CHAPTER 1

# An Introduction to Business Statistics

## LEARNING OBJECTIVES

*After reading this chapter, you should be able to*

- understand how and why research samples are collected
- know the difference between a sample and a population
- explain what is meant by the term *random sample* and explain how a random sample may be generated
- describe the four levels of measurement
- list some of the potential problems associated with surveys

## CHAPTER OUTLINE



The subject of **statistics** involves the study of how to collect, summarize, and interpret data. **Data** are numerical facts and figures from which conclusions can be drawn. Such conclusions are important to the decision-making processes of many professions and organizations. For example, government officials use conclusions drawn from the latest data on unemployment and inflation to make policy decisions. Financial planners use recent trends in stock market prices to make investment decisions. Businesses decide which products to develop and market by using data that reveal consumer preferences. Production supervisors use manufacturing data to evaluate, control, and improve product quality. Politicians rely on data from public opinion polls to formulate legislation and to devise campaign strategies. Physicians and hospitals use data on the effectiveness of drugs and surgical procedures to provide patients with the best possible treatment.

In this chapter, we begin to see how we collect and analyze data. As we proceed through the chapter, we introduce several case studies. These case studies (and others to be introduced later, many from Statistics Canada) are revisited throughout later chapters as we learn the statistical methods needed to analyze the cases. Briefly, we begin to study four cases:

**The Cell Phone Case.** A bank estimates its cellular phone costs and decides whether to outsource management of its wireless resources by studying the calling patterns of its employees.

**The Marketing Research Case.** A bottling company investigates consumer reaction to a new bottle design for one of its popular soft drinks.

**The Coffee Temperature Case.** A fast-food restaurant studies and monitors the temperature of the coffee it serves.

**The Mass of the Loonie.** A researcher examines the overall distribution of the masses (in grams) of the 1989 Canadian dollar coin (nicknamed the "loonie") to determine the average mass and range of masses.

## 1.1 Populations and Samples

Statistical methods are very useful for learning about populations. Populations can be defined in various ways, including the following:

A **population** is a set of existing units (usually people, objects, or events).

Examples of populations include (1) all of last year's graduates of Sauder School of Business at UBC, (2) all consumers who bought a cellular phone last year, (3) all accounts receivable invoices accumulated last year by Procter & Gamble, (4) all Toyota Corollas that were produced last year, and (5) all fires reported last month to the Ottawa fire department.

We usually focus on studying one or more characteristics of the population units.

Any characteristic of a population unit is called a **variable**.

For instance, if we study the starting salaries of last year's graduates of an MBA program, the variable of interest is starting salary. If we study the fuel efficiency obtained in city driving by last year's Toyota Corolla, the variable of interest is litres per 100 km in city driving.

We carry out a **measurement** to assign a **value** of a variable to each population unit. For example, we might measure the starting salary of an MBA graduate to the nearest dollar. Or we might measure the fuel efficiency obtained by a car in city driving to the nearest litre per 100 km by conducting a test on a driving course prescribed by the Ministry of Transportation. If the possible measurements are numbers that represent quantities (that is, "how much" or "how many"), then the variable is said to be **quantitative**. For example, starting salary and fuel efficiency are both quantitative. However, if we simply record into which of several categories a population unit falls, then the variable is said to be **qualitative** or **categorical**. Examples of categorical variables include (1) a person's sex, (2) the make of an automobile, and (3) whether a person who purchases a product is satisfied with the product.[1]

If we measure each and every population unit, we have a **population of measurements** (sometimes called **observations**). If the population is small, it is reasonable to do this. For instance, if 150 students graduated last year from an MBA program, it might be feasible to survey the graduates and to record all of their starting salaries. In general:

If we examine all of the population measurements, we say that we are conducting a **census** of the population.

Often the population that we wish to study is very large, and it is too time-consuming or costly to conduct a census. In such a situation, we select and analyze a subset (or portion) of the population.

A **sample** is a subset of the units in a population.

For example, suppose that 8742 students graduated last year from a large university. It would probably be too time-consuming to take a census of the population of all of their starting salaries. Therefore, we would select a sample of graduates, and we would obtain and record their starting salaries. When we measure the units in a sample, we say that we have a **sample of measurements**.

We often wish to describe a population or sample.

**Descriptive statistics** is the science of describing the important aspects of a set of measurements.

As an example, if we are studying a set of starting salaries, we might wish to describe (1) how large or small they tend to be, (2) what a typical salary might be, and (3) how much the salaries differ from each other.

When the population of interest is small and we can conduct a census of the population, we will be able to directly describe the important aspects of the population measurements. However, if the population is large and we need to select a sample from it, then we use what we call **statistical inference**.

[1]In Section 1.4, we discuss two types of quantitative variables (ratio and interval) and two types of qualitative variables (ordinal and nominative). Study Hint: To remember the difference between quantitative and qualitative, remember that quantitative has the letter "n" and "n is for number." Qualitative has an "l" and "l is for letter," so you have to use words to describe the data.

**Statistical inference** is the science of using a sample of measurements to make generalizations about the important aspects of a population of measurements.

For instance, we might use a sample of starting salaries to **estimate** the important aspects of a population of starting salaries. In the next section, we begin to look at how statistical inference is carried out.

## 1.2 Sampling a Population of Existing Units

**Random samples** If the information contained in a sample is to accurately reflect the population under study, the sample should be **randomly selected** from the population. To intuitively illustrate random sampling, suppose that a small company employs 15 people and wishes to randomly select two of them to attend a convention. To make the random selections, we number the employees from 1 to 15, and we place in a hat 15 identical slips of paper numbered from 1 to 15. We thoroughly mix the slips of paper in the hat and, blindfolded, choose one. The number on the chosen slip of paper identifies the first randomly selected employee. Then, still blindfolded, we choose another slip of paper from the hat. The number on the second slip identifies the second randomly selected employee.

Of course, it is impractical to carry out such a procedure when the population is very large. It is easier to use a **random number table** or a computerized random number generator. To show how to use such a table, we must more formally define a random sample.[2]

A **random sample** is selected so that, on each selection from the population, every unit remaining in the population on that selection has the same chance of being chosen.

To understand this definition, first note that we can randomly select a sample **with or without replacement**. If we **sample with replacement**, we place the unit chosen on any particular selection back into the population. Thus, we give this unit a chance to be chosen on any succeeding selection. In such a case, all of the units in the population remain as candidates to be chosen for each and every selection. Randomly choosing two employees with replacement to attend a convention would make no sense because we wish to send two different employees to the convention. If we **sample without replacement**, we do not place the unit chosen on a particular selection back into the population. Thus, we do not give this unit a chance to be selected on any succeeding selection. In this case, the units remaining as candidates for a particular selection are all of the units in the population except for those that have previously been selected. **It is best to sample without replacement.** Intuitively, because we will use the sample to learn about the population, sampling without replacement will give us the fullest possible look at the population. This is true because choosing the sample without replacement guarantees that all of the units in the sample will be different (and that we are looking at as many different units from the population as possible).

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In the following example, we illustrate how to use a random number table, or computer-generated random numbers, to select a random sample.

### Example 1.1 The Cell Phone Case: Estimating Cell Phone Costs[3]

Businesses and students have at least two things in common—both find cellular phones to be nearly indispensable because of their convenience and mobility, and both often rack up unpleasantly high cell phone bills. Students' high bills are usually the result of **overage**—a student uses more minutes than their plan allows. Businesses also lose money due to overage and, in addition, lose money due to **underage** when some employees do not use all of the (already-paid-for) minutes allowed by their plans. Because cellular carriers offer a very large number of rate plans,

[2]Actually, there are several different kinds of random samples. The type we will define is sometimes called a **simple random sample**. For brevity's sake, however, we will use the term **random sample**.

[3]The authors would like to thank Mr. Doug L. Stevens, Vice President of Sales and Marketing, at MobileSense Inc., Westlake Village, California, for his help in developing this case.

it is nearly impossible for a business to intelligently choose calling plans that will meet its needs at a reasonable cost.

Rising cell phone costs have forced companies with large numbers of cellular users to hire services to manage their cellular and other wireless resources. These cellular management services use sophisticated software and mathematical models to choose cost-efficient cell phone plans for their clients. One such firm, MobileSense Inc. of Westlake Village, California, specializes in automated wireless cost management. According to Doug L. Stevens, Vice President of Sales and Marketing at MobileSense, cell phone carriers count on overage and underage to deliver almost half of their revenues. As a result, a company's typical cost of cell phone use can easily exceed 25 cents per minute. However, Mr. Stevens explains that by using MobileSense automated cost management to select calling plans, this cost can be reduced to 12 cents per minute or less.

In this case, we will demonstrate how a bank can use a random sample of cell phone users to study its cellular phone costs. Based on this cost information, the bank will decide whether to hire a cellular management service to choose calling plans for the bank's employees. While the bank has over 10,000 employees on a variety of calling plans, the cellular management service suggests that by studying the calling patterns of cellular users on 500-minute plans, the bank can accurately assess whether its cell phone costs can be substantially reduced.

The bank has 2,136 employees on a 500-minute-per-month plan with a monthly cost of $50. The overage charge is 40 cents per minute, and there are additional charges for long distance and roaming. The bank will estimate its cellular cost per minute for this plan by examining the number of minutes used last month by each of 100 randomly selected employees on this 500-minute plan. According to the cellular management service, if the cellular cost per minute for the random sample of 100 employees is over 18 cents per minute, the bank should benefit from automated cellular management of its calling plans.

In order to randomly select the sample of 100 cell phone users, the bank will make a numbered list of the 2,136 users on the 500-minute plan. This list is called a **frame**. The bank can then use a **random number table**, such as Table 1.1(a), to select the needed sample. To see how this is done, note that any single-digit number in the table is assumed to have been randomly selected from the digits 0 to 9. Any two-digit number in the table is assumed to have been randomly selected from the numbers 00 to 99. Any three-digit number is assumed to have been randomly selected from the numbers 000 to 999, and so forth. Note that the table entries are segmented into groups of five to make the table easier to read. Because the total number of cell phone users on the 500-minute plan (2,136) is a four-digit number, we arbitrarily select any set of four digits

**TABLE 1.1 Random Numbers**

**(a) A portion of a random number table**

| | | | | | | |
|---|---|---|---|---|---|---|
| 33276 | 85590 | 79936 | 56865 | 05859 | 90106 | 78188 |
| 03427 | 90511 | 69445 | 18663 | 72695 | 52180 | 90322 |
| 92737 | 27156 | 33488 | 36320 | 17617 | 30015 | 74952 |
| 85689 | 20285 | 52267 | 67689 | 93394 | 01511 | 89868 |
| 08178 | 74461 | 13916 | 47564 | 81056 | 97735 | 90707 |
| 51259 | 63990 | 16308 | 60756 | 92144 | 49442 | 40719 |
| 60268 | 44919 | 19885 | 55322 | 44819 | 01188 | 55157 |
| 94904 | 01915 | 04146 | 18594 | 29852 | 71585 | 64951 |
| 58586 | 17752 | 14513 | 83149 | 98736 | 23495 | 35749 |
| 09998 | 19509 | 06691 | 76988 | 13602 | 51851 | 58104 |
| 14346 | 61666 | 30168 | 90229 | 04734 | 59193 | 32812 |
| 74103 | 15227 | 25306 | 76468 | 26384 | 58151 | 44592 |
| 24200 | 64161 | 38005 | 94342 | 28728 | 35806 | 22851 |
| 87308 | 07684 | 00256 | 45834 | 15398 | 46557 | 18510 |
| 07351 | 86679 | 92420 | 60952 | 61280 | 50001 | 94953 |

**(b) MINITAB output of 100 different four-digit random numbers between 1 and 2136**

| | | | | | |
|---|---|---|---|---|---|
| 705 | 1131 | 169 | 1703 | 1709 | 609 |
| 1990 | 766 | 1286 | 1977 | 222 | 43 |
| 1007 | 1902 | 1209 | 2091 | 1742 | 1152 |
| 111 | 69 | 2049 | 1448 | 659 | 338 |
| 1732 | 1650 | 7 | 388 | 613 | 1477 |
| 838 | 272 | 1227 | 154 | 18 | 320 |
| 1053 | 1466 | 2087 | 265 | 2107 | 1992 |
| 582 | 1787 | 2098 | 1581 | 397 | 1099 |
| 757 | 1699 | 567 | 1255 | 1959 | 407 |
| 354 | 1567 | 1533 | 1097 | 1299 | 277 |
| 663 | 40 | 585 | 1486 | 1021 | 532 |
| 1629 | 182 | 372 | 1144 | 1569 | 1981 |
| 1332 | 1500 | 743 | 1262 | 1759 | 955 |
| 1832 | 378 | 728 | 1102 | 667 | 1885 |
| 514 | 1128 | 1046 | 116 | 1160 | 1333 |
| 831 | 2036 | 918 | 1535 | 660 | |
| 928 | 1257 | 1468 | 503 | 468 | |

**TABLE 1.2 A Sample of Cellular Usages (in minutes) for 100 Randomly Selected Employees**
CellUse

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 75 | 485 | 37 | 547 | 753 | 93 | 897 | 694 | 797 | 477 |
| 654 | 578 | 504 | 670 | 490 | 225 | 509 | 247 | 597 | 173 |
| 496 | 553 | 0 | 198 | 507 | 157 | 672 | 296 | 774 | 479 |
| 0 | 822 | 705 | 814 | 20 | 513 | 546 | 801 | 721 | 273 |
| 879 | 433 | 420 | 521 | 648 | 41 | 528 | 359 | 367 | 948 |
| 511 | 704 | 535 | 585 | 341 | 530 | 216 | 512 | 491 | 0 |
| 542 | 562 | 49 | 505 | 461 | 496 | 241 | 624 | 885 | 259 |
| 571 | 338 | 503 | 529 | 737 | 444 | 372 | 555 | 290 | 830 |
| 719 | 120 | 468 | 730 | 853 | 18 | 479 | 144 | 24 | 513 |
| 482 | 683 | 212 | 418 | 399 | 376 | 323 | 173 | 669 | 611 |

in the table (we have circled these digits). This number, which is 0511, identifies the first randomly selected user. Then, moving in any direction from the 0511 (up, down, right, or left—it does not matter which), we select additional sets of four digits. These succeeding sets of digits identify additional randomly selected users. Here we arbitrarily move down from 0511 in the table. The first seven sets of four digits we obtain are

0511 7156 0285 4461 3990 4919 1915

(See Table 1.1(a)—these numbers are enclosed in a rectangle.) Since there are no users numbered 7156, 4461, 3990, or 4919 (remember only 2,136 users are on the 500-minute plan), we ignore these numbers. This implies that the first three randomly selected users are those numbered 0511, 0285, and 1915. Continuing this procedure, we can obtain the entire random sample of 100 users. Notice that, because we are sampling without replacement, we should ignore any set of four digits previously selected from the random number table.

While using a random number table is one way to select a random sample, this approach has a disadvantage that is illustrated by the current situation. Specifically, since most four-digit random numbers are not between 0001 and 2136, obtaining 100 different four-digit random numbers between 0001 and 2136 will require ignoring a large number of random numbers in the random number table, and we will in fact need to use a random number table that is larger than Table 1.1(a). Although larger random number tables are readily available in books of mathematical and statistical tables, a good alternative is to use a computer software package, which can generate random numbers that are between whatever values we specify. For example, Table 1.1(b) gives the MINITAB output of 100 different four-digit random numbers that are between 0001 and 2136 (note that the "leading 0s" are not included in these four-digit numbers). If used, the random numbers in Table 1.1(b) identify the 100 employees that should form the random sample.

After the random sample of 100 employees is selected, the number of cellular minutes used by each employee during the month (the employee's **cellular usage**) is found and recorded. The 100 cellular-usage figures are given in Table 1.2. Looking at this table, we can see that there is substantial overage and underage—many employees used far more than 500 minutes, while many others failed to use all of the 500 minutes allowed by their plan. In Chapter 2, we will use these 100 usage figures to estimate the cellular cost per minute for the 500-minute plan.

**Approximately random samples** In general, to take a random sample we must have a list, or **frame**, of all the population units. This is needed because we must be able to number the population units in order to make random selections from them (by, for example, using a random number table). In Example 1.1, where we wished to study a population of 2,136 cell phone users who were on the bank's 500-minute cellular plan, we were able to produce a frame (list) of the population units. Therefore, we were able to select a random sample. Sometimes, however, it is not possible to list and thus number all the units in a population. In such a situation, we often select a **systematic sample**, which approximates a random sample.

## Example 1.2 The Marketing Research Case: Rating a New Bottle Design[4]

The design of a package or bottle can have an important effect on a company's bottom line. For example, an article in the September 16, 2004, issue of *USA Today* reported that the introduction of a contoured 1.5-L bottle for Coke drinks played a major role in Coca-Cola's failure to meet third-quarter earnings forecasts in 2004. According to the article, Coke's biggest bottler, Coca-Cola Enterprises, "said it would miss expectations because of the 1.5-liter bottle and the absence of common 2-liter and 12-pack sizes . . . in supermarkets."[5]

In this case, a brand group is studying whether changes should be made in the bottle design for a popular soft drink. To research consumer reaction to a new design, the brand group will use the "mall intercept method," in which shoppers at a large metropolitan shopping mall are intercepted and asked to participate in a consumer survey. Each shopper will be exposed to the new bottle design and asked to rate the bottle image. Bottle image will be measured by combining consumers' responses to five items, with each response measured using a seven-point "Likert scale." The five items and the scale of possible responses are shown in Figure 1.1. Here, since we describe the least favourable response and the most favourable response (and we do not describe the responses between them), we say that the scale is "anchored" at its ends. Responses to the five items will be summed to obtain a composite score for each respondent. It follows that the minimum composite score possible is 5 and the maximum composite score possible is 35. Furthermore, experience has shown that the smallest acceptable composite score for a successful bottle design is 25.

In this situation, it is not possible to list and number each and every shopper at the mall while the study is being conducted. Consequently, we cannot use random numbers (as we did in the cell phone case) to obtain a random sample of shoppers. Instead, we can select a **systematic sample**. To do this, every 100th shopper passing a specified location in the mall will be invited to participate in the survey. Here, selecting every 100th shopper is arbitrary—we could select every 200th, every 300th, and so forth. If we select every 100th shopper, it is probably reasonable to believe that the responses of the survey participants are not related. Therefore, it is reasonable to assume that the sampled shoppers obtained by the systematic sampling process make up an **approximate** random sample.

During a Tuesday afternoon and evening, a sample of 60 shoppers is selected by using the systematic sampling process. Each shopper is asked to rate the bottle design by responding to the five items in Figure 1.1, and a composite score is calculated for each shopper. The 60 composite scores obtained are given in Table 1.3. Since these scores range from 20 to 35, we might infer that **most** of the shoppers at the mall on the Tuesday afternoon and evening of the study would rate the new bottle design between 20 and 35. Furthermore, since 57 of the 60 composite

**FIGURE 1.1 The Bottle Design Survey Instrument**

**Please circle** the response that most accurately describes whether you agree or disagree with each statement about the bottle you have examined.

| Statement | Strongly Disagree | | | | | | Strongly Agree |
|---|---|---|---|---|---|---|---|
| The size of this bottle is convenient. | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| The contoured shape of this bottle is easy to handle. | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| The label on this bottle is easy to read. | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| This bottle is easy to open. | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| Based on its overall appeal, I like this bottle design. | 1 | 2 | 3 | 4 | 5 | 6 | 7 |

[4] This case was motivated by an example in the book *Essentials of Marketing Research,* by W. R. Dillon, T. J. Madden, and N. H. Firtle (Burr Ridge, IL: Richard D. Irwin, 1993). The authors also wish to thank Professor L. Unger of the Department of Marketing at Miami University for helpful discussions concerning how this type of marketing study would be carried out.

[5] **Source:** "Coke says earnings will come up short," by Theresa Howard, *USA Today,* September 16, 2004, p. 801.

**TABLE 1.3 A Sample of Bottle Design Ratings (Composite Scores for a Systematic Sample of 60 Shoppers)** Design

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 34 | 33 | 33 | 29 | 26 | 33 | 28 | 25 | 32 | 33 |
| 32 | 25 | 27 | 33 | 22 | 27 | 32 | 33 | 32 | 29 |
| 24 | 30 | 20 | 34 | 31 | 32 | 30 | 35 | 33 | 31 |
| 32 | 28 | 30 | 31 | 31 | 33 | 29 | 27 | 34 | 31 |
| 31 | 28 | 33 | 31 | 32 | 28 | 26 | 29 | 32 | 34 |
| 32 | 30 | 34 | 32 | 30 | 30 | 32 | 31 | 29 | 33 |

scores are at least 25, we might estimate that the proportion of all shoppers at the mall on the Tuesday afternoon and evening who would give the bottle design a composite score of at least 25 is $57/60 = 0.95$. That is, we estimate that 95 percent of the shoppers would give the bottle design a composite score of at least 25.

In Chapter 2, we will see how to estimate a typical composite score and we will further analyze the composite scores in Table 1.3.

In some situations, we need to decide whether a sample taken from one population can be employed to make statistical inferences about another, related, population. Often logical reasoning is used to do this. For instance, we might reason that the bottle design ratings given by shoppers at the mall on the Tuesday afternoon and evening of the research study would be representative of the ratings given by (1) shoppers at the same mall at other times, (2) shoppers at other malls, and (3) consumers in general. However, if we have no data or other information to back up this reasoning, making such generalizations is dangerous. In practice, marketing research firms choose locations and sampling times that data and experience indicate will produce a representative cross-section of consumers. To simplify our presentation, we will assume that this has been done in the bottle design case. Therefore, we will suppose that it is reasonable to use the 60 bottle design ratings in Table 1.3 to make statistical inferences about **all consumers**.

To conclude this section, we emphasize the importance of taking a random (or approximately random) sample. Statistical theory tells us that, when we select a random (or approximately random) sample, we can use the sample to make valid statistical inferences about the sampled population. However, if the sample is not random, we cannot do this. A classic example occurred prior to the U.S. presidential election of 1936, when the *Literary Digest* predicted that Alf Landon would defeat Franklin D. Roosevelt by a margin of 57 percent to 43 percent. Instead, Roosevelt won the election in a landslide. *Literary Digest*'s error was to sample names from telephone books and club membership rosters. In 1936, the United States had not yet recovered from the Great Depression, and many unemployed and low-income people did not have phones or belong to clubs. The *Literary Digest*'s sampling procedure excluded these people, who overwhelmingly voted for Roosevelt. At this time, George Gallup, founder of the Gallup Poll, was beginning to establish his survey business. He used an approximately random sample to correctly predict Roosevelt's victory.

As another example, today's television and radio stations, as well as newspaper columnists and Web sites, use **voluntary response samples**. In such samples, participants self-select—that is, whoever wishes to participate does so (usually expressing some opinion). These samples overrepresent people with strong (usually negative) opinions. For example, the advice columnist Ann Landers once asked her readers, "If you had it to do over again, would you have children?" Of the nearly 10,000 parents who **voluntarily** responded, 70 percent said that they would not. An approximately random sample taken a few months later found that 91 percent of parents would have children again. We further discuss random sampling in Section 1.5.

# Exercises for Sections 1.1 and 1.2

## CONCEPTS

**1.1** Define a **population**. Give an example of a population that you might study when you start your career after graduating from university.

**1.2** Define what we mean by a **variable**, and explain the difference between a quantitative variable and a qualitative (categorical) variable.

**1.3** Below we list several variables. Which of these variables are quantitative and which are qualitative? Explain.

**a.** The dollar amount on an accounts receivable invoice.
**b.** The net profit for a company in 2007.
**c.** The stock exchange on which a company's stock is traded.
**d.** The national debt of Canada in 2007.
**e.** The advertising medium (radio, television, Internet, or print) used to promote a product.

**1.4** Explain the difference between a census and a sample.

**1.5** Explain each of the following terms:

**a.** Descriptive statistics.
**b.** Statistical inference.
**c.** Random sample.
**d.** Systematic sample.

**1.6** Explain why sampling without replacement is preferred to sampling with replacement.

## METHODS AND APPLICATIONS

**1.7** *Business News Network (BNN)* has a link on its Web site http://www.bnn.ca to the top 1,000 Canadian companies (ROB Top 1000, 2006 edition). Below we have listed the top 50 best-performing companies in terms of revenue and profit from the *BNN* Web site. Top50

The companies listed here are the **50 largest publicly traded Canadian corporations, measured by assets**.

ROB's explanation of the criteria used for these rankings is as follows:

> They are ranked according to their after-tax profits in their most recent fiscal year, excluding extraordinary gains or losses.
>
> When companies state their results in U.S. dollars, we do the same, but rankings are made based on the Canadian dollar equivalent.

| Rank | | Company and Year-End | Profit | Revenue | |
|---|---|---|---|---|---|
| **2004** | **2003** | | **$1000s** | **$1000s** | **Rank** |
| **1** | 1 | EnCana Corp. (De04)[1] | 3,513,000 | 12,241,000 | 21 |
| **2** | 3 | Bank of Nova Scotia (Oc04) | 2,931,000 | 16,497,000 | 19 |
| **3** | 2 | Royal Bank of Canada (Oc04) | 2,817,000 | 25,204,000 | 6 |
| **4** | 11 | Manulife Financial (De04) | 2,564,000 | 27,265,000 | 3 |
| **5** | 7 | Bank of Montreal (Oc04) | 2,351,000 | 13,208,000 | 23 |
| **6** | 19 | Toronto-Dominion Bank (Oc04) | 2,310,000 | 16,015,000 | 20 |
| **7** | 5 | Cdn. Imp. Bank of Commerce (Oc04) | 2,199,000 | 16,705,000 | 17 |
| **8** | 9 | Imperial Oil (De04) | 2,033,000 | 21,206,000 | 11 |
| **9** | nr | Manufacturers Life Insurance (De04) | 2,015,000 | 19,991,000 | 13 |
| **10** | 10 | Petro-Canada (De04) | 1,757,000 | 14,442,000 | 22 |
| **11** | 14 | Sun Life Financial (De04) | 1,681,000 | 21,769,000 | 10 |
| **12** | 16 | Great-West Lifeco (De04) | 1,660,000 | 21,871,000 | 9 |
| **13** | 6 | Power Financial (De04) | 1,558,000 | 24,077,000 | 8 |
| **14** | 8 | BCE Inc. (De04) | 1,524,000 | 19,285,000 | 14 |
| **15** | 4 | Bell Canada (De04) | 1,518,000 | 16,972,000 | 16 |
| **16** | 12 | Cdn. Natural Resources (De04) | 1,405,000 | 7,179,000 | 39 |
| **17** | 17 | Thomson Corp. (De04)[1] | 1,011,000 | 8,132,000 | 30 |
| **18** | 28 | Canadian National Railway Co. (De04) | 1,297,000 | 6,581,000 | 44 |
| **19** | 26 | Shell Canada (De04) | 1,286,000 | 11,288,000 | 27 |
| **20** | 25 | Great-West Life Assurance (De04) | 1,226,000 | 17,353,000 | 15 |
| **21** | 18 | Suncor Energy (De04) | 1,100,000 | 8,699,000 | 33 |
| **22** | 22 | TransCanada PipeLines (De04) | 1,083,000 | 5,343,000 | 55 |
| **23** | 23 | TransCanada Corp. (De04) | 1,032,000 | 5,343,000 | 55 |
| **24** | 13 | Husky Energy (De04) | 1,006,000 | 8,540,000 | 35 |
| **25** | 24 | Loblaw Companies (Ja05) | 968,000 | 26,209,000 | 5 |
| **26** | 15 | Power Corp. (De04) | 949,000 | 24,470,000 | 7 |
| **27** | 29 | Magna International (De04)[1] | 692,000 | 20,672,000 | 4 |
| **28** | 33 | Brascan Corp. (De04)[1] | 688,000 | 4,372,000 | 52 |
| **29** | 49 | Falconbridge Ltd. (De04)[1] | 672,000 | 3,070,000 | 67 |
| **30** | 62 | Inco Ltd. (De04)[1] | 612,000 | 4,320,000 | 53 |
| **31** | 31 | Nexen Inc. (De04) | 793,000 | 3,884,000 | 69 |
| **32** | 32 | National Bank of Canada (Oc04) | 725,000 | 4,771,000 | 59 |

| | | | | | |
|---|---|---|---|---|---|
| **33** | 180 | Noranda Inc. (De04)[1] | 551,000 | 7,002,000 | 32 |
| **34** | 21 | Talisman Energy (De04) | 663,000 | 6,479,000 | 45 |
| **35** | 30 | Enbridge Inc. (De04) | 652,200 | 6,843,700 | 43 |
| **36** | 36 | PetroKazakhstan Inc. (De04)[1] | 500,668 | 1,652,346 | 102 |
| **37** | nr | ING Canada (De04) | 624,152 | 3,780,886 | 71 |
| **38** | 34 | IGM Financial (De04) | 617,096 | 2,119,071 | 104 |
| **39** | 82 | Teck Cominco (De04) | 617,000 | 3,452,000 | 78 |
| **40** | 314 | IPSCO Inc. (De04)[1] | 438,610 | 2,458,893 | 82 |
| **41** | 41 | Telus Corp. (De04) | 565,800 | 7,623,400 | 37 |
| **42** | 43 | Canadian Oil Sands Trust (De04) | 509,200 | 1,480,200 | 130 |
| **43** | 931 | Gerdau Ameristeel (De04)[1] | 337,669 | 3,154,390 | 65 |
| **44** | 27 | George Weston (De04) | 428,000 | 29,723,000 | 2 |
| **45** | 70 | Norbord Inc. (De04)[1] | 326,000 | 1,492,000 | 110 |
| **46** | 79 | Canfor Corp. (De04) | 420,900 | 4,120,000 | 64 |
| **47** | 38 | Canadian Pacific Railway Ltd. (De04) | 413,000 | 3,990,900 | 68 |
| **48** | 994 | Potash Corp. of Saskatchewan (De04)[1] | 298,600 | 3,328,200 | 61 |
| **49** | 46 | Placer Dome (De04)[1] | 291,000 | 1,946,000 | 90 |
| **50** | 100 | Dofasco Inc. (De04) | 376,900 | 4,235,400 | 62 |

[1]Figures are reported in U.S. dollars.
nr: not ranked
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Consider the random numbers given in the random number table of Table 1.1(a) on page 4. Starting in the upper left corner of Table 1.1(a) and moving down the two leftmost columns, we see that the first three two-digit numbers obtained are

33 03 92

Starting with these three random numbers, and moving down the two leftmost columns of Table 1.1(a) to find more two-digit random numbers, use Table 1.1 to randomly select five of these companies to be interviewed in detail about their business strategies. Hint: Note that the companies in the *BNN* list are numbered from 1 to 50.

**1.8 THE VIDEO GAME SATISFACTION RATING CASE**
VideoGame

A company that produces and markets video game systems wishes to assess its customers' level of satisfaction with a relatively new model, the XYZ-Box. In the six months since the introduction of the model, the company has received 73,219 warranty registrations from purchasers. The company will randomly select 65 of these registrations and will conduct telephone interviews with the purchasers. Specifically, each purchaser will be asked to state their level of agreement with each of the seven statements listed on the survey instrument given in Figure 1.2. Here the level of agreement for each statement is measured on a seven-point Likert scale.[7] Purchaser

**FIGURE 1.2 The Video Game Satisfaction Survey Instrument**

| Statement | Strongly Disagree | | | | | | Strongly Agree |
|---|---|---|---|---|---|---|---|
| The game console of the XYZ-Box is well designed. | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| The game controller of the XYZ-Box is easy to handle. | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| The XYZ-Box has high-quality graphics capabilities. | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| The XYZ-Box has high-quality audio capabilities. | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| The XYZ-Box serves as a complete entertainment centre. | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| There is a large selection of XYZ-Box games to choose from. | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| I am totally satisfied with my XYZ-Box game system. | 1 | 2 | 3 | 4 | 5 | 6 | 7 |

[7] Historical Trivia: The Likert scale is named after Rensis Likert (1903–1981), who originally developed this numerical scale for measuring attitudes in his PhD dissertation in 1932. [Source: *Psychology in America: A Historical Survey,* by E. R. Hilgard (San Diego, CA: Harcourt Brace Jovanovich, 1987).]

**TABLE 1.4 Composite Scores for the Video Game Satisfaction Rating Case**
VideoGame

| | | | | |
|---|---|---|---|---|
| 39 | 44 | 46 | 44 | 44 |
| 45 | 42 | 45 | 44 | 42 |
| 38 | 46 | 45 | 45 | 47 |
| 42 | 40 | 46 | 44 | 43 |
| 42 | 47 | 43 | 46 | 45 |
| 41 | 44 | 47 | 48 | |
| 38 | 43 | 43 | 44 | |
| 42 | 45 | 41 | 41 | |
| 46 | 45 | 40 | 45 | |
| 44 | 40 | 43 | 44 | |
| 40 | 46 | 44 | 44 | |
| 39 | 41 | 41 | 44 | |
| 40 | 43 | 38 | 46 | |
| 42 | 39 | 43 | 39 | |
| 45 | 43 | 36 | 41 | |

**TABLE 1.5 Waiting Times (in Minutes) for the Bank Customer Waiting Time Case**
WaitTime

| | | | | | | |
|---|---|---|---|---|---|---|
| 1.6 | 6.2 | 3.2 | 5.6 | 7.9 | 6.1 | 7.2 |
| 6.6 | 5.4 | 6.5 | 4.4 | 1.1 | 3.8 | 7.3 |
| 5.6 | 4.9 | 2.3 | 4.5 | 7.2 | 10.7 | 4.1 |
| 5.1 | 5.4 | 8.7 | 6.7 | 2.9 | 7.5 | 6.7 |
| 3.9 | 0.8 | 4.7 | 8.1 | 9.1 | 7.0 | 3.5 |
| 4.6 | 2.5 | 3.6 | 4.3 | 7.7 | 5.3 | 6.3 |
| 6.5 | 8.3 | 2.7 | 2.2 | 4.0 | 4.5 | 4.3 |
| 6.4 | 6.1 | 3.7 | 5.8 | 1.4 | 4.5 | 3.8 |
| 8.6 | 6.3 | 0.4 | 8.6 | 7.8 | 1.8 | 5.1 |
| 4.2 | 6.8 | 10.2 | 2.0 | 5.2 | 3.7 | 5.5 |
| 5.8 | 9.8 | 2.8 | 8.0 | 8.4 | 4.0 | |
| 3.4 | 2.9 | 11.6 | 9.5 | 6.3 | 5.7 | |
| 9.3 | 10.9 | 4.3 | 1.3 | 4.4 | 2.4 | |
| 7.4 | 4.7 | 3.1 | 4.8 | 5.2 | 9.2 | |
| 1.8 | 3.9 | 5.8 | 9.9 | 7.4 | 5.0 | |

satisfaction will be measured by adding the purchaser's responses to the seven statements. It follows that for each consumer the minimum composite score possible is 7 and the maximum is 49. Furthermore, experience has shown that a purchaser of a video game system is "very satisfied" if their composite score is at least 42.

**a.** Assume that the warranty registrations are numbered from 1 to 73,219 on a computer. Starting in the upper left corner of Table 1.1(a) and moving down the five leftmost columns, we see that the first three five-digit numbers obtained are

33276 03427 92737

Starting with these three random numbers and moving down the five leftmost columns of Table 1.1(a) to find more five-digit random numbers, randomly select the numbers of the first 10 warranty registrations to be included in the sample of 65 registrations.

**b.** Suppose that when the 65 customers are interviewed, their composite scores are obtained and are as given in Table 1.4. Using the data, estimate limits between which most of the 73,219 composite scores would fall. Also estimate the proportion of the 73,219 composite scores that would be at least 42.

**1.9 THE BANK CUSTOMER WAITING TIME CASE** WaitTime

A bank manager has developed a new system to reduce the time customers spend waiting to be served by tellers during peak business hours. Typical waiting times during peak business hours under the current system are roughly 9 to 10 minutes. The bank manager hopes that the new system will lower typical waiting times to less than six minutes.

A 30-day trial of the new system is conducted. During the trial run, every 150th customer who arrives during peak business hours is selected until a systematic sample of 100 customers is obtained. Each of the sampled customers is observed, and the time spent waiting for teller service is recorded. The 100 waiting times obtained are given in Table 1.5. Moreover, the bank manager feels that this systematic sample is as representative as a random sample of waiting times would be. Using the data, estimate limits between which the waiting times of most of the customers arriving during peak business hours would be. Also estimate the proportion of waiting times of customers arriving during peak business hours that are less than six minutes.

**1.10** In an article titled "Turned off" in the June 2–4, 1995, issue of *USA Weekend,* Olmsted and Anders report on the results of a survey conducted by the magazine. Readers were invited to write in and answer several questions about sex and vulgarity on television. Olmsted and Anders summarized the survey results as follows:

> Nearly all of the 65,000 readers responding to our write-in survey say TV is too vulgar, too violent, and too racy. TV execs call it reality.

Some of the key survey results were as follows:

**SURVEY RESULTS**
- 96 percent are very or somewhat concerned about SEX on TV.
- 97 percent are very or somewhat concerned about VULGAR LANGUAGE on TV.
- 97 percent are very or somewhat concerned about VIOLENCE on TV.

Note: Because participants were not chosen at random, the results of the write-in survey may not be scientific.

**a.** Note the disclaimer at the bottom of the survey results. In a write-in survey, anyone who wishes to participate may respond to the survey questions. Therefore, the sample is not random and we say that the survey is "not scientific." What kind of people would be most likely to respond to a survey about TV sex and violence? Do the survey results agree with your answer?

**b.** If a random sample of the general population were taken, do you think that its results would be the same? Why or why not? Similarly, for instance, do you think that 97 percent of the general population is "very or somewhat concerned about violence on TV"?

**c.** Another result obtained in the write-in survey is as follows:

- Should "V-chips" be installed on TV sets so parents could easily block violent programming?

YES 90%    NO 10%

If you planned to start a business manufacturing and marketing such V-chips (at a reasonable price), would you expect 90 percent of the general population to desire a V-chip? Why or why not?

## 1.3 Sampling a Process

A population is not always defined to be a set of **existing** units. Often we are interested in studying the population of all of the units that will be or could potentially be produced by a process.

A **process** is a sequence of operations that takes inputs (labour, materials, methods, machines, and so on) and turns them into outputs (products, services, and the like).

Processes produce output **over time**. For example, this year's Toyota Corolla manufacturing process produces Toyota Corollas over time. Early in the model year, Toyota Canada might wish to study the population of the city fuel efficiency of all Toyota Corollas that will be produced during the model year. Or, even more hypothetically, Toyota Canada might wish to study the population of the city fuel efficiency of all Toyota Corollas that could **potentially** be produced by this model year's manufacturing process. The first population is called a **finite population** because only a finite number of cars will be produced during the year. Any population of existing units is also finite. The second population is called an **infinite population** because the manufacturing process that produces this year's model could in theory always be used to build one more car. That is, theoretically there is no limit to the number of cars that could be produced by this year's process. There are a multitude of other examples of finite or infinite hypothetical populations. For instance, we might study the population of all waiting times that will or could potentially be experienced by patients of a hospital emergency room. Or we might study the population of all the amounts of raspberry jam that will be or could potentially be dispensed into 500-mL jars by an automated filling machine. To study a population of potential process observations, we sample the process—usually at equally spaced time points—over time. This is illustrated in the following case.

### Example 1.3 The Coffee Temperature Case: Monitoring Coffee Temperatures

According to the Web site of the Association of Trial Lawyers of America,[8] Stella Liebeck of Albuquerque, New Mexico, was severely burned by McDonald's coffee in February 1992. Liebeck, who received third-degree burns over 6 percent of her body, was awarded $160,000 ($US) in compensatory damages and $480,000 ($US) in punitive damages. A postverdict investigation revealed that the coffee temperature at the local Albuquerque McDonald's had dropped from about 85°C before the trial to about 70°C after the trial.

This case concerns coffee temperatures at a fast-food restaurant. Because of the possibility of future litigation and to possibly improve the coffee's taste, the restaurant wishes to study and monitor the temperature of the coffee it serves. To do this, the restaurant personnel measure the temperature of the coffee being dispensed (in degrees Celsius) at half-hour intervals from 10 A.M. to 9:30 P.M. on a given day. Table 1.6 gives the 24 temperature measurements obtained in the time order that they were observed. Here time equals 1 at 10 A.M. and 24 at 9:30 P.M.

[8] http://www.atla.org/pressroom/FACTS/frivolous/McdonaldsCoffeecase.aspx, Association of Trial Lawyers of America, January 25, 2005.

**TABLE 1.6** 24 Coffee Temperatures Observed in Time Order (°C) CoffeeTemp

| Time | | Coffee Temperature | Time | | Coffee Temperature | Time | | Coffee Temperature |
|---|---|---|---|---|---|---|---|---|
| (10:00 A.M.) | 1 | 73°C | (2:00 P.M.) | 9 | 71°C | (6:00 P.M.) | 17 | 70°C |
| | 2 | 76 | | 10 | 68 | | 18 | 77 |
| | 3 | 69 | | 11 | 75 | | 19 | 68 |
| | 4 | 67 | | 12 | 72 | | 20 | 72 |
| (12:00 noon) | 5 | 74 | (4:00 P.M.) | 13 | 67 | (8:00 P.M.) | 21 | 69 |
| | 6 | 70 | | 14 | 74 | | 22 | 75 |
| | 7 | 69 | | 15 | 72 | | 23 | 68 |
| | 8 | 72 | | 16 | 68 | | 24 | 73 |

Examining Table 1.6, we see that the coffee temperatures range from 67°C to 77°C. Based on this, is it reasonable to conclude that the temperature of most of the coffee that will or could potentially be served by the restaurant will be between 67°C and 77°C? The answer is yes if the restaurant's coffee-making process operates consistently over time. That is, this process must be in a state of **statistical control**.

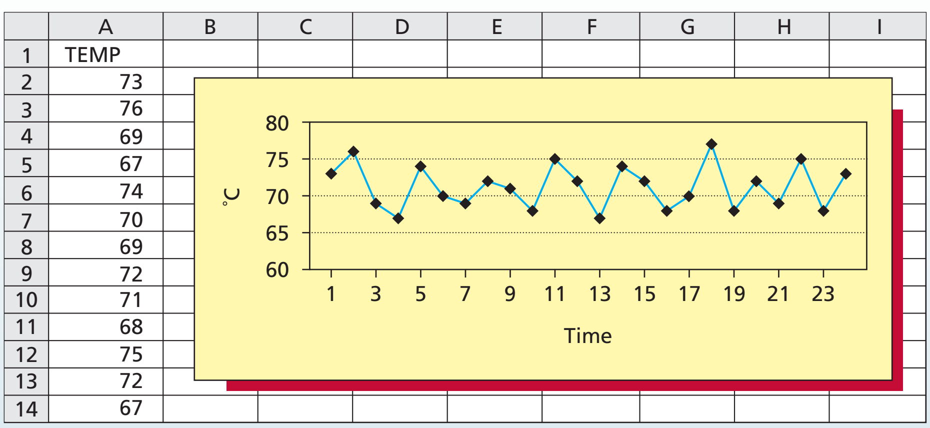
A process is in **statistical control** if it does not exhibit any unusual process variations. Often this means that the process displays a **constant amount of variation** around a **constant**, or horizontal, **level**.

To assess whether a process is in statistical control, we sample the process often enough to detect unusual variations or instabilities. The fast-food restaurant has sampled the coffee-making process every half hour. In other situations, we sample processes with other frequencies—for example, every minute, every hour, or every day. Using the observed process measurements, we can then construct a **runs plot** (sometimes called a **time series plot**).

A **runs plot** is a graph of individual process measurements versus time.

Figure 1.3 shows the Excel outputs of a runs plot of the temperature data. (Some people call such a plot a **line chart** when the plot points are connected by line segments as in the Excel output.) Here we plot each coffee temperature on the vertical scale versus its corresponding

**FIGURE 1.3** Excel Runs Plots of Coffee Temperatures: The Process Is in Statistical Control



time index on the horizontal scale. For instance, the first temperature (73°C) is plotted versus time equals 1, the second temperature (76°C) is plotted versus time equals 2, and so forth. The runs plot suggests that the temperatures exhibit a relatively constant amount of variation around a relatively constant level. That is, the centre of the temperatures can be pretty much represented by a horizontal line (constant level), and the spread of the points around the line stays about the same (constant variation). Note that the plot points tend to form a horizontal band. Therefore, the temperatures are in statistical control.

In general, assume that we have sampled a process at different (usually equally spaced) time points and made a runs plot of the resulting sample measurements. If the plot indicates that the process is in statistical control, and if it is reasonable to believe that the process will remain in control, then it is probably reasonable to regard the sample measurements as an approximately random sample from the population of all possible process measurements. Furthermore, since the process remains in statistical control, the process performance is **predictable**. This allows us to make statistical inferences about the population of all possible process measurements that will or potentially could result from using the process. For example, assuming that the coffee-making process will remain in statistical control, it is reasonable to conclude that the temperature of most of the coffee that will be or could potentially be served will be between 67°C and 77°C.

To emphasize the importance of statistical control, suppose that another fast-food restaurant observes the 24 coffee temperatures that are plotted versus time in Figure 1.4. These temperatures range between 67°C and 80°C. However, we cannot infer from this that the temperature of most of the coffee that will be or could potentially be served by this other restaurant will be between 67°C and 80°C. This is because the downward trend in the runs plot of Figure 1.4 indicates that the coffee-making process is out of control and will soon produce temperatures below 67°C. Another example of an out-of-control process is illustrated in Figure 1.5. Here the coffee temperatures seem to fluctuate around a constant level but with increasing variation (notice that the plotted temperatures fan out as time advances). In general, the specific pattern of out-of-control behaviour can suggest the reason for this behaviour. For example, the downward trend in the runs plot of Figure 1.4 might suggest that the restaurant's coffeemaker has a defective heating element.

Visually inspecting a runs plot to check for statistical control can be tricky. One reason is that the scale of measurements on the vertical axis can influence whether the data appear to form a horizontal band. For now, we will simply emphasize that a process must be in statistical control in order to make valid statistical inferences about the population of all possible process observations. Also, note that being in statistical control does not necessarily imply that a process is **capable** of producing output that meets our requirements. For example, suppose that marketing research suggests that the fast-food restaurant's customers feel that coffee tastes best if its temperature is between 67°C and 75°C. Since Table 1.6 indicates that the temperature of some of the coffee it serves is not in this range (note that two of the temperatures are 67°C, one is 76°C, and another is 77°C), the restaurant might take action to reduce the variation of the coffee temperatures.

**FIGURE 1.4** A Runs Plot of Coffee Temperatures: The Process Level Is Decreasing

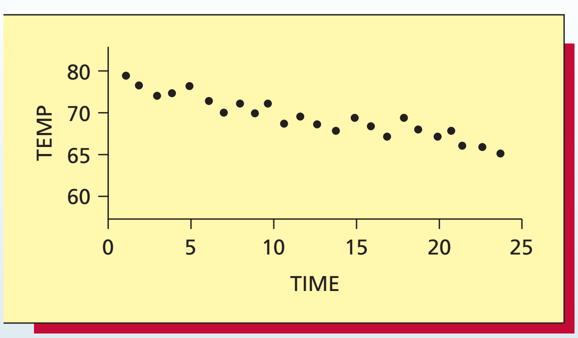
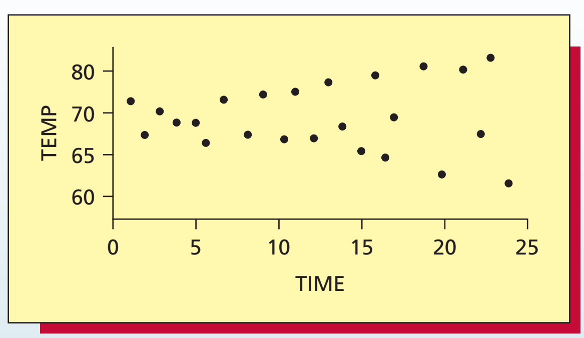



**FIGURE 1.5** A Runs Plot of Coffee Temperatures: The Process Variation Is Increasing



The marketing research and coffee temperature cases are both examples of using the **statistical process** to make a statistical inference. In the next case, we formally describe and illustrate this process.

## Example 1.4 The Mass of the Loonie

Source: Coin image © 2007 Royal Canadian Mint—All Rights Reserved.

In 1989, Mr. Steve Kopp, a lecturer in the Department of Statistical and Actuarial Sciences at the University of Western Ontario, decided to weigh 200 one dollar coins (loonies) that were minted in that year. He was curious about the distribution of the mass of the loonie.

From a production standpoint, the loonie would have to be minted within strict specifications. The loonie does in fact have specific minting requirements:

**Composition:** 91.5% nickel with 8.5% bronze plating
**Mass (g):** 7
**Diameter (mm):** 26.5
**Thickness (mm):** 1.75

A person at the Royal Canadian Mint might be interested in knowing if the minted coins fall within an acceptable tolerance. Remember, these loonies cannot be too light or too heavy, as vending machines are set to accept coins according to mass and size. As a statistician, you may be interested in testing a hypothesis about the mass of the coin. We will use this sample of 200 loonies to ultimately draw conclusions about the entire population of loonies minted in 1989. The steps used in the **statistical process** for making a statistical inference are as follows:

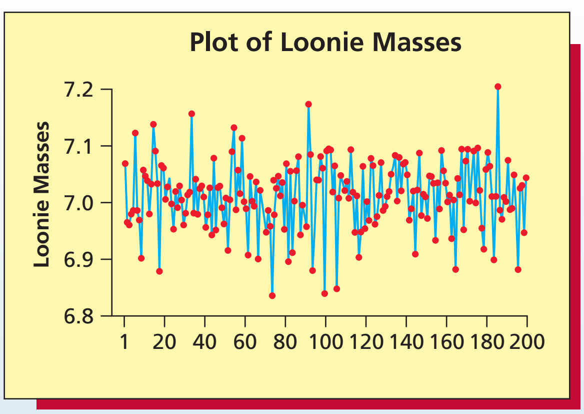
1. **Describe the practical problem of interest and the associated population or process to be studied.** We wish to ultimately determine whether or not the coins are being minted in an acceptable and consistent manner. The coin minter will use statistical processes on a sample to study the population of coins that were minted during that year.
2. **Describe the variable of interest and how it will be measured.** The variable of interest is the mass of the loonie (in grams). The mass was obtained using a highly sensitive scale that gives masses in grams to four decimal places.
3. **Describe the sampling procedure.** A sample of 200 coins was obtained from a local bank in London, Ontario. The coins are packaged in rolls of 25, so eight rolls were obtained at random. These coins may or may not have come from the same production run, but we do know that they were minted in the same year (1989). Each coin was carefully weighed, and the masses are given in Table 1.7.
4. **Describe the statistical inference of interest.** The sample of 200 loonies will be used to determine if the distribution of masses follows any specific type of distribution, and we are also interested in knowing if the coins are being minted within the required mass specifications.
5. **Describe how the statistical inference will be made and evaluate the reliability of the inference.** Figure 1.6 gives the MINITAB output of a plot of the 200 masses. Remember, we do not know if the coins were minted in different production runs, but we do know that they could not have all been minted at the same time, so this sample of coins was in fact minted over a period of time. If it's reasonable to believe that loonie masses will remain in control, we can make statistical inferences about the mass of the coin. For example, in Table 1.7 we see that the masses of the coins range between 6.8358 g and 7.2046 g, so we might infer that most loonies would be somewhere between these two masses. In order to determine the "typical" mass of the loonie population, we might try to determine the midpoint of this sample range. When we do this, we get 7.0202 g. Therefore, we might conclude that the typical mass for the entire population of loonies minted is around 7.0202 g. According to specifications outlined on http://www.mint.ca, our sample of coins appears to meet the required standard of a mass of 7 g for each loonie. More analysis would have to be done to arrive at this conclusion, however. This estimate is intuitive, so we do not

**TABLE 1.7** Loonie Mass Data Loonies

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 7.0688 | 7.0196 | 7.008 | 7.0252 | 7.0912 | 6.9753 | 6.9720 | 7.0963 |
| 6.9651 | 6.9911 | 6.9156 | 7.0466 | 7.0948 | 7.0127 | 7.0470 | 7.0215 |
| 6.9605 | 7.0294 | 7.0050 | 7.0119 | 7.0929 | 7.0706 | 7.0459 | 6.9549 |
| 6.9797 | 7.0045 | 7.0898 | 7.0354 | 7.0186 | 6.9861 | 7.0339 | 6.9178 |
| 6.9861 | 6.9605 | 7.1322 | 6.9528 | 7.0648 | 6.9920 | 6.9334 | 7.0584 |
| 7.1227 | 6.9812 | 6.9873 | 7.0686 | 6.8479 | 7.0106 | 7.0340 | 7.0884 |
| 6.9861 | 7.0136 | 7.0572 | 6.8959 | 7.0079 | 7.0195 | 6.9888 | 7.0641 |
| 6.9692 | 7.0185 | 7.0158 | 7.0552 | 7.0478 | 7.0500 | 7.0919 | 7.0107 |
| 6.9018 | 7.1567 | 7.1135 | 6.9117 | 7.0346 | 7.0627 | 7.0561 | 6.8990 |
| 7.0574 | 6.9814 | 7.0016 | 7.0026 | 7.0212 | 7.0833 | 7.0343 | 7.0111 |
| 7.0467 | 7.0413 | 6.9892 | 7.0563 | 7.0374 | 7.0027 | 7.0012 | 7.2046 |
| 7.0386 | 6.9793 | 6.9074 | 7.0810 | 7.0076 | 7.0797 | 7.0132 | 6.9867 |
| 6.9799 | 7.0245 | 7.0461 | 6.9430 | 7.0934 | 7.0207 | 6.9364 | 6.9705 |
| 7.0326 | 7.0295 | 7.0024 | 6.9955 | 7.0184 | 7.0681 | 7.0046 | 7.0092 |
| 7.1380 | 7.0099 | 6.9936 | 6.9784 | 6.9475 | 7.0708 | 6.8821 | 7.0009 |
| 7.0908 | 6.9563 | 7.0364 | 6.9575 | 7.0118 | 7.0490 | 7.0426 | 7.0746 |
| 7.0335 | 6.9785 | 6.9005 | 7.1735 | 6.9034 | 6.9690 | 7.0137 | 6.9876 |
| 6.8788 | 7.0260 | 7.0216 | 7.0847 | 6.9481 | 6.9891 | 7.0943 | 6.9898 |
| 7.0654 | 6.9428 | 6.9986 | 6.8801 | 7.0640 | 7.0203 | 6.9521 | 7.0489 |
| 7.0610 | 7.0784 | 6.9741 | 6.9491 | 6.9541 | 6.9091 | 7.0732 | 6.9874 |
| 7.0057 | 6.9516 | 6.9477 | 7.0401 | 7.0017 | 7.0222 | 7.0941 | 6.8818 |
| 7.0277 | 7.0264 | 6.9862 | 7.0396 | 6.9685 | 7.0874 | 7.0024 | 7.0253 |
| 7.0438 | 7.0291 | 6.9582 | 7.0812 | 7.0780 | 6.9771 | 7.0463 | 7.0304 |
| 6.9977 | 6.9909 | 6.8358 | 7.0607 | 7.0652 | 7.0148 | 7.0909 | 6.9469 |
| 6.9531 | 6.9623 | 6.9785 | 6.8395 | 6.9618 | 7.0401 | 6.9994 | 7.0438 |

have any information about its **reliability**. In Chapter 2, we will study more precise ways to both define and estimate a “typical” population value. In Chapters 3 through 7, we will study tools for assessing the reliability of estimation procedures and for estimating “with confidence.”

**FIGURE 1.6**



## Exercises for Section 1.3

### CONCEPTS

**1.11** Define a **process**. Then give an example of a process you might study when you start your career after graduating from university.

**1.12** Explain what it means to say that a process is in statistical control.

**1.13** What is a runs plot? What does a runs plot look like when we sample and plot a process that is in statistical control?

### METHODS AND APPLICATIONS

**1.14** The data below give 18 measurements of a critical dimension for an automobile part (measurements in centimetres). Here one part has been randomly selected each hour from the previous hour's production, and the measurements are given in time order. AutoPart1

| Hour | Measurement | Hour | Measurement |
|---|---|---|---|
| 1 | 3.005 | 10 | 3.005 |
| 2 | 3.020 | 11 | 3.015 |
| 3 | 2.980 | 12 | 2.995 |
| 4 | 3.015 | 13 | 3.020 |
| 5 | 2.995 | 14 | 3.000 |
| 6 | 3.010 | 15 | 2.990 |
| 7 | 3.000 | 16 | 2.985 |
| 8 | 2.985 | 17 | 3.020 |
| 9 | 3.025 | 18 | 2.985 |

Construct a runs plot and determine if the process appears to be in statistical control.

**1.15** Table 1.8 presents the time (in days) needed to settle the 67 homeowners' insurance claims handled by an insurance agent over a year. The claims are given in time order by loss date. ClaimSet

**a.** Figure 1.7 shows a MINITAB runs plot of the claims data in Table 1.8. Does the claims-handling process seem to be in statistical control? Why or why not?

**b.** In March of 2005, the region covered by the insurance company was hit by a widespread ice storm that caused heavy damage to homes in the area. Did this ice storm have a significant impact on the time needed to settle homeowners' claims? Should the agent consider improving procedures for handling claims in emergency situations? Why or why not?

**1.16** In the article "Accelerating improvement" published in *Quality Progress* (October 1991), Gaudard, Coates, and Freeman describe a restaurant that caters to business travellers and has a self-service breakfast buffet. Interested in customer satisfaction, the manager conducts a survey over a three-week period and finds that the main customer complaint is having to wait too long to be seated. On each day from September 11, 1989, to October 1, 1989, a problem-solving team records the percentage of patrons who must wait more than one minute to be seated. A runs plot of the daily

**TABLE 1.8 Number of Days Required to Settle Homeowners' Insurance Claims (Claims Made from July 2, 2004 to June 25, 2005)** ClaimSet

| Claim | Loss Date | Days to Settle | Claim | Loss Date | Days to Settle | Claim | Loss Date | Days to Settle |
|---|---|---|---|---|---|---|---|---|
| 1 | 2004-07-02 | 111 | 24 | 2004-11-05 | 34 | 47 | 2005-03-05 | 70 |
| 2 | 2004-07-06 | 35 | 25 | 2004-11-13 | 25 | 48 | 2005-03-05 | 67 |
| 3 | 2004-07-11 | 23 | 26 | 2004-11-21 | 22 | 49 | 2005-03-06 | 81 |
| 4 | 2004-07-12 | 42 | 27 | 2004-11-23 | 14 | 50 | 2005-03-06 | 92 |
| 5 | 2004-07-16 | 54 | 28 | 2004-11-25 | 20 | 51 | 2005-03-06 | 96 |
| 6 | 2004-07-27 | 50 | 29 | 2004-12-01 | 32 | 52 | 2005-03-06 | 85 |
| 7 | 2004-08-01 | 41 | 30 | 2004-12-08 | 27 | 53 | 2005-03-07 | 83 |
| 8 | 2004-08-13 | 12 | 31 | 2004-12-10 | 23 | 54 | 2005-03-07 | 102 |
| 9 | 2004-08-20 | 8 | 32 | 2004-12-20 | 35 | 55 | 2005-03-19 | 23 |
| 10 | 2004-08-20 | 11 | 33 | 2004-12-23 | 29 | 56 | 2005-03-27 | 11 |
| 11 | 2004-08-28 | 11 | 34 | 2004-12-31 | 25 | 57 | 2005-04-01 | 8 |
| 12 | 2004-09-03 | 31 | 35 | 2004-12-31 | 18 | 58 | 2005-04-11 | 11 |
| 13 | 2004-09-10 | 35 | 36 | 2004-12-31 | 16 | 59 | 2005-04-15 | 35 |
| 14 | 2004-09-17 | 14 | 37 | 2005-01-05 | 23 | 60 | 2005-04-19 | 29 |
| 15 | 2004-09-18 | 14 | 38 | 2005-01-08 | 26 | 61 | 2005-05-02 | 80 |
| 16 | 2004-09-29 | 27 | 39 | 2005-01-16 | 30 | 62 | 2005-05-15 | 18 |
| 17 | 2004-10-04 | 14 | 40 | 2005-01-18 | 36 | 63 | 2005-05-25 | 58 |
| 18 | 2004-10-06 | 23 | 41 | 2005-01-22 | 42 | 64 | 2005-06-06 | 4 |
| 19 | 2004-10-15 | 47 | 42 | 2005-01-25 | 45 | 65 | 2005-06-12 | 5 |
| 20 | 2004-10-23 | 17 | 43 | 2005-01-27 | 43 | 66 | 2005-06-24 | 15 |
| 21 | 2004-10-25 | 21 | 44 | 2005-02-05 | 39 | 67 | 2005-06-25 | 19 |
| 22 | 2004-10-30 | 18 | 45 | 2005-02-09 | 53 | | | |
| 23 | 2004-11-02 | 31 | 46 | 2005-02-23 | 64 | | | |

**FIGURE 1.7** MINITAB Runs Plot of the Insurance Claims Data for Exercise 1.15

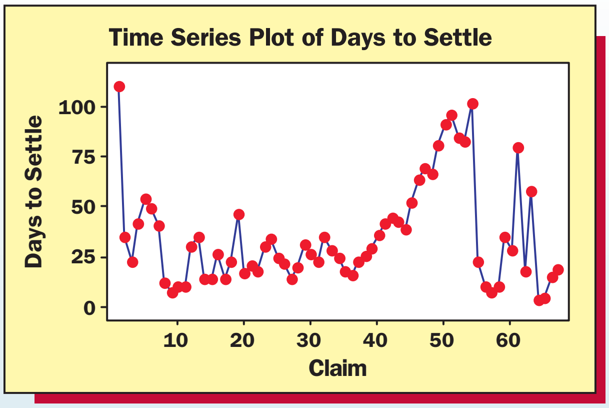
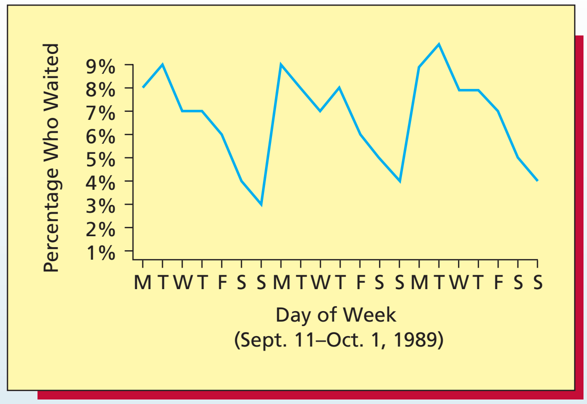



**FIGURE 1.8** Runs Plot of Daily Percentages of Customers Waiting More Than One Minute to Be Seated (for Exercise 1.16)




percentages is shown in Figure 1.8.[9] What does the runs plot suggest?

**1.17 THE TRASH BAG CASE**[10] TrashBag

A company that produces and markets trash bags has developed an improved 130-L bag. The new bag is produced using a specially formulated plastic that is both stronger and more biodegradable than previously used plastics, and the company wishes to evaluate the strength of this bag. The **breaking strength** of a trash bag is considered to be the mass (in kilograms) of a representative trash mix that when loaded into a bag suspended in the air will cause the bag to sustain significant damage (such as ripping or tearing). The company has decided to carry out a 40-hour pilot production run of the new bags. Each hour, at a randomly selected time during the hour, a bag is taken off the production line. The bag is then subjected to a breaking strength test. The 40 breaking strengths obtained during the pilot production run are given in Table 1.9, and an Excel runs plot of these breaking strengths is given in Figure 1.9.

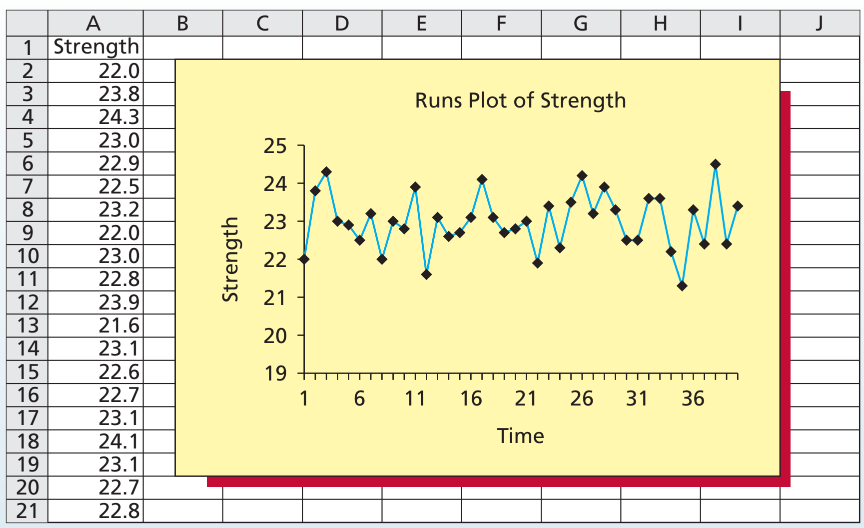
**a.** Do the 40 breaking strengths appear to be in statistical control? Explain.

**b.** Estimate limits between which most of the breaking strengths of all trash bags would fall.

**TABLE 1.9** Breaking Strengths TrashBag

| | | | |
|---|---|---|---|
| 22.0 | 23.9 | 23.0 | 22.5 |
| 23.8 | 21.6 | 21.9 | 23.6 |
| 24.3 | 23.1 | 23.4 | 23.6 |
| 23.0 | 22.6 | 22.3 | 22.2 |
| 22.9 | 22.7 | 23.5 | 21.3 |
| 22.5 | 23.1 | 24.2 | 23.3 |
| 23.2 | 24.1 | 23.2 | 22.4 |
| 22.0 | 23.1 | 23.9 | 24.5 |
| 23.0 | 22.7 | 23.3 | 22.4 |
| 22.8 | 22.8 | 22.5 | 23.4 |

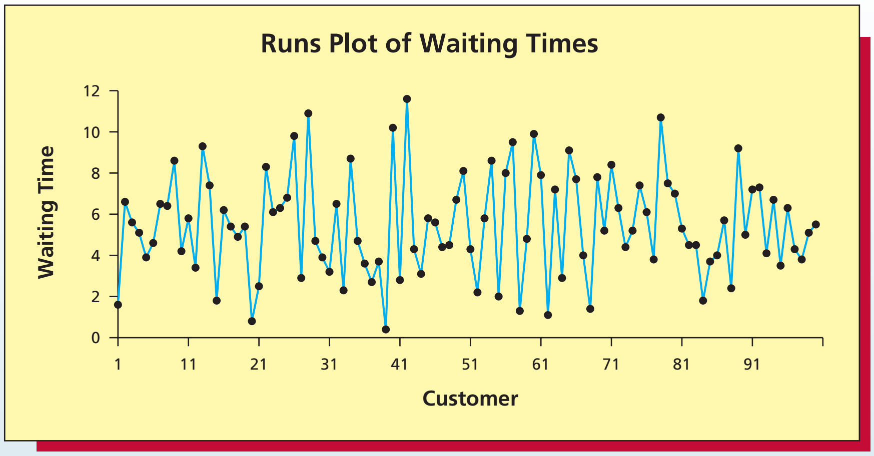
**FIGURE 1.9** Excel Runs Plot of Breaking Strengths for Exercise 1.17




[9]The source of Figure 1.8 is "Accelerating improvement," by M. Gaudard, R. Coates, and L. Freeman, *Quality Progress*, October 1991, pp. 81–88. Copyright © 1991 American Society for Quality Control. Used with permission.

[10]This case is based on conversations by the authors with several employees working for a leading producer of trash bags. For purposes of confidentiality, we have withheld the company's name.

**FIGURE 1.10** MegaStat Runs Plot of Waiting Times for Exercise 1.18




**1.18 THE BANK CUSTOMER WAITING TIME CASE** WaitTime

Recall that every 150th customer arriving during peak business hours was sampled until a systematic sample of 100 customers was obtained. This systematic sampling procedure is equivalent to sampling from a process. Figure 1.10 shows a MegaStat runs plot of the 100 waiting times in Table 1.5. Does the process appear to be in statistical control? Explain.

## 1.4 Levels of Measurement: Nominal, Ordinal, Interval, and Ratio

In Section 1.1, we said that a variable is **quantitative** if its possible values are **numbers that represent quantities** (that is, "how much" or "how many"). In general, a quantitative variable is measured on a scale with a **fixed unit of measurement** between its possible values. For example, if we measure employees' salaries to the nearest dollar, then one dollar is the fixed unit of measurement between different employees' salaries. There are two types of quantitative variables: **ratio** and **interval**. A **ratio variable** is a quantitative variable measured on a scale such that ratios of its values are meaningful and there is an inherently defined zero value. Variables such as salary, height, weight, time, and distance are ratio variables. For example, a distance of zero kilometres is "no distance at all," and a town that is 30 km away is "twice as far" as a town that is 15 km away.

An **interval variable** is a quantitative variable where ratios of its values are not meaningful and there is no meaningful zero. The 1 to 7 Likert scale example given earlier is an example of an interval scale. The distance from 2 to 3 is the same as that from 5 to 6. The scale could also have been

$$-3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3$$

Here the zero is the midpoint and represents the same concept as the number 4 in the 1 to 7 scale.

In Section 1.1, we also said that if we simply record into which of several categories a population (or sample) unit falls, then the variable is **qualitative** (or **categorical**). There are two types of qualitative variables: **ordinal** and **nominative** (or **nominal**). An **ordinal variable** is a qualitative variable for which there is a meaningful **ordering**, or **ranking**, of the categories. The measurements of an ordinal variable may be nonnumerical or numerical. For example, a student may be asked to rank their four favourite colours. The person may say that yellow (#1) is their most favourite colour, then green (#2), red (#3), and blue (#4). If asked further, the person may say they really adore yellow and green and that red and blue are "so-so" but that red is slightly better than blue. This ranking does not have equal distances between points in that 1 to 2 is not the same as 2 to 3. Only the order (of preference) is meaningful. In Chapter 13, we will learn how to use **nonparametric statistics** to analyze an ordinal variable without considering the variable to be somewhat quantitative and performing such arithmetic operations. In addition to these four types of variables, data may also take the form of continuous or discrete values. Continuous variables are typically interval or ratio scale numbers and fall along a continuum so that decimals make sense (such as salaries, age, mass, and height). In contrast, discrete variables are nominal or ordinal and represent distinct groups in which decimals do not make sense (such as sex categorization, living in urban or rural communities, and number of children in a household).

To conclude this section, we consider the second type of qualitative variable. A **nominative** (or **nominal) variable** is a qualitative variable for which there is no meaningful ordering, or ranking, of the categories. A person's sex, the colour of a car, and an employee's city of residence are nominative (or nominal) variables.[11]

# Exercises for Section 1.4

### CONCEPTS

**1.19** Discuss the difference between a ratio variable and an interval variable.

**1.20** Discuss the difference between an ordinal variable and a nominative (or nominal) variable.

### METHODS AND APPLICATIONS

**1.21** Classify each of the following qualitative variables as ordinal or nominative (or nominal). Explain your answers.

| Qualitative Variable | Categories |
|---|---|
| Statistics course letter mark | A B C D F |
| Door choice on *Let's Make a Deal* | Door #1 Door #2 |
| Television show classifications | C C8 G PG 14+ 18+ |
| Personal computer ownership | Yes No |
| Restaurant rating | ***** **** *** ** * |
| Income tax filing status | Married Living common-law Widowed Divorced Separated Single |

**1.22** Classify each of the following qualitative variables as ordinal or nominative (or nominal). Explain your answers.

| Qualitative Variable | Categories |
|---|---|
| Personal computer operating system | Windows XP Mac 05-X Windows Unix Linux Other Vista |
| Motion picture classifications | G PG 14A 18A R A |
| Level of education | Elementary Middle school High school University Graduate school |
| Rankings of top 10 university hockey teams | 1 2 3 4 5 6 7 8 9 10 |
| Exchange on which a stock is traded | S&P/TSX DJIA S&P 500 NASDAQ |
| First three characters of postal code | B3J M1J T2K V7E |

## 1.5 A Brief Introduction to Surveys

The Likert scale was introduced in Section 1.2 and has proven to be a valuable method of measuring topics such as attitudes (such as job satisfaction), values (such as organizational commitment), personality traits, and market research feedback. This section is a brief introduction to survey types and some issues that arise with surveys.

Surveys are also known as questionnaires. The purpose of surveys is typically to elicit responses from the participants. There are typically four steps involved in creating a survey. The first involves deciding upon the content (what is being studied and how the questions will be asked). Question types can vary. For example, the surveyor may want to know factual information (such as demographics of age, sex, and income). The variable of interest might be behavioural (such as what the person does on their holidays). The questions may also be opinion based (such as what fragrance a person prefers in their laundry detergent). Basically, the questions can be about anything of interest to the surveyor.

After the content has been decided upon, the questionnaire creator generates the questions. It is ideal if these questions are as short as possible and are easy to read and understand. Following

[11]Study Hint: To remember the levels of measurement, simply remember the French word for "black" (NOIR). This acronym is useful since it also puts the levels into order from simplest level of measurement (nominal or nominative) to most complex (ratio).

the question creation, the response key has to be decided upon. Here there are two options: open and closed. Open-ended questions are ones in which the respondent can answer the question in any manner they wish. These types of responses provide rich information but are difficult to score or code. The closed-ended questions represent those that give the respondent a choice of answers. These responses are typically much easier to code and quantify.

Once the questions and the response system are determined, the questionnaire is compiled. The order of the questions is important, as questions themselves may influence people's responses to following questions. To address the quality of the survey created, the surveyor must complete the fourth step, which is to pilot test the questionnaire and address issues such as stability (reliability) and validity (do the questions actually measure what they were intended to measure?). Following the creation of the survey, the delivery of the questionnaire must be determined.

In general, surveys are delivered using one of three methods: mailed (direct or mass/bulk), telephone, and in-person. Mailed surveys are relatively inexpensive and unobtrusive, but tend to have low response rates (the number of people who complete the survey compared to the number of surveys sent out). Other concerns with mailing surveys is that you are never certain that the person who completed the survey is the person you wanted to complete the survey. As a researcher you also are never certain that the person completing the survey fully understood your questions. In general, if you plan to use mailed surveys, pretest the survey with members of your target audience. A recent trend and variation on the mailed survey is online surveys, but the same concerns with mailed surveys hold true for these as well.

Telephone surveys have increased in popularity with the increase in the number of telephones in people's homes. Using the telephone is less expensive than in-person interviews and tends to be faster. For example, surveys are conducted using telephones by organizations such as Environics Research Group and Ipsos Canada, which has offices across the country. Results from telephone surveys can be conveyed to the public almost immediately. Surveyors can cover a wide geographical region without having to travel. Historically, surveyors used telephone directories to contact people. Most surveyors now use random digit dialling (RDD), which uses the same logic underlying the random number table presented near the start of this chapter. When a surveyor uses RDD, there is an equal probability of any telephone number appearing (including unlisted numbers). The drawbacks are that RDD will also produce telephone numbers that are not in use, fax machine numbers, and nonresidential numbers. The other concerns that telephone surveyors have are the growing public wariness of telemarketers and reluctance to participate in telephone surveys.

A common type of survey is the in-person interview. The face-to-face method is the richest form of communication. The participant in the survey can ask for clarification of the questions. But the in-person method is costly and may be perceived as more intrusive.

In general, there are three types of in-person interviews. The first is the structured interview, in which each respondent is given the same questions in the same order. Many businesses now use this method when interviewing job candidates. The interviewer is trained to act in the same manner for each interviewee. Answers given by respondents are then scored. The second in-person interview type is the intensive interview. Here the style is unstructured and informal. Interviewees are not given the same questions in the same order as in the structured method. This method is typically used in career counselling, performance appraisal feedback, and clinical settings. The third method is the focus group. The logic behind the focus group is that a group of people will provide more information than will individuals. The groups typically range in size from 4 to 15 people, and they will discuss approximately 10 issues. This method is common for market research. In it, responses are coded by a moderator and by observers of the group.

## Exercises for Section 1.5

**1.23** Describe the steps involved in creating a questionnaire.

**1.24** Give an example of how the content of an item might influence responses to subsequent items.

**1.25** What are the benefits and drawbacks of using each of the three methods of surveying?

**a.** In-person. **b.** Mailed. **c.** Telephone.

**1.26** Explain what is meant by a "focus group." When would a researcher use a focus group?

**1.27** Explain how you would go about requesting that people complete an online survey. How would you contact the people? How would you deal with the question of whether or not the person you contacted was the person who completed the survey?

## 1.6 An Introduction to Survey Sampling

Random sampling is not the only type of sampling. Methods for obtaining a sample are called **sampling designs**, and the sample we take is sometimes called a **sample survey**. In this section, we explain three sampling designs that are alternatives to random sampling—**stratified random sampling**, **cluster sampling**, and **systematic sampling**.

One common sampling design involves separately sampling important groups within a population. Then the samples are combined to form the entire sample. This approach is the idea behind **stratified random sampling**.

In order to select a **stratified random sample**, we divide the population into nonoverlapping groups of similar units (people, objects, etc.). These groups are called **strata**. Then a random sample is selected from each stratum, and these samples are combined to form the full sample.

It is wise to stratify when the population consists of two or more groups that differ with respect to the variable of interest. For instance, consumers could be divided into strata based on sex, age, language (e.g., speaks English, French, or other), or income.

As an example, suppose that a department store chain proposes to open a new store in a location that would serve customers who live in a geographical region that consists of (1) an industrial city, (2) a suburban community, and (3) a rural area. In order to assess the potential profitability of the proposed store, the chain wishes to study the incomes of all households in the region. In addition, the chain wishes to estimate the proportion and the total number of households whose members would be likely to shop at the store. The department store chain feels that the industrial city, the suburban community, and the rural area differ with respect to income and the store's potential desirability. Therefore, it uses these subpopulations as strata and takes a stratified random sample.

Taking a stratified sample can be advantageous because such a sample takes advantage of the fact that units in the same stratum are similar to each other. It follows that a stratified sample can provide more accurate information than a random sample of the same size. As a simple example, if all of the units in each stratum were exactly the same, then examining only one unit in each stratum would allow us to describe the entire population. Furthermore, stratification can make a sample easier (or possible) to select. Recall that, in order to take a random sample, we must have a frame, or list, of all of the population units. Although a frame might not exist for the overall population, a frame might exist for each stratum. For example, suppose nearly all the households in the department store's geographical region have telephones. Although there might not be a telephone directory for the overall geographical region, there might be separate telephone directories for the industrial city, the suburb, and the rural area from which samples could be drawn (although recall some of the drawbacks of telephone surveying listed in the previous section).

Sometimes it is advantageous to select a sample in stages. This is a common practice when selecting a sample from a very large geographical region. In such a case, a frame often does not exist. For instance, there is no single list of all households in Canada. In this situation, we can use **multistage cluster sampling**. To illustrate this procedure, suppose we wish to take a sample of households from all households in Canada. We might proceed as follows:

**Stage 1:** Randomly select a sample of counties from all of the counties in Canada.
**Stage 2:** Randomly select a sample of townships in each county.
**Stage 3:** Randomly select a sample of households from each township.

We use the term *cluster sampling* to describe this type of sampling because at each stage we "cluster" the households into subpopulations. For instance, in Stage 1 we cluster the households into counties, and in Stage 2 we cluster the households in each county into townships. Also, notice that the random sampling at each stage can be carried out because there are lists of (1) all counties in Canada, (2) all townships in Canada, and (3) all households in each township.

As another example, consider another way of sampling the households in Canada. We might use Stages 1 and 2 above to select counties and townships within the selected counties. Then, if there is a telephone directory of the households in each township, we can randomly sample households from each selected township by using its telephone directory. Because most households today have telephones, and telephone directories are readily available, most national polls are now conducted by telephone.

It is sometimes a good idea to combine stratification with multistage cluster sampling. For example, suppose a national polling organization wants to estimate the proportion of all registered voters who favour a particular federal party. Because the federal party preferences of voters might tend to vary by geographical region, the polling organization might divide Canada into regions (say, Atlantic Canada, Québec, Ontario, and Western Canada). The polling organization might then use these regions as strata and might take a multistage cluster sample from each stratum (region).[12]

In order to select a random sample, we must number the units in a frame of all the population units. Then we use a random number table (or a random number generator on a computer) to make the selections. However, numbering all the population units can be quite time-consuming. Moreover, random sampling is used in the various stages of many complex sampling designs (requiring the numbering of numerous populations). Therefore, it is useful to have an alternative to random sampling. One such alternative is called **systematic sampling**. In order to systematically select a sample of $n$ units without replacement from a frame of $N$ units, we divide $N$ by $n$ and round the result down to the nearest whole number. Calling the rounded result $\ell$, we then randomly select one unit from the first $\ell$ units in the frame—this is the first unit in the systematic sample. The remaining units in the sample are obtained by selecting every $\ell$th unit following the first (randomly selected) unit. For example, suppose we wish to sample a population of $N = 14{,}327$ members of an international allergists' association to investigate how often they have prescribed a particular drug during the last year. The association has a directory listing the 14,327 allergists, and we wish to draw a systematic sample of 500 allergists from this frame. Here we compute $14{,}327/500 = 28.654$, which is 28 when rounded down. Therefore, we number the first 28 allergists in the directory from 1 to 28, and we use a random number table to randomly select one of the first 28 allergists. Suppose we select allergist number 19. We interview allergist 19 and every 28th allergist in the frame thereafter, so we choose allergists 19, 47, 75, and so forth until we obtain our sample of 500 allergists. In this scheme, we must number the first 28 allergists, but we do not have to number the rest because we can "count off" every 28th allergist in the directory. Alternatively, we can measure the approximate amount of space in the directory that it takes to list 28 allergists. This measurement can then be used to select every 28th allergist.

In this book, we concentrate on showing how to analyze data produced by random sampling. However, if the order of the population units in a frame is random with respect to the characteristic under study, then a systematic sample should be (approximately) a random sample and we can analyze the data produced by the systematic sample by using the same methods employed to analyze random samples. For instance, it would seem reasonable to assume that the alphabetically ordered allergists in a medical directory would be random (that is, have nothing to do with the number of times the allergists prescribed a particular drug). Similarly, the alphabetically ordered people in a telephone directory would probably be random with respect to many of the people's characteristics that we might wish to study.

When we employ random sampling, we eliminate bias in the choice of the sample from a frame. However, a proper sampling design does not guarantee that the sample will produce accurate information. One potential problem is **undercoverage**.

**Undercoverage** occurs when some population units are excluded from the process of selecting the sample.

This problem occurs when we do not have a complete, accurate list of all the population units. For example, although telephone polls today are common, some people in Canada do not have telephones. In general, undercoverage usually causes some people to be underrepresented. If underrepresented groups differ from the rest of the population with respect to the characteristic under study, the survey results will be biased. A second potentially serious problem is **nonresponse**.

[12]The analysis of data produced by multistage cluster sampling can be quite complicated. We explain how to analyze data produced by one- and two-stage cluster sampling in Appendix E (Part 2). This appendix also includes a discussion of an additional survey sampling technique called **ratio estimation**. For a more detailed discussion of cluster sampling and ratio estimation, see Mendenhall, Schaeffer, and Ott (1986).

**Nonresponse** occurs when a population unit selected as part of the sample cannot be contacted or refuses to participate.

In some surveys, 35 percent or more of the selected individuals cannot be contacted—even when several callbacks are made. In such a case, other participants are often substituted for the people who cannot be contacted. If the substitute participants differ from the originally selected participants with respect to the characteristic under study, the survey will again be biased. Third, when people are asked potentially embarrassing questions, their responses might not be truthful. We then have what we call **response bias**. Fourth, the wording of the questions asked can influence the answers received. Slanted questions often evoke biased responses. For example, consider the following question:

Which of the following best describes your views on gun control?

1. The government should take away our guns, leaving us defenceless against heavily armed criminals.
2. We have the right to keep guns.

## Exercises for Section 1.6

### CONCEPTS

**1.28** When is it appropriate to use stratified random sampling? What are strata, and how should strata be selected?

**1.29** When is cluster sampling used? Why do we describe this type of sampling by using the term *cluster?*

**1.30** Explain each of the following terms:
- **a.** Undercoverage.
- **b.** Nonresponse.
- **c.** Response bias.

**1.31** Explain how to take a systematic sample of 100 companies from the 1,853 companies that are members of an industry trade association.

**1.32** Explain how a stratified random sample is selected. Discuss how you might define the strata to survey student opinion on a proposal to charge all students a $100 fee for a new university-run bus system that will provide transportation between off-campus apartments and campus locations.

**1.33** Marketing researchers often use city blocks as clusters in cluster sampling. Using this fact, explain how a market researcher might use multistage cluster sampling to select a sample of consumers from all cities with a population of more than 10,000 in a region having many such cities.

# CHAPTER SUMMARY

This chapter has introduced the idea of using **sample data** to make **statistical inferences**—that is, drawing conclusions about populations and processes by using sample data. We began by learning that a **population** is a set of existing units that we wish to study. We saw that, since many populations are too large to examine in their entirety, we often study a population by selecting a **sample**, which is a subset of the population units. Next we learned that, if the information contained in a sample is to accurately represent the population, then the sample should be **randomly selected** from the population, and we saw how **random numbers** (obtained from a **random number table**) can be used to select a **random sample**. We also learned that selecting a random sample requires a **frame** (that is, a list of all of the population units) and that, since a frame does not always exist, we sometimes select a **systematic sample**.

We continued this chapter by studying **processes**. We learned that to make statistical inferences about the population of all possible values of a variable that could be observed when using a process, the process must be in **statistical control**. We learned that a process is in statistical control if it does not exhibit any unusual process variations, and we demonstrated how we might sample a process and how to use a runs plot to try to judge whether a process is in control.

Next, in Section 1.4, we studied different types of quantitative and qualitative variables. We learned that there are two types of **quantitative variables**—**ratio variables**, which are measured on a scale such that ratios of its values are meaningful and there is an inherently defined zero value, and **interval variables**, for which ratios are not meaningful and there is no inherently defined zero value. We also saw that there are two types of **qualitative variables**—**ordinal variables**, for which there is a meaningful ordering of the categories, and **nominative** (or **nominal**) **variables**, for which there is no meaningful ordering of the categories.

We concluded this chapter with Sections 1.5 and 1.6, which discuss **survey construction**, **types of survey methods**, and **survey sampling**. We introduced **stratified random sampling**, in which we divide a population into groups (**strata**) and then select a random sample from each group. We also introduced **multistage cluster sampling**, which involves selecting a sample in stages, and we explained how to select a **systematic sample**. Finally, we discussed some potential problems encountered when conducting a sample survey—**undercoverage**, **nonresponse**, **response bias**, and slanted questions.

## GLOSSARY OF TERMS

**categorical (qualitative) variable:** A variable with values that indicate into which of several categories a population unit belongs. (page 2)
**census:** An examination of all the units in a population. (page 2)
**cluster sampling (multistage cluster sampling):** A sampling design in which we sequentially cluster population units into subpopulations. (page 21)
**descriptive statistics:** The science of describing the important aspects of a set of measurements. (page 2)
**finite population:** A population that contains a finite number of units. (page 11)
**frame:** A list of all of the units in a population. This is needed in order to select a random sample. (page 4)
**infinite population:** A population that is defined so that there is no limit to the number of units that could potentially belong to the population. (page 11)
**interval variable:** A quantitative variable such that ratios of its values are not meaningful and for which there is not an inherently defined zero value. (page 18)
**measurement:** The process of assigning a value of a variable to each of the units in a population or sample. (page 2)
**nominative (or nominal) variable:** A qualitative variable for which there is no meaningful ordering, or ranking, of the categories. (page 19)
**nonresponse:** A situation in which population units selected to participate in a survey do not respond to the survey instrument. (page 23)
**ordinal variable:** A qualitative variable for which there is a meaningful ordering or ranking of the categories. (page 18)
**population:** A set of existing or potential units (people, objects, events, or the like) that we wish to study. (page 2)
**process:** A sequence of operations that takes inputs and turns them into outputs. (page 11)
**qualitative (categorical) variable:** A variable with values that indicate in which of several categories a population unit belongs. (page 2)
**quantitative variable:** A variable with values that are numbers representing quantities. (page 2)
**random number table:** A table containing random digits that is often used to select a random sample. (page 4)
**random sample:** A sample selected so that, on each selection from the population, every unit remaining in the population on that selection has the same chance of being chosen. (page 3)
**ratio variable:** A quantitative variable such that ratios of its values are meaningful and for which there is an inherently defined zero value. (page 18)
**response bias:** A situation in which survey participants do not respond truthfully to the survey questions. (page 23)
**runs plot:** A graph of individual process measurements versus time. (page 12)
**sample:** A subset of the units in a population. (page 2)
**sampling with replacement:** A sampling procedure in which we place any unit that has been chosen back into the population to give the unit a chance to be chosen on succeeding selections. (page 3)
**sampling without replacement:** A sampling procedure in which we do not place previously selected units back into the population and, therefore, do not give these units a chance to be chosen on succeeding selections. (page 3)
**statistical control:** A state in which a process does not exhibit any unusual variations. Often this means that the process displays a uniform amount of variation around a constant, or horizontal, level. (page 12)
**statistical inference:** The science of using a sample of measurements to make generalizations about the important aspects of a population. (page 3)
**statistical process control (SPC):** A method of analyzing process data in which we monitor and study the process variation. The goal is to stabilize (and reduce) the amount of process variation. (page 12)
**strata:** The subpopulations in a stratified sampling design. (page 21)
**stratified random sampling:** A sampling design in which we divide a population into nonoverlapping subpopulations and then select a random sample from each subpopulation (stratum). (page 21)
**systematic sample:** A sample taken by moving systematically through the population. For instance, we might randomly select one of the first 200 population units and then systematically sample every 200th population unit thereafter. (page 6)
**undercoverage:** A situation in sampling in which some groups of population units are underrepresented. (page 22)
**variable:** A characteristic of a population unit. (page 2)



## SUPPLEMENTARY EXERCISES

**1.34** Some television stations attempt to gauge public opinion by posing a question on the air and asking viewers to call to give their opinions. Suppose that a particular television station asks viewers whether they support or oppose the federal gun registry. Viewers are to call one of two toll-free numbers to register support or opposition. When the results are tabulated, the station reports that 78 percent of those who called are opposed to the registry. What do you think of the sampling method used by the station? Do you think that the percentage of the entire population that opposes the registry is as high as the 78 percent of the sample that was opposed?

**1.35** Table 1.10 gives the "35 best companies to work for" as rated on the *Fortune* magazine Web site on March 14, 2005. Use random numbers to select a random sample of 10 of these companies. Justify that your sample is random by carefully explaining how you obtained it. List the random numbers you used and show how they gave your random sample.

**1.36** A bank wishes to study the amount of time it takes to complete a withdrawal transaction from one of its ABMs (automated banking machines). On a particular day, 63 withdrawal transactions are observed between 10 A.M. and noon. The time required to complete each transaction is given in Table 1.11. Figure 1.11 shows an Excel runs plot of the 63 transaction times. Do the transaction times seem to be in statistical control? Why or why not? ABMTime

**1.37** Figure 1.12 gives a runs plot of the Edmonton Oilers' point percentages (number of points divided by total number of points available for the regular season) from the 1979/1980 season until the 2006/2007 season. Note that no games were played in the 2004/2005 season due to the lockout. The Oilers were one of the best hockey teams of the 1980s. However, many longtime Oilers fans believe that the 1987 trade of Paul Coffey to the Pittsburgh Penguins was the beginning of the team's decline. That supposedly signalled the beginning of the end of the Stanley Cup Championship "dynasty" in Edmonton. Does the runs plot provide any evidence to support this opinion? Why or why not? What else do you notice about the team's point percentage starting in the 1999/2000 season? Can you give any reasons for the sudden change?

**1.38 THE TRASH BAG CASE** TrashBag

Recall that the company will carry out a 40-hour pilot production run of the new bags and will randomly select one bag each hour to be subjected to a breaking strength test.

**a.** Explain how the company can use random numbers to randomly select the times during the 40 hours of the pilot production run at which bags will be tested. Hint: Suppose that a randomly selected time will be determined to the nearest minute.

**b.** Use the following random numbers (obtained from Table 1.1) to select the times during the first five hours at which the first five bags to be tested will be taken from the production line: 61, 15, 64, 07, 86, 87, 57, 64, 66, 42, 59, 51.

**TABLE 1.10** *Fortune's* **35 Best Companies to Work for in March 2005 (for Exercise 1.35)**

| Rank | Company | Rank | Company |
|---|---|---|---|
| 1 | Wegmans Food Markets | 20 | HomeBanc Mortgage |
| 2 | W.L. Gore | 21 | David Weekley Homes |
| 3 | Republic Bancorp | 22 | TD Industries |
| 4 | Genentech | 23 | Valero Energy |
| 5 | Xilinx | 24 | Network Appliance |
| 6 | J.M. Smucker | 25 | JM Family Enterprises |
| 7 | S.C. Johnson & Son | 26 | American Century Investments |
| 8 | Griffin Hospital | 27 | Cisco Systems |
| 9 | Alston & Bird | 28 | American Cast Iron Pipe |
| 10 | Vision Service Plan | 29 | Stew Leonard's |
| 11 | Starbucks | 30 | Whole Foods Market |
| 12 | Quicken Loans | 31 | Baptist Health South Florida |
| 13 | Adobe Systems | 32 | Arnold & Porter |
| 14 | CDW | 33 | Amgen |
| 15 | Container Store | 34 | American Fidelity Assurance |
| 16 | SAS Institute | 35 | Goldman Sachs Group |
| 17 | Qualcomm | | |
| 18 | Robert W. Baird | | |
| 19 | QuikTrip | | |

Source: *Fortune,* March 14, 2005. 

**TABLE 1.11 ABM Transaction Times (in Seconds) for 63 Withdrawals** ABMTime

| Transaction | Time | Transaction | Time | Transaction | Time |
|---|---|---|---|---|---|
| 1 | 32 | 22 | 34 | 43 | 37 |
| 2 | 32 | 23 | 32 | 44 | 32 |
| 3 | 41 | 24 | 34 | 45 | 33 |
| 4 | 51 | 25 | 35 | 46 | 33 |
| 5 | 42 | 26 | 33 | 47 | 40 |
| 6 | 39 | 27 | 42 | 48 | 35 |
| 7 | 33 | 28 | 46 | 49 | 33 |
| 8 | 43 | 29 | 52 | 50 | 39 |
| 9 | 35 | 30 | 36 | 51 | 34 |
| 10 | 33 | 31 | 37 | 52 | 34 |
| 11 | 33 | 32 | 32 | 53 | 33 |
| 12 | 32 | 33 | 39 | 54 | 38 |
| 13 | 42 | 34 | 36 | 55 | 41 |
| 14 | 34 | 35 | 41 | 56 | 34 |
| 15 | 37 | 36 | 32 | 57 | 35 |
| 16 | 37 | 37 | 33 | 58 | 35 |
| 17 | 33 | 38 | 34 | 59 | 37 |
| 18 | 35 | 39 | 38 | 60 | 39 |
| 19 | 40 | 40 | 32 | 61 | 44 |
| 20 | 36 | 41 | 35 | 62 | 40 |
| 21 | 32 | 42 | 33 | 63 | 39 |

**FIGURE 1.11** Excel Runs Plot of ABM Transaction Times for Exercise 1.36

| | A |
|---|---|
| 1 | ABM TIME |
| 2 | 32 |
| 3 | 32 |
| 4 | 41 |
| 5 | 51 |
| 6 | 42 |
| 7 | 39 |
| 8 | 33 |
| 9 | 43 |
| 10 | 35 |
| 11 | 33 |
| 12 | 33 |
| 13 | 32 |
| 14 | 42 |

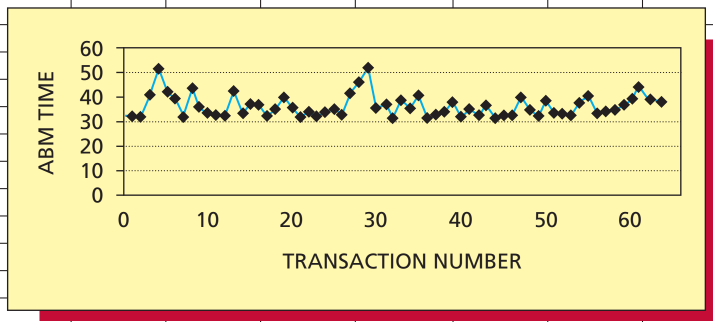



**FIGURE 1.12** Runs Plot of the Edmonton Oilers' Point Percentages from 1979/80 to 2006/07 (for Exercise 1.37)