CHAPTER 7 Geometric Relationships
7.5 Midpoints and Diagonals in Quadrilaterals
Midpoint and Diagonal Properties of Quadrilaterals

Joining the midpoints of the sides of any quadrilateral produces a parallelogram. The diagonals of a parallelogram bisect each other.

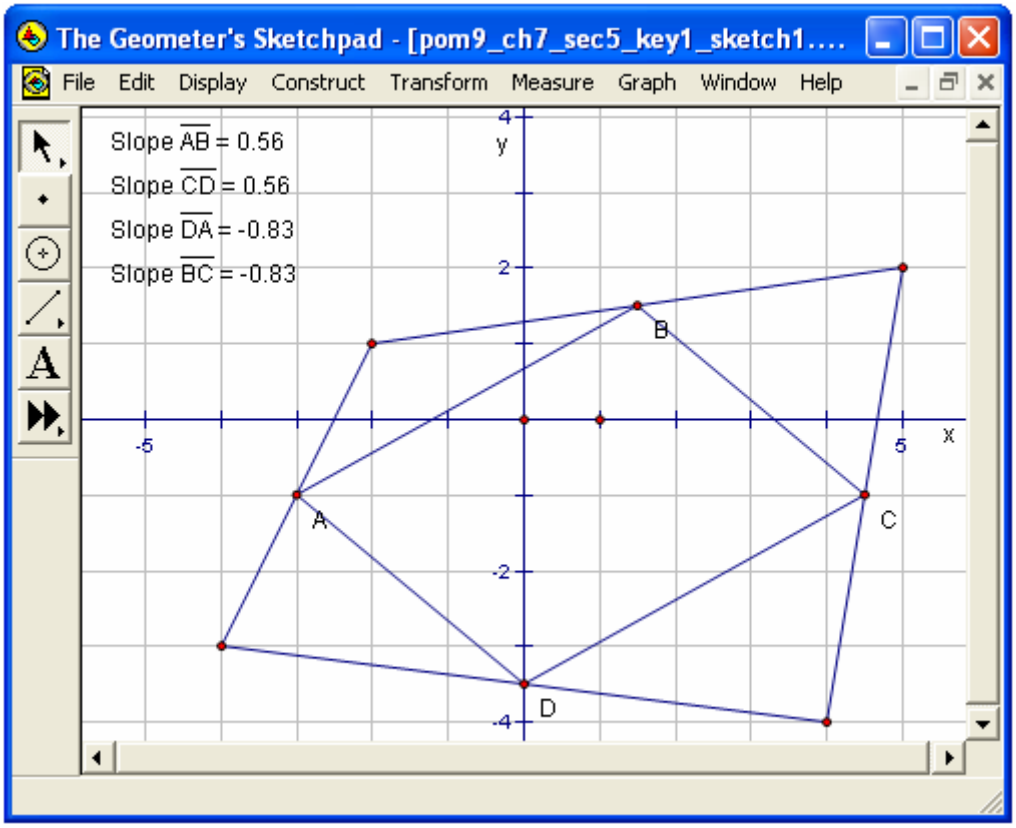
Example:

a) A quadrilateral has vertices at (–2, 1), (5, 2), (4, –4), and (–4, –3). Use dynamic geometry software to show that the quadrilateral formed by joining the midpoints of the sides is a parallelogram.

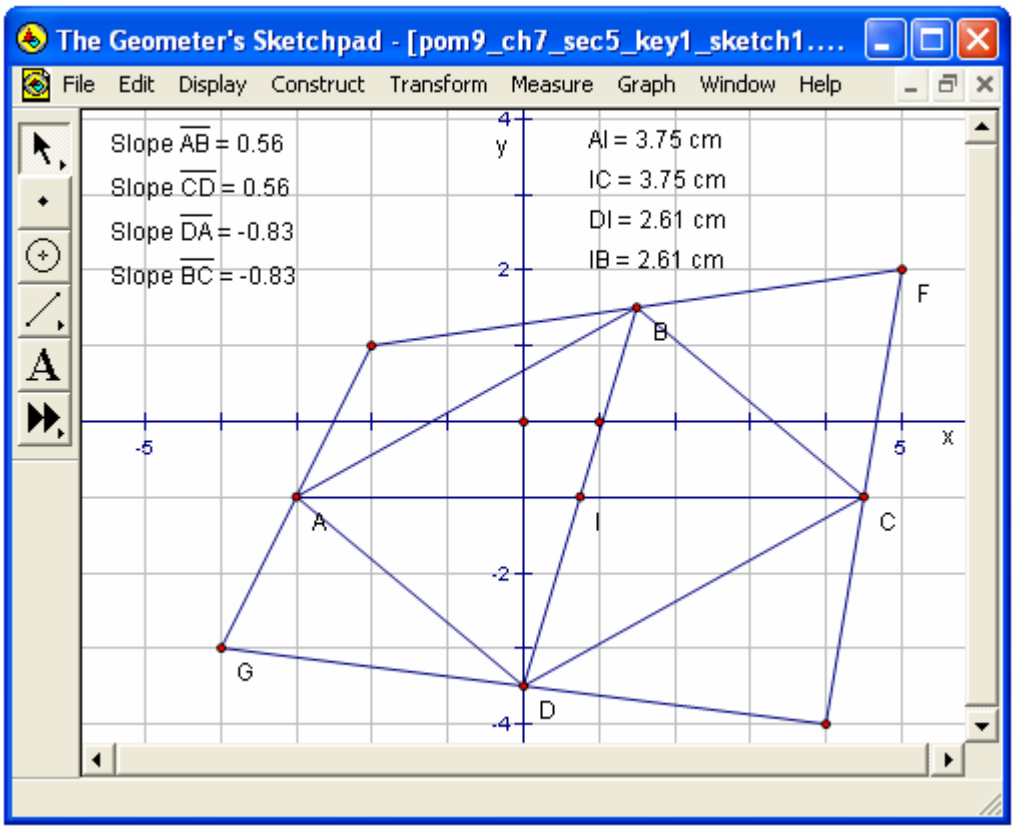
b) Show that the diagonals of the parallelogram in part a) bisect each other.

Solution:

a) Plot the points that form the quadrilateral, and then, draw line segments. Measure the midpoint of each of the sides. Join the midpoints to form the interior quadrilateral. Find the slopes of each pair of opposite sides. Note that the slopes of each pair are equal, indicating that the sides are parallel. Therefore, ABCD is a parallelogram.



b) Draw the diagonals of the parallelogram. Construct the point of intersection of the diagonals. Measure the length of each of the four line segments formed by the diagonals. Note that the diagonals bisect each other.



Practice:

1. a) Use dynamic geometry software to show that either diagonal of a parallelogram bisects its area.

b) Use dynamic geometry software to demonstrate that the diagonals of a parallelogram meet at right angles only if the parallelogram is a rhombus.

Answers:

1. a) Construct a parallelogram. Measure its area. Draw one of the diagonals. Measure the area of one of the triangles formed. Draw the other diagonal. Measure the area of one of the triangles formed.

b) Construct a parallelogram. Measure the lengths of the sides to show that it is not a rhombus. If it is, drag vertices until it is not, but is still a parallelogram. Draw the diagonals. Construct the point of intersection. Measure the angle between the diagonals. Drag vertices until the angle becomes 90º. Note the relation among the sides of the parallelogram.