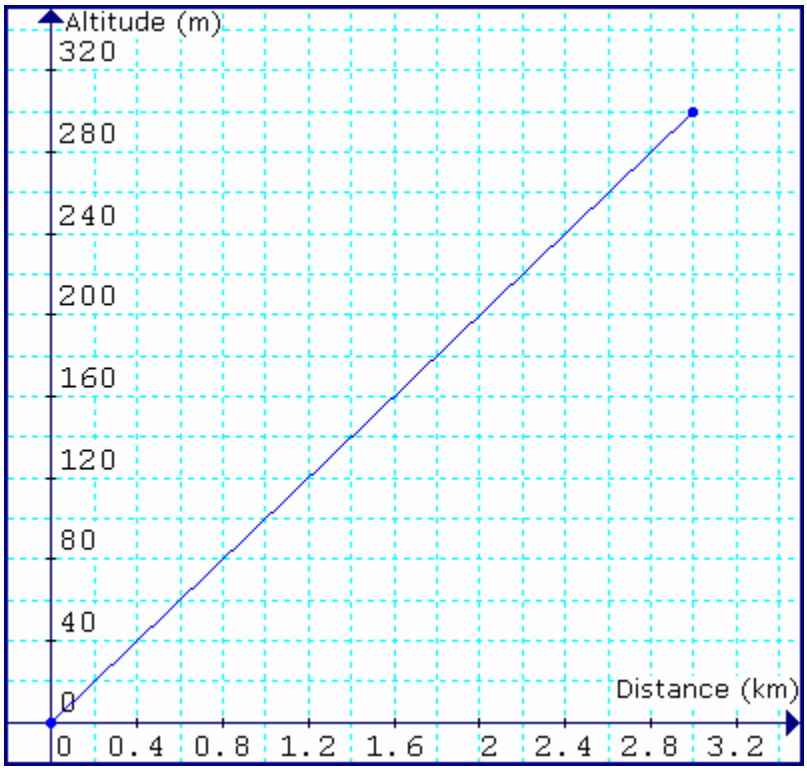
CHAPTER 5 Modelling With Graphs
5.4. Slope as a Rate of Change
Solving Problems Involving Slope as a Rate of Change

Example:

a) The pilot's handbook for a small aircraft includes the graph shown. It indicates the vertical distance that the aircraft climbs in relation to its horizontal movement over the ground. This information is important when there are obstructions on the flight path after takeoff, such as trees, buildings, or rising terrain. Calculate the slope of the graph.



b) Interpret the slope as a rate of change.

c) Trees rising to a height of 30 m lie 0.4 km from the end of the runway. Will the aircraft clear the trees? Explain.

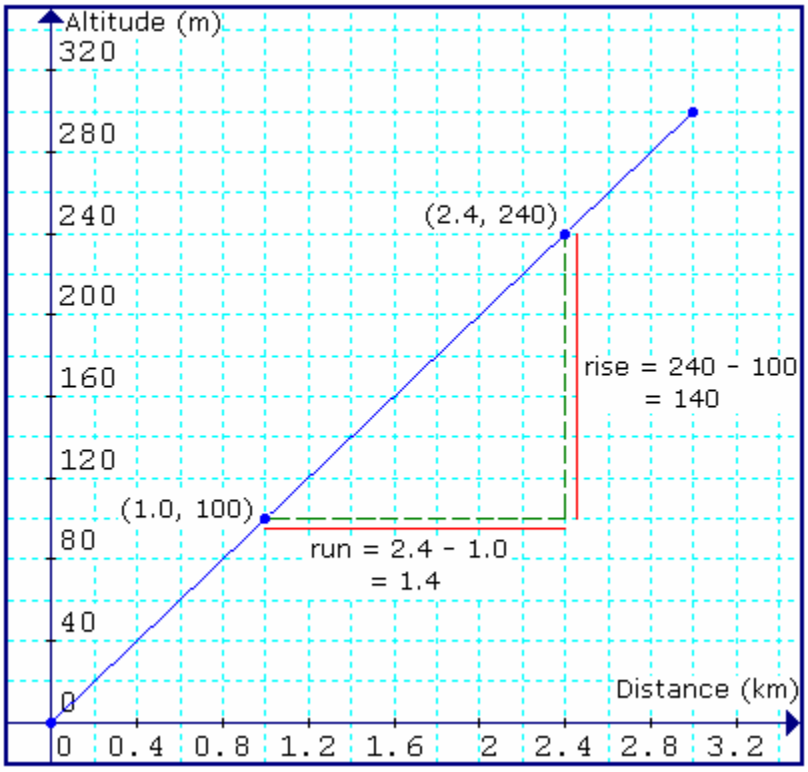
d) A radio broadcasting antenna rising to a height of 120 m is 1.0 km from the end of the runway. Will the aircraft clear the antenna? Explain.

Solution:

a) $m = \dfrac{\text{rise}}{\text{run}}$

$= \dfrac{140}{1.4}$

$= 100$



b) The slope indicates that the rate of change of altitude is 100 m for every kilometre of horizontal travel.

c) Use the graph to find that, at a distance of 0.4 km, the aircraft will have climbed 40 m. This allows it to clear the 30-m high trees.

d) Use the graph to find that, at a distance of 1.0 km, the aircraft will have climbed 100 m. This does not allow it to clear the 120-m high radio antenna.

Practice:

1. a) A scuba tank holds 800 L of compressed air. At a depth of 10 m, the air will last a diver 50 min. Calculate the rate of change of the volume of air left in the tank.

b) Sketch a graph of the volume of air left in the tank as it relates to time.

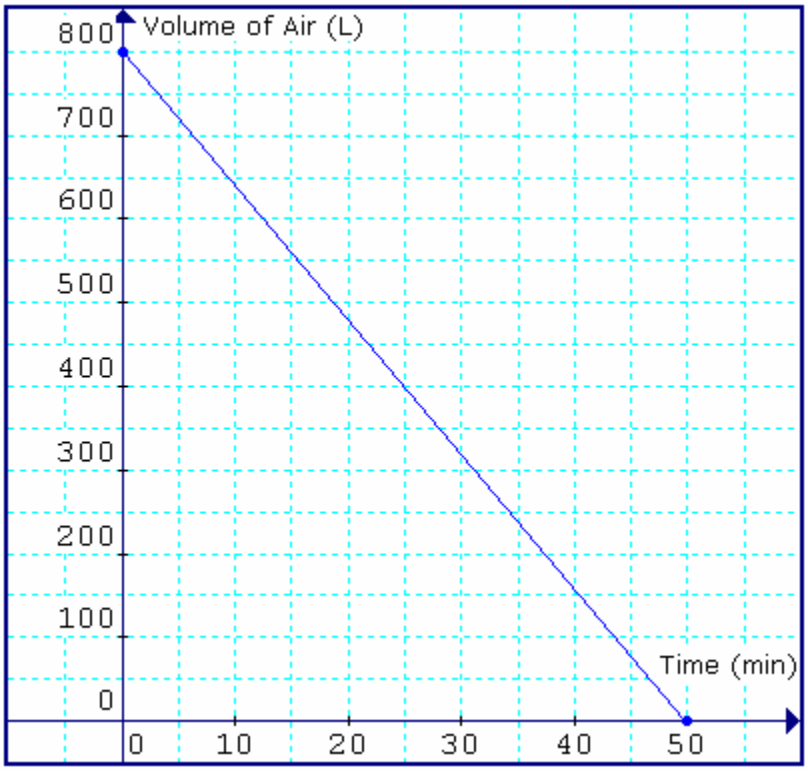
c) How does the rate of change relate to the graph?

d) At a depth of 20 m, the diver breathes about 24 L of air per minute. How long will the air in the tank last?

Answers:

1. a) The rate of change is –16 L/min.

b) The graph is shown.



c) The rate of change is equal to the slope of the graph. The slope is negative, since the volume of air left in the tank is decreasing.

d) To calculate how long the air will last, divide the volume of air by the rate of change:

$$\text{time} = \frac{800}{24}$$
$$= 33.3$$

The air will last about 33.3 min.