAMPLE 2 Calculate the discounted value of $1000 due in 1 year: a) at a simple interest rate of 7%; b) at a simple discount rate of 7%.

Solution a We have $S = 1000$, $r = 7\%$, $t = 1$ and calculate the discounted value P by formula **(2)**

$$P = \frac{S}{1+rt} = \frac{\$1000}{1+(0.07)(1)} = \$934.58$$

Solution b We have $S = 1000$, $d = 7\%$, $t = 1$ and calculate the discounted value P by formula **(5)**

$$P = S(1-dt) = \$1000[1-(0.07)(1)] = \$1000(0.93) = \$930.00$$

Note the difference of $4.58 between the discounted value at a simple interest rate and the discounted value at a simple discount rate. We can conclude that a given simple discount rate results in a larger money return to a lender than the same simple interest rate. Note also that in **a)**, the borrower is taking out a loan for $934.58. At the end of 1 year, he/she pays back the original principal of $934.58 plus interest of $65.42 for a total of $1000. In **b)**, the borrower is taking out a loan for $1000. He/she pays $70 in interest up front, receives $930 and repays the original principal of $1000 at the end of 1 year.

EXAMPLE 3 If $1200 is the present value of $1260 due at the end of 9 months, determine a) the annual simple interest rate, and b) the annual simple discount rate.

Solution a We have $P = 1200$, $I = 60$, $t = \frac{9}{12}$, and calculate

$$r = \frac{I}{Pt} = \frac{60}{1200(\frac{9}{12})} \doteq 6.67\%$$

Solution b We have $S = 1260$, $D = 60$, $t = \frac{9}{12}$, and calculate

$$d = \frac{D}{St} = \frac{60}{1260(\frac{9}{12})} \doteq 6.35\%$$

In general, we can calculate **equivalent rates** of interest or discount by using the following definition:

> Two rates are equivalent if they have the same effect on money over the same period of time.

The above definition can be used to determine simple (or compound) equivalent rates of interest or discount.

EXAMPLE 4 What simple rate of discount d is equivalent to a simple rate of interest $r = 6\%$ if money is invested for a) 1 year; b) 2 years?

Solution a Compare the discounted values of $1 due at the end of 1 year under simple discount and simple interest:

$$[1 - d(1)] = [1 + (0.06)(1)]^{-1}$$
$$1 - d = (1.06)^{-1}$$
$$d \doteq 5.66\%$$

Solution b Compare the discounted values of $1 due at the end of 2 years under simple discount and simple interest:

$$[1 - d(2)] = [1 + (0.06)(2)]^{-1}$$
$$1 - 2d = (1.12)^{-1}$$
$$d \doteq 5.36\%$$

OBSERVATION:

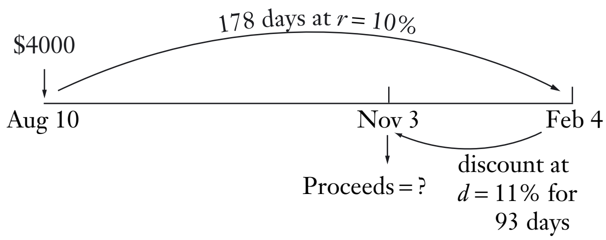
Notice that for simple rates of interest and discount, the equivalent rates are dependent on the period of time.

Application of Simple Discount — Promissory Notes

Promissory notes are occasionally sold based on simple discount from the maturity value, as illustrated in the following example.

EXAMPLE 5 On August 10 Smith borrows $4000 from Jones and gives Jones a promissory note at a simple interest rate of 10% with a maturity date of February 4 in the following year. Brown buys the note from Jones on November 3, based on a simple discount rate of 11%. Determine Brown's purchase price.

Solution We arrange the dated values on a time diagram.



$$\text{Maturity value of the note, } S = \$4000[1 + 0.10(\tfrac{178}{365})] = \$4195.07$$

Since Brown is buying the note using a simple discount rate, the proceeds are determined using formula **(5)**:

$$\text{Proceeds on November 3, } P = S[1 - dt] = \$4195.07[1 - 0.11(\tfrac{93}{365})] = \$4077.49$$

The amount paid by Brown to Jones is $4077.49.

Exercise 1.5

1. Determine the discounted value of \$2000 due in 130 days
   a) at a simple interest rate of 8.5%;
   b) at a simple discount rate of 8.5%.

2. A bank offers a 272-day discounted loan at a simple discount rate of 12%.
   a) How much money would a borrower receive if he asked for a \$5000 loan?
   b) What size loan should the borrower ask for in order to actually receive \$5000?
   c) What is the equivalent simple interest rate that is being charged on the loan?

3. A note with a maturity value of \$700 is sold at a discount rate of 8%, 45 days before maturity. Determine the discount and the proceeds.

4. A promissory note with face value \$2000 is due in 175 days and bears 7% simple interest. After 60 days it is sold for \$2030. What simple discount rate can the purchaser expect to earn?

5. A storekeeper buys goods costing \$800. He asks his creditor to accept a 60-day noninterest-bearing note, which, if his creditor discounts immediately at a 10% discount rate, will result in proceeds of \$800. For what amount should he make the note?

6. A company borrowed \$50 000 on May 1, 2007, and signed a promissory note bearing interest at 11% for 3 months. On the maturity date, the company paid the interest in full and gave a second note for 3 months without interest and for such an amount that when it was discounted at a 12% discount rate on the day it was signed, the proceeds were just sufficient to pay the debt. Determine the amount of interest paid on the first note and the face value of the second note.

7. A discounted loan of \$3000 at a simple discount rate of 6.5% is offered to Mr. Jones. If the actual amount of money that Mr. Jones receives is \$2869.11, when is the \$3000 due to be paid back?

8. A borrower receives \$1500 today and must pay back \$1580 in 200 days. If this is a discounted loan, what rate of simple discount is assumed?

9. If a borrower actually wanted to receive \$1800 today, what size discounted loan should they ask for if $d = 8.7\%$ and the loan is due in 1 year?

Section 1.6 Summary and Review Exercises

- Accumulated value of P at the end of t years at a simple interest rate r is given by $P(1 + rt)$ where $(1 + rt)$ is called an accumulation factor at simple interest rate r.
- Discounted value of S due in t years at a simple interest rate r is given by $S(1 + rt)^{-1}$ where $(1 + rt)^{-1}$ is called a discount factor at a simple interest rate r.
- Discounted value of S due in t years at a simple discount rate d is given by $S(1 - dt)$ where $(1 - dt)$ is called a discount factor at a simple discount rate d.
- Accumulated value of P at the end of t years at a simple discount rate d is given by $P(1 - dt)^{-1}$ where $(1 - dt)^{-1}$ is called an accumulation factor at a simple discount rate d.
- Equivalence of dated values at a simple interest rate r:
  X due on a given date is equivalent to Y due t years later,
  if $Y = X(1 + rt)$ or $X = Y(1 + rt)^{-1}$.

Note: The above definition of equivalence of dated values at a simple interest rate r does not satisfy the property of transitivity. The lack of transitivity leads to different answers when different comparison dates are used for equations of value.

- Equivalent rates:
  Two rates are equivalent if they have the same effect on money over the same period of time.

Note: At simple rates of interest and discount, the equivalent rates are dependent on the period of time.

Review Exercises 1.6

1. How long will it take $1000
  a) to earn $100 at 6% simple interest?
  b) to accumulate to $1200 at $13\frac{1}{2}$% simple interest?

2. Determine the accumulated and the discounted value of $1000 over 55 days at 7% using both ordinary and exact simple interest.

3. A taxpayer expects an income tax refund of $380 on May 1. On March 10, a tax discounter offers 85% of the full refund in cash. What rate of simple interest will the tax discounter earn?

4. A retailer receives an invoice for $8000 for a shipment of furniture with terms 3/10, *n*/40.
  a) What is the highest simple interest rate at which he can afford to borrow money in order to take advantage of the cash discount?
  b) If the retailer can borrow at a simple interest rate of 12%, calculate his savings resulting from the cash discount, when he pays the invoice within 10 days.

5. A cash discount of 3% is given if a bill is paid 40 days in advance of its due date.
  a) What is the highest simple interest rate at which you can afford to borrow money if you wanted to take advantage of this discount?
  b) If you can borrow money at a simple interest rate 8%, how much can you save by paying an invoice for $5000 forty days in advance of its due date?

6. A loan of $2500 taken out on April 2 requires equal payments on May 25, July 20, September 10, and a final payment of $500 on October 15. If the focal date is October 15, what is the size of the equal payments at 9% simple interest?

7. Last night Marion won $5000 in a lottery. She was given two options. She can take $5000 today or $$X$ every six months (beginning six months from now) for 2 years. If the options are equivalent and the simple interest rate is 10%, determine X using a focal date of 2 years.

8. Today Mr. Mueller borrowed $2400 and arranged to repay the loan with three equal payments of $$X$ at the end of 4, 8, and 12 months respectively. If the lender charges a simple interest rate of 6%, find X using today as a focal date.

9. Roger borrows $4500 at 9% simple interest on July 3, 2006. He pays $1250 on October 27, 2006, and $2500 on January 7, 2007. Determine the balance due on May 1, 2007, using the Declining Balance Method and the Merchant's Rule.

10. Paul borrows $4000 at a 7.5% simple interest rate. He is to repay the loan by paying $1000 at the end of 3 months and two equal payments at the end of 6 months and 9 months. Determine the size of the equal payments using
  a) the end of 6 months as a focal date.
  b) the present time as a focal date.

11. Melissa borrows $1000 on May 8, 2006 at an 8% simple interest rate. She pays $500 on July 17, 2006 and $400 on September 29, 2006. What is the balance due on October 31, 2006 using the Declining Balance Method and the Merchant's Rule?

12. Robert borrows $1000 for 8 months from a lender who charges an 11% discount rate.
  a) How much money does Robert receive?
  b) What size loan should Robert ask for in order to receive $1000 cash?

13. Consider the following promissory note:

> $1500.00 Vancouver, May 11, 2006
>
> Ninety days after date, I promise to pay to the order of J.D. Prasad
>
> Fifteen hundred and $\frac{00}{100}$ dollars
>
> for value received with interest at 12% per annum.
>
> (Signed) J.B. Lau

Mr. Prasad sells the note on July 2, 2006, to a bank that discounts notes at a 13% discount rate.

a) How much money does Mr. Prasad receive?
b) What rate of interest does Mr. Prasad realize on his investment?
c) What rate of interest does the bank realize on its investment if it holds the note until maturity?
d) What rate of interest does the bank realize if Mr. Lau pays off the loan as due in exactly 90 days?

14. A 180-day promissory note for $2000 bears 7% simple interest. After 60 days it is sold to a bank that charges a discount rate of 7%.

a) Calculate the price paid by the bank for the note.
b) What simple interest rate did the original owner of the note actually earn?
c) If the note is actually paid on the **due** date, what discount rate will the bank have realized?

15. A note for $800 is due in 90 days with simple interest at 6%. On the maturity date, the maker of the note paid the interest in full and gave a second note for 60 days without interest and for such an amount that when it was discounted at a 7% discount rate on the day it was signed, the proceeds were just sufficient to pay the debt. Determine the interest paid on the first note and the face value of the second note.

16. A debt of $1200 is to be paid off by payments of $500 in 45 days, $300 in 100 days, and a final payment of $436.92. Interest is at $r = 11\%$ and the Merchant's Rule was used to calculate the final payment. In how many days should the final payment be made?

17. Determine the difference in simple interest on a $20 000 loan at a simple interest rate of 9% from April 15 to the next February 4, using the Banker's Rule and Canadian practice.

18. A person borrows $3000 at a 14% simple interest rate. He is to repay the debt with 3 equal payments, the first at the end of 3 months, the second at the end of 7 months, and the third at the end of 12 months. Determine the difference between payments resulting from the selection of the focal date at the present time and at the end of 12 months.

19. A promissory note for $1500 is due in 170 days with simple interest at 9.5%. After 40 days it is sold to a finance company that charges a simple discount rate of 14%. How much does the finance company pay for the note?

20. You borrow $1000 now and $2000 in 4 months. You agree to pay $$X$ in 6 months and 2X$ in 8 months (from now). Determine X using a focal date 8 months from now

a) at simple interest rate $r = 8\%$;
b) at simple discount rate $d = 8\%$.