## Index Numbers

## Introduction

In this chapter we will examine a useful descriptive tool called an index. No doubt you are familiar with indexes such as the Consumer Price Index, which is released monthly by Statistics Canada. There are many other indexes, such as the Dow Jones Industrial Average and the S\&P/TSX Composite Index. Indexes are published on a regular basis by the federal government, by business publications such as Business Week and Forbes, and in most daily newspapers.

Of what importance is an index? Why is the Consumer Price Index so important and so widely reported? As the name implies, it measures the change in the price of a large group of items consumers purchase. Governments, consumer groups, unions, management, senior citizens organizations, and others in business and economics are very concerned about changes in prices. These groups closely monitor the Consumer Price Index as well as other indexes. To combat sharp price increases, the Bank of Canada often raises the interest rate to "cool down" the economy. Likewise, the S\&P/TSX Composite Index measures the overall daily performance of more than 200 of the largest publicly traded companies in Canada.

A few stock market indexes appear daily in the financial section of most newspapers. They are updated every 15 minutes on many Web sites.

## Simple Index Numbers

What is an index number?

INDEX NUMBER A number that expresses the relative change in price, quantity, or value compared to a base period.

If the index number is used to measure the relative change in just one variable, such as hourly wages in manufacturing, we refer to this as a simple index. It is the ratio of two values of the variable and that ratio converted to a percentage. The following four examples will serve to illustrate the use of index numbers.

## LEARNING OBJECTIVES

When you have completed this chapter, you will be able to:

1
Describe the term index.

Understand the difference between a weighted and an unweighted index.
3 Construct and interpret a Laspeyers price index.

Construct and interpret a Paasche price index.

## 5 Construct and interpret a value index.

Explain how the Consumer Price Index is constructed and interpreted.

## Example

## Solution

According to Statistics Canada, in 1995 the average salary of wage earners 15 years and older in Newfoundland and Labrador was \$20 828 per year. In 2001, it was $\$ 24165$ per year. What is the index of yearly earnings of workers over age 15 in Newfoundland and Labrador for 2001 based on 1995?

It is 116.0, found by:

$$
\begin{aligned}
I & =\frac{\text { Average yearly income of wage earners over } 15 \text { in } 2001}{\text { Average yearly income of wage earners over } 15 \text { in } 1995}(100) \\
& =\frac{24165}{20828}(100)=116.0
\end{aligned}
$$

Thus, the yearly salaries in 2001 compared to 1995 were 116.0 percent. This means that there was a 16 percent increase in yearly salaries during the six years from 1995 to 2001, found by $116.0-100.0=16.0$.

## EXAMPLE

Solution
Statistics Canada results show that the number of farms in Canada dropped from 276548 in 1996, to an estimated 246923 in 2001. What is the index for the number of farms in 2001 based on the number in 1996 ?

The index is 89.3 , found by:

$$
I=\frac{\text { Number of farms in } 2001}{\text { Number of farms in } 1996}(100)=\frac{246923}{276547}(100)=89.3
$$

This indicates that the number of farms in 2001 compared with 1996 was 89.3 percent. To put it another way, the number of farms in Canada decreased by 10.7 percent (100.0-89.3 = 10.7) during the five-year period.

## Example

## Solution

An index can also compare one item with another. The population of British Columbia in 2003 was 4146580 and for Ontario it was 12238300 . What is the population of British Columbia compared to Ontario?

The index of population for British Columbia is 33.9, found by:

$$
I=\frac{\text { Population of British Columbia }}{\text { Population of Ontario }}(100)=\frac{4146580}{12238300}(100)=33.9
$$

This indicates that the population of British Columbia is 33.9 percent (about one third) of the population of Ontario, or the population of British Columbia is 66.1 percent less than the population of Ontario ( $100-33.9=66.1$ ).
Source: Statistics Canada CANSIM database, http://cansim2.statcan. Table 051=001, March 3, 2005.


## Solution

To find the four indexes, we divide the yearly salaries for Nova Scotia, Alberta, Ontario, and British Columbia by the yearly income for Canada. We conclude that the average yearly income is $10.8 \%$ higher in Ontario than for Canada.

|  | Average Salary (\$) | Index | Found by |
| :--- | :---: | ---: | :---: |
| Canada | 31757 | 100.0 | $(31757 / 31757) \times 100$ |
| Nova Scotia | 26632 | 83.9 | $(26632 / 31757) \times 100$ |
| Ontario | 35185 | 110.8 | $(35185 / 31757) \times 100$ |
| Alberta | 32603 | 102.7 | $(32603 / 31757) \times 100$ |
| British Columbia | 31544 | 99.3 | $(31544 / 31757) \times 100$ |

Source: Adapted from the Statistics Canada Web site www.statcan.ca/english/pgdb/labor50a.htm; March 29, 2005.

Note from the previous discussion that:

1. Index numbers are actually percentages because they are based on the number 100. However, the percent symbol is usually omitted.
2. Each index number has a base period. The current base period for the Consumer Price Index is 1992 = 100, changed from $1986=100$ in January 1998.
3. Most business and economic indexes are computed to the nearest whole number, such as 214 or 96 , or to the nearest tenth of a percent, such as 83.4 or 118.7.

## Why Convert Data to Indexes?

Indexes allow us to express a change in price, quantity, or value as a percent

Compiling index numbers is not a recent innovation. An Italian, G. R. Carli, is credited with originating index numbers in 1764. They were incorporated in a report he made regarding price fluctuations in Europe from 1500 to 1750. No systematic approach to collecting and reporting data in index form was evident until about 1900. The cost-of-living index (now called the Consumer Price Index) was introduced in 1913, and a long list of indexes has been compiled since then.

Why convert data to indexes? An index is a convenient way to express a change in a diverse group of items. The Consumer Price Index (CPI), for example, encompasses many items-including gasoline, golf balls, lawn mowers, hamburgers, funeral services, and dentists' fees. Prices are expressed in dollars per kilogram, box, yard, and many other different units. Only by converting the prices of these many diverse goods and services to one index number can the federal government and others concerned with inflation keep informed of the overall movement of consumer prices.

Converting data to indexes also makes it easier to assess the trend in a series composed of exceptionally large numbers. For example, the total retail trade in Canada for 2004 was $\$ 346721498$ and $\$ 331146620$ for 2003. The increase of $\$ 15574878$ appears significant. Yet, if the 2004 sales were expressed as an index based on 2003 sales, the increase would be approximately $4.7 \%$.
$\frac{\text { Retail trade in } 2004}{\text { Retail trade in } 2003}=\frac{346721498}{331146620} \times 100=104.7$

## Construction of Index Numbers

We already discussed the construction of a simple price index. The price in a selected year (such as 2003) is divided by the price in the base year. The base-period price is designated as $p_{0}$, and a price other than the base period is often referred to as the given period or selected period and designated $p_{t}$. To calculate the simple price index $P$ using 100 as the base value for any given period use the formula:
SIMPLE INDEX

$$
P=\frac{p_{t}}{p_{0}} \times 100
$$

[15-1]

Suppose that the price of a standard lot at the Shady Rest Cemetery in 1998 was $\$ 600$. The price rose to $\$ 1000$ in 2004. What is the price index for 2004 using 1998 as the base period and 100 as the base value? It is 166.7, found by:

$$
P=\frac{p_{t}}{p_{0}}(100)=\frac{\$ 1000}{\$ 600}(100)=166.7
$$

Interpreting this result, the price of a cemetery lot increased 66.7 percent from 1998 to 2004.
The base period need not be a single year. Note in Table 15-1 that if we use 1990-91 = 100, the base price for the stapler would be $\$ 21$ [found by determining the mean price of 1990 and 1991, $(\$ 20+\$ 22) / 2=\$ 21]$. The prices $\$ 20, \$ 22$, and $\$ 23$ are averaged if 1990-92 had been selected as the base. The mean price would be \$21.67. The indexes constructed using the three different base periods are presented in Table 15-1. (Note that when 1990-92 = 100, the index numbers for 1990, 1991, and 1992 average 100.0, as we would expect.) Logically, the index numbers for 2001 using the three different bases are not the same.

TABLE 15-1 Prices of a Benson Automatic Stapler, Model 3, Converted to Indexes Using Three Different Base Periods

| Year | Price of <br> Stapler (\$) | Price Index <br> $(\mathbf{1 9 9 0}=\mathbf{1 0 0})$ | Price index <br> $(\mathbf{1 9 9 0}-\mathbf{9 1}=\mathbf{1 0 0})$ | Price index <br> $(\mathbf{1 9 9 0} \mathbf{- 9 2}=\mathbf{1 0 0})$ |
| :---: | :---: | :---: | :---: | :---: |
| 1985 | 18 | 90.0 | $\frac{18}{21} \times 100=85.7$ | $\frac{18}{21.67} \times 100=83.1$ |
| 1990 | 20 | 100.0 | $\frac{20}{21} \times 100=95.2$ | $\frac{20}{21.67} \times 100=92.3$ |
| 1991 | 22 | 110.0 | $\frac{22}{21} \times 100=104.8$ | $\frac{22}{21.67} \times 100=101.5$ |
| 1992 | 23 | 115.0 | $\frac{23}{21} \times 100=109.5$ | $\frac{23}{21.67} \times 100=106.1$ |
| 2001 | 38 | 190.0 | $\frac{38}{21} \times 100=181.0$ | $\frac{38}{21.67} \times 100=175.4$ |

## Self-Review 15-1



1. The revenue in 2003 for a few selected companies is:

| Company | Revenue (\$ thousands) |
| :--- | :---: |
| Globelive Communinations Inc | 35058 |
| Swift Trade Inc | 35130 |
| Gram Precision Inc | 9543 |
| Trafford Publishing | 6658 |
| Diversinet Corp | 10258 |

Use Diversinet Corp as the base 2003 revenue and 100 as the base value. Express the 2003 revenue of the other four companies as an index. Interpret.
2. The average hourly earnings of production workers for selected periods are given below.

| Year | Average Hourly Earnings (\$) |
| :--- | :---: |
| 1997 | 10.32 |
| 1998 | 10.57 |
| 1999 | 10.83 |
| 2001 | 11.43 |
| 2003 | 12.28 |
| 2005 | 13.24 |
| 2006 | $13.74^{\star}$ |
| *preliminary estimate |  |

(a) Using 1997 as the base period and 100 as the base value, determine the indexes for 2005 and for the preliminary 2006 data. Interpret the index.
(b) Use the average of 1997, 1998, and 1999 as the base and determine indexes for 2005 and the preliminary 2006 data using 100 as the base value. Interpret the index.
(c) What is the index for the preliminary 2006 data using 2001 as the base?

## Exercises

1. Gasoline prices in cents per litre for Winnipeg, Manitoba from 1996 to 2003 are listed below.

|  | $\mathbf{2 0 0 3}$ | $\mathbf{2 0 0 2}$ | $\mathbf{2 0 0 1}$ | $\mathbf{2 0 0 0}$ | $\mathbf{1 9 9 9}$ | $\mathbf{1 9 9 8}$ | $\mathbf{1 9 9 7}$ | $\mathbf{1 9 9 6}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Winnipeg, Manitoba | 68.2 | 64.1 | 66.4 | 67.4 | 58.5 | 54.1 | 58.1 | 57.8 |

Develop a simple index for the change in price per litre based on the average of years 1998-2000.
2. The following table shows the average amount received in employment benefits from 1999 to 2004.

|  | $\mathbf{1 9 9 9}$ | $\mathbf{2 0 0 0}$ | $\mathbf{2 0 0 1}$ | $\mathbf{2 0 0 2}$ | $\mathbf{2 0 0 3}$ | $\mathbf{2 0 0 4}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| All benefits | 263.69 | 268.71 | 279.89 | 288.78 | 293.63 | 296.87 |

Develop a simple index with 2001 as the base year.
3. Listed below is the change in Internet use of a single family with unmarried children under age 18 from 1999 to 2003. Develop a simple index with 1999 as the base year to show the increase in Internet use. By what percent did Internet use increase over the five years?

| $\mathbf{1 9 9 9}$ | $\mathbf{2 0 0 0}$ | $\mathbf{2 0 0 1}$ | $\mathbf{2 0 0 2}$ | $\mathbf{2 0 0 3}$ |
| :---: | :---: | :---: | :---: | :---: |
| 59.0 | 71.2 | 80.5 | 81.2 | 83.7 |

4. In January 1994 the price for a whole fresh chicken was $\$ 1.99$ per kilogram. In September 2005 the price for the same chicken was $\$ 5.49$. Use the January 1994 price as the base period and 100 as the base value to develop a simple index. By what percent has the cost of chicken increased?

## Unweighted Indexes

In many situations we wish to combine several items and develop an index to compare the cost of this aggregation of items in two different time periods. For example, we might be interested in an index for items that relate to the expense of running and maintaining an automobile. The items in the index might include tires, oil changes, and gasoline prices. Or we might be interested in a college student index. This index might include the cost of books, tuition, housing, meals, and entertainment. There are several ways we can combine the items to determine the index.

## Simple Average of the Price Indexes

Table 15-2 reports the prices for several food items for the years 1995 and 2005. We would like to develop an index for this group of food items for 2005, using 1995 as the base. This is written in the abbreviated code $1995=100$.

TABLE 15-2 Computation of Index for Food Price 2005, $1995=100$

| Item | $\mathbf{1 9 9 5}$ Price (\$) | 2005 Price (\$) | Simple Index |
| :--- | :---: | :---: | :---: |
| Bread white (loaf) | 0.77 | 1.98 | 257.1 |
| Eggs (dozen) | 1.85 | 1.84 | 99.5 |
| Milk (litre) white | 0.88 | 1.98 | 225.0 |
| Apples, red delicious $(500 \mathrm{~g})$ | 1.46 | 1.75 | 119.9 |
| Orange juice $(355 \mathrm{ml}$ concentrate) | 1.58 | 1.70 | 107.6 |
| Coffee, $100 \%$ ground roast $(400 \mathrm{~g})$ | $\underline{4.40}$ | $\underline{3.99}$ | 90.7 |
| $\quad$ Total | $\mathbf{1 0 . 9 4}$ | $\mathbf{1 3 . 2 4}$ |  |

We could begin by computing a simple average of the price indexes for each item, using 1995 as the base year and 2005 as the given year. The simple index for bread is 257.1, found by using formula (15-1).

$$
P=\frac{p_{t}}{p_{0}}(100)=\frac{\$ 1.98}{\$ 0.77}(100)=257.1
$$

We compute the simple index for the other items in Table 15-2 similarly. The largest price increase is for bread, 157.1 percent $(257.1-100=157.1)$, and milk was a close second with 125 percent. The price of eggs dropped by half a percent in the period, found by $100.0-99.5=0.5$. Then it would be natural to average the simple indexes. The formula is:

$$
\text { SIMPLE AVERAGE OF THE PRICE RELATIVES } \quad P=\frac{\Sigma P_{i}}{n}
$$

where $P_{i}$ refers to the simple index for each of the items and $n$ the number of items. In our example the index is 150.0 , found by:

$$
P=\frac{\Sigma P_{i}}{n}=\frac{257.1+\cdots+90.7}{6}=\frac{899.7}{6}=150.0
$$

This indicates that the mean of the group of indexes increased 50 percent from 1995 to 2005.

A positive feature of the simple average of price indexes is that we would obtain the same value for the index regardless of the units of measure. In the above index, if apples were priced in tonnes, instead of kilograms, the impact of apples on the combined index would not change. That is, the commodity "apples" represents one of six items in the index, so the impact of the item is not related to the units. A negative feature of this index is that it fails to consider the relative importance of the items included in the index. For example, milk and eggs receive the same weight, even though a typical family might spend far more over the year on milk than on eggs.

## Simple Aggregate Index

A second possibility is to sum the prices (rather than the indexes) for the two periods and then determine the index based on the totals. The formula is

SIMPLE AGGREGATE INDEX

$$
\begin{equation*}
P=\frac{\Sigma P_{t}}{\Sigma P_{0}} \times 100 \tag{15-3}
\end{equation*}
$$

This is called a simple aggregate index. The index for the above food items is found by summing the prices in 1995 and 2005. The sum of the prices for the base period is $\$ 10.94$ and for the given period it is $\$ 13.24$. The simple aggregate index is 121.0 . This means that the aggregate group of prices had increased 21 percent in the ten-year period.

$$
P=\frac{\Sigma p_{t}}{\Sigma p_{0}}(100)=\frac{\$ 13.24}{\$ 10.94}(100)=121.0
$$

Because the value of a simple aggregate index can be influenced by the units of measurement, it is not used frequently. In our example the value of the index would differ significantly if we were to report the price of apples in tonnes rather than kilograms. Also, note the effect of coffee on the total index. For both the current year and the base year, the value of coffee is about 40 percent of the total index, so a change in the price of coffee will drive the index much more than any other item. So we need a way to appropriately "weight" the items according to their relative importance.

## Weighted Indexes

Two methods of computing a weighted price index are the Laspeyres method and the Paasche method. They differ only in the period used for weighting. The Laspeyres method uses base-period weights; that is, the original prices and quantities of the items bought are used to find the percent change over a period of time in either price or quantity consumed, depending on the problem. The Paasche method uses current-year weights for the denominator of the weighted index.

## Laspeyres' Price Index

Etienne Laspeyres developed a method in the latter part of the 18th century to determine a weighted index using base-period weights. Applying his method, a weighted price index is computed by:

$$
\begin{equation*}
\text { LASPEYRES' PRICE INDEX } \quad P=\frac{\sum p_{t} q_{0}}{\sum p_{0} q_{0}} \times 100 \tag{15-4}
\end{equation*}
$$

where:
$P$ is the price index.
$p_{t}$ is the current price.
$p_{0}$ is the price in the base period.
$q_{0}$ is the quantity used in the base period.

## Example

The prices for the six food items from Table 15-2 are repeated below in Table 15-3. Also included is the number of units of each consumed by a typical family in 1995 and 2005.

TABLE 15-3 Computation of Laspeyres and Paasche Indexes of Food Price, $1995=100$

|  | 1995 |  |  | 2005 |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Item | Price (\$) | Quantity |  | Price (\$) | Quantity |
| Bread white (loaf) | 0.77 | 50 |  | 1.98 | 55 |
| Eggs (dozen) | 1.85 | 26 |  | 2.98 | 20 |
| Milk (litre) white | 0.88 | 102 |  | 1.98 | 130 |
| Apples, red delicious (500 g) | 1.46 | 30 |  | 1.75 | 40 |
| Orange juice, (355 ml concentrate) | 1.58 | 40 |  | 1.70 | 41 |
| Coffee, 100\% ground roast (400 g) | 4.40 | 12 |  | 4.75 | 12 |

Determine a weighted price index using the Laspeyres method. Interpret the result.

First we determine the total amount spent for the six items in the base period, 1995. To find this value we multiply the base period price for bread (\$0.77) by the base period quantity of 50. The result is $\$ 38.50$. This indicates that a total of $\$ 38.50$ was spent in the base period on bread. We continue that for all items and total the results. The base period total is $\$ 336.16$. The current period total is computed in a similar fashion. For the first item, bread, we multiply the quantity in 1995 by the price of bread in 2005, that is, $\$ 1.98(50)$. The result is $\$ 99.00$. We make the same calculation for each item and total the result. The total is $\$ 555.94$. Because of the repetitive nature of these calculations, a spreadsheet is effective for carrying out the calculations. Following is a copy of the Excel output showing the calculations.

| Item | 1995 |  | 2005 |  | $P_{0} \mathrm{O}_{0}$ | $P_{t} Q_{0}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Price (\$) | Quantity | Price (\$) | Quantity |  |  |
| Bread, white (loaf) | 0.77 | 50 | 1.98 | 55 | 38.50 | 99.00 |
| Eggs (dozen) | 1.85 | 26 | 2.98 | 20 | 48.10 | 77.48 |
| Milk (litre) white | 0.88 | 102 | 1.98 | 130 | 89.76 | 201.96 |
| Apples, red delicious ( 500 g ) | 1.46 | 30 | 1.75 | 40 | 43.80 | 52.50 |
| Orange juice ( 355 ml , concentrate) | 1.58 | 40 | 1.70 | 41 | 63.20 | 68.00 |
| Coffee, 100\% ground roast (400 g) | 4.40 | 12 | 4.75 | 12 | 52.80 | 57.00 |
|  |  |  |  |  | 336.16 | 555.94 |
| Laspeyres: | 165.4 |  |  |  |  |  |

The weighted price index for 2005 is 165.4 , found by

$$
P=\frac{\sum p_{t} q_{0}}{\sum p_{0} q_{0}}(100)=\frac{\$ 555.94}{\$ 336.16}(100)=165.4
$$

Based on this analysis we conclude that the price of this group of items has increased 65.4 percent in the ten year period. The advantage of this method over the simple aggregate index is that the weight of each of the items is considered. In the simple aggregate index coffee had about 40 percent of the weight in determining the index. In the Laspeyres index the item with the most weight is milk, because the product of the price and the units sold is the largest.

## Paasche's Price Index

The major disadvantage of the Laspeyres index is it assumes that the base-period quantities are still realistic in the given period. That is, the quantities used for the six items are about the same in 1995 as 2005. In this case notice that the quantity of eggs purchased declined by 23 percent, the quantity of milk increased by nearly 28 percent, and the number of apples increased by 33 percent.

The Paasche index is an alternative. The procedure is similar, but instead of using base period weights, we use current period weights. We use the sum of the products of the 1995 prices and the 2005 quantities. This has the advantage of using the more recent quantities. If there has been a change in the quantities consumed since the base period, such a change is reflected in the Paasche index.

PAASCHE'S PRICE INDEX

$$
\begin{equation*}
P=\frac{\Sigma p_{t} q_{t}}{\sum p_{0} q_{t}} \times 100 \tag{15-5}
\end{equation*}
$$

## Example

Solution

Use the information from Table 15-3 to determine the Paasche index. Discuss which of the indexes should be used.

Again, because of the repetitive nature of the calculations, Excel is used to perform the calculations. The results are shown in the following output.

| Item | 1995 |  | 2005 |  | $P_{0} \mathrm{O}_{t}$ | $P_{t} Q_{t}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Price (\$) | Quantity | Price (\$) | Quantity |  |  |
| Bread, white (loaf) | 0.77 | 50 | 1.98 | 55 | 42.35 | 108.90 |
| Eggs (dozen) | 1.85 | 26 | 2.98 | 20 | 37.00 | 59.60 |
| Milk (litre) white | 0.88 | 102 | 1.98 | 130 | 114.40 | 257.40 |
| Apples, red delicious ( 500 g ) | 1.46 | 30 | 1.75 | 40 | 58.40 | 70.00 |
| Orange juice, ( 355 ml concentrate) | 1.58 | 40 | 1.70 | 41 | 64.78 | 69.70 |
| Coffee, 100\% ground roast (400 g) | 4.40 | 12 | 4.75 | 12 | 52.80 | 57.00 |
|  |  |  |  |  | 369.73 | 622.6 |
| Paasche | 168.4 |  |  |  |  |  |

The Paasche index is 168.4 , found by

$$
P=\frac{\sum p_{t} q_{0}}{\sum p_{0} q_{0}}(100)=\frac{\$ 622.6}{\$ 369.73}(100)=168.4
$$

This result indicates that there has been an increase of 68.4 percent in the price of this market basket of goods between 1995 and 2005. That is, it costs 68.4 percent more to purchase these items in 2005 than it did in 1995. All things considered, because of the change in the quantities purchased between 1995 and 2005, the Paasche index is more reflective of the current situation. It should be noted that the Laspeyres index is more widely used. The Consumer Price Index, the most widely reported index, is an example of a Laspeyres index.

How do we decide which index to use? When is Laspeyres' most appropriate and when is Paasche's the better choice?

## Laspeyres'

Advantages Requires quantity data from only the base period. This allows a more meaningful comparison over time. The changes in the index can be attributed to changes in the price.
Disadvantages Does not reflect changes in buying patterns over time. Also, it may overweight goods whose prices increase.

## Paasche's

Advantages Because it uses quantities from the current period, it reflects current buying habits.
Disadvantages It requires quantity data for each year, which may be difficult to obtain. Because different quantities are used each year, it is impossible to attribute changes in the index to changes in price alone. It tends to overweight the goods whose prices have declined. It requires the prices to be recomputed each year.

## Fisher's Ideal Index

As noted above, Laspeyres' index tends to overweight goods whose prices have increased. Paasche's index, on the other hand, tends to overweight goods whose prices have gone down. In an attempt to offset these shortcomings, Irving Fisher, in his book The Making of Index Numbers, published in 1922, proposed an index called Fisher's ideal index. It is the geometric mean of the Laspeyres and Paasche indexes. We described the geometric mean in Chapter 3. It is determined by taking the $k$ th root of the product of $k$ positive numbers.

$$
\text { Fisher's ideal index }=\sqrt{(\text { Laspeyres' index)(Paasche's index) }}
$$

Fisher's index seems to be theoretically ideal because it combines the best features of both Laspeyres and Paasche. That is, it balances the effects of the two indexes. However, it is rarely used in practice because it has the same basic set of problems as the Paasche index. It requires that a new set of quantities be determined for each year.

## Example

Solution
Determine Fisher's ideal index for the data in Table 15-3.
Fisher's ideal index is 166.9.

$$
\begin{aligned}
\text { Fisher's ideal index } & =\sqrt{(\text { Laspeyres' index)(Paasche's index) }} \\
& =\sqrt{(165.4)(168.4)}=166.9
\end{aligned}
$$

## Self-Review 15-2



An index of clothing prices for 2005 based on 1998 is to be constructed. The clothing items considered are shoes and dresses. The information for prices and quantities for both years is given below. Use 1998 as the base period and 100 as the base value.

|  | 1998 |  |  | 2005 |  |
| :--- | :---: | :---: | :--- | :---: | :---: | :---: |
| Item | Price (\$) | Quantity |  | Price (\$) | Quantity |
| Dress (each) | 75 | 500 |  | 85 | 520 |
| Shoes (pair) | 40 | 1200 |  | 45 | 1300 |

(a) Determine the simple average of the price indexes.
(b) Determine the aggregate price indexes for the two years.
(c) Determine Laspeyres' price index.
(d) Determine the Paasche price index.
(e) Determine Fisher's ideal index.

## Exercises

For exercises 5-8:
a. Determine the simple price indexes.
b. Determine the simple aggregate price indexes for index the two years.
c. Determine Laspeyres' price index.
5. Below are the prices of toothpaste ( 100 ml ), shampoo ( 500 ml ), cough tablets (package of 100 ), and antiperspirant ( 45 g ) for August 2001 and August 2005. Also included are the quantities purchased. Use August 2001 as the base.

|  | August 2001 |  |  | August 2005 |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Item | Price (\$) | Quantity |  | Price (\$) | Quantity |
| Toothpaste | 2.49 | 6 |  | 2.69 | 6 |
| Shampoo | 3.29 | 4 |  | 3.59 | 5 |
| Cough tablets | 1.79 | 2 |  | 2.79 | 3 |
| Antiperspirant | 2.29 | 3 |  | 3.79 | 4 |

6. Fruit prices and the amounts consumed for 1995 and 2005 are below. Use 1995 as the base.

|  | 1995 |  |  | $\mathbf{2 0 0 5}$ |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Item | Price (\$) | Quantity |  | Price (\$) | Quantity |
| Bananas (lb) | 0.23 | 100 |  | 0.49 | 120 |
| Grapefruit (each) | 0.29 | 50 |  | 0.27 | 55 |
| Apples | 0.35 | 85 |  | 0.35 | 85 |
| Strawberries (basket) | 1.02 | 8 |  | 1.99 | 10 |
| Oranges (bag) | 0.89 | 6 |  | 2.99 | 8 |

7. The prices and the numbers of various items produced by a small machine and stamping plant are reported below. Use 2000 as the base.

|  | $\mathbf{2 0 0 0}$ |  |  | $\mathbf{2 0 0 5}$ |  |
| :--- | :---: | ---: | :--- | :---: | ---: |
| Item | Price (\$) | Quantity |  | Price (\$) | Quantity |
| Washer | 0.07 | 17000 |  | 0.10 | 20000 |
| Cotter pin | 0.04 | 125000 |  | 0.10 | 130000 |
| Stove bolt | 0.15 | 40000 |  | 0.18 | 42000 |
| Hex nut | 0.08 | 62000 |  | 0.10 | 65000 |

8. Following are the quantities and prices for the years 1998 and 2005 for Sam's Student Centre.

|  | $\mathbf{1 9 9 8}$ |  |  | 2005 |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Item | Price (\$) | Quantity |  | Price (\$) | Quantity |
| Pens (dozen) | 0.90 | 50 |  | 1.10 | 55 |
| Pencils (dozen) | 0.65 | 50 |  | 0.80 | 60 |
| Erasers (each) | 0.45 | 250 |  | 0.55 | 275 |
| Paper, lined (pkg) | 0.89 | 500 |  | 1.09 | 750 |
| Paper, printer (pkg) | 5.99 | 300 |  | 4.99 | 450 |
| Printer (cartridges) | 15.99 | 150 |  | 19.99 | 200 |

## Value Index

Value index measures percent change in value

A value index measures changes in both the price and quantities involved. A value index, such as the index of department store sales, needs the original base-year prices, the original base-year quantities, the present-year prices, and the present-year quantities for its construction. Its formula is:

$$
\text { VALUE INDEX } \quad V=\frac{\Sigma p_{t} q_{t}}{\Sigma p_{0} q_{0}} \times 100
$$

## EXAMPLE

## Solution

The prices and quantities sold at the Waleska Department Store for various items of apparel for May 2000 and May 2005 are:

|  | $\mathbf{2 0 0 0}$ <br> Quantity |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | $\mathbf{2 0 0 0}$ <br> Price, | Sold <br> (thousands), | $\mathbf{2 0 0 5}$ <br> Price, <br> $\boldsymbol{p}_{\boldsymbol{t}} \mathbf{( \$ )}$ | Quantity <br> Sold <br> (thousands), <br> $\boldsymbol{q}_{\boldsymbol{t}}$ |
| Item | $\boldsymbol{p}_{\mathbf{0}} \mathbf{( \$ )}$ | $\boldsymbol{q}_{\mathbf{0}}$ | 12 | 900 |
| Ties (each) | 10 | 1000 | 400 | 120 |
| Suits (each) | 300 | 100 | 120 | 500 |
| Shoes (pair) | 100 | 500 | 120 |  |

What is the index of value for May 2005 using May 2000 as the base period?
Total sales in May 2005 were $\$ 118800000$, and the comparable figure for 2000 is $\$ 90000000$. (See Table 15-4.) Thus, the index of value for May 2005 using $2000=100$ is 132.0. The value of apparel sales in 2005 was 132.0 percent of the 2000 sales. To put it another way, the value of apparel sales increased 32.0 percent from May 2000 to May 2005.

$$
V=\frac{\Sigma p_{t} q_{t}}{\Sigma p_{0} q_{0}}(100)=\frac{118800}{90000}(100)=132.0
$$

TABLE 15-4 Construction of a Value Index for $2005(2000=100)$

| Item | 2000 <br> Price, <br> $p_{0}$ (\$) | 2000 Quantity Sold (thousands), $q_{0}$ | $\begin{gathered} p_{0} q_{0} \\ \text { (\$ thousands) } \end{gathered}$ | $\begin{aligned} & 2005 \\ & \text { Price, } \\ & p_{t}(\$) \end{aligned}$ | $\begin{gathered} 2005 \\ \text { Quantity } \\ \text { Sold } \\ \text { (thousands), } \\ q_{t} \\ \hline \end{gathered}$ | $\begin{gathered} p_{t} q_{t} \\ \text { (\$ thousands) } \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Ties (each) | 10 | 1000 | 10000 | 12 | 900 | 10800 |
| Suits (each) | 300 | 100 | 30000 | 400 | 120 | 48000 |
| Shoes (pair) | 100 | 500 | 50000 | 120 | 500 | 60000 |
|  |  |  | 90000 |  |  | 118800 |


| Self-Review 15-3 | The number of items produced by Houghton Products for 1999 and 2005 and the for the two periods are: |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Item Produced | Price (\$) |  | Number Produced |  |
| - |  | 1999 | 2005 | 1999 | 2005 |
|  | Shear pins (box) | 3 | 4 | 10000 | 9000 |
|  | Cutting compound (500 g) | 1 | 5 | 600 | 200 |
|  | Tie rods (each) | 10 | 8 | 3000 | 5000 |

(a) Find the index of the value of production for 2005 using 1999 as the base period.
(b) Interpret the index.

## Exercises

9. The prices and production of grains for August 1998 and August 2005 are:

|  |  | $\mathbf{1 9 9 8}$ <br> Quantity <br> Produced <br> (millions of <br> bushels) | 2005 <br> Praice (\$) | $\mathbf{1 9 9 8}$ <br> Price (\$) |
| :--- | :---: | :---: | :---: | :---: |
| Oats | 1.52 | 200 | 1.87 | $\mathbf{2 0 0 5}$ <br> Quantity <br> Produced <br> (millions of <br> bushels) |
| Wheat | 2.10 | 565 | 2.05 | 489 |
| Corn | 1.48 | 291 | 1.48 | 203 |
| Barley | 3.05 | 87 | 3.29 | 106 |

Using 1998 as the base period, find the value index of grains produced for August 2005.
10. The Johnson Wholesale Company manufactures a variety of products. The prices and quantities produced for April 1997 and April 2005 are:

|  | $\mathbf{1 9 9 7}$ <br> Price (\$) | 2005 <br> Price (\$) | $\mathbf{1 9 9 7}$ <br> Quantity <br> Produced | 2005 <br> Quantity <br> Produced |
| :--- | :---: | :---: | :---: | :---: |
| Product | 23.60 | 28.80 | 1760 | 4259 |
| Small motor (each) | 2.96 | 3.08 | 86450 | 62949 |
| Scrubbing compound (litre) | 0.40 | 0.48 | 9460 | 22370 |
| Nails (pound) |  |  |  |  |

Using April 1997 as the base period, find the index of the value of goods produced for April 2005.

## Special-Purpose Indexes

Many important indexes are prepared and published by private organizations. Financial institutions, utility companies, and university bureaus of research often prepare indexes on employment, factory hours and wages, and retail sales for the regions they serve. Many trade associations prepare indexes of price and quantity that are vital to their particular area of interest. As well, there are many special purpose indexes. Here are a few examples.

The Consumer Price Index (CPI) Statistics Canada reports this index monthly. It describes the changes in prices from one period to another for a "market basket" of goods and services. The base year for the CPI as of 2005 is $1992=100.0$. A historical summary of the CPI for Canada from 1973 to 2003 follows. A listing of the CPIs for the provinces and territories is on the CD-ROM. We present some applications later in the chapter.

S\&P/TSX Composite Index Introduced in 1977 as The TSE 300 Composite Index, the Toronto Stock Exchange's composite index represented the average performance of 300 of Canada's largest public companies traded on the Toronto Stock Exchange. Effective May 2002, the index was renamed S\&P/TSX, and is no longer restricted to 300 companies.

Dow Jones Industrial Average (DJIA) This is an index of stock prices, but perhaps it would be better to say it is an "indicator" rather than an index. It is supposed to be the mean price of 30 specific industrial stocks. However, summing the 30 stocks and dividing by 30 does not calculate its value. This is because of stock splits, mergers, and stocks being added or dropped. When changes occur, adjustments are made in the denominator used with the average. Today the DJIA is more of a psychological indicator than a representation of the general price movement on the New York Stock Exchange. The lack of representativeness of the stocks on the DJIA is one of the reasons for the development of the New York Stock Exchange Index.

There are many other indexes that track business and economic behavior, such as the Nasdaq Composite and the Russell 2000.

## Consumer Price Index

Frequent mention has been made of the Consumer Price Index (CPI) in the preceding pages. It measures the change in price of a fixed market basket of goods and services from one period to another.

In brief, the CPI serves several major functions. It allows consumers to determine the degree to which their purchasing power is being eroded by price increases. In that respect, it is a yardstick for revising wages, pensions, and other income payments to keep pace with changes in price. Equally important, it is an economic indicator of the rate of inflation and is used by business analysts and governments for evaluating and forecasting trends in interest rates, etc. The CPI is also used as a deflator to show the trend in "real" increases. As reported by Statistics Canada, a historical summary of the Consumer Price Index from 1973 to 2003 follows. The current base year is $1992=100$.

| Year | All items | Year | All items | Year | All items |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1973 | 28.1 | 1984 | 72.1 | 1995 | 104.2 |
| 1974 | 31.1 | 1985 | 75.0 | 1996 | 105.9 |
| 1975 | 34.5 | 1986 | 78.1 | 1997 | 107.6 |
| 1976 | 37.1 | 1987 | 81.5 | 1998 | 108.6 |
| 1977 | 40.0 | 1988 | 84.8 | 1999 | 110.5 |
| 1978 | 43.6 | 1989 | 89.0 | 2000 | 113.5 |
| 1979 | 47.6 | 1990 | 93.3 | 2001 | 116.4 |
| 1980 | 52.4 | 1991 | 98.5 | 2002 | 119.0 |
| 1981 | 58.9 | 1992 | 100.0 | 2003 | 122.3 |
| 1982 | 65.3 | 1993 | 101.8 |  |  |
| 1983 | 69.1 | 1994 | 102.0 |  |  |

Real income

Money income

Deflated income and real income are the same

## Special Uses of the Consumer Price Index

In addition to measuring changes in the prices of goods and services, the Consumer Price Index has a number of other applications. The CPI is used to determine real disposable personal income, to deflate sales or other series, to find the purchasing power of the dollar, and to establish cost-of-living increases. We first discuss the use of the CPI in determining real income.

Real Income As an example of the meaning and computation of real income, assume the Consumer Price Index is presently 122.3 with $1992=100$. Also, assume that Ms. Watts earned $\$ 25000$ in the base period of 1992. She has a current income of $\$ 30575$. Note that although her money income has increased by $22.3 \%$ since the base period of 1992, the prices she paid for food, gasoline, clothing, and other items have also increased by $22.3 \%$. Thus, Ms. Watts's standard of living has remained the same from the base period to the present time. Price increases have exactly offset an increase in income, so her present buying power (real income) is still \$25,000. (See Table 15-6 for computations.) In general:

REAL INCOME

$$
\begin{equation*}
\text { Real income }=\frac{\text { Money income }}{\mathrm{CPI}} \times 100 \tag{15-8}
\end{equation*}
$$

TABLE 15-6 Computation of Real Income for 1992 and Present Year

|  | Money Income | Consumer <br> Price Index <br> $(\mathbf{1 9 9 2}=\mathbf{1 0 0 )}$ | Real Income | Computation <br> of Real Income |
| :--- | :---: | :---: | :---: | :---: |
| 1992 | $\$ 25000$ | 100 | $\$ 25000$ | $\frac{\$ 25000}{100}(100)$ |
| Present year | 30575 | 122.3 | 25000 | $\frac{\$ 30575}{122.3}(100)$ |

The concept of real income is sometimes called deflated income, and the CPI is called the deflator. Also, a popular term for deflated income is income expressed in constant dollars. Thus, in Table 15-6, to determine whether Ms. Watts's standard of living changed, her money income was converted to constant dollars. We found that her purchasing power, expressed in 1992 dollars (constant dollars), remained at \$25000.

## Self-Review 15-4



The take-home pay of Jon Greene and the CPI for 1998 and 2003 are:

| Year | Take-Home <br> Pay (\$) | CPI <br> $(\mathbf{1 9 9 2}=\mathbf{1 0 0})$ |
| :--- | :---: | :---: |
| 1998 | 25000 | 107.6 |
| 2003 | 41200 | 119.0 |

(a) What was Jon's real income in 1998?
(b) What was his real income in 2003?
(c) Interpret your findings.

Deflating Sales A price index can also be used to "deflate" sales or similar money series. Deflated sales are determined by

$$
\text { Deflated sales }=\frac{\text { Actual sales }}{\text { An appropriate index }} \times 100
$$

Sam's Enterprises has retail stores in Victoria and Collingwood. Sales in 1992 were \$445 873 and \$775 995 respectively. Last year, sales were \$773998 and \$973545 respectively. Sam wants to know how much sales have increased over the last eleven years, so he decides to deflate the sales for last year to the 1992 levels. Given that the CPI increase for all items is 122.3, express Sam's sales last year in constant 1992 dollars.

## Solution

The results are shown in the following Excel output.

| Sam's Enterprises |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :--- | :--- | :--- |
| CPI 2003 $=$ | 122.3 |  |  |  |  |  |
| Sales |  |  |  |  |  |  |
|  | 1992 | 2003 | Constant Dollar (1992) | Found by |  |  |
| Collingwood | 775995 | 973545 | 796030 | $=973545 / 122.3^{* 100}$ |  |  |
| Victoria | 445873 | 773998 | 632868 | $=773998 / 122.3^{* 1} 100$ |  |  |
|  |  |  |  |  |  |  |

Comparing the sales for 1992 to the constant dollars, we see that sales grew in both locations from 1992 to 2003.

Purchasing Power of the Dollar The Consumer Price Index is also used to determine the purchasing power of the dollar.

$$
\begin{align*}
& \text { USING AN INDEX TO FIND } \quad \text { Purchasing power of dollar }=\frac{\$ 1}{\mathrm{CPI}} \times 100 \\
& \text { PURCHASING POWER }
\end{align*}
$$

## Example

## Solution

Suppose the Consumer Price Index this month is $125.0(1992=100)$. What is the purchasing power of the dollar?

Using formula 15-10, it is 80 cents, found by:

$$
\text { Purchasing power of dollar }=\frac{\$ 1}{125.0}(100)=\$ 0.80
$$

The CPI of 125.0 indicates that prices have increased by $25 \%$ from the years 1992 to this month. Thus, the purchasing power of a dollar has been cut. That is, a 1992 dollar is worth only 80 cents this month. To put it another way, if you lost $\$ 1000$ in 1992 and just found it, the $\$ 1000$ could only buy $\$ 800$ worth of goods that could have been bought in 1992.

Cost-of-LivingAdjustments The Consumer Price Index is also the basis for cost-of-living adjustments in many management-union contracts. The specific clause in the contract is often referred to as the "escalator clause" or COLA. Many workers have their incomes or pensions pegged to the Consumer Price Index.

The CPI is also used to adjust alimony and child support payments; attorneys' fees; workers' compensation payments; rentals on apartments, homes, and office buildings; welfare payments; and so on. In brief, say a retiree receives a pension of \$500 a month and the Consumer Price Index increases 5 points from 165 to 170. Suppose for each point that the CPI increases the pension benefits increase 1.0 percent, so the monthly increase in benefits will be $\$ 25$, found by $\$ 500$ ( 5 points)(.01). Now the retiree will receive $\$ 525$ per month.

Self-Review 15-5 Suppose the Consumer Price Index for the latest month is $134.0(1992=100)$. What is the purchasing power of the dollar? Interpret.

## Shifting the Base

If two or more time series have the same base period, they can be compared directly. As an example, suppose we are interested in the trend in the prices of food, shelter, clothing and footwear, and health and personal care over the last four years compared to the base year $(1992=100)$. Note in Table 15-8 that all of the consumer price indexes use the same base. Thus, it can be said that the price of all consumer items combined increased $19 \%$ from the base period (1992). Likewise, shelter increased $13.8 \%$, clothing and footwear $5.2 \%$, and so on.

TABLE 15-8 Trend in Consumer Prices to March 2005 (1992 = 100)

| Year | All <br> Items | Food | Shelter | Clothing <br> and Footwear | Health and <br> Personal Care |
| :--- | :---: | :---: | :---: | :---: | :---: |
| 1992 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 |
| 1998 | 108.6 | 109.3 | 103.7 | 103.9 | 108.1 |
| 2002 | 119.0 | 120.3 | 113.8 | 105.2 | 115.5 |
| March 2005 | 126.5 | 127.1 | 123.0 | 106.0 | 120.0 |

A problem arises, however, when two or more series being compared do not have the same base period. The following example compares the stock price changes of Nortel Network Corporation Common Stock, which are listed on both the New York Stock Exchange in \$US and on the Toronto Stock Exchange in \$Cdn.

We want to compare the price changes of Nortel Networks Corporation common stock prices which are listed on the New York Stock Exchange in \$US and on the Toronto Stock Exchange in \$Cdn. The information follows.

|  | \$US |  |  | \$Cdn |  |
| :--- | ---: | ---: | :--- | :--- | :--- |
| 2004 | High | Low |  | High | Low |
| First Quarter | 11.94 | 5.53 |  | 8.50 | 3.98 |
| Second Quarter | 8.35 | 4.16 |  | 6.33 | 4.30 |
| Third Quarter | 6.40 | 4.11 |  | 5.05 | 3.01 |
| Fourth Quarter | 4.80 | 3.49 |  | 3.91 | 3.16 |

Source: Nortel Networks Corporation, 2004 Annual Report.
From the information given, we are not sure the base periods are the same, so a direct comparison is not appropriate. Because we want to compare the changes in the stock prices in the two stock markets, the logical thing to do is to let a particular period, say the first quarter, be the base for both periods. For the Toronto Stock Exchange, $\$ 8.50$ becomes the base for the high price, and $\$ 3.98$ for the low price. For the New York stock market, $\$ 11.94$ becomes the base for the high price, and $\$ 5.53$ for the low price.

The calculations for the \$US High, fourth quarter using \$11.94 = 100 are:

$$
\text { Index }=\frac{4.80}{11.94}(100)=40.2
$$

The following Excel output reports the complete set of indexes.

| 2004 | \$US |  | \$US | \$Cdn |  | \$Cdn |  |
| :--- | ---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | High | Index | Low | Index | High | Index | Low | Index

We conclude that all four indexes have decreased over the year, but the US indexes have decreased more than the Canadian, and so, we can conclude that the US stock prices have decreased more than the Canadian stock prices.

## Self-Review 15-6



The following are the fourth quarter high stock prices for Nortel Networks Communications on the New York Stock Exchange and the Toronto Stock Exchange. Develop indexes for both markets using 2000 as the base period. Interpret your results.

|  | $\mathbf{2 0 0 4}$ | $\mathbf{2 0 0 3}$ | $\mathbf{2 0 0 2}$ | $\mathbf{2 0 0 1}$ | $\mathbf{2 0 0 0}$ |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Toronto Stock Exchange (\$Cdn) | 4.80 | 6.37 | 3.61 | 14.24 | 105.70 |
| New York Stock Exchange (\$US) | 3.91 | 4.80 | 2.75 | 9.05 | 70.00 |

## Exercises

11. In 1992, Marilyn started working for $\$ 400$ per week. How much would she have to earn in March 2005 to have the same purchasing power if the CPI is 126.5 in March 2005? Use 1992 as the base year.
12. The price of a pair of boots in 1992 was $\$ 125$, and $\$ 150$ in 2004. During the same period, the CPI for clothing and footwear increased by $3.1 \%$. Did the price of the boots increase more than, the same, or less than the CPI?
13. At the end of 2004, the average salary for a senior customer service representative at Mercury Distribution Inc was $\$ 45000$. The Consumer Price Index for December, 2004, was 124.6 $(1992=100.0)$. The mean salary for the same position in the base period of 1992 was $\$ 34000$. What was the real income of the customer service representative in 2004? How much had the average salary increased?
14. The Trade Union Association maintains indexes on the hourly wages for a number of the trades. Unfortunately, the indexes do not all have the same base periods. Listed below is information on plumbers and electricians. Shift the base periods to 1995 and compare the hourly wage increases.

| Year | Plumbers $(\mathbf{1 9 9 0}=\mathbf{1 0 0})$ | Electricians $(\mathbf{1 9 9 2}=\mathbf{1 0 0})$ |
| :--- | :---: | :---: |
| 1995 | 133.8 | 126.0 |
| 2000 | 159.4 | 158.7 |

15. In 1990, the mean salary of plant workers at Mercury Distribution Inc. was $\$ 23650$. The salary included bonuses and overtime. By 1995, the mean salary increased to $\$ 28972$, and further increased to $\$ 32382$ in 2000 and $\$ 34269$ in 2005. The company maintains information on employment trends throughout their industry. Their industry index, which has a base of 1990, was 122.5 for 1995, 136.9 for 2000 and 144.9 for 2005. Compare Mercury Distribution Inc.'s plant workers salaries to the industry trends.
16. Sam Steward is a freelance computer programmer. Listed below are his yearly wages for the years 2000 through 2005. Also included is an industry index for computer programmers that reports the rate of wage inflation in the industry. This index has a base of 1990.

| Year | Wage (\$ thousands) | Index $(\mathbf{1 9 9 0}=\mathbf{1 0 0})$ |
| :---: | :---: | :---: |
| 2000 | 175 | 148.3 |
| 2001 | 175 | 140.6 |
| 2002 | 150 | 120.9 |
| 2003 | 120 | 110.2 |
| 2004 | 120 | 105.3 |
| 2005 | 130 | 105.0 |

Compute Sam's real income for the period. Did his wages match the increase/decline in the industry?

## Chapter Outline

I. An index number measures the relative change from one period to another.
A. The major characteristics of an index are:

1. It is a percentage, but the percent sign is usually omitted.
2. It has a base period.
3. Most indexes are reported to the nearest tenth of a percent, such as 153.1.
4. The base of most indexes is 100 .
B. The reasons for computing an index are:
5. It facilitates the comparison of unlike series.
6. If the numbers are very large, often it is easier to comprehend the change of the index than the actual numbers.
II. There are two types of price indexes, unweighted and weighted.
A. In an unweighted index we do not consider the quantities.
7. In a simple index we compare the base period to the given period.

$$
\begin{equation*}
I=\frac{p_{t}}{p_{0}} \times 100 \tag{15-1}
\end{equation*}
$$

where $p_{t}$ refers to the price in the current period, and $p_{0}$ is the price in the base period.
2. In the simple average of price indexes, we add the simple indexes for each item and divide by the number of items.

$$
\begin{equation*}
P=\frac{\Sigma p_{i}}{n} \tag{15-2}
\end{equation*}
$$

3. In a simple aggregate price index the price of the items in the group are totaled for both periods and compared.

$$
\begin{equation*}
P=\frac{\Sigma p_{t}}{\Sigma p_{0}} \times 100 \tag{15-3}
\end{equation*}
$$

B. In a weighted index the quantities are considered.

1. In the Laspeyres method the base period quantities are used in both the base period and the given period.

$$
\begin{equation*}
P=\frac{\Sigma p_{t} q_{0}}{\Sigma p_{0} q_{0}} \times 100 \tag{15-4}
\end{equation*}
$$

2. In the Paasche method current period quantities are used.

$$
\begin{equation*}
P=\frac{\Sigma p_{t} q_{t}}{\Sigma p_{0} q_{t}} \times 100 \tag{15-5}
\end{equation*}
$$

3. Fisher's ideal index is the geometric mean of Laspeyres' index and Paasche's index.

$$
\text { Fisher's ideal index }=\sqrt{(\text { Laspeyres' index)(Paasche's index) }}
$$



## Statistics in Action

In the 1920s wholesale prices in Germany increased dramatically. In 1920 wholesale prices increased about 80 percent, in 1921 the rate of increase was 140 percent, and in 1922 it was a whopping 4100 percent! Between December 1922 and November 1923 wholesale prices increased another 4100 percent. By that time government printing presses could not keep up, even by printing notes as large as 500 million marks. Stories are told that workers were paid daily, then twice daily, so their wives could shop for necessities before the wages became too devalued.
C. A value index uses both base period and current period prices and quantities.

$$
\begin{equation*}
V=\frac{\Sigma p_{t} q_{t}}{\Sigma p_{0} q_{0}} \tag{15-7}
\end{equation*}
$$

III. The most widely reported index is the Consumer Price Index (CPI).
A. It is often used to show the rate of inflation.
B. It is reported monthly by Statistics Canada.
C. The base year for 2003 is $1992=100.0$, changed from $1986=100.0$ in January 1998.

## Chapter Exercises

The following information was taken from Statistics Canada (2001 Census). The complete file is on the CD-ROM, Data Sets, Earnings (average) by level of schooling.

|  |  | Less Than High <br> School Graduation <br> Certificate (\$) | College Certificate <br> or Diploma (\$) | University Certificate, <br> Diploma or Degree (\$) |
| :--- | :---: | :---: | :---: | :---: |
| Canada | $\mathbf{3 1 7 5 7}$ | $\mathbf{2 1 2 3 0}$ | $\mathbf{3 2 7 3 6}$ | $\mathbf{4 8} \mathbf{6 4 8}$ |
| Newfoundland \& Labrador | $\mathbf{2 4 1 6 5}$ | 15922 | 28196 | 41942 |
| Prince Edward Island | $\mathbf{2 2 3 0 3}$ | 15058 | 25613 | 37063 |
| Nova Scotia | $\mathbf{2 6 6 3 2}$ | 18251 | 26930 | 41146 |
| New Brunswick | $\mathbf{2 4 9 7 1}$ | 17074 | 27178 | 40375 |
| Quebec | $\mathbf{2 9 3 8 5}$ | 20553 | 28742 | 45834 |
| Ontario | $\mathbf{3 5 1 8 5}$ | 22691 | 36309 | 53525 |
| Manitoba | $\mathbf{2 7 1 7 8}$ | 19201 | 29351 | 41856 |
| Saskatchewan | $\mathbf{2 5 6 9 1}$ | 18288 | 27742 | 40279 |
| Alberta | $\mathbf{3 2 6 0 3}$ | 22196 | 33572 | 50069 |
| British Columbia | $\mathbf{3 1 5 4 4}$ | 21971 | 33159 | 44066 |
| Yukon | $\mathbf{3 1 5 2 6}$ | 19265 | 33817 | 45982 |

17. Refer to the table above. Use Canada as the base period and compute a simple index of less than high school graduation certificate for each province and territory. Interpret your findings.
18. Refer to the table above. Use Canada, all levels $(\$ 31757)$ as the base period and compute a simple index of less than high school graduation certificate for each province and territory. Interpret your findings.
19. Refer to the table above. Use Canada as the base period and compute a simple index of college certificate or diploma for each province and territory. Interpret your findings.
20. Refer to the table above. Use Canada, all levels $(\$ 31757)$ as the base period and compute a simple index of college certificate or diploma for each province and territory. Interpret your findings.
21. Refer to the table above. Use Canada as the base period and compute a simple index of university certificate, diploma or degree for each province and territory. Interpret your findings.
22. Refer to the table above. Use Canada, all levels $(\$ 31757)$ as the base period and compute a simple index of university certificate, diploma or degree for each province and territory. Interpret your findings.

The following information is from the Nortel Networks Corporation (millions of US dollars).

| Year | $\mathbf{2 0 0 4}$ | $\mathbf{2 0 0 3}$ | $\mathbf{2 0 0 2}$ | $\mathbf{2 0 0 1}$ | $\mathbf{2 0 0 0}$ |
| :--- | ---: | ---: | ---: | ---: | :---: |
| Revenues | 9828 | 10193 | 11008 | 17511 | 27948 |
| Total Assets | 16984 | 16591 | 16961 | 21137 | 42180 |
| Total Shareholders' Equity | 3987 | 3945 | 3053 | 4824 | 29109 |

23. Compute a simple index for the revenue of Nortel Networks Corporation. Use 2000 as the base year. What can you conclude about the change in revenue over the period?
24. Compute a simple index for the revenue of Nortel Networks Corporation using 2002 as the base year. What can you conclude about the change in revenue over the period?
25. Compute a simple index for the total assets of Nortel Networks Corporation. Use 2000 as the base year. What can you conclude about the change in total assets revenue over the period?
26. Compute a simple index for the total shareholders' equity of Nortel Networks Corporation. Use 2000 as the base year. What can you conclude about the change in total shareholders' equity over the period?

The following table lists the common share prices of Nortel Networks Corporation as traded on the Toronto Stock Exchange.

| Year | 2004 |  | 2003 |  | 2002 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | High | Low | High | Low | High | Low |
| First Quarter | 11.94 | 5.53 | 4.13 | 2.59 | 13.99 | 6.75 |
| Second Quarter | 8.35 | 4.16 | 4.81 | 3.04 | 7.54 | 2.01 |
| Third Quarter | 6.40 | 4.11 | 6.50 | 3.84 | 2.60 | 0.70 |
| Fourth Quarter | 4.80 | 3.49 | 6.37 | 5.17 | 3.61 | 0.67 |

27. Compute a simple price index for each quarter with 2002 as the base year. What can you conclude about the change in stock price over the period?
28. Compute a simple price index for each quarter with 2003 as the base year. What can you conclude about the change in stock price over the period?

The following information was reported on food items for the years 1995 and 2005.

|  | 1995 |  |  | 2005 |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Item | Price (\$) | Quantity |  | Price (\$) | Quantity |
| Margarine $(454 \mathrm{~g})$ | 0.81 | 18 |  | 2.39 | 27 |
| Shortening $(454 \mathrm{~g})$ | .84 | 5 |  | 1.49 | 9 |
| Milk $(2 \mathrm{~L}$ ) | 1.44 | 70 |  | 3.79 | 65 |
| Potato chips $(454 \mathrm{~g})$ | 2.91 | 27 |  | 3.99 | 33 |

29. Compute a simple price index for each of the four items. Use 1995 as the base period.
30. Compute a simple aggregate price index. Use 1995 as the base period.
31. Compute Laspeyres' price index for 2005 using 1995 as the base period.
32. Compute Paasche's index for 2005 using 1995 as the base period.
33. Determine Fisher's ideal index using the values for the Laspeyres and Paasche indexes computed in the two previous problems.
34. Determine a value index for 2005 using 1995 as the base period.

Betts Electronics purchases three replacement parts for robotic machines used in their manufacturing process. Information on the price of the replacement parts and the quantity purchased is given below.

|  | Price (\$) |  |  | Quantity |  |
| :--- | :---: | :---: | :--- | :---: | :---: |
| Part | $\mathbf{1 9 9 9}$ | $\mathbf{2 0 0 5}$ |  | $\mathbf{1 9 9 9}$ | $\mathbf{2 0 0 5}$ |
| RC-33 | 0.50 | 0.60 |  | 320 | 340 |
| SM-14 | 1.20 | 0.90 |  | 110 | 130 |
| WC50 | 0.85 | 1.00 |  | 230 | 250 |

35. Compute a simple price index for each of the three items. Use 1999 as the base period.
36. Compute a simple aggregate price index for 2005. Use 1999 as the base period.
37. Compute Laspeyres' price index for 2005 using 1999 as the base period.
38. Compute Paasche's index for 2005 using 1999 as the base period.
39. Determine Fisher's ideal index using the values for the Laspeyres and Paasche indexes computed in the two previous problems.
40. Determine a value index for 2005 using 1999 as the base period.

Prices for selected foods for 1999 and 2005 are given in the following table.

|  | Price (\$) |  |  | Quantity |  |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Part | $\mathbf{1 9 9 9}$ | $\mathbf{2 0 0 5}$ |  | $\mathbf{1 9 9 9}$ | $\mathbf{2 0 0 5}$ |
| Cabbage (500 g) | 0.60 | 0.90 |  | 2000 | 1500 |
| Carrots (bunch) | 0.49 | 0.69 |  | 200 | 200 |
| Peas (kg) | 1.99 | 2.99 |  | 400 | 500 |
| Endive (bunch) | 0.89 | 1.29 |  | 100 | 200 |

41. Compute a simple price index for each of the four items. Use 1999 as the base period.
42. Compute a simple aggregate price index. Use 1999 as the base period.
43. Compute Laspeyres' price index for 2005 using 1999 as the base period.
44. Compute Paasche's index for 2005 using 1999 as the base period.
45. Determine Fisher's ideal index using the values for the Laspeyres and Paasche indexes computed in the two previous problems.
46. Determine a value index for 2005 using 1999 as the base period.

The prices of selected items for 2000 and 2005 follow. Quantity purchased is also listed.

|  | 2000 |  |  | 2005 |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Item | Price (\$) | Quantity |  | Price (\$) | Quantity |
| Paper, computer (pkg) | 4.99 | 400 |  | 5.99 | 500 |
| Paper, lined (pkg) | 0.89 | 1000 |  | 0.99 | 1200 |
| Paper, plain (pkg) | 0.99 | 850 |  | 1.19 | 1000 |
| Paper, coloured (pkg) | 1.49 | 350 |  | 1.79 | 350 |

47. Compute a simple price index for each of the four items. Use 2000 as the base period.
48. Compute a simple aggregate price index. Use 2000 as the base period.
49. Compute Laspeyres' price index for 2005 using 2000 as the base period.
50. Compute Paasche's index for 2005 using 2000 as the base period.
51. Determine Fisher's ideal index using the values for the Laspeyres and Paasche indexes computed in the two previous problems.
52. Determine a value index for 2005 using 2000 as the base period.
53. A special-purpose index is to be designed to monitor the overall economy of the region. Four key series were selected. After considerable deliberation it was decided to weight retail sales 20 percent, total bank deposits 10 percent, industrial production in the region 40 percent, and nonagricultural employment 30 percent. The data for 2000 and 2005 are:

| Year | Retail Sales <br> (\$ millions) | Bank <br> Deposits <br> (\$ billions) | Industrial <br> Production <br> $\mathbf{( 1 9 9 4}=\mathbf{1 0 0 )}$ | Employment |
| :---: | :---: | :---: | :---: | :---: |
| 2000 | 1159.0 | 87 | 110.6 | 1214000 |
| 2005 | 1971.0 | 91 | 114.7 | 1501000 |

Construct a special-purpose index for 2005 using 2000 as the base period and interpret.
54. M Studios is studying its revenue to determine where its greatest growth has been. The business started ten years ago and a summary of sales is below.
a. Make whatever calculations are necessary to compare the trend in revenue from 1992 to 2002. b. Interpret.

|  | Consumer <br> Price <br> Index <br> $(\mathbf{1 9 9 2}=\mathbf{1 0 0 )}$ | Photographic <br> Supplies <br> (in thousands) | Index of <br> Photographic <br> Services <br> (in thousands) |
| :---: | :---: | :---: | :---: |
| 1994 | 102.0 | 175 | 65 |
| 1996 | 105.9 | 205 | 70 |
| 1998 | 108.6 | 300 | 72 |
| 2000 | 113.5 | 310 | 86 |
| 2002 | 119.0 | 315 | 92 |

55. The management of Ingalls Super Discount stores wants to construct an index of economic activity for its metropolitan area. Management contends that if the index reveals that the economy is slowing down, inventory should be kept at a low level.

Three series seem to hold promise as predictors of economic activity-area retail sales, bank deposits, and employment. All of these data can be secured monthly from the government. Retail sales is to be weighted 40 percent, bank deposits 35 percent, and employment 25 percent. Seasonally adjusted data for the first three months of the year are:

| Month | Retail Sales <br> (\$ millions) | Bank Deposits <br> (\$ billions) | Employment <br> (thousands) |
| :--- | :---: | :---: | :---: |
| January | 8.0 | 20 | 300 |
| February | 6.8 | 23 | 303 |
| March | 6.4 | 21 | 297 |

Construct an index of economic activity for each of the three months, using January as the base period.
56. The following table gives information on the Consumer Price Index and the monthly takehome pay of Bill Martin, an employee at the Jeep Corporation.

| Year | Consumer Price Index <br> $(\mathbf{1 9 9 2}=\mathbf{1 0 0})$ | Mr. Martin's Monthly <br> Take-Home Pay |
| :--- | :---: | :---: |
| 1992 | 100.0 | $\$ 1200$ |
| 2003 (Dec.) | 122.3 | 3200 |

a. What is the purchasing power of the dollar for December 2003 based on the period 1992 ?
b. Determine Mr. Martin's "real" monthly income for December 2003.

## Chapter 15 Answers to Self-Reviews

15-1 1.

| Company | Revenue | Index |
| :--- | ---: | ---: |
| Globelive Communinations Inc | 35058 | 341.8 |
| Swift Trade Inc | 35130 | 342.5 |
| Gram Precision Inc | 9543 | 93.0 |
| Trafford Publishing | 6658 | 64.9 |
| Diversinet Corp | 10258 | 100.0 |

Revenue for Gram Precision Inc is $93 \%$ of that of Diversinet Corp, while revenue of Trafford Publishing is $64.9 \%$. Revenue for Globelive is $241.8 \%$ more than that of Diversinet, while revenue for Swift Trade is $242.5 \%$ more.
2. (a) $P=(\$ 13.24 / \$ 10.32)(100)=128.3$
$P=(\$ 13.74 / \$ 10.32)(100)=133.1$
(b) $X=(\$ 10.32 \times \$ 10.57 \times \$ 10.83) / 3=\$ 10.573$
$P=(\$ 13.24 / \$ 10.573)(100)=125.2$
$P=(\$ 13.74 / \$ 10.573)(100)=130.0$
(c) $P=(\$ 13.74 / \$ 11.43)(100)=120.2$

15-2 (a) $P_{1}=(\$ 85 / \$ 75)(100)=113.3$
$P_{2}=(\$ 45 / \$ 40)(100)=112.5$
$P=(113.3+112.5) / 2=112.9$
(b) $P=(\$ 130 / \$ 115)(100)=113.0$
(c) $P=\frac{\$ 85(500)+\$ 45(1200)}{\$ 75(500)+\$ 40(1200)}(100)$

$$
=\frac{\$ 96500}{85500}(100)=112.9
$$

(d) $P=\frac{\$ 85(520)+\$ 45(1300)}{\$ 75(520)+\$ 40(1300)}(100)$
$=\frac{\$ 102700}{\$ 91000}(100)=112.9$
(e) $P=\sqrt{(112.9)(112.9)}=112.9$

15-3 (a) $P=\frac{\$ 4(9000)+\$ 5(200)+\$ 8(5000)}{\$ 3(10000)+\$ 1(600)+\$ 10(3000)}(100)$

$$
=\frac{\$ 77000}{60600}(100)=127.1
$$

(b) The value of sales has gone up 27.1 percent from 1999 to 2005.
15-4 (a) \$23 234.20, found by (\$25000/107.6)(100).
(b) $\$ 34621.85$, found by $(\$ 41200 / 119.0)(100)$.
(c) In terms of the base period, Jon's salary was $\$ 23234$ in 1998 and \$34 622 in 2003. This indicates his take-home pay increased at a faster rate than the price paid for food, transportation, etc.
15-5 \$0.75, found by (\$1.00/134.0)(100). A 1992 dollar is worth only 75 cents this month.
15-6

|  | (\$Cdn) | (\$US) |
| ---: | ---: | ---: |
| $\mathbf{2 0 0 4}$ | 4.5 | 5.6 |
| $\mathbf{2 0 0 3}$ | 6.0 | 6.9 |
| $\mathbf{2 0 0 2}$ | 3.4 | 3.9 |
| $\mathbf{2 0 0 1}$ | 13.5 | 12.9 |
| $\mathbf{2 0 0 0}$ | 100.0 | 100.0 |

The indexes in both markets have decreased dramatically.

