CHAPTER 7

RISK AND RETURN

LEARNING OBJECTIVES

After studying this chapter you should be able to:

1. understand how return and risk are defined and measured
2. understand the concept of risk aversion by investors
3. explain how diversification reduces risk
4. understand the importance of covariance between returns on risky assets in determining the risk of a portfolio
5. explain the concept of efficient portfolios
6. explain the distinction between systematic and unsystematic risk
7. explain why systematic risk is important to investors
8. explain the relationship between returns and risk proposed by the capital asset pricing model
9. understand the relationship between the capital asset pricing model and models that include additional factors
10. explain the development of models that include additional factors
11. distinguish between alternative methods of appraising the performance of an investment portfolio.

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7.7 Additional factors that explain returns
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INTRODUCTION

7.1

A financial decision typically involves risk. For example, a company that borrows money faces the risk that interest rates may change, and a company that builds a new factory faces the risk that product sales may be lower than expected. These and many other decisions involve future cash flows that are risky. Investors generally dislike risk, but they are also unable to avoid it. The valuation formulae for shares and debt securities outlined in Chapter 4 showed that the price of a risky asset depends on its expected future cash flows, the time value of money, and risk. However, little attention was paid to the causes of risk or to how risk should be defined and measured.

To make effective financial decisions, managers need to understand what causes risk, how it should be measured and the effect of risk on the rate of return required by investors. These issues are discussed in this chapter using the framework of portfolio theory, which shows how investors can maximise the expected return on a portfolio of risky assets for a given level of risk. The relationship between risk and expected return is first described by the capital asset pricing model (CAPM), which links expected return to a single source of risk, and second, by models that include additional factors to explain returns.

To understand the material in this chapter it is necessary to understand what is meant by *return* and *risk*. Therefore, we begin by discussing these concepts.

RETURN AND RISK

7.2

The return on an investment and the risk of an investment are basic concepts in finance. Return on an investment is the financial outcome for the investor. For example, if someone invests $100 in an asset and subsequently sells that asset for $111, the *dollar return* is $11. Usually an investment's dollar return is converted to a *rate of return* by calculating the proportion or percentage represented by the dollar return. For example, a dollar return of $11 on an investment of $100 is a rate of return of $11/$100, which is 0.11, or 11 per cent. In the remainder of this chapter the word *return* is used to mean *rate of return*.

LEARNING OBJECTIVE 1

Understand how return and risk are defined and measured

Risk is present whenever investors are not certain about the outcomes an investment will produce. Suppose, however, that investors can attach a probability to each possible dollar return that may occur. Investors can then draw up a probability distribution for the dollar returns from the investment. A *probability distribution* is a list of the possible dollar returns from the investment together with the probability of each return. For example, assume that the probability distribution in Table 7.1 is an investor's assessment of the dollar returns R_i that may be received from holding a share in a company for 1 year.

TABLE 7.1

DOLLAR RETURN, R_i/($)	PROBABILITY, P_i
9	0.1
10	0.2
11	0.4
12	0.2
13	0.1

Suppose the investor wishes to summarise this distribution by calculating two measures, one to represent the size of the dollar returns and the other to represent the risk involved. The size of the dollar returns may be measured by the expected value of the distribution. The expected value $E(R)$ of the dollar returns is given by the weighted average of all the possible dollar returns, using the probabilities as weights—that is:

$$E(R) = (\$9)(0.1) + (\$10)(0.2) + (\$11)(0.4) + (\$12)(0.2) + (\$13)(0.1)$$
$$= \$11$$

In general, the expected return on an investment can be calculated as:

$$E(R) = R_1P_1 + R_2P_2 + \ldots + R_nP_n$$

which can be written as follows:

$$E(R) = \sum_{i=1}^{n} R_iP_i$$

The choice of a measure for risk is less obvious. In this example, risk is present because any one of five outcomes ($9, $10, $11, $12 or $13) might result from the investment. If the investor had perfect foresight, then only one possible outcome would be involved, and there would not be a probability distribution to be considered. This suggests that risk is related to the dispersion of the distribution. The more dispersed or widespread the distribution, the greater the risk involved. Statisticians have developed a number of measures to represent dispersion. These measures include the range, the mean absolute deviation and the variance. However, it is generally accepted that in most instances the **variance** (or its square root, the **standard deviation**, σ) is the most useful measure. Accordingly, this measure of dispersion is the one we will use to represent the risk of a single investment. The variance of a distribution of dollar returns is the weighted average of the square of each dollar return's deviation from the expected dollar return, again using the probabilities as the weights. For the share considered opposite, the variance is:

variance
measure of variability; the mean of the squared deviations from the mean or expected value

$$\sigma^2 = (9 - 11)^2(0.1) + (10 - 11)^2(0.2) + (11 - 11)^2(0.4) + (12 - 11)^2(0.2) + (13 - 11)^2(0.1)$$
$$= 1.2$$

standard deviation
square root of the variance

In general the variance can be calculated as:

$$\sigma^2 = [R_1 - E(R)]^2P_1 + [R_2 - E(R)]^2P_2 + \ldots + [R_n - E(R)]^2P_n$$

which can be written as follows:

$$\sigma^2 = \sum_{i=1}^{n} \left[R_i - E(R)\right]^2 P_i$$

In this case the variance is 1.2 so the standard deviation is:

$$\sigma = \sqrt{1.2}$$
$$= \$1.095$$

In these calculations we have used dollar returns rather than returns measured in the form of a rate. This is because it is generally easier to visualise dollars than rates, and because it avoids calculations with a large number of zeros following the decimal point. However, there is no difference in substance, as may be seen from reworking the example using returns in rate form. If the sum invested is $100, then a dollar return of $9, for example, is a return of 0.09 when expressed as a rate. Table 7.2 shows rates of return that correspond to the dollar returns in Table 7.1.

TABLE 7.2

RETURN, R_i	PROBABILITY, P_i
0.09	0.1
0.10	0.2
0.11	0.4
0.12	0.2
0.13	0.1

Using rates, the expected return $E(R)$ is:

$$\begin{aligned} E(R) &= (0.09)(0.1) + (0.10)(0.2) + (0.11)(0.4) + (0.12)(0.2) + (0.13)(0.1) \\ &= 0.11 \\ &= 11\% \end{aligned}$$

The variance of returns is:

$$\begin{aligned} \sigma^2 &= (0.09 - 0.11)^2(0.1) + (0.10 - 0.11)^2(0.2) + (0.11 - 0.11)^2(0.4) + (0.12 - 0.11)^2(0.2) \\ &\quad + (0.13 - 0.11)^2(0.1) \\ &= 0.000\,12 \end{aligned}$$

The standard deviation of returns is therefore:

$$\begin{aligned} \sigma &= \sqrt{0.000\,12} \\ &= 0.010\,95 \\ &= 1.095\% \end{aligned}$$

It is often assumed that an investment's distribution of returns follows a normal distribution. This is a convenient assumption because a normal distribution can be fully described by its expected value and standard deviation. Therefore, an investment's distribution of returns can be fully described by its expected return and risk. Assuming that returns follow a normal probability distribution, the table of areas under the standard normal curve (see Table 5 of Appendix A page xxx) can be used to calculate the probability that the investment will generate a return greater than or less than any specified return. For example, suppose that the returns on an investment in Company A are normally distributed, with an expected return of 13 per cent per annum and a standard deviation of 10 per cent per annum. Suppose an investor in the company wishes to calculate the probability of a loss—that is, the investor wishes to calculate the probability of a return of less than zero per cent. A return of zero per cent is 1.3 standard deviations below the expected return (because 0.13/0.10 = 1.3). Figure 7.1, overleaf, illustrates this case. The shaded area represents the probability of a loss. The table of areas under the standard normal curve (Table 5, Appendix A page xxx or the NORMSDIST function in Microsoft Excel®) indicates that the probability of a loss occurring is 0.0968 or almost 9.7 per cent.

To highlight the importance of the standard deviation of the return distribution, assume that the same investor also has the opportunity of investing in Company B with an expected return of 13 per cent and a standard deviation of 6.91 per cent. The probability distributions of the returns on investments in companies A and B are shown in Figure 7.2, overleaf.

Both investments have the same expected return but, on the basis of the dispersion of the returns, an investment in Company A (with a standard deviation of 10 per cent) is riskier than an investment in Company B (with a standard deviation of 6.91 per cent).

Suppose that the investor decides that a return of zero per cent or less is unsatisfactory. A return of zero per cent on an investment in Company B is 1.88 standard deviations below the expected return

FIGURE 7.1

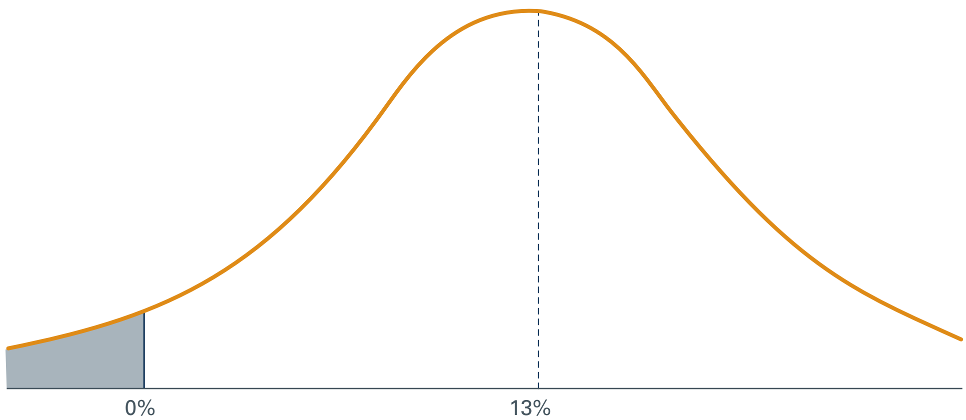
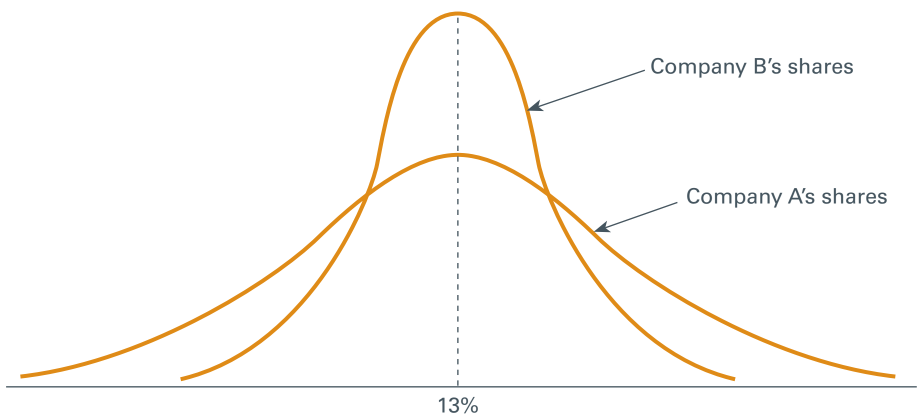


FIGURE 7.2



(because 0.13/0.0691 = 1.88). The probability of this occurring is 0.03. Therefore, the probability that an investment in one of these companies will generate a negative return is 3 per cent for Company B compared with 9.7 per cent for Company A. However, when the investor considers returns at the upper end of the distributions it is found that an investment in Company A offers a 9.7 per cent chance of a return in excess of 26 per cent, compared with only a 3 per cent chance for an investment in Company B. In summary the probability of both very low returns and very high returns is much greater in the case of Company A. The fact that the investor is more uncertain about the return from an investment in Company A does not mean that the investor will necessarily prefer to invest in Company B. The choice depends on the investor's attitude to risk.

risk-averse investor one who dislikes risk

Alternative attitudes to risk and the effects of risk are considered in the next section, which can safely be omitted by readers who are prepared to accept that investors are generally **risk averse**. Risk aversion does *not* mean that an investor will refuse to bear any risk at all. Rather it means that an investor regards risk as something undesirable, but which may be worth tolerating if the expected return is sufficient to compensate for the risk. Therefore, a risk-averse investor would prefer to invest in Company B because A and B offer the same expected return, but B is less risky.

THE INVESTOR'S UTILITY FUNCTION

7.3

LEARNING OBJECTIVE 2

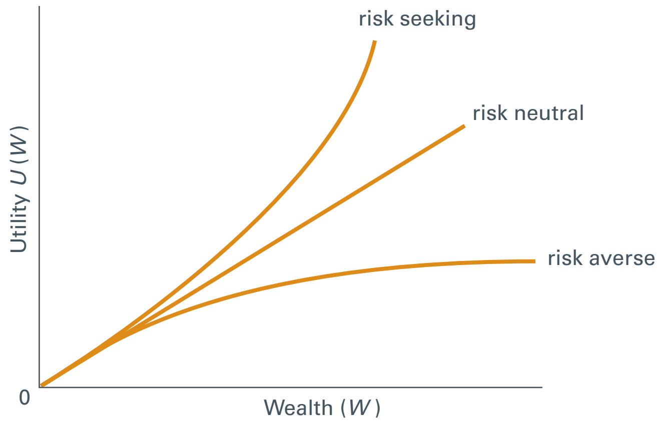
Understand the concept of risk aversion by investors

Consider the decision to invest in either Company A or Company B. As discussed in Section 7.2, both companies offer the same expected return, but differ in risk. A preference for investing in either Company A or Company B will depend on the investor's attitude to risk. An investor may be risk averse, **risk neutral** or **risk seeking**. A risk-averse investor attaches decreasing utility to each increment in wealth; a risk-neutral investor attaches equal utility to each increment in wealth; while a risk-seeking investor attaches increasing utility to each increment in wealth. Typical utility-to-wealth functions for each type of investor are illustrated in Figure 7.3.

risk-neutral investor one who neither likes nor dislikes risk

risk-seeking investor one who prefers risk

FIGURE 7.3 Utility-to-wealth functions for different types of investors

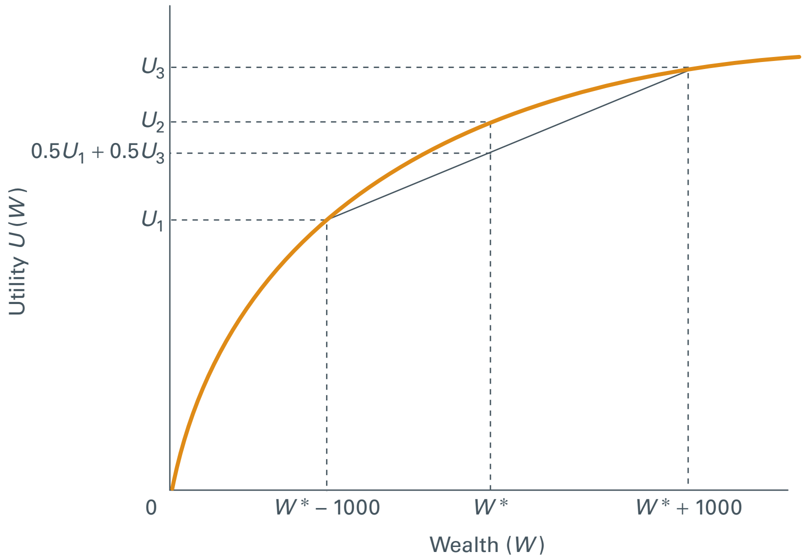


The characteristics of a risk-averse investor warrant closer examination, as risk aversion is the standard assumption in finance theory. Assume that a risk-averse investor has wealth of $\$W^*$ and has the opportunity of participating in the following game: a fair coin is tossed and if it falls tails (probability 0.5), then \$1000 is won; if it falls heads (probability 0.5), then \$1000 is lost. The expected value of the game is \$0 and it is, therefore, described as a 'fair game'. Would a risk-averse investor participate in such a game? If he or she participates and wins, wealth will increase to $\$(W^* + 1000)$, but if he or she loses, wealth will fall to $\$(W^* - 1000)$. The results of this game are shown in Figure 7.4, overleaf.

The investor's current level of utility is U_2. The investor's utility will increase to U_3 if he or she wins the game and will decrease to U_1 in the event of a loss. What is the expected utility if the investor decides to participate in the game? There is a 50 per cent chance that his or her utility will increase to U_3, and a 50 per cent chance that it will decrease to U_1. Therefore, the expected utility is $0.5U_1 + 0.5U_3$. As shown in Figure 7.4, the investor's expected utility with the gamble $(0.5U_1 + 0.5U_3)$ is lower than the utility obtained without the gamble (U_2). As it is assumed that investors maximise their expected utility, a risk-averse investor would refuse to participate in this game. In fact, a risk-averse investor may be defined as someone who would not participate in a fair game. Similarly, it can be shown that a risk-neutral investor would be indifferent to participation, and a risk-seeking investor would be prepared to pay for the right to participate in a fair game.

Now consider the preferences of a risk-averse investor with respect to an investment in either Company A or Company B. As we have seen, the expected return from each investment is the same but the investment in A is riskier. An investment in A offers the possibility of making either higher returns or lower returns, compared with an investment in B. However, from Figure 7.2, the increased spread of returns above the expected return tends to increase expected utility. But this increase will be outweighed by the decrease in expected utility resulting from the greater spread of returns below the expected return. Therefore, the risk-averse investor's expected utility would be greater if he or she invests in B.

FIGURE 7.4



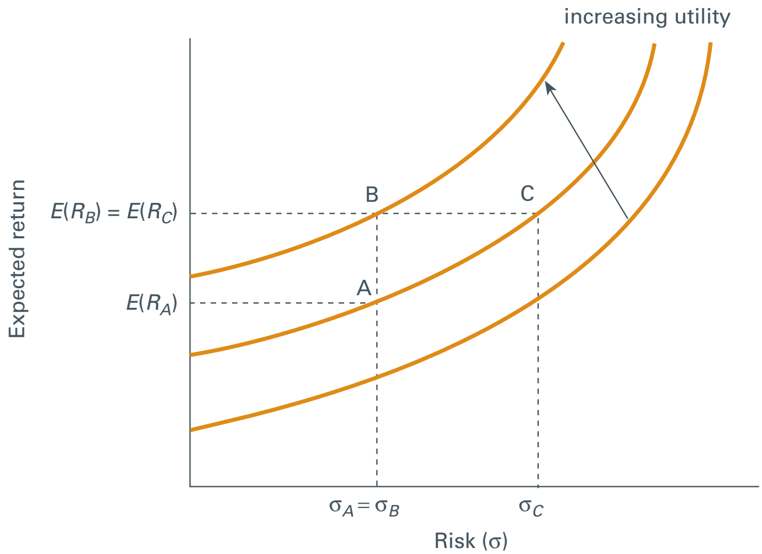
As both investments offer the same expected return, the risk-averse investor's choice implies that the increased dispersion of returns makes an investment riskier. This suggests that the standard deviation of the return distribution may be a useful measure of risk for a risk-averse investor. Similarly, it can be argued that the risk-neutral investor would be indifferent between these two investments. For any given amount to be invested, such an investor will always choose the investment that offers the higher return, irrespective of the relative risk of other investments—that is, the standard deviation is ignored. The risk-seeking investor would choose to invest in A. If a given amount is to be invested, and the investor has the choice of two investments that offer the same expected return, the risk-seeking investor will always choose the investment with the higher risk.

An investor's preferences regarding expected return and risk can be illustrated using indifference curves. For a given amount invested, an indifference curve traces out all those combinations of expected return and risk that provide a particular investor with the same level of utility. Because the level of utility is the same, the investor is indifferent between all points on the curve. A risk-averse investor has a positive attitude towards expected return and a negative attitude towards risk. By this, we mean that a risk-averse investor will prefer an investment to have a higher expected return (for a given risk level) and lower risk (for a given expected return).

Risk aversion does *not* mean that an investor will refuse to bear any risk at all. Rather it means that an investor regards risk as something undesirable, but which may be worth tolerating if the expected return is sufficient to compensate for the risk. In graphical terms, indifference curves for a risk-averse investor must be upward sloping as shown in Figure 7.5, opposite.

The risk–return coordinates for a risk-averse investor are shown in Figure 7.5 for three investments—A, B and C. It is apparent that this investor would prefer Investment B to Investment A, and would also prefer Investment B to Investment C. This investor prefers a higher expected return at any given level of risk (compare investments B and A) and a lower level of risk at any given expected return (compare investments B and C). However, this investor would be indifferent between investments A and C. The higher expected return on investment C compensates this investor exactly for the higher risk. In addition, for a given expected return the expected utility of a risk-averse investor falls at an increasing rate as the dispersion of the distribution of returns increases. As a result, the rate of increase in expected return required to compensate for every

FIGURE 7.5



increment in the standard deviation increases faster as the risk becomes larger. Note that indifference curves for a risk-averse investor are not only upward sloping, but also convex, as shown in Figure 7.5.

So far we have concentrated on the characteristics and behaviour of a risk-averse investor. However, there are instances where individuals behave in a way contrary to risk aversion. For example, a risk-averse person will never purchase a lottery ticket, as the expected value of the gamble is less than the price of the ticket. However, many individuals whose current level of wealth is quite low relative to the lottery prize are prepared to purchase lottery tickets because, while only a small outlay is required, there is the small chance of achieving a relatively large increase in wealth. In decisions that involve larger outlays, risk aversion is much more likely. As the financial decisions considered in this book generally involve large investments and small rates of return (at least relative to winning a lottery prize), it is assumed throughout that investors behave as if they are risk averse.

7.4 THE RISK OF ASSETS

If investors' expectations of the returns from an investment can be represented by a normal probability distribution, then the standard deviation is a relevant measure of risk for a risk-averse investor. If two investments offer the same expected return, but differ in risk, then a risk-averse investor will prefer the less risky investment. Further, it has been shown that a risk-averse investor is prepared to accept higher risk for higher expected return, with the result that the required return on a particular investment increases with the investor's perception of its risk.

The standard deviation of the return from a single investment is a relevant measure of its riskiness in cases where an individual is considering the investment of all available funds in one asset. However, it is exceptional to limit investments in this way. Most people invest in a number of assets; they may invest in a house, a car, their human capital and numerous other assets. In addition, where they invest in shares, it is likely that they will hold shares in a number of companies. In other words, people typically invest their wealth in a **portfolio** of assets and will be concerned about the risk of their overall portfolio. This risk can be measured by the standard deviation of the returns on the portfolio. Therefore, when an individual asset is considered, an investor will be concerned about the risk of that asset as a *component* of a portfolio

portfolio
combined holding of more than one asset

of assets. What we need to know is how individual portfolio components (assets) contribute to the risk of the portfolio as a whole. An apparently plausible guess would be that the contribution of each asset is proportional to the asset's standard deviation. However, portfolio theory, which is discussed in the next section, shows that this guess turns out to be almost always incorrect.

7.5 PORTFOLIO THEORY AND DIVERSIFICATION

LEARNING OBJECTIVE 3

Explain how diversification reduces risk

Portfolio theory was initially developed by Markowitz (1952) as a normative approach to investment choice under uncertainty.[1] Two important assumptions of portfolio theory have already been discussed. These are:

(a) The returns from investments are normally distributed. Therefore, two parameters, the expected return and the standard deviation, are sufficient to describe the distribution of returns.[2]
(b) Investors are risk averse. Therefore, investors prefer the highest expected return for a given standard deviation and the lowest standard deviation for a given expected return.

Given these assumptions, it can be shown that it is rational for a utility-maximising investor to hold a well-diversified portfolio of investments. Suppose that an investor holds a portfolio of securities. This investor will be concerned about the expected return and risk of the portfolio. The expected return on a portfolio is a weighted average of the expected returns on the securities in the portfolio. Let $E(R_i)$ be the expected return on the ith security and $E(R_p)$ the expected return on a portfolio of securities. Then, using the notation introduced earlier:

$$E(R_p) = \sum_{i=1}^{n} w_i E(R_i) \qquad [7.1]$$

where w_i = the proportion of the total current market value of the portfolio constituted by the current market value of the ith security—that is, it is the 'weight' attached to the security
n = the number of securities in the portfolio

Calculation of the expected return on a portfolio is illustrated in Example 7.1.

EXAMPLE 7.1

Assume that there are only two securities (1 and 2) in a portfolio and $E(R_1) = 0.08$ and $E(R_2) = 0.12$. Also assume that the current market value of Security 1 is 60 per cent of the total current market value of the portfolio (that is, $w_1 = 0.6$ and $w_2 = 0.4$). Then:

$$E(R_p) = (0.6)(0.08) + (0.4)(0.12)$$
$$= 0.096 \text{ or } 9.6\%$$

Example 7.1 illustrates the fact that the expected return on a portfolio is simply the weighted average of the expected returns on the securities in the portfolio. However, the standard deviation of the return on the portfolio (σ_p) is *not* simply a weighted average of the standard deviations of the securities in the portfolio. This is because the riskiness of a portfolio depends not only on the riskiness of the individual securities but also on the relationship between the returns on those securities. The variance of the return on a portfolio of two securities is given by:

$$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2 \operatorname{Cov}(R_1, R_2) \qquad [7.2]$$

where $\operatorname{Cov}(R_1,R_2)$ = the covariance between the returns on securities 1 and 2

1 For a more extensive treatment, see Markowitz (1959).
2 Other parameters may exist if the distribution is non-normal. In this case it is assumed that investors base decisions on expected return and standard deviation and ignore other features such as skewness.

The covariance between the returns on any pair of securities is a measure of the extent to which the returns on those securities tend to move together or 'covary'. This tendency is more commonly measured using the correlation coefficient ρ, which is found by dividing the covariance between the returns on the two securities by the standard deviations of their returns. Therefore, the correlation coefficient for securities 1 and 2 is:

$$\rho_{1,2} = \frac{\text{Cov}(R_1, R_2)}{\sigma_1 \sigma_2} \qquad [7.3]$$

The correlation coefficient is essentially a scaled measure of covariance and it is a very convenient measure because it can only have values between +1 and –1. If the correlation coefficient between the returns on two securities is +1, the returns are said to be *perfectly positively correlated*. This means that if the return on security i^1 is 'high' (compared with its expected level), then the return on security j^2 will, unfailingly, also be 'high' (compared with *its* expected level) to precisely the same degree. If the correlation coefficient is –1, the returns are *perfectly negatively correlated*; high (low) returns on security i^1 will always be paired with low (high) returns on security j^2. A correlation coefficient of zero indicates the absence of a systematic relationship between the returns on the two securities. Using Equation 7.3 to substitute for the covariance, Equation 7.2 can be expressed as:

$$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho_{1,2}\sigma_1\sigma_2 \qquad [7.4]$$

As may be seen from Equation 7.4, the variance of a portfolio depends on:

(a) the composition of the portfolio—that is, the proportion of the current market value of the portfolio constituted by each security
(b) the standard deviation of the returns for each security
(c) the correlation between the returns on the securities held in the portfolio.

The effect of changing the composition of a portfolio of two securities is illustrated in Example 7.2.

EXAMPLE 7.2

An investor wishes to construct a portfolio consisting of Security 1 and Security 2. The expected returns on the two securities are $E(R_1)$ = 8% p.a. and $E(R_2)$ = 12% p.a. and the standard deviations are σ_1 = 20% p.a. and σ_2 = 30% p.a. The correlation coefficient between their returns is $\rho_{1,2}$ = –0.5. The investor is free to choose the investment proportions w_1 and w_2, subject only to the requirements that $w_1 + w_2 = 1$ and that both w_1 and w_2 are positive.[3] There is no limit to the number of portfolios that meet these requirements, since there is no limit to the number of proportions that sum to 1. Therefore, a representative selection of values is considered for w_1: 0, 0.2, 0.4, 0.6, 0.8 and 1.

Using Equation 7.1, the expected return on a two-security portfolio is:

$$\begin{aligned} E(R_p) &= w_1 E(R_1) + w_2 E(R_2) \\ &= w_1(0.08) + w_2(0.12) \end{aligned}$$

Using Equation 7.4, the variance of the return on a two-security portfolio is:

$$\begin{aligned} \sigma_p^2 &= w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho_{1,2}\sigma_1\sigma_2 \\ &= w_1^2(0.20)^2 + w_2^2(0.30)^2 + 2w_1w_2(-0.5)(0.20)(0.30) \\ &= 0.04w_1^2 + 0.09w_2^2 - 0.06w_1w_2 \end{aligned}$$

continued →

3 Negative investment proportions would indicate a 'short sale', which means that the asset is first sold and later purchased. Therefore, a short-seller benefits from price decreases.

Example 7.2 ***continued***

The standard deviation of the portfolio returns is found by taking the square root of σ. Each pair of proportions is now considered in turn:

(a) $w_1 = 0$ and $w_2 = 1$

$E(R_p) = (0.08)(0) + (0.12)(1)$
$= 0.12$ or 12% p.a.
$\sigma_p^2 = (0.04)(0)^2 + (0.09)(1)^2 - (0.06)(0)(1)$
$\sigma_p^2 = 0.09$
$\therefore \sigma_p = 0.30$ or 30% p.a.

(b) $w_1 = 0.2$ and $w_2 = 0.8$ $E(R_p) = (0.08)(0.2) + (0.12)(0.8) = 0.112$ or 11.2% p.a.

$\sigma_p^2 = (0.04)(0.2)^2 + (0.09)(0.8)^2 - (0.06)(0.2)(0.8)$
$\sigma_p^2 = 0.0496$
$\therefore \sigma_p = 0.2227$ or 22.27% p.a.

(c) $w_1 = 0.4$ and $w_2 = 0.6$ $E(R_p) = (0.08)(0.4) + (0.12)(0.6) = 0.104$ or 10.4% p.a.

$\sigma_p^2 = (0.04)(0.4)^2 + (0.09)(0.6)^2 - (0.06)(0.4)(0.6)$
$\sigma_p^2 = 0.0244$
$\therefore \sigma_p = 0.1562$ or 15.62% p.a.

(d) $w_1 = 0.6$ and $w_2 = 0.4$ $E(R_p) = (0.08)(0.6) + (0.12)(0.4) = 0.096$ or 9.6% p.a.

$\sigma_p^2 = (0.04)(0.6)^2 + (0.09)(0.4)^2 - (0.06)(0.6)(0.4)$
$\sigma_p^2 = 0.0144$
$\therefore \sigma_p = 0.12$ or 12% p.a.

(e) $w_1 = 0.8$ and $w_2 = 0.2$ $E(R_p) = (0.08)(0.8) + (0.12)(0.2) = 0.088$ or 8.8% p.a.

$\sigma_p^2 = (0.04)(0.8)^2 + (0.09)(0.2)^2 - (0.06)(0.8)(0.2)$
$\sigma_p^2 = 0.0196$
$\therefore \sigma_p = 0.14$ or 14% p.a.

(f) $w_1 = 1.0$ and $w_2 = 0$ $E(R_p) = (0.08)(1) + (0.12)(0) = 0.08$ or 8% p.a.

$\sigma_p^2 = (0.04)(1)^2 + (0.09)(0)^2 - (0.06)(1)(0)$
$\sigma_p^2 = 0.04$
$\therefore \sigma_p = 0.20$ or 20% p.a.

These results are summarised in Table 7.3.

TABLE 7.3

	PORTFOLIO					
	(a)	(b)	(c)	(d)	(e)	(f)
Proportion in Security 1 (w_1)	0.0000	0.2000	0.4000	0.6000	0.8000	1.0000
Proportion in Security 2 (w_2)	1.0000	0.8000	0.6000	0.4000	0.2000	0.0000
Expected return E (R_p)	0.1200	0.1120	0.1040	0.0960	0.0880	0.0800
Standard deviation σ	0.3000	0.2227	0.1562	0.1200	0.1400	0.2000

Reading across Table 7.3, the investor places more wealth in the low-return Security 1 and less in the high-return Security 2. Consequently, the expected return on the portfolio declines with each step.

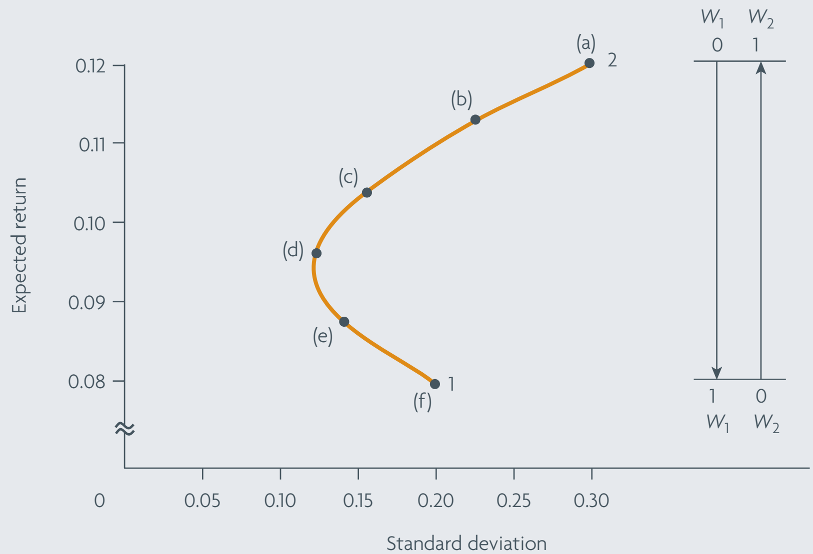
The behaviour of the standard deviation is more complicated. It declines over the first four portfolios, reaching a minimum value of 0.1200 when w_1 = 0.6, but then rises to 0.2000 at the sixth portfolio, which consists entirely of Security 1.[4] This is an important finding as it implies that some portfolios would never be held by risk-averse investors. For example, no risk-averse investor would choose Portfolio (e) because Portfolio (d) offers both a higher expected return and a lower risk than Portfolio (e). Portfolios that offer the highest expected return at a given level of risk are referred to as 'efficient' portfolios. The data in Table 7.3 are plotted in Figure 7.6.

As can be seen from Figure 7.6, portfolios (e) and (f) are not efficient.

LEARNING OBJECTIVE 5

Explain the concept of efficient portfolios

FIGURE 7.6



7.5.1 GAINS FROM DIVERSIFICATION

Example 7.2 shows that some portfolios enable an investor to achieve simultaneously higher expected return and lower risk; for example, compare portfolios (d) and (f) in Figure 7.6. It should be noted that Portfolio (d) consists of both securities, whereas Portfolio (f) consists of only Security 1—that is, Portfolio (d) is diversified, whereas Portfolio (f) is not. This illustrates the general principle that investors can gain from diversification.

The magnitude of the gain from diversification is closely related to the value of the correlation coefficient, $\rho_{1,2}ij$. To show the importance of the correlation coefficient, securities 1 and 2 are again considered. This time, however, the investment proportions are held constant at w_1 = 0.6 and w_2 = 0.4 and different values of the correlation coefficient are considered. Portfolio variance is given by:

$$\begin{aligned}
\sigma_p^2 &= w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho_{1,2}\sigma_1\sigma_2 \\
&= (0.6)^2(0.20)^2 + (0.4)^2(0.30)^2 + 2(0.6)(0.4)\rho_{1,2}(0.20)(0.30) \\
&= 0.0144 + 0.0144 + 0.0288\rho_{1,2}
\end{aligned}$$

$$\sigma_p = \sqrt{0.0288 + 0.0288\rho_{1,2}}$$

4 The minimum value of the standard deviation actually occurs slightly beyond Portfolio (d) at proportions w_1 = 0.6333, and w_2 = 0.3667. The standard deviation for this portfolio is 0.11.92% p.a. and its expected return is 9.48% p.a.

(a) $\rho_{1,2} = +1.00$

$$\sigma_p = \sqrt{0.0288 + 0.0288\rho_{1,2}}$$
$$\sigma_p = 0.2400$$

(b) $\rho_{1,2} = +0.50$

$$\sigma_p = \sqrt{0.0288 + 0.0288\rho_{1,2}}$$
$$\sigma_p = 0.2079$$

(c) $\rho_{1,2} = 0.00$

$$\sigma_p = \sqrt{0.0288 + 0.0288\rho_{1,2}}$$
$$\sigma_p = 0.1697$$

(d) $\rho_{1,2} = -0.50$

$$\sigma_p = \sqrt{0.0288 + 0.0288\rho_{1,2}}$$
$$\sigma_p = 0.1200$$

(e) $\rho_{1,2} = -1.00$

$$\sigma_p = \sqrt{0.0288 + 0.0288\rho_{1,2}}$$
$$\sigma_p = 0$$

These results are summarised in Table 7.4.

TABLE 7.4 Effect of correlation coefficient on portfolio standard deviation

CORRELATION COEFFICIENT	STANDARD DEVIATION
$\rho^{1,2} = +1.00$	0.2400
$\rho^{1,2} = +0.50$	0.2079
$\rho^{1,2} = \ \ 0.00$	0.1697
$\rho^{1,2} = -0.50$	0.1200
$\rho^{1,2} = -1.00$	0.0000

Table 7.4 shows three important facts about portfolio construction:

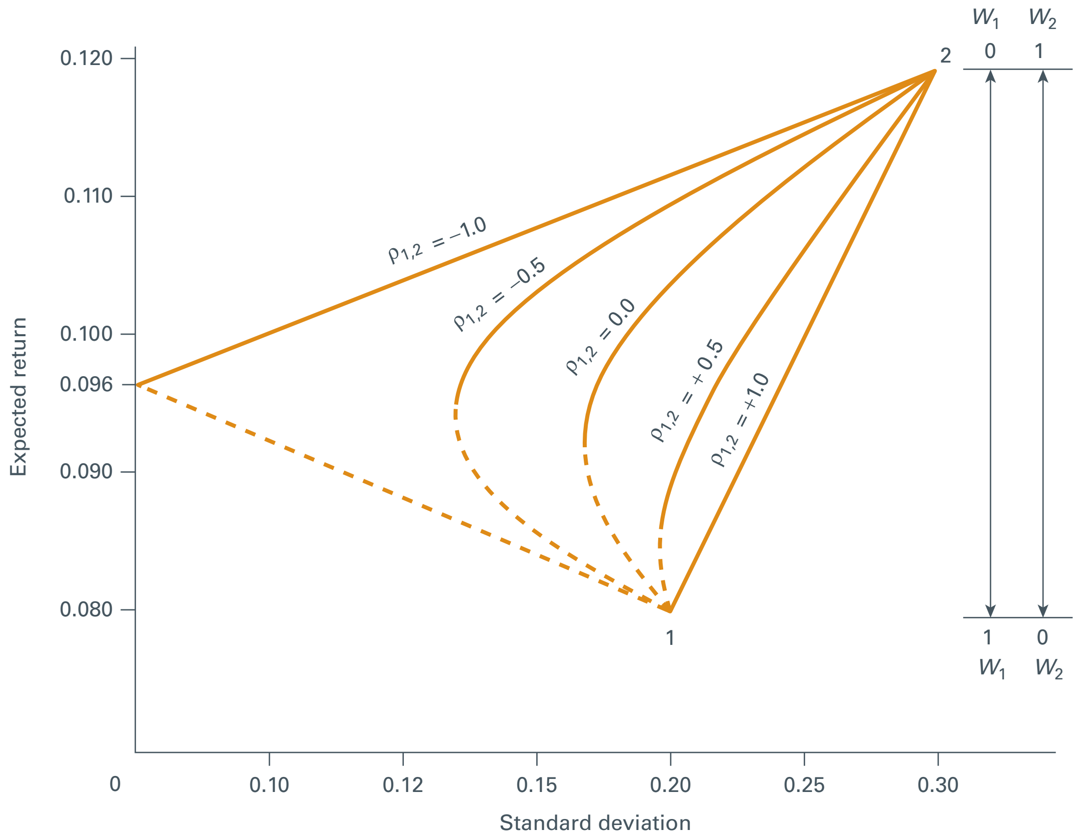
(a) Combining two securities whose returns are perfectly positively correlated (that is, the correlation coefficient is +1) results only in risk averaging, and does not provide any risk reduction. In this case the portfolio standard deviation is the weighted average of the two standard deviations, which is (0.6)(0.20) + (0.4)(0.30) = 0.2400.

(b) The real advantages of diversification result from the risk reduction caused by combining securities whose returns are less than perfectly positively correlated.

(c) The degree of risk reduction increases as the correlation coefficient between the returns on the two securities decreases. The largest risk reduction available is where the returns are perfectly negatively correlated, so the two risky securities can be combined to form a portfolio that has zero risk ($\sigma_p = 0$).

By considering different investment proportions w_1 and w_2, a curve similar to that shown in Figure 7.6 can be plotted for each assumed value of the correlation coefficient. These curves are shown together in Figure 7.7.

FIGURE 7.7



It can be seen that the lower the correlation coefficient, the higher the expected return for any given level of risk (or the lower the level of risk for any given expected return). This shows that the benefits of diversification increase as the correlation coefficient decreases, and when the correlation coefficient is –1, risk can be eliminated completely. The significance of the dotted lines in Figure 7.7 is that a risk-averse investor would never hold combinations of the two securities represented by points on the dotted lines. At any given level of correlation these combinations of the two securities are always dominated by other combinations that offer a higher expected return for the same level of risk.

7.5.2 DIVERSIFICATION WITH MULTIPLE ASSETS

LEARNING OBJECTIVE 4

Understand the importance of covariance between returns on risky assets in determining the risk of a portfolio

While the above discussion relates to the two-security case, even stronger conclusions can be drawn for larger portfolios. To examine the relationship between the risk of a large portfolio and the riskiness of the individual assets in the portfolio, we start by considering two assets. Using Equation 7.2, the portfolio variance is:

$$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\text{Cov}(R_1,R_2)$$

The variances and covariances on the right-hand side of this equation can be arranged in a matrix as follows:

	1	2
1	σ_1^2	$\text{Cov}(R_1,R_2)$
2	$\text{Cov}(R_2,R_1)$	σ_2^2

With two assets the variances and covariances form a 2 × 2 matrix; three assets will result in a 3 × 3 matrix; and in general with n assets there will be an $n \times n$ matrix. Regardless of the number of assets involved, the variance–covariance matrix will always have the following properties:

(a) The matrix will contain a total of n^2 terms. Of these terms, n are the variances of the individual assets and the remaining $(n^2 - n)$ terms are the covariances between the various pairs of assets in the portfolio.
(b) The two covariance terms for each pair of assets are identical. For example, in the 2 × 2 matrix above, $\text{Cov}(R_1,R_2) = \text{Cov}(R_2,R_1)$.
(c) Since the covariance terms form identical pairs, the matrix is symmetrical about the main diagonal, which contains the n variance terms.

Remember that the significance of the variance–covariance matrix is that it can be used to calculate the portfolio variance. The portfolio variance is a weighted sum of the terms in the matrix, where the weights depend on the proportions of the various assets in the portfolio.

The first property of the matrix listed above shows that as the number of assets increases, the number of covariance terms increases much more rapidly than the number of variance terms. For a portfolio of n assets there are n variances and $(n^2 - n)$ covariances in the matrix. This suggests that as a portfolio becomes larger, the effect of the covariance terms on the risk of the portfolio will be greater than the effect of the variance terms.

To illustrate the effects of diversification and the significance of the covariance between assets, consider a portfolio of n assets. Assume that each of these assets has the same variance (σ_1^2). Also assume, initially, that the returns on these assets are independent—that is, the correlation between the returns on the assets is assumed to be zero in all cases. If we form an equally weighted portfolio of these assets, the proportion invested in each asset will be $(1/n)$. Given the assumption of zero correlation between all the asset returns, the covariance terms will all be zero, so the variance of the portfolio will depend only on the variance terms.

Since there are n variance terms and each such term is $\left(\frac{\sigma_1}{n}\right)^2$, the variance of the portfolio will be:

$$\sigma_p^2 = n\left(\frac{\sigma_1}{n}\right)^2 = \frac{\sigma_1^2}{n} \qquad [7.5]$$

Equation 7.5 shows that as n increases, the portfolio variance will decrease and as n becomes large, the variance of the portfolio will approach zero; that is, *if* the returns between all risky assets were independent, then it would be possible to eliminate all risk by diversification.

However, in practice, the returns between risky assets are *not* independent and the covariance between returns on most risky assets is positive. For example, the correlation coefficients between the returns on company shares are mostly in the range 0.5 to 0.7. This positive correlation reflects the fact that the returns on most risky assets are related to each other. For example, if the economy were growing strongly we would expect sales of new cars and construction of houses and other buildings to be increasing strongly. In turn, the demand for steel and other building materials would also increase. Therefore, the profits and share prices of steel and building material manufacturers should have a tendency to increase at the same time as the profits and share prices of car manufacturers and construction companies.

To reflect the relationships among the returns on individual assets, we relax the assumption that the returns between assets are independent. Instead, we now assume that the correlation between the returns on all assets in the portfolio is ρ^*. If the portfolio is again equally weighted, the portfolio variance will now be equal to the sum of the variance terms shown in Equation 7.5, plus $(n^2 - n)$

covariance terms where each such term will be $\left(\frac{1}{n}\right)^2 \rho^* \sigma_1^2$. Therefore, the variance of the portfolio will be:

$$\sigma_p^2 = \frac{\sigma_1^2}{n} + \frac{(n^2 - n)\rho^* \sigma_1^2}{n^2} \qquad [7.6]$$
$$= \frac{\sigma_1^2}{n} + \left(1 - \frac{1}{n}\right)\rho^* \sigma_1^2$$

Equation 7.6 illustrates an important result: with identical positively correlated assets, risk cannot be completely eliminated, no matter how many such assets are included in a portfolio. As n becomes large, $(1/n)$ will approach zero so the first term in Equation 7.6 will approach zero, but the second term will approach $\rho^*\sigma_1^2$; that is, the variance of the portfolio will approach $\rho^*\sigma_1^2$ which is the *covariance* between the returns on the assets in the portfolio. Thus, the positive correlation between the assets in a portfolio imposes a limit on the extent to which risk can be reduced by diversification.

In practice, the assets in a portfolio will not be identical and the correlations between the assets will differ rather than being equal as we have assumed. However, the essential results illustrated in Equation 7.6 remain the same—that is, in a diversified portfolio the variances of the individual assets will contribute little to the risk of the portfolio. Rather, the risk of a diversified portfolio will depend largely on the covariances between the returns on the assets. For example, Fama (1976, pp. 245–52) found that in an equally weighted portfolio of 50 randomly selected securities, 90 per cent of the portfolio standard deviation was due to the covariance terms.

7.5.3 SYSTEMATIC AND UNSYSTEMATIC RISK

LEARNING OBJECTIVE 6

Explain the distinction between systematic and unsystematic risk

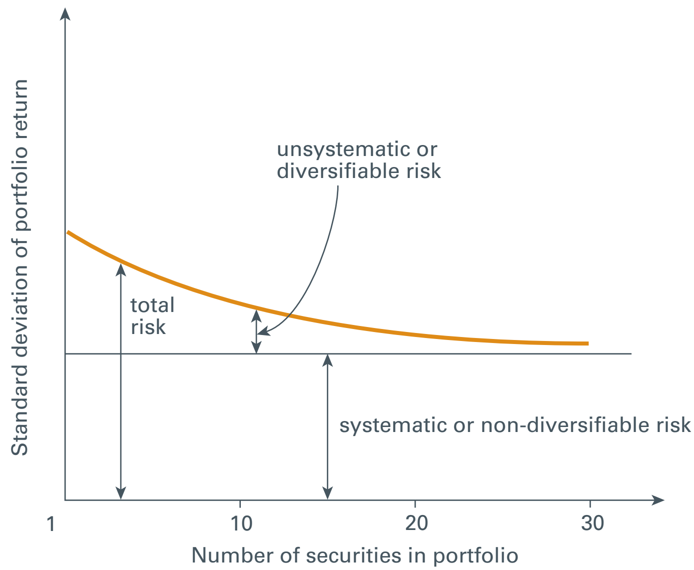
As discussed in Section 7.5.2, if we diversify by combining risky assets in a portfolio, the risk of the portfolio returns will decrease. Diversification is most effective if the returns on the individual assets are negatively correlated, but it still works with positive correlation, provided that the correlation coefficient is less than +1. We have noted that, in practice, the correlation coefficients between the returns on company shares are mostly in the range 0.5 to 0.7. We also noted that this positive correlation reflects the fact that the returns on the shares of most companies are economically related to each other. However, the correlation is less than perfect, which reflects the fact that much of the variability in the returns on shares is due to factors that are specific to each company. For example, the price of a company's shares may change due to an exploration success, an important research discovery or a change of chief executive. Over any given period, the effects of these company-specific factors will be positive for some companies and negative for others. Therefore, when shares of different companies are combined in a portfolio, the effects of the company-specific factors will tend to offset each other, which will, of course, be reflected in reduced risk for the portfolio. In other words, part of the risk of an individual security can be eliminated by diversification and is referred to as **unsystematic risk** or **diversifiable risk**. However, no matter how much we diversify, there is always some risk that cannot be eliminated because the returns on all risky assets are related to each other. This part of the risk is referred to as **systematic risk** or **non-diversifiable risk**. These two types of risk are illustrated in Figure 7.8 on the next page.

unsystematic (diversifiable) risk that component of total risk that is unique to the firm and may be eliminated by diversification

Figure 7.8 shows that most unsystematic risk is removed by holding a portfolio of about 25 to 30 securities. In other words, the returns on a well-diversified portfolio will not be significantly affected by the events that are specific to individual companies. Rather, the returns on a well-diversified portfolio will vary due to the effects of market-wide or economy-wide factors such as changes in interest rates, changes in tax laws and variations in commodity prices. The systematic risk of a security or portfolio will depend on its sensitivity to the effects of these market-wide factors. The distinction between systematic and unsystematic risk is important when we consider the risk of individual assets in a portfolio context, which is discussed in Section 7.5.4, and the pricing of risky assets, which is discussed in Section 7.6.

systematic (market-related or non-diversifiable) risk that component of total risk which is due to economy-wide factors

FIGURE 7.8



LEARNING OBJECTIVE 7

Explain why systematic risk is important to investors

7.5.4 THE RISK OF AN INDIVIDUAL ASSET

The reasoning used above can be extended to explain the factors that will determine the risk of an individual asset as a *component of a diversified portfolio*. Suppose that an investor holds a portfolio of 50 assets and is considering the addition of an extra asset to the portfolio. The investor is concerned with the effect that this extra asset will have on the standard deviation of the portfolio. The effect is determined by the portfolio proportions, the extra asset's variance and the 50 covariances between the extra asset and the assets already in the portfolio. As discussed above, the covariance terms are the dominant influence—that is, to the holder of a large portfolio the risk of an asset is largely determined by the covariance between the return on that asset and the return on the holder's existing portfolio. The variance of the return on the extra asset is of little importance. Therefore, the risk of an asset when it is held in a large portfolio is determined by the covariance between the return on the asset and the return on the portfolio. The covariance of a security i with a portfolio P is given by:

$$\text{Cov}(R_i, R_p) = \rho_{ip}\sigma_i\sigma_p \qquad [7.7]$$

The holders of large portfolios of securities can still achieve risk reduction by adding a new security to their portfolios, provided that the returns on the new security are not perfectly positively correlated with the returns on the existing portfolio. However, the incremental risk reduction due to adding a new security to a portfolio decreases as the size of the portfolio increases and, as shown in Figure 7.8, the additional benefits from diversification are very small for portfolios that include more than 30 securities (Statman 1987).

beta
measure of a security's systematic risk, describing the amount of risk contributed by the security to the market portfolio

If investors are well diversified, their portfolios will be representative of the market as a whole. Therefore, the relevant measure of risk is the covariance between the return on the asset and the return on the market or Cov(R_i,R_M). The covariance can then be scaled by dividing it by the variance of the return on the market that gives a convenient measure of risk, the **beta** factor, β_i, of the asset—that is, for any asset i, the beta is:

$$\beta_i = \frac{\text{Cov}(R_i, R_M)}{\sigma_M^2}$$

Beta is a very useful measure of the risk of an asset and it will be shown in Section 7.6.2 that the capital asset pricing model proposes that the expected rates of return on risky assets are directly related to their betas.

WWW

Value Line (**www.valueline.com**) is a US website based on the Value Line Investment Survey and contains information to help determine a share's level of risk.

FINANCE IN ACTION

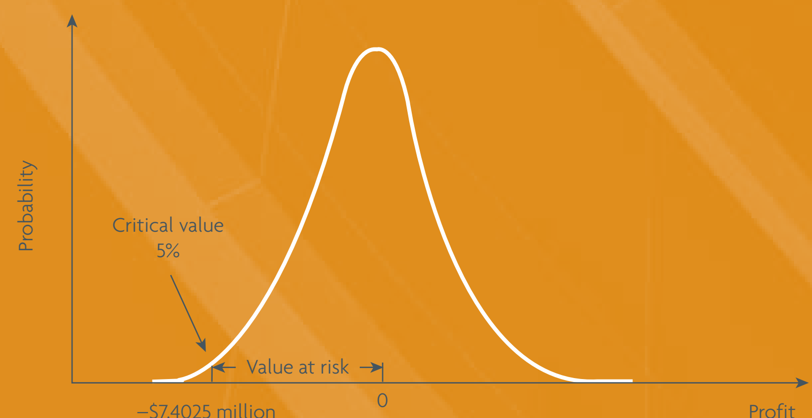
VALUE AT RISK (VaR)—ANOTHER WAY OF LOOKING AT RISK

Since the mid-1990s, a new measure of risk exposure has become popular. This measure was developed by the investment bank J.P. Morgan and is known as value at risk (VaR).[5] It is defined as the worst loss that is possible under normal market conditions during a given time period. It is therefore determined by what are estimated to be normal market conditions and by the time period under consideration. For a given set of market conditions, the longer the time horizon the greater is the value at risk. This measure of risk is being increasingly used by corporate treasurers, fund managers and financial institutions as a summary measure of the total risk of a portfolio.

value at risk
worst loss possible under normal market conditions for a given time horizon

To illustrate how value at risk is measured, suppose that $15 million is invested in shares in Gradstarts Ltd. Shares in Gradstarts have an estimated return of zero and a standard deviation of 30 per cent per annum.[6] The standard deviation on the investment of $15 million is therefore $4.5 million. Suppose also that returns follow a normal probability distribution. This means that the table of areas under the standard normal curve (see Table 5 of Appendix A page xxx, or the NORMSDIST function in Microsoft Excel®) can be used to calculate the probability that the return will be greater than a specified number. Suppose also that abnormally bad market conditions are expected 5 per cent of the time. The table of areas under the standard normal curve indicates that there is a 5 per cent chance of a loss of greater than $7.4025 million per annum. This figure is equal to 1.645 multiplied by the standard deviation of $4.5 million. As shown in Figure 7.9, the value at risk of the investment in Gradstarts is therefore $7.4025 million per annum.

FIGURE 7.9 Value of Gradstarts Ltd



Suppose that $10 million is also invested in shares in Curzon Creative Ideas Ltd. These Curzon Creative Ideas shares have an estimated return of zero and have a standard deviation of 20 per cent per annum.

5 A detailed examination of value at risk is provided by Jorion (2006), while an excellent online resource for those interested in the topic is provided at **www.gloriamundi.org.**

WWW

6 It is usual in calculating value at risk to assume an expected return of zero. This is a reasonable assumption where the expected return is small compared with the standard deviation of the expected return.

continued ↓

FINANCE IN ACTION *continued*

The standard deviation on the investment of $10 million is therefore $2 million per annum. It is again assumed that returns follow a normal probability distribution and that abnormally bad market conditions are expected 5 per cent of the time. A similar calculation to that for Gradstarts provides a value at risk of the investment in Curzon Creative Ideas of $2 million multiplied by 1.645 or $3.29 million per annum.

The benefits of diversification may be demonstrated by calculating the value at risk of a portfolio comprising a $15 million investment in Gradstarts and a $10 million investment in Curzon Creative Ideas. The weight of the investment in Gradstarts is $15 million of $25 million or 0.6 of the portfolio. The weight of the investment in Curzon Creative Ideas is 0.4. Suppose that the correlation between the returns on the shares is 0.65. Using Equation 7.4, the variance of the returns on the portfolio is:

$$\sigma_p^2 = (0.6)^2 (0.3)^2 + (0.4)^2 (0.2)^2 + 2(0.6)(0.4)(0.3)(0.2)(0.65)$$
$$= 0.05752$$

The standard deviation of portfolio returns, σ_p, is therefore 0.239 833 or 23.9833 per cent and the standard deviation on the investment is $25 million × 0.239 833 = $5.9958 million. The value at risk of the portfolio is $5.9958 multiplied by 1.645 or $9.8631 million per annum.

The total value at risk of the individual investments in Gradstarts and Curzon Creative Ideas was $7.4025 million plus $3.29 million or $10.6925 million per annum. The difference between that amount and the value at risk of the portfolio of $9.8631 million is due to the benefits of diversification. If, however, the returns on the shares of the two companies were perfectly correlated, the value at risk of the portfolio would equal the value at risk for the investment in Gradstarts plus the value at risk of the investment in Curzon Creative Ideas.

VaR is a technique that is commonly used by financial institutions to monitor their exposure to losses through adverse changes in market conditions. A pertinent example of the use of VaR is provided by the January 2004 announcement of a $360 million foreign exchange loss by the National Australia Bank. While an independent investigation by PricewaterhouseCoopers attributed most of the blame for the loss to dishonesty on the part of the currency traders involved and the lack of suitable control mechanisms in place to uncover such behaviour, the report also made some interesting comments on the bank's use of VaR. The National Australia Bank's board of directors had authorised a VaR market risk exposure limit of $80 million per day for the banking group as a whole. This limit was divided between the various divisions of the bank. The currency options desk had a VaR limit of $3.25 million per day. This limit was persistently breached over the 12-month period prior to the announcement of the $360 million loss. In relation to the implementation of a flawed VaR system the PricewaterhouseCoopers report commented that:

> *… management had little confidence in the VaR numbers due to systems and data issues, and effectively ignored VaR and other limit breaches. There was no sense of urgency in resolving the VaR calculation issues which had been a problem for a period of two or more years.*[7]

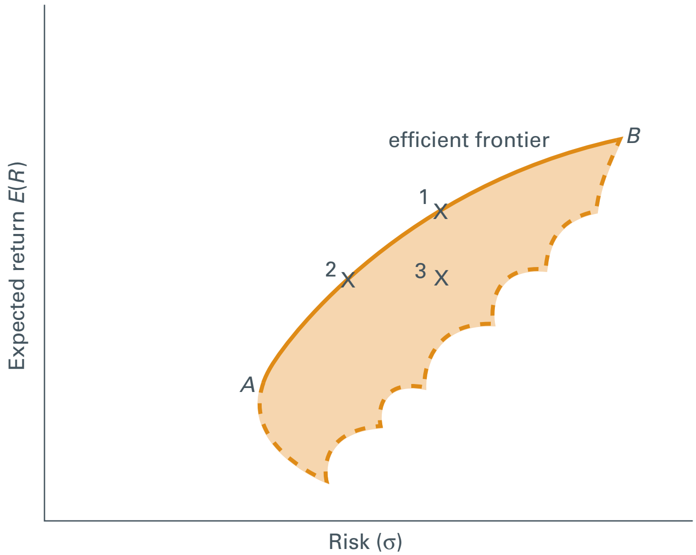
7.5.5 THE EFFICIENT FRONTIER

When all risky assets are considered, there is no limit to the number of portfolios that can be formed, and the expected return and standard deviation of the return can be calculated for each portfolio. The coordinates for all possible portfolios are represented by the shaded area in Figure 7.10, overleaf.

Only portfolios on the curve between points *A* and *B* are relevant since all portfolios below this curve yield lower expected return and/or greater risk. The curve *AB* is referred to as the *efficient frontier* and it includes those portfolios that are efficient in that they offer the maximum expected return for a

7 See PricewaterhouseCoopers (2004, p. 4).

FIGURE 7.10



given level of risk. For example, Portfolio 1 is preferred to an internal point such as Portfolio 3 because Portfolio 1 offers a higher expected return for the same level of risk. Similarly, Portfolio 2 is preferred to Portfolio 3 because it offers the same expected return for a lower level of risk. No such 'dominance' relationship exists between efficient portfolios—that is, between portfolios whose risk–return coordinates plot on the efficient frontier.

Given risk aversion, each investor will want to hold a portfolio somewhere on the efficient frontier. Risk-averse investors will choose the portfolio that suits their preference for risk. As investors are a diverse group there is no reason to believe that they will have identical risk preferences. Each investor may therefore prefer a different point (portfolio) along the efficient frontier. For example, a conservative investor would choose a portfolio near point *A* while a more risk-tolerant investor would choose a portfolio near point *B*.

In summary, the main points established in this section are that:

(a) diversification reduces risk
(b) the effectiveness of diversification depends on the correlation or covariance between returns on the individual assets combined into a portfolio
(c) the positive correlation that exists between the returns on most risky assets imposes a limit on the degree of risk reduction that can be achieved by diversification
(d) the total risk of an asset can be divided into two parts: systematic risk that cannot be eliminated by diversification and unsystematic risk that can be eliminated by diversification
(e) the only risk that remains in a well-diversified portfolio is systematic risk
(f) for investors who diversify, the relevant measure of the risk of an individual asset is its systematic risk, which is usually measured by the beta of the asset
(g) risk-averse investors will aim to hold portfolios that are efficient in that they provide the highest expected return for a given level of risk.

The concepts discussed in this section can be extended to model the relationship between risk and expected return for individual risky assets. This extension of portfolio theory is discussed in Section 7.6 and we discuss below an alternative technique to measuring risk that focuses on the maximum dollar losses that would be expected during normal trading conditions.

7.6 THE PRICING OF RISKY ASSETS

Section 7.5 focused on investment decision making by individuals. We now shift the focus from the behaviour of individuals to the pricing of risky assets and we introduce the assumption that investors can also invest in an asset that has no default risk. The return on this risk-free asset is the risk-free interest rate, R_f. Typically, this is regarded as the interest rate on a government security, such as Treasury notes.

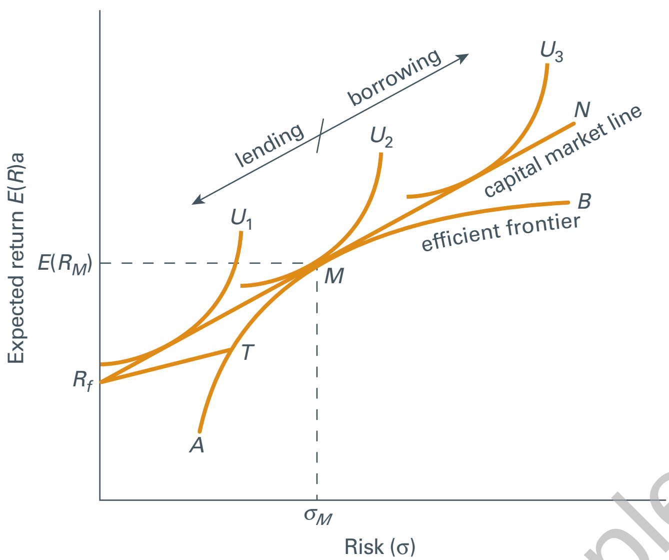
We continue to assume that all investors in a particular market behave according to portfolio theory, and ask: How would prices of individual securities in that market be determined? Intuitively, we would expect risky assets to provide a higher expected rate of return than the risk-free asset. In other words, the expected return on a risky asset could be viewed as consisting of the risk-free rate plus a premium for risk and this premium should be related to the risk of the asset.

However, as discussed in Section 7.5.3, part of the risk of any risky asset—unsystematic risk—can be eliminated by diversification. It seems reasonable to suggest that in a competitive market, assets should be priced so that investors are not rewarded for bearing risk that could easily be eliminated by diversification. On the other hand, some risk—systematic risk—cannot be eliminated by diversification so it is reasonable to suggest that investors will expect to be compensated for bearing that type of risk. In summary, intuition suggests that risky assets will be priced such that there is a relationship between returns and systematic risk. The remaining question is: What sort of relationship will there be between returns and systematic risk? The work of Sharpe (1964), Lintner (1965), Fama (1968) and Mossin (1969) provides an answer to this question.[8]

7.6.1 THE CAPITAL MARKET LINE

With the opportunity to borrow and lend at the risk-free rate, an investor is no longer restricted to holding a portfolio that is on the efficient frontier *AB*. Investors can now invest in combinations of risky assets and the risk-free asset in accordance with their risk preferences. This is illustrated in Figure 7.11.

FIGURE 7.11



8 Although we have referred to the 'pricing' of assets, much of this work deals with expected returns, rather than asset prices. However, there is a simple relationship between expected return and price in that the expected rate of return can be used to discount an asset's expected net cash flows to obtain an estimate of its current price.

The line R_fT represents portfolios that consist of an investment in a portfolio of risky assets T and an investment in the risk-free asset. Investors can achieve any combination of risk and return on the line R_fT by investing in the risk-free asset and Portfolio T. Each point on the line corresponds to different proportions of the total funds being invested in the risk-free asset and Portfolio T. However, it would not be rational for investors to hold portfolios that plot on the line R_fT, because they can achieve higher returns for any given level of risk by combining the risk-free asset with other portfolios that plot above T on the efficient frontier (AB). This approach suggests that investors will achieve the best possible return for any level of risk by holding Portfolio M rather than any other portfolio of risky assets.

The line R_fMN is tangential at the point M to the efficient frontier (AB) of portfolios of risky assets. This line represents portfolios that consist of an investment in Portfolio M and an investment in the risk-free asset. Points on the line to the left of M require a positive amount to be invested in the risk-free asset—that is, they require the investor to lend at the risk-free rate. Points on the line to the right of M require a negative amount to be invested in the risk-free asset—that is, they require the investor to borrow at the risk-free rate.

It is apparent that the line R_fMN dominates the efficient frontier AB since at any given level of risk a portfolio on the line offers an expected return at least as great as that available from the efficient frontier (curve AB). Risk-averse investors will therefore choose a portfolio on the line R_fMN—that is, some combination of the risk-free asset and Portfolio M. This is true for all risk-averse investors who conform to the assumptions of portfolio theory. The portfolios that might be chosen by three investors are shown in Figure 7.11. Having chosen to invest in Portfolio M, each investor combines this risky investment with a position in the risk-free asset. In Figure 7.11, Investor 1 will invest partly in Portfolio M and partly in the risk-free asset. Investor 2 will invest all funds in Portfolio M, while Investor 3 will borrow at the risk-free rate and invest his or her own funds, plus the borrowed funds, in Portfolio M. A fourth strategy, not shown in Figure 7.11, is to invest only in the risk-free asset. This is the least risky strategy, whereas the strategy pursued by Investor 3 is the riskiest.

If all investors in a particular market behave according to portfolio theory, *all investors hold Portfolio M* as at least a part of their total portfolio.[9] In turn, this implies that Portfolio M must consist of all risky assets. In other words, under these assumptions, a given risky asset, X, is either held by all investors as part of Portfolio M or it is not held by any investor. In the latter case, Asset X does not exist. Therefore, Portfolio M is often called the **market portfolio** because it comprises all risky assets available in the market. For example, if the total market value of all shares in Company X represents 1 per cent of the total market value of all assets, then shares in Company X will represent 1 per cent of every investor's total investment in risky assets.

market portfolio
portfolio of all risky assets, weighted according to their market capitalisation

capital market line
efficient set of all portfolios that provides the investor with the best possible investment opportunities when a risk-free asset is available. It describes the equilibrium risk–return relationship for efficient portfolios, where the expected return is a function of the risk-free interest rate, the expected market risk premium and the proportionate risk of the efficient portfolio to the risk of the market portfolio

The line R_fMN is called the **capital market line** because it shows all the total portfolios in which investors in the capital market might choose to invest. Since investors will choose only efficient portfolios, it follows that the market portfolio is predicted to be 'efficient' in the sense that it will provide the maximum expected return for that particular level of risk. The capital market line, therefore, shows the trade-off between expected return and risk for all efficient portfolios. The equation of the capital market line is given by:[10]

$$E(R_p) = R_f + \left(\frac{E(R_M) - R_f}{\sigma_M}\right)\sigma_p \qquad [7.8]$$

where σ_M is the standard deviation of the return on the market Portfolio M

9 This ignores the extreme case of investors who hold only the risk-free asset.

10 The fact that Equation 7.8 is the equation for the capital market line can be shown as follows: let Portfolio p consist of an investment in the risk-free asset and the market portfolio. The investment proportions are w_f in the risk-free asset and $w_M = 1 - w_f$ in the market portfolio. Therefore, Portfolio p is, in effect, a two-security portfolio and its expected return is given by:

$$E(R_p) = w_f R_f + (1 - w_f)E(R_M)$$

and the variance of its return is:

$$\sigma_P^2 = w_f^2\sigma_f^2 + (1 - w_f)^2\,\sigma_M^2 + 2w_f(1 - w_f)\rho_{fM}\,\sigma_f\sigma_M$$

(*continued*)

The slope of this line is $\frac{E(R_M) - R_f}{\sigma_M}$, and this measures the market price of risk. It represents the additional expected return that investors would require to compensate them for incurring additional risk, as measured by the standard deviation of the portfolio.

7.6.2 THE CAPITAL ASSET PRICING MODEL (CAPM) AND THE SECURITY MARKET LINE

LEARNING OBJECTIVE 8

Explain the relationship between returns and risk proposed by the capital asset pricing model

Although the capital market line holds for efficient portfolios, it does not describe the relationship between expected return and risk for individual assets or inefficient portfolios. In equilibrium, the expected return on a risky asset (or inefficient portfolio), *i*, can be shown to be:[11]

$$E(R_i) = R_f + \left(\frac{E(R_M) - R_f}{\sigma_M^2}\right)\text{Cov}(R_i, R_M) \qquad [7.9]$$

where $E(R_i)$ = the expected return on the *i*th risky asset

$\text{Cov}(R_i, R_M)$ = the covariance between the returns on the *i*th risky asset and the market portfolio

Equation 7.9 is often called the *CAPM equation*. An equivalent version is given in Equation 7.11. The CAPM equation shows that the expected return demanded by investors on a risky asset depends on the risk-free rate of interest, the expected return on the market portfolio, the variance of the return on the market portfolio, and the *covariance* of the return on the risky asset with the return on the market portfolio.

The covariance term $\text{Cov}(R_i, R_M)$ is the only explanatory factor in the CAPM equation specific to asset *i*. The other explanatory factors (R_f, $E(R_M)$ and σ_M^2) are the same, regardless of which asset *i* is being considered. Therefore, according to the CAPM equation, if two assets have different expected returns, this is because they have different covariances with the market portfolio. In other words, the measure of risk relevant to pricing a risky asset is $\text{Cov}(R_i, R_M)$, the covariance of its returns with returns on the market portfolio, as this measures the *contribution* of the risky asset to the riskiness of an efficient portfolio. In contrast, for the efficient portfolio itself the standard deviation of the portfolio's return is the relevant measure of risk (see Figure 7.11).

As discussed in Section 7.5.4, the measure of risk for an investment in a risky asset *i* is often referred to as its beta factor, β_i, where:

$$\beta_i = \frac{\text{Cov}(R_i, R_M)}{\sigma_M^2} \qquad [7.10]$$

By definition, $\sigma_p^2 = 0$ so the variance reduces to:

$$\sigma_p^2 = (1 - w_f)^2 \sigma_M^2$$

Therefore:

$$\sigma_p = (1 - w_f)\sigma_M$$

Since the expected return and standard deviation of Portfolio *p* are linear functions of w_f, it follows that $R_f M$ in Figure 7.11 is a straight line. This result is not specific to portfolios consisting of the risk-free asset and Portfolio *M*: rather it applies to *all* portfolios that include the risk-free asset.

The equation for a straight line can be expressed as $y = mx + c$ where m is the slope of the line and c is the intercept on the y axis. Referring to Figure 7.11, it can be seen that:

$$c = R_f \quad \text{and} \quad m = \frac{E(R_M - R_f)}{\sigma_M}$$

Therefore, the equation for the line $r_f MN$ is Equation 7.8.

11 This is a purely mathematical problem. For a derivation see Levy and Sarnat (1990) or Brailsford and Faff (1993).

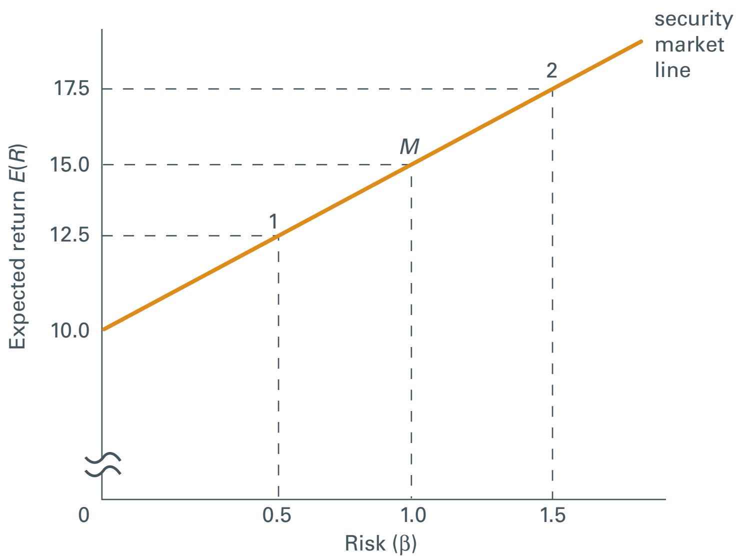
Because $\mathrm{Cov}(R_i,R_M)$ is the risk of an asset held *as part of* the market portfolio, while σ_M^2 is the risk (in terms of variance) of the market portfolio, it follows that β_i measures the risk of i relative to the risk of the market as a whole. Using beta as the measure of risk, the CAPM equation can be rewritten:

$$E(R_i) = R_f + \beta_i[E(R_M) - R_f] \quad [7.11]$$

When graphed, Equation 7.11 is called the **security market line** and is illustrated in Figure 7.12.

security market line graphical representation of the capital asset pricing model

FIGURE 7.12 Security market line



The significance of the security market line is that in equilibrium each risky asset should be priced so that it plots exactly on the line. Equation 7.11 shows that according to the capital asset pricing model, the expected return on a risky asset consists of two components: the risk-free rate of interest plus a premium for risk. The risk premium for each asset depends on the asset's beta and on the market risk premium $[E(R_M) - R_f]$. The betas of individual assets will be distributed around the beta value of the market portfolio, which is 1.[12] A risky asset with a beta value greater than 1 (that is, higher risk) will have an expected return greater than $E(R_M)$, while the expected return on a risky asset with a beta value of less than 1 (that is, lower risk) will be less than $E(R_M)$. Assuming that the risk-free rate of interest is 10 per cent and the market risk premium $[E(R_M) - R_f]$ is 5 per cent, the expected return on risky Asset 1 with a beta value of 0.5 will be 12.5 per cent. The expected return on risky Asset 2 with a beta value of 1.5 will be 17.5 per cent.

The capital asset pricing model applies to individual assets and to portfolios. The beta factor for a portfolio p is simply:

$$\beta_p = \frac{\mathrm{Cov}(R_p, R_M)}{\sigma_M^2} \quad [7.12]$$

where $\mathrm{Cov}(R_p,R_M)$ = the covariance between the returns on portfolio p and the market portfolio.

12 Since

$$\beta_i = \frac{\mathrm{Cov}(R_i, R_M)}{\sigma_M^2}$$

we have

$$\beta_M = \frac{\mathrm{Cov}(R_M, R_M)}{\sigma_M^2} = \frac{\sigma_M^2}{\sigma_M^2} = 1 .$$

Equation 7.12 is simply Equation 7.10 rewritten in terms of a portfolio p, instead of a particular asset i. Fortunately, there is a simple relationship between a portfolio's beta (β_p) and the betas of the individual assets that make up the portfolio. This relationship is:

$$\beta_p = \sum_{i=1}^{n} w_i \beta_i \qquad [7.13]$$

where n = the number of assets in the portfolio

w_i = the proportion of the current market value of portfolio p constituted by the ith asset

Equation 7.13 states that the beta factor for a portfolio is simply a weighted average of the betas of the assets in the portfolio.[13] One useful application of Equation 7.13 is to guide investors in choosing the investment proportions w_i to achieve some target portfolio *beta*, β^*_p. An important special case is to construct such a portfolio using only the market portfolio (β = 1) and a position in the risk-free asset (β = 0). In this case, investors place a proportion w_M of their total funds in the market portfolio, and a proportion $w_f = (1 - w_M)$ in the risk-free asset. Using Equation 7.13, the target beta is given by:

$$\beta^*_p = w_f \beta_f + w_M \beta_M$$

Substituting $\beta_f = 0$ and $\beta_M = 1$ gives:

$$w_M = \beta^*_p$$

and $$w_f = 1 - \beta^*_p$$

For example, if $\beta^*_p = 0.75$, investors should invest 75 per cent of their funds in the market portfolio and lend 25 per cent of their funds at the risk-free rate. If $\beta^*_p = 1.3$, investors should borrow an amount equal to 30 per cent of their own investment funds and invest the total amount (130 per cent) in the market portfolio.

7.6.3 IMPLEMENTATION OF THE CAPM

Use of the CAPM requires estimation of the risk-free interest rate, R_f, the systematic risk of equity, β_e and the market risk premium, $E(R_M) - R_f$. Each of these variables is discussed in turn.

The risk-free interest rate (R_f)

The assets closest to being risk free are government debt securities, so interest rates on these securities are normally used as a measure of the risk-free rate. However, as discussed in Section 4.6.1, unless the term structure of interest rates is flat, the various government securities will offer different interest rates. The appropriate risk-free rate is the current yield on a government security whose term to maturity matches the life of the proposed projects to be undertaken by the company. Since these activities undertaken by the company typically provide returns over many years, the rate on long-term securities is generally used.

13. Our discussion has omitted the steps between Equations 7.12 and 7.13. For the interested reader, these steps are as follows. Since:

$$R_p = \sum_{i=1}^{n} w_i R_i$$

it follows that:

$$\begin{aligned}\text{Cov}(R_p, R_M) &= \text{Cov}\left(\sum_{i=1}^{n} w_i R_i, R_M\right)\\ &= \sum_{i=1}^{n} w_i \text{Cov}(R_i, R_M)\end{aligned}$$

Substituting in Equation 7.12:

$$\begin{aligned}\beta_i &= \frac{\sum_{i=1}^{n} w_i \text{Cov}(R_i, R_M)}{\sigma^2_M}\\ &= \sum_{i=1}^{n} w_i \beta_i\end{aligned}$$

The share's systematic risk (β_e)

The betas of securities are usually estimated by applying regression analysis to estimate the following equation from time series data:

$$R_{it} = \alpha_i + \beta_i R_M + e_{it} \qquad [7.14]$$

where α_i = a constant, specific to asset i

e_{it} = an error term

Equation 7.14 is generally called the **market model**. Its relationship to the security market line can be readily seen by rewriting Equation 7.11 as follows:

market model time series regression of an asset's returns

$$E(R_i) = R_f + \beta_i\, E(R_M) - \beta_i R_f$$

$$\therefore\ E(R_i) = R_f\,(1 - \beta_i) + \beta_i\, E(R_M) \qquad [7.15]$$

Therefore, the market model is a counterpart (or analogue) of Equation 7.15. The magnitude of the betas that result from using this model when it is applied to returns on shares is illustrated in Table 7.5, which contains a sample of betas for the shares of selected listed firms. The values are calculated using ordinary least squares (OLS) regression.

TABLE 7.5 Betas of selected Australian listed firms calculated using daily share price and index data for the period July 2007–June 2010

NAME OF FIRM	MAIN INDUSTRIAL ACTIVITY	BETA
ANZ Banking Group	Banking	1.18
Amcor	Packaging	0.62
BHP Billiton	Minerals exploration, production and processing	1.41
Qantas Airways	Airline transportation	0.96
Telstra Corporation	Telecommunication services	0.40
Toll Holdings	Transportation	0.87
Wesfarmers	Diversified operations across multiple industries	0.97
Woolworths	Food and staples retailing	0.61

The market model, as specified in Equation 7.14, is often used to obtain an estimate of ex-post systematic risk. To use the market model, it is necessary to obtain time series data on the rates of return on the share and on the market portfolio—that is, a series of observations for both R_{it} and R_{Mt} is needed. However, when using the market model, choices must be made about two factors. First, the model may be estimated over periods of different length. For example, data for the past 1, 2, 3 or more years may be used. Five years of data are commonly used, but the choice is somewhat arbitrary. Second, the returns used in the market model may be calculated over periods of different length. For example, daily, weekly, monthly, quarterly or yearly returns may be used. Again this choice is subject to a considerable degree of judgement.

From a statistical perspective, it is generally better to have more rather than fewer observations, because using more observations generally lead to greater statistical confidence. However, the greater the number of years of data that are used, the more likely it is that the firm's riskiness will have changed. This fact highlights a fundamental problem of using the market model. The market model provides a measure of how risky a firm's equity was in the past. What we are seeking to obtain is an estimate of future risk. Therefore, the choice of both the number of years of data and the length of the period over which returns are calculated involves a trade-off between the desire to have many observations and the need to have recent and consequently more relevant data.[14]

The market risk premium $[E(R_M) - R_f]$

The market portfolio specified in the CAPM consists of every risky asset in existence. Consequently, it is impossible in practice to calculate its expected rate of return and hence impossible to also calculate the market risk premium. Instead, a share market index is generally used as a substitute for the market portfolio. As the rate of return on a share market index is highly variable from year to year, it is usual to calculate the average return on the index over a relatively long period. Suppose that the average rate of return on a share market index such as the All-Ordinaries Accumulation Index over the past 10 years was 18.5 per cent per annum. If this rate were used as the estimate of $E(R_M)$ and today's risk-free rate is 8.5 per cent, the market risk premium $[E(R_M) - R_f]$ would be 10 per cent.

A problem with using this approach is that the estimate of R_f reflects the market's current expectations of the future, whereas $E(R_M)$ is an average of past returns. In other words, the two values may not match, and some unacceptable estimates may result. For example, $[E(R_M) - R_f]$ estimated in this way may be negative if the rate of inflation expected now, which should be reflected in R_f, is greater than the realised rate of inflation during the period used to estimate $E(R_M)$.

A better approach is to estimate the market risk premium directly, over a relatively long period. For example, Ibbotson and Goetzmann (2005) compare the returns on equities with the returns on bonds in the US between 1792 and 1925 and report an average difference of approximately 3.8 per cent per annum. Similarly, Dimson, Marsh and Staunton (2003) found over the 103 years from 1900 to 2002 that the long-term average premium in the US was 7.2 per cent per annum. They also examined 15 other countries over this time period, finding that the country with the lowest premium was Denmark at 3.8 per cent per annum, and the country with the highest premium was Italy at 10.3 per cent per annum. Brailsford, Handley and Maheswaran (2008) estimate that in Australia the premium on the market portfolio over the 123 years from 1883 to 2005 was approximately 6.2 per cent per annum. Using a shorter time period during which the quality of the data is higher, they estimate that the premium from 1958 to 2005 was approximately 6.3 per cent per annum.

However, estimating the market risk premium directly also has some problems. Ritter (2002) uses the example of Japan at the end of 1989 to illustrate that historical estimates can result in nonsensical numbers. He notes that estimating the market risk premium at the end of 1989 using historical data starting when the Japanese stock market reopened after World War II would have provided a market risk premium of over 10 per cent per annum. The Japanese economy was booming, corporate profits were high and average price–earnings (P–E) ratios were over 60. It was considered that the cost of equity for Japanese firms was low. However, it is not possible for the cost of equity to be low and the market risk premium to be high. Of course, it is possible for the *historical* market risk premium to be high and the *expected* market risk premium (and therefore the expected cost of equity capital) to be low.

In an important theoretical paper, Mehra and Prescott (1985) showed that a long-term risk premium such as that found in the US, Canada, the UK and Australia cannot be explained by standard models of risk and return. This finding has led to arguments that historical measures of the risk premium are subject to errors in their measurement. For example, Jorion and Goetzmann (1999) argue that estimates of the market risk premium based solely on data obtained from the US will be biased upwards simply as a result of the outperformance of the US market relative to other equity markets over the twentieth century. Others, such as Heaton and Lucas (2000), argue that increased opportunities for portfolio diversification mean that the market risk premium has fallen.

14 For a discussion of the issues associated with calculating systematic risk from historical data, see Brailsford, Faff and Oliver (1997).

These concerns have led to new techniques being employed to estimate the market risk premium. Fama and French (2002), among others, use the dividend growth model and conclude that the market risk premium is now of the order of 1 per cent per annum. Claus and Thomas (2001) use forecasts by security analysts and conclude that the market risk premium is approximately 3 per cent per annum. Duke University and CFO magazine have conducted a quarterly survey of chief financial officers since 1996 (see **www.cfosurvey.org**). The average estimated risk premium for the US over that time has been approximately 4 per cent per annum. For the third quarter of 2010, when asked how much they expect returns in the equity market in the US to exceed the returns on government bonds over the next 10 years, the average response was 3.2 per cent per annum. In summary, the disparity of estimates of the market risk premium is considerable, ranging from 1 to in excess of 6 per cent per annum.

WWW

7.6.4 RISK, RETURN AND THE CAPM

The distinction between systematic and unsystematic risk is important in explaining why the CAPM should represent the risk–return relationship for assets such as shares. This issue was discussed in Section 7.5.3 but is reiterated here because of its importance in understanding the CAPM. The returns on a firm's shares can vary for many reasons: for example, interest rates may change, or the firm may develop a new product, attract important new customers or change its chief executive. These factors can be divided into two categories: those related only to an individual firm (firm-specific factors) and those that affect all firms (market-wide factors). As the shares of different firms are combined in a portfolio, the effects of the firm-specific factors will tend to cancel each other out; this is how diversification reduces risk. However, the effects of the market-wide factors will remain, no matter how many different shares are included in the portfolio. Therefore, systematic risk reflects the influence of market-wide factors, while unsystematic risk reflects the influence of firm-specific factors.

Because unsystematic risk can be eliminated by diversification, the capital market will not reward investors for bearing this type of risk. The capital market will only reward investors for bearing risk that cannot be eliminated by diversification—that is, the risk inherent in the market portfolio. There are cases when, with hindsight, we can identify investors who have reaped large rewards from taking on unsystematic risk. These cases do not imply that the CAPM is invalid: the model simply says that such rewards cannot be *expected* in a competitive market. The reward for bearing systematic risk is a higher expected return and, according to the CAPM, there is a simple linear relationship between expected return and systematic risk as measured by beta.

7.7 ADDITIONAL FACTORS THAT EXPLAIN RETURNS

LEARNING OBJECTIVE 9

Understand the relationship between the capital asset pricing model and models that include additional factors

In 1977 Richard Roll published an important article that pointed out that while the CAPM has strong theoretical foundations, there is a range of difficulties that researchers face in testing it empirically. For example, in testing for a positive relationship between an asset's beta and realised returns, a researcher first needs to measure the correlation between the asset's returns and the returns on the market portfolio. The market portfolio theoretically consists of *all* assets in existence and is therefore unobservable in practice—implying that ultimately the CAPM itself is untestable.

Aside from the problems associated with testing for a relationship between estimates of beta and realised returns, voluminous empirical research has shown that there are other factors that also explain returns. These factors include a company's dividend yield, its price–earnings (P–R) ratio, its size (as measured by the market value of its shares), and the ratio of the book value of its equity to the market value of its equity. This last ratio is often called the company's book-to-market ratio. In a detailed study, Fama and French (1992) show that the size and book-to-market ratio were dominant and that dividend yield and the price–earnings ratio were not useful in explaining returns after allowing for these more dominant factors.

In another important paper, Fama and French (1993) tested the following three-factor model of expected returns:

$$E(R_{it}) - R_{ft} = \beta_{iM}[E(R_{Mt}) - R_{ft}] + \beta_{iS}\,E(SMB_t) + \beta_{iH}\,E(HML_t) \quad [7.16]$$

LEARNING OBJECTIVE 10

Explain the development of models that include additional factors

In Equation 7.16, the first factor is the market risk premium, which is the basis of the CAPM discussed earlier in this chapter. The next factor, SMB, refers to the difference between the returns of a diversified portfolio of small and large firms, while HML reflects the differences between the returns of a diversified portfolio of firms with high versus low book-to-market values. β_{iM}, β_{iS} and β_{iH} are the risk parameters reflecting the sensitivity of the asset to the three sources of risk. All three factors were found to have strong explanatory power. O'Brien, Brailsford and Gaunt (2008) found that in Australia, over the period 1982 to 2006, all three factors provided strong explanatory power.

It is possible that both the size and book-to-market ratio factors might be explicable by risk. For example, Fama and French (1996) argue that smaller companies are more likely to default than larger companies. Further, they argue that this risk is likely to be systematic in that small companies as a group are more exposed to default during economic downturns. As a result, investors in small companies will require a risk premium. Similarly, Zhang (2005) argues that companies with high book-to-market ratios will on average have higher levels of physical capacity. Much of this physical capacity will represent excess capacity during economic downturns and therefore expose such companies to increased risk.

However, as discussed in detail in Chapter 16, the relationship between these additional factors and returns may not be due to risk. Further, Carhart (1997) added a fourth factor to the three described in Equation 7.16 to explain returns earned by mutual funds. In an earlier paper Jegadeesh and Titman (1993), using US data from 1963 to 1989, identified better-performing shares (the winners) and poorer-performing shares (the losers) over a period of 6 months. They then tracked the performance of the shares over the following 6 months. On average, the biggest winners outperformed the biggest losers by 10 per cent per annum. When Carhart added this **momentum effect** to the three-factor model, he found that it too explained returns. Unlike the size and book-to-market ratio factors, it is difficult to construct a simple risk-based explanation for this factor.

AQ: 1

While the CAPM is clearly an incomplete explanation of the relationship between risk and returns, it is important to note that it is still widely applied. This point is perhaps best demonstrated by the Coleman, Maheswaran and Pinder (2010) survey of the financial practices adopted by senior financial managers in Australia. Financial managers employ asset pricing models to estimate the discount rate used in project evaluation techniques such as the net present value approach. Coleman, Maheswaran and Pinder reported that more than twice as many respondents used the traditional single-factor CAPM compared with models that used additional factors.

7.8 PORTFOLIO PERFORMANCE APPRAISAL

LEARNING OBJECTIVE 11

Distinguish between alternative methods of appraising the performance of an investment portfolio

A fundamental issue that faces investors is how to measure the performance of their investment portfolio. To illustrate the problem, assume that an investor observes that during the past 12 months, his or her portfolio has generated a return of 15 per cent. Is this a good, bad or indifferent result? The answer to that question depends, of course, on the *expected* return of the portfolio given the portfolio's risk. That is, in order to answer the question, we need a measure of the risk of the investor's portfolio, and then compare its performance with the performance of a benchmark portfolio of similar risk. However, even after accounting for the specific risk of the portfolio, the performance of a portfolio may differ from that of the benchmark for four reasons:

- *Asset allocation*. Investors must decide how much of their wealth should be allocated between alternative categories of assets such as corporate bonds, government bonds, domestic shares, international shares and property. This decision will ultimately affect the performance of the portfolio because in any given period a particular asset class may outperform other asset classes on a risk-adjusted basis.

- *Market timing*. In establishing and administering a portfolio, investors need to make decisions about when to buy and sell the assets held in a portfolio. For example, investors might choose to move out of domestic shares and into corporate bonds or alternatively sell the shares of companies that operate in the telecommunication industry and invest these funds in the shares of companies operating in the retail industry. Clearly, the performance of a portfolio will be affected by an investor's success in selling assets before their prices fall and buying assets before their prices rise.
- *Security selection*. Having made a decision about the desired mix of different asset classes within a portfolio, and when that desired mix should be implemented, investors must then choose between many different individual assets within each class. For example, having determined that they wish to hold half of their portfolio in domestic shares, investors must then decide which of the more than 2000 shares listed on the Australian Securities Exchange they should buy. The art of security selection requires the investors to identify those individual assets that they believe are currently underpriced by the market and hence whose values are expected to rise over the holding period. Similarly, if investors believe that any of the assets held in the portfolio are currently overpriced, they would sell these assets so as to avoid any future losses associated with a reduction in their market value.
- *Random influences*. Ultimately, investing is an uncertain activity and in any given period the performance of a portfolio may not reflect the skills of the investor who makes the investment decisions. That is, good decisions might yield poor outcomes and poor decisions might yield good outcomes in what we would label as 'bad luck' or 'good luck', respectively. Over enough time, though, we would expect the influence of good luck and bad luck to average out.

7.8.1 ALTERNATIVE MEASURES OF PORTFOLIO PERFORMANCE

We now consider four commonly used ways of measuring the performance of a portfolio. Each of these measures has a different approach to trying to determine the 'expected' performance of the benchmark portfolio in order to determine whether the portfolio has met, exceeded or failed to meet expectations.

Simple benchmark index

This is probably the most commonly used approach to appraising the performance of a portfolio and involves a simple comparison between the portfolio's return and the return on a benchmark index that has (or is assumed to have) similar risk to the portfolio being measured. For example, a well-diversified portfolio of domestic shares might be benchmarked against the S&P/ASX 200 Index, which measures the performance of the shares in the 200 largest companies listed on the Australian Securities Exchange.

The advantages associated with using this approach to performance appraisal are that it is easy to implement and to understand. The main problem with this approach is that it implies that the risk of the portfolio is identical to the risk of the benchmark index, whereas, with the exception of so-called passive funds, which are specifically established to mimic (or track) the performance of benchmark indices, this will rarely be the case.

The Sharpe ratio

The Sharpe ratio, developed by William Sharpe[15], is a measure of the excess return of the portfolio per unit of total risk and is calculated using the following formula:

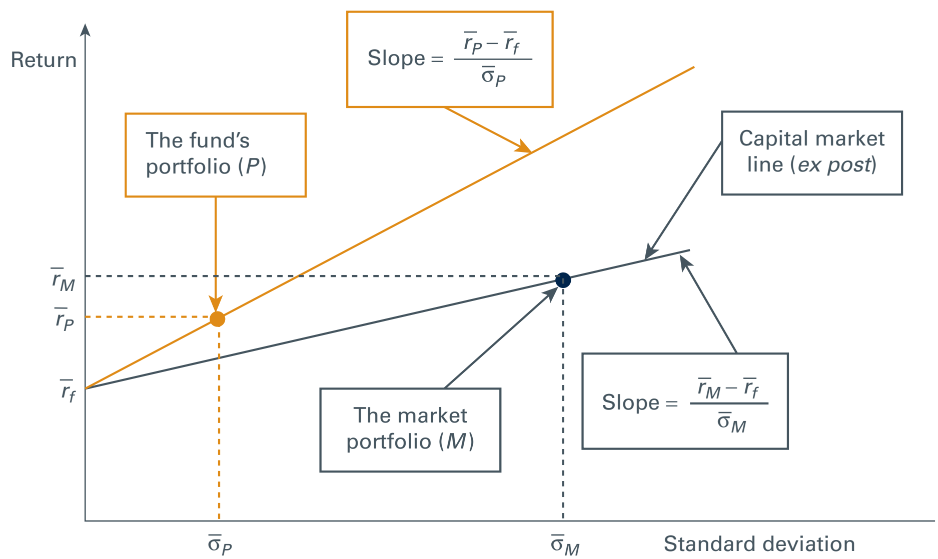
$$S = \frac{\overline{r_P} - \overline{r_f}}{\overline{\sigma_P}} \qquad [7.17]$$

15 See Sharpe (1966).

where $\bar{r}_p$ is the average return achieved on the portfolio over the time period, $\bar{r}_f$ is the average risk-free rate of return over the same time period and $\bar{\sigma}_p$ is the standard deviation of the returns on the portfolio over the time period and is a measure of the total risk of the portfolio. If the Sharpe ratio of the investor's portfolio exceeds the Sharpe ratio of the market portfolio, then the investor's portfolio has generated a greater excess return per unit of total risk and hence is regarded as exhibiting superior performance to the market portfolio. Conversely, if the portfolio's Sharpe ratio is less than that of the market portfolio then the portfolio has generated less excess return per unit of total risk than the market portfolio and the portfolio can be seen as having underperformed that benchmark.

The rationale behind the use of the Sharpe ratio is best demonstrated by considering the ratio's links with the risk–return trade-off described by the capital market line discussed in section 7.6.1. Consider Figure 7.13, which illustrates the risk and return profile for a superannuation fund's portfolio relative to the market portfolio.

FIGURE 7.13 The Sharpe ratio



Note from Figure 7.13 that the superannuation fund's portfolio has generated a lower rate of return than the market portfolio but has also generated a lower level of total risk. That is, while $\bar{r}_p$ is less than $\bar{r}_M$, $\bar{\sigma}_P$ is also less than $\bar{\sigma}_M$. The key point, however, is that the realised excess return *per unit of risk* is higher for the fund's portfolio compared with the market portfolio and hence the fund's portfolio is regarded as having exhibited superior performance. This is illustrated in Figure 7.13 by the fund's portfolio plotting *above* the capital market line. If the fund's portfolio had generated a lower excess return per unit of risk than the market portfolio, then it would have plotted below the capital market line and this would have implied that the portfolio had underperformed the benchmark on a total risk-adjusted basis.

Note that the Sharpe ratio assumes that in determining the risk-adjusted performance of a portfolio the appropriate measure of risk is total risk. Following on from our discussion earlier in the chapter, it is clear that total risk is an appropriate measure only when we are dealing with well-diversified portfolios rather than individual assets or undiversified portfolios.

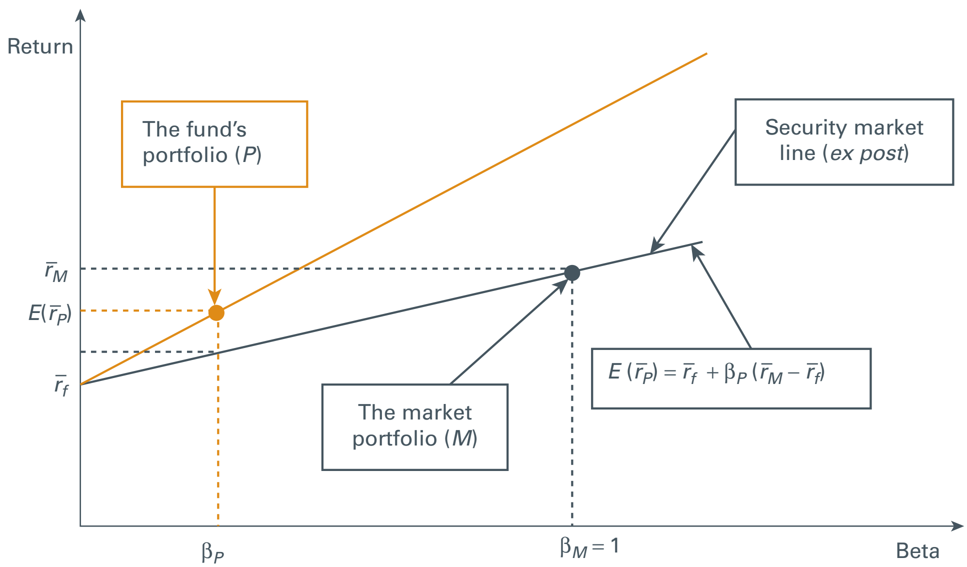
The Treynor ratio

The Treynor ratio, named after Jack Treynor[16], is a measure that is related to the Sharpe ratio of performance measurement, in that it measures excess returns per unit of risk, but differs in that it defines risk as non-diversifiable (or systematic) risk instead of total risk. It can be calculated using the following formula:

$$T = \frac{\bar{r}_p - \bar{r}_f}{\bar{\beta}_p} \qquad [7.18]$$

where σ_p^2 and $\bar{r}_f$ are the returns on the portfolio and the risk-free asset as defined earlier, and $\bar{\beta}_p$ is an estimate of the systematic risk of the portfolio over the period in which the returns were generated, as measured by beta and defined in Section 7.6.2. As with the Sharpe ratio, insights into the rationale behind the use of the Treynor ratio are provided by considering the link between risk and expected return—but this time, instead of considering the trade-off for efficient portfolios implied by the capital market line, we turn instead to the security market line, which applies to individual assets and inefficient portfolios. In Figure 7.14 we compare the ex-post systematic risk and excess returns of a superannuation fund relative to the market portfolio over the same period of time.

FIGURE 7.14 The Treynor ratio



Recall that the security market line is simply the graphical representation of the CAPM. The slope of the security market line describes the extra return, in excess of the risk-free rate, that is expected for each additional unit of systematic risk (as measured by beta) and is what we have previously defined as the market risk premium $(\bar{r}_M - \bar{r}_f)$. The slope of the line that intersects the realised systematic risk and return of the fund's portfolio is in turn the Treynor ratio. Hence, the decision rule used in assessing the performance of a portfolio using this technique requires a comparison of the Treynor ratio calculated for the portfolio over a specified interval with the market risk premium generated over that same interval. Example 7.3 illustrates the three approaches to portfolio appraisal discussed above.

16 See Treynor (1966).

EXAMPLE 7.3

An investor holds a portfolio that consists of shares in 15 different companies and wants to assess the performance using a simple benchmark index as well as calculating the portfolio's Sharpe and Treynor ratios. She estimates the following parameters for the financial year ended 30 June 2011.

TABLE 7.6

	REALISED RETURN (% p. a.)	STANDARD DEVIATION OF RETURNS (σ) (% p.a.)	SYSTEMATIC RISK ESTIMATE (β)
Portfolio	13	30	1.2
S&P/ASX 200 share price index	11	20	1.0
Government bonds	5	0	0

Based solely on the benchmark index approach, the portfolio appears to have performed well in that it has generated an additional 2 per cent return above the proxy for the market (S&P/ASX 200). As discussed earlier, however, this assessment fails to account for differences in the risk profiles of the two portfolios.

The Sharpe ratio is estimated using Equation 7.17 for both the investor's portfolio and the ASX 200 as follows:

$$S = \frac{\bar{r}_P - \bar{r}_f}{\bar{\sigma}_P}$$

$$S_{Portfolio} = \frac{13 - 5}{30} = 0.27$$

$$S_{ASX\ 200} = \frac{11 - 5}{20} = 0.30$$

As the Sharpe ratio for the portfolio is less than that of the S&P/ASX 200, the investor concludes that the portfolio has underperformed the market on a risk-adjusted basis. A possible problem with this conclusion is that, as described above, the Sharpe ratio assumes that the relevant measure of risk for the investor is total risk, as measured by the standard deviation of returns. This is not the case where, for example, the portfolio of shares represents only one component of the investor's overall set of assets.

The Treynor ratios for the portfolio and for the ASX 200 are measured as follows:

$$T = \frac{\bar{r}_P - \bar{r}_f}{\bar{\beta}_P}$$

$$T_{Portfolio} = \frac{13 - 5}{1.2} = 6.67$$

$$T_{ASX\ 200} = \frac{11 - 5}{1.0} = 6$$

Note that the Treynor ratio for the S&P/ASX 200 is simply equal to the market risk premium of 6 per cent. As the Treynor ratio of the portfolio exceeds this amount the investor concludes that the portfolio has outperformed the market on a systematic risk-adjusted basis. We can reconcile this result with the seemingly contrary results provided by the Sharpe ratio by acknowledging that some of the portfolio risk that is accounted for in the Sharpe ratio may actually be diversified away once we account for the other assets in the investor's portfolio. Therefore, in this case, the Treynor ratio provides the more suitable assessment of the performance of the portfolio relative to the market generally, as it considers only that risk that cannot be eliminated by diversification.

Jensen's alpha

Jensen's alpha is a measure that was a pioneered by Michael Jensen[17] and that relies on a multi-period analysis of the performance of an investment portfolio relative to some proxy for the market generally. Recall that the CAPM suggests that the relationship between systematic risk and return is fully described by the following equation:

$$E(R_i) = R_f + \beta_i[E(R_M) - R_f]$$

The CAPM is an *ex-ante* single-period model, in the sense that it is concerned with the returns that might be *expected* over the next time period. Its conclusion is relatively simple: the return in excess of the risk-free rate that we expect any asset *i* to generate is determined only by the level of systematic risk reflected in the asset's β. We compute Jensen's alpha by implementing an *ex-post* multi-period regression analysis of the returns on the portfolio and the returns on the market and ask the question: Is there any evidence of systematic abnormal return performance that cannot be explained by the portfolio's systematic risk? The regression equation estimated is as follows:

$$r_{P,t} - r_{f,t} = \alpha_P + \beta_P\left[r_{M,t} - r_{f,t}\right] + e_t \qquad [7.19]$$

where $r_{P,t}$, $r_{f,t}$ and $r_{M,t}$ are the returns from the portfolio, the risk-free asset and the proxy for the market portfolio that have been observed in period *t*. β_P is an estimate of the portfolio's beta over the entire period in which returns were collected. α_P is an estimate of Jensen's alpha and reflects the incremental performance of the portfolio after accounting for the variation in portfolio returns that can be explained by market-wide returns.

If α_P is positive, and statistically significant, then this is an indication that the portfolio has outperformed the market, on a risk-adjusted basis, and may be interpreted as evidence of a portfolio manager's skill in managing the portfolio. Conversely, a statistically significant negative estimate of α_P might be interpreted as evidence that the portfolio manager's actions in managing the portfolio are actually destroying value!

There are many other techniques that have been developed by academics and practitioners to try to assess the performance of investment portfolios and each technique brings with it both advantages and disadvantages over the alternative approaches.[18] While much of the preceding discussion has been concerned with measuring the relative performance of a portfolio, another issue facing managers and investors is how much of the performance of a portfolio may be attributed to the different decisions made by the investment manager. Specifically, as described at the beginning of Section 7.8, an investor may be concerned with how the performance has been affected by the manager's decisions with respect to asset allocation, market timing and security selection as well as the possible interactions between each of these decisions.

SUMMARY

This chapter discussed two main issues. The first, portfolio theory, concerns the approach that can be used by risk-averse investors to secure the best trade-off between risk and return. Second, the chapter dealt with the pricing of risky assets, which involves the relationship between risk and return in the market for risky assets.

17 See Jensen (1968 & 1969).

18 See Chapter 24 of Bodie, Kane and Marcus (2011) for an excellent review of some of these alternative techniques, and a comprehensive description of other issues faced when assessing portfolio performance.

- The essential message of portfolio theory is that diversification reduces risk. It is also shown that the effectiveness of diversification depends on the correlation or covariance between returns on the individual assets combined into a portfolio. The gains from diversification are largest when there is negative correlation between asset returns, but they still exist when there is positive correlation between asset returns, provided that the correlation is less than perfect. In practice, the positive correlation that exists between the returns on most risky assets imposes a limit on the degree of risk reduction that can be achieved by diversification.
- The total risk of an asset can be divided into two parts: systematic risk that *cannot* be eliminated by diversification, and unsystematic risk that *can* be eliminated by diversification. It follows that the only risk that remains in a well-diversified portfolio is systematic risk.
  - The risk of a well-diversified portfolio can be measured by the standard deviation of portfolio returns. However, analysis of the factors that contribute to this standard deviation shows that, for investors who diversify, the relevant measure of risk for an individual asset is its systematic risk.
  - Systematic risk depends on the covariance between the returns on the asset and returns on the market portfolio, which contains all risky assets. The systematic risk of an asset is usually measured by the asset's beta factor, which measures the risk of the asset relative to the risk of the market as a whole. Risk-averse investors will aim to hold portfolios that are efficient in that they provide the highest expected return for a given level of risk. The set of efficient portfolios forms the efficient frontier, and in a market where only risky assets are available, each investor will aim to hold a portfolio somewhere on the efficient frontier.
- Introduction of a risk-free asset allows the analysis to be extended to model the relationship between risk and expected return for individual risky assets. The main result is the CAPM, which proposes that there is a linear relationship between the expected rate of return on an asset and its risk as measured by its beta factor.
- Alternative asset pricing models propose expected returns are linearly related to multiple factors rather than the single market factor proposed by the CAPM.
- Assessment of the performance of an investment portfolio requires the specification of the 'expected' performance of a benchmark portfolio.

WWW

An excellent site with a wealth of information relating to this topic is **www.wsharpe.com**. Professor William Sharpe's work was recognised with a Nobel Prize in 1990. Financial advisory information can also be found at **www.fido.asic.gov.au**.

KEY TERMS

beta 184
capital market line 189
market model 193
market portfolio 189
portfolio 175
risk-averse investor 172
risk-neutral investor 173
risk-seeking investor 173
security market line 191
standard deviation 170
systematic (market-related or non-diversifiable) risk 183
unsystematic (diversifiable) risk 183
value at risk 185
variance 170

SELF-TEST PROBLEMS

1. An investor places 30 per cent of his funds in Security X and the balance in Security Y. The expected returns on X and Y are 12 and 18 per cent, respectively. The standard deviations of returns on X and Y are 20 and 15 per cent, respectively.

(a) Calculate the expected return on the portfolio.

(b) Calculate the variance of returns on the portfolio assuming that the correlation between the returns on the two securities is:

(i) +1.0 **(iii)** 0
(ii) +0.7 **(iv)** –0.7

2. An investor holds a portfolio that comprises 20 per cent X, 30 per cent Y and 50 per cent Z. The standard deviations of returns on X, Y and Z are

22, 15 and 10 per cent, respectively, and the correlation between returns on each pair of securities is 0.6. Prepare a variance–covariance matrix for these three securities and use the matrix to calculate the variance and standard deviation of returns for the portfolio.

3. The risk-free rate of return is currently 8 per cent and the market risk premium is estimated to be 6 per cent. The expected returns and betas of four shares are as follows:

Share	Expected return (%)	Beta
Carltown	13.0	0.7
Pivot	17.6	1.6
Forresters	14.0	1.1
Brunswick	10.4	0.4

Which shares are undervalued, overvalued or correctly valued based on the CAPM?

Solutions to self-test problems are available in Appendix B, page XXX.

REFERENCES

Bodie, Z., Kane, A. & Marcus, A.J., *Investments*, 9th edn, McGraw-Hill, New York, 2011.

Brailsford, T. & Faff, R., 'A derivation of the CAPM for pedagogical use', *Accounting and Finance*, May 1993, pp. 53–60.

Brailsford, T., Faff, R. & Oliver, B., *Research Design Issues in the Estimation of Beta*, McGraw-Hill, Sydney, 1997.

Brailsford, T., Handley, J. & Maheswaran, K., 'Re-examination of the historical equity premium in Australia', *Accounting and Finance*, March 2008, pp. 73–97.

Carhart, M. M., 'On persistence in mutual fund performance', *Journal of Finance*, March 1997, pp. 57–82.

Claus, J. & Thomas, J., 'Equity premia as low as three per cent? Evidence from analysts' earnings forecasts for domestic and international stock markets', *Journal of Finance,* October 2001, pp. 1629–66.

Coleman, L., Maheswaran, K. & Pinder, S., 'Narratives in managers' corporate finance decisions', *Accounting and Finance*, September 2010, pp. 605–33.

Dimson, E., Marsh, P.R. & Staunton, M., 'Global evidence on the equity risk premium', *Journal of Applied Corporate Finance*, Fall 2003, pp. 27–38.

Fama, E.F., 'Risk, return and equilibrium: some clarifying comments', *Journal of Finance*, March 1968, pp. 29–40.

——, *Foundations of Finance*, Basic Books, New York, 1976.

Fama, E.F. & French, K.R., 'The cross-section of expected stock returns', *Journal of Finance*, June 1992, pp. 427–65.

——, ——, 'Common risk factors in the returns on stocks and bonds', *Journal of Financial Economics*, February 1993, pp. 3–56.

——, ——, 'Multifactor explanations of asset pricing anomalies', *Journal of Finance*, March 1996, pp. 55–84.

——, ——, 'The equity premium', *Journal of Finance,* April 2002, pp. 637–59.

Heaton, J. & Lucas, D., 'Portfolio choice and asset prices: the importance of entrepreneurial risk', *Journal of Finance*, June 2000, pp. 1163–98.

Ibbotson, R.G. & Goetzmann, W. N., 'History and the equity risk premium', April 2005, Yale ICF Working Paper No. 05–04. **http://ssrn.com/abstract=702341**

Jegadeesh, N. & Titman, S., 'Returns to buying winners and selling losers: implications for market efficiency', *Journal of Finance,* 1993, pp. 65–91.

Jensen, M., 'The performance of mutual funds in the period 1945–1964', *Journal of Finance*, May 1968, pp. 389–416.

——, 'Risk, the pricing of capital assets, and the evaluation of investment portfolios', *Journal of Business*, April 1969, pp. 167–247.

Jensen, M., 'Capital markets: theory and evidence', *Bell Journal of Economics and Management Science*, Autumn 1972, pp. 357–98.

Jorion, P., *Value at Risk: The New Benchmark for Controlling Market Risk*, 3rd edn, McGraw-Hill, Chicago, 2006.

Jorion, P. & Goetzmann, W.N., 'Global stock markets in the twentieth century', *Journal of Finance*, June 1999, pp. 953–80.

Levy, H. & Sarnat, M., *Capital Investment and Financial Decisions*, 4th edn, Prentice-Hall, New Jersey, 1990, pp. 319–22.

Lintner, J., 'The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets', *Review of Economics and Statistics*, February 1965, pp. 13–37.

Markowitz, H.M., 'Portfolio selection', *Journal of Finance*, March 1952, pp. 77–91.

——, *Portfolio Selection: Efficient Diversification of Investments*, John Wiley & Sons, New York, 1959.

WWW

Mehra, R. & Prescott, E.C., 'The equity premium: a puzzle', *Journal of Monetary Economics*, March 1985, pp. 145–61.

Mossin, J., 'Security pricing and investment criteria in competitive markets', *American Economic Review*, December 1969, pp. 749–56.

O'Brien, M., Brailsford, T. & Gaunt, C., 'Size and book-to-market factors in Australia', 21st Australasian Finance and Banking Conference Paper, 2008 **http://ssrn.com/abstract=1206542**.

PricewaterhouseCoopers, *Investigation into Foreign Exchange Losses at the National Australia Bank*, Melbourne, 2004.

Ritter, J.R., 'The biggest mistakes we teach', *Journal of Financial Research,* Summer 2002, pp. 159–68.

Roll, R., 'A critique of the asset pricing theory's tests; Part 1: On the past and potential testability of the theory', *Journal of Financial Economics,* March 1977, pp. 126–76.

Sharpe, W.F., 'Capital asset prices: a theory of market equilibrium under conditions of risk', *Journal of Finance*, September 1964, pp. 425–42.

——, 'Mutual fund performance', *Journal of Business*, January 1966, pp. 119–38

Statman, M., 'How many stocks make a diversified portfolio?', *Journal of Financial and Quantitative Analysis*, September 1987, pp. 353–63.

Treynor, J.L, 'How to rate management investment funds', January 1966, *Harvard Business Review*, pp. 63–75.

Zhang, L., 'The value premium', *Journal of Finance*, February 2005, pp. 67–103.

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QUESTIONS

1. **[LO X]** Is risk aversion a reasonable assumption? What is the relevant measure of risk for a risk-averse investor?
2. **[LO X]** What are the benefits of diversification to an investor? What is the key factor determining the extent of these benefits?
3. **[LO X]** *Risky assets can be combined to form a riskless asset.* Discuss.
4. **[LO X]** *Whenever an asset is added to a portfolio, the total risk of the portfolio will be reduced provided the returns of the asset and the portfolio are less than perfectly correlated.* Discuss.
5. **[LO X]** Explain each of the following:
   (a) the efficient frontier
   (b) the capital market line
   (c) the security market line.
6. **[LO X]** *Total risk can be decomposed into systematic and unsystematic risk.* Explain each component of risk, and how each is affected by increasing the number of securities in a portfolio.
7. **[LO X]** *An important conclusion of the CAPM is that the relevant measure of an asset's risk is its systematic risk.* Outline the significance of this conclusion for a manager making financial decisions.
8. **[LO X]** For investors who aim to diversify, shares with negative betas would be very useful investments, but such shares are very rare. Explain why few shares have negative betas.
9. **[LO X]** *Diversification is certainly good for investors. Therefore investors should be prepared to pay a premium for the shares of companies that operate in several lines of business.* Explain why this statement is true or false.
10. **[LO X]** Minco Ltd, a large mining company, provides a superannuation fund for its employees. The fund's manager says: 'We know the mining industry well, so we feel comfortable investing most of the fund in a portfolio of mining company shares'. Advise Minco's employees on whether to endorse the fund's investment policy.
11. **[LO X]** Farmers can insure their crops against damage by hailstorms at reasonable rates. However, the same insurance companies refuse to provide flood insurance at any price. Explain why this situation exists.
12. **[LO X]** Compare and contrast the capital asset pricing model and models that include additional factors.
13. **[LO X]** In what situations would it be appropriate to use a simple benchmark index, such as the S&P/ASX 200 share price index, to assess the performance of a portfolio?
14. **[LO X]** *When assessing the performance of a set of portfolios it does not really matter if you choose the Sharpe ratio or the Treynor ratio to do so as both approaches account for the risk inherent in the portfolios.* Discuss.

PROBLEMS

1. **Investment and risk [LO X]**
   Mr Barlin is considering a 1-year investment in shares in one of the following three companies:
   Company X: expected return = 15% with a standard deviation of 15%
   Company Y: expected return = 15% with a standard deviation of 20%

Company Z: expected return = 20% with a standard deviation of 20%

Rank the investments in order of preference for each of the cases where it is assumed that Mr Barlin is:

(a) risk averse

(b) risk neutral

(c) risk seeking.

Give reasons.

2. **Variance of return [LO X]**

An investor places 40 per cent of her funds in Company A's shares and the remainder in Company B's shares. The standard deviation of the returns on A is 20 per cent and on B is 10 per cent. Calculate the variance of return on the portfolio, assuming that the correlation between the returns on the two securities is:

(a) +1.0

(b) +0.5

(c) 0

(d) –0.5

3. **Portfolio standard deviation and diversification [LO X]**

The standard deviations of returns on assets A and B are 8 per cent and 12 per cent, respectively. A portfolio is constructed consisting of 40 per cent in Asset A and 60 per cent in Asset B. Calculate the portfolio standard deviation if the correlation of returns between the two assets is:

(a) 1

(b) 0.4

(c) 0

(d) –1

Comment on your answers.

4. **Expected return, variance and risk [LO X]**

You believe that there is a 50 per cent chance that the share price of Company L will decrease by 12 per cent and a 50 per cent chance that it will increase by 24 per cent. Further, there is a 40 per cent chance that the share price of Company M will decrease by 12 per cent and a 60 per cent chance that it will increase by 24 per cent. The correlation coefficient of the returns on shares in the two companies is 0.75. Calculate:

(a) the expected return, variance and standard deviation for each company's shares

(b) the covariance between their returns.

5. **Expected return, risk and diversification [LO X]**

Harry Jones has invested one-third of his funds in Share 1 and two-thirds of his funds in Share 2. His assessment of each investment is as follows:

Item	Share 1	Share 2
Expected return (%)	15.0	21.0
Standard deviation (%)	18.0	25.0
Correlation between the returns	0.5	

(a) What are the expected return and the standard deviation of return on Harry's portfolio?

(b) Recalculate the expected return and the standard deviation where the correlation between the returns is 0 and 1.0, respectively.

(c) Is Harry better or worse off as a result of investing in two securities rather than in one security?

6. **Expected return, risk and diversification [LO X]**

The table gives information on three risky assets: A, B and C.

			Correlations		
Asset	**Expected return**	**Standard deviation of return**	**A**	**B**	**C**
A	12.5	40	1.00	0.20	0.35
B	16.0	45	0.20	1.00	0.10
C	20.0	60	0.35	0.10	1.00

There is also a risk-free Asset F whose expected return is 9.9 per cent.

(a) Portfolio 1 consists of 40 per cent Asset A and 60 per cent Asset B. Calculate its expected return and standard deviation.

(b) Portfolio 2 consists of 60 per cent Asset A, 22.5 per cent Asset B and 17.5 per cent Asset C. Calculate its expected return and standard deviation. Compare your answers to (a) and comment.

(c) Portfolio 3 consists of 4.8 per cent Asset A, 75 per cent Asset B and 20.2 per cent in the risk-free asset. Calculate its expected return and standard deviation. Compare your answers to (a) and (b) and comment.

(d) Portfolio 4 is an equally weighted portfolio of the three risky assets A, B and C. Calculate its expected return and standard deviation and comment on these results.

(e) Portfolio 5 is an equally weighted portfolio of all four assets. Calculate its expected return and standard deviation and comment on these results.

7. **Expected return and systematic risk [LO X]**

The expected return on the ith asset is given by:

$$E(R_i) = R_f + \beta_i\,(E(R_M) - R_f)$$

(a) What is the expected return on the *i*th asset where $R_f = 0.08$, $\beta_i = 1.25$ and $E(R_M) = 0.14$?

(b) What is the expected return on the market portfolio where $E(R_i) = 0.11$, $R_f = 0.08$ and $\beta_i = 0.75$?

(c) What is the systematic risk of the *i*th asset where $E(R_i) = 0.14$, $R_f = 0.10$ and $E(R_M) = 0.15$?

8. Portfolio weights systematic risk, unsystematic risk [LO X]

The table provides data on two risky assets, A and B, the market portfolio M and the risk-free asset F.

Asset	Expected return (%)	A	B	M	F
A	10.8	324	60	48	0
B	15.6	60	289	96	0
M	14.0	48	96	80	0
F	6.0	0	0	0	0

An investor wishes to achieve an expected return of 12 per cent and is considering three ways this may be done:

(a) invest in A and B

(b) invest in B and F

(c) invest in M and F.

For each of these options, calculate the portfolio weights required and the portfolio standard deviation. Show that assets A and B are priced according to the capital asset pricing model and, in the light of this result, comment on your findings.

9. Assessing diversification benefits [LO X]

You are a share analyst employed by a large multinational investment fund and have been supplied with the following information:

Asset	Expected return (%)	Standard deviation (%)
BHZ Ltd	9	8
ANB Ltd	13	48

You are also told that the correlation coefficient between the returns of the two firms is 0.8. A client currently has all of her wealth invested in BHZ shares. She wishes to diversify her portfolio by redistributing her wealth such that 30 per cent is invested in BHZ shares and 70 per cent in ANB shares.

(a) What will be the expected return of the new portfolio?

(b) What will be the standard deviation of returns for the new portfolio?

After constructing the portfolio and reporting the results to your client, she is quite upset, saying, 'I thought the whole purpose of diversification was to reduce risk? Yet you have just told me that the variability of my portfolio has actually been increased from what it was when I invested only in BHZ.'

(c) Provide a response to your client that demonstrates that the new portfolio does (not) reflect the benefits of diversification. Show all necessary calculations.

10. Value at risk [LO X]

Consider a portfolio comprising a \$3 million investment in Outlook Publishing and a \$5 million investment in Russell Computing. Assume that the standard deviations of the returns for shares in these companies are 0.4 and 0.25 per cent per annum, respectively. Assume also that the correlation between the returns on the shares in these companies is 0.7. Assuming a 1 per cent chance of abnormally bad market conditions, calculate the value at risk of this portfolio. State any assumptions that you make in your calculations.

11. Portfolio performance appaisal [LO X]

In 2011 the return on the Fort Knox Fund was 10 per cent, while the return on the market portfolio was 12 per cent and the risk-free return was 3 per cent. Comparative statistics are shown in the table below.

Statistic	Fort Knox fund	S&P/ASX 200 share price index
Standard deviation of return	15%	30%
Beta	0.75	1.00

Calculate and comment on the performance of the fund using the following three approaches:

(a) the simple benchmark index

(b) the Sharpe ratio

(c) the Treynor ratio.