

Preface

In writing this book, I was guided by my long-standing experience and interest in teaching discrete mathematics. For the student, my purpose was to present material in a precise, readable manner, with the concepts and techniques of discrete mathematics clearly presented and demonstrated. My goal was to show the relevance and practicality of discrete mathematics to students, who are often skeptical. I wanted to give students studying computer science all of the mathematical foundations they need for their future studies. I wanted to give mathematics students an understanding of important mathematical concepts together with a sense of why these concepts are important for applications. And most importantly, I wanted to accomplish these goals without watering down the material.

For the instructor, my purpose was to design a flexible, comprehensive teaching tool using proven pedagogical techniques in mathematics. I wanted to provide instructors with a package of materials that they could use to teach discrete mathematics effectively and efficiently in the most appropriate manner for their particular set of students. I hope that I have achieved these goals.

I have been extremely gratified by the tremendous success of this text. The many improvements in the seventh edition have been made possible by the feedback and suggestions of a large number of instructors and students at many of the more than 600 North American schools, and at any many universities in parts of the world, where this book has been successfully used.

This text is designed for a one- or two-term introductory discrete mathematics course taken by students in a wide variety of majors, including mathematics, computer science, and engineering. College algebra is the only explicit prerequisite, although a certain degree of mathematical maturity is needed to study discrete mathematics in a meaningful way. This book has been designed to meet the needs of almost all types of introductory discrete mathematics courses. It is highly flexible and extremely comprehensive. The book is designed not only to be a successful textbook, but also to serve as valuable resource students can consult throughout their studies and professional life.

Goals of a Discrete Mathematics Course

A discrete mathematics course has more than one purpose. Students should learn a particular set of mathematical facts and how to apply them; more importantly, such a course should teach students how to think logically and mathematically. To achieve these goals, this text stresses mathematical reasoning and the different ways problems are solved. Five important themes are interwoven in this text: mathematical reasoning, combinatorial analysis, discrete structures, algorithmic thinking, and applications and modeling. A successful discrete mathematics course should carefully blend and balance all five themes.

1. *Mathematical Reasoning:* Students must understand mathematical reasoning in order to read, comprehend, and construct mathematical arguments. This text starts with a discussion of mathematical logic, which serves as the foundation for the subsequent discussions of methods of proof. Both the science and the art of constructing proofs are addressed. The technique of mathematical induction is stressed through many different types of examples of such proofs and a careful explanation of why mathematical induction is a valid proof technique.

2. *Combinatorial Analysis*: An important problem-solving skill is the ability to count or enumerate objects. The discussion of enumeration in this book begins with the basic techniques of counting. The stress is on performing combinatorial analysis to solve counting problems and analyze algorithms, not on applying formulae.
3. *Discrete Structures*: A course in discrete mathematics should teach students how to work with discrete structures, which are the abstract mathematical structures used to represent discrete objects and relationships between these objects. These discrete structures include sets, permutations, relations, graphs, trees, and finite-state machines.
4. *Algorithmic Thinking*: Certain classes of problems are solved by the specification of an algorithm. After an algorithm has been described, a computer program can be constructed implementing it. The mathematical portions of this activity, which include the specification of the algorithm, the verification that it works properly, and the analysis of the computer memory and time required to perform it, are all covered in this text. Algorithms are described using both English and an easily understood form of pseudocode.
5. *Applications and Modeling*: Discrete mathematics has applications to almost every conceivable area of study. There are many applications to computer science and data networking in this text, as well as applications to such diverse areas as chemistry, biology, linguistics, geography, business, and the Internet. These applications are natural and important uses of discrete mathematics and are not contrived. Modeling with discrete mathematics is an extremely important problem-solving skill, which students have the opportunity to develop by constructing their own models in some of the exercises.

Changes in the Seventh Edition

Although the sixth edition has been an extremely effective text, many instructors, including longtime users, have requested changes designed to make this book more effective. I have devoted a significant amount of time and energy to satisfy their requests and I have worked hard to find my own ways to make the book more effective and more compelling to students.

The seventh edition is a major revision, with changes based on input from more than 40 formal reviewers, feedback from students and instructors, and author insights. The result is a new edition that offers an improved organization of topics making the book a more effective teaching tool. Substantial enhancements to the material devoted to logic, algorithms, number theory, and graph theory make this book more flexible and comprehensive. Numerous changes in the seventh edition have been designed to help students more easily learn the material. Additional explanations and examples have been added to clarify material where students often have difficulty. New exercises, both routine and challenging, have been added. Highly relevant applications, including many related to the Internet, to computer science, and to mathematical biology, have been added. The companion website has benefited from extensive development activity and now provides tools students can use to master key concepts and explore the world of discrete mathematics, and many new tools under development will be released in the year following publication of this book.

I hope that instructors will closely examine this new edition to discover how it might meet their needs. Although it is impractical to list all the changes in this edition, a brief list that highlights some key changes, listed by the benefits they provide, may be useful.

More Flexible Organization

- Applications of propositional logic are found in a new dedicated section, which briefly introduces logic circuits.

- Recurrence relations are now covered in Chapter 2.
- Expanded coverage of countability is now found in a dedicated section in Chapter 2.
- Separate chapters now provide expanded coverage of algorithms (Chapter 3) and number theory and cryptography (Chapter 4).
- More second and third level heads have been used to break sections into smaller coherent parts.

Tools for Easier Learning

- Difficult discussions and proofs have been marked with the famous Bourbaki dangerous bend symbol in the margin.
- New marginal notes make connections, add interesting notes, and provide advice to students.
- More details and added explanations, in both proofs and exposition, make it easier for students to read the book.
- Many new exercises, both routine and challenging, have been added, while many existing exercises have been improved.

Enhanced Coverage of Logic, Sets, and Proof

- The satisfiability problem is addressed in greater depth, with Sudoku modeled in terms of satisfiability.
- Hilbert's Grand Hotel is used to help explain uncountability.
- Proofs throughout the book have been made more accessible by adding steps and reasons behind these steps.
- A template for proofs by mathematical induction has been added.
- The step that applies the inductive hypothesis in mathematical induction proof is now explicitly noted.

Algorithms

- The pseudocode used in the book has been updated.
- Explicit coverage of algorithmic paradigms, including brute force, greedy algorithms, and dynamic programming, is now provided.
- Useful rules for big- O estimates of logarithms, powers, and exponential functions have been added.

Number Theory and Cryptography

- Expanded coverage allows instructors to include just a little or a lot of number theory in their courses.
- The relationship between the **mod** function and congruences has been explained more fully.
- The sieve of Eratosthenes is now introduced earlier in the book.
- Linear congruences and modular inverses are now covered in more detail.
- Applications of number theory, including check digits and hash functions, are covered in great depth.

- A new section on cryptography integrates previous coverage, and the notion of a cryptosystem has been introduced.
- Cryptographic protocols, including digital signatures and key sharing, are now covered.

Graph Theory

- A structured introduction to graph theory applications has been added.
- More coverage has been devoted to the notion of social networks.
- Applications to the biological sciences and motivating applications for graph isomorphism and planarity have been added.
- Matchings in bipartite graphs are now covered, including Hall's theorem and its proof.
- Coverage of vertex connectivity, edge connectivity, and n -connectedness has been added, providing more insight into the connectedness of graphs.

Enrichment Material

- Many biographies have been expanded and updated, and new biographies of Bellman, Bézout Bienyamé, Cardano, Catalan, Cocks, Cook, Dirac, Hall, Hilbert, Ore, and Tao have been added.
- Historical information has been added throughout the text.
- Numerous updates for latest discoveries have been made.

Media Resources

- Extensive effort has been devoted to producing valuable web resources for this book.
- Extra examples in key parts of the text have been provided on companion website.
- Interactive algorithms have been developed, with tools for using them to explore topics and for classroom use.
- Student assessment modules are available for key concepts.
- Powerpoint transparencies for instructor use have been developed.
- An extensive collection of external web links is provided.
- Instructor's Resource Guide available on the website for instructors contains full solutions to even-numbered exercises in the text. It also offers sample tests for each chapter and a test bank containing over 1500 exam questions to choose from. Answers to all sample tests and test bank questions are included.

Features of the Book

ACCESSIBILITY This text has proved to be easily read and understood by beginning students. There are no mathematical prerequisites beyond college algebra for almost all the content of the text. Students needing extra help will find tools on the companion website for bringing their mathematical maturity up to the level of the text. The few places in the book where calculus is referred to are explicitly noted. Most students should easily understand the pseudocode used in the text to express algorithms, regardless of whether they have formally studied programming languages. There is no formal computer science prerequisite.

Each chapter begins at an easily understood and accessible level. Once basic mathematical concepts have been carefully developed, more difficult material and applications to other areas of study are presented.

FLEXIBILITY This text has been carefully designed for flexible use. The dependence of chapters on previous material has been minimized. Each chapter is divided into sections of approximately the same length, and each section is divided into subsections that form natural blocks of material for teaching. Instructors can easily pace their lectures using these blocks.

WRITING STYLE The writing style in this book is direct and pragmatic. Precise mathematical language is used without excessive formalism and abstraction. Care has been taken to balance the mix of notation and words in mathematical statements.

MATHEMATICAL RIGOR AND PRECISION All definitions and theorems in this text are stated extremely carefully so that students will appreciate the precision of language and rigor needed in mathematics. Proofs are motivated and developed slowly; their steps are all carefully justified. The axioms used in proofs and the basic properties that follow from them are explicitly described in an appendix, giving students a clear idea of what they can assume in a proof. Recursive definitions are explained and used extensively.

WORKED EXAMPLES Extensive examples are used to illustrate concepts, relate different topics, and introduce applications. In most examples, a question is first posed, then its solution is presented with the appropriate amount of detail.

APPLICATIONS The applications included in this text demonstrate the utility of discrete mathematics in the solution of real-world problems. This text includes applications to a wide variety of areas, including computer science, data networking, psychology, chemistry, engineering, linguistics, biology, business, and the Internet.

ALGORITHMS Results in discrete mathematics are often expressed in terms of algorithms; hence, key algorithms are introduced in each chapter of the book. These algorithms are expressed in words and in an easily understood form of structured pseudocode, which is described and specified in Appendix 3. The computational complexity of the algorithms in the text is also analyzed at an elementary level.

HISTORICAL INFORMATION The background of many topics is succinctly described in the text. Brief biographies of 83 mathematicians and computer scientists are included as footnotes. These biographies include information about the lives, careers, and accomplishments of these important contributors to discrete mathematics and images, when available, are displayed.

In addition, numerous historical footnotes are included that supplement the historical information in the main body of the text. Efforts have been made to keep the book up-to-date by reflecting the latest discoveries.

KEY TERMS AND RESULTS A list of key terms and results follows each chapter. The key terms include only the most important that students should learn, and not every term defined in the chapter.

EXERCISES There are thousands of exercises in the text, with many different types of questions posed. There is an ample supply of straightforward exercises that develop basic skills, a large number of intermediate exercises, and many challenging exercises. Exercises are stated clearly and unambiguously, and all are carefully graded for level of difficulty. Exercise sets contain special discussions that develop new concepts not covered in the text, enabling students to discover new ideas through their own work.

Exercises that are somewhat more difficult than average are marked with a single star *; those that are much more challenging are marked with two stars **. Exercises whose solutions require calculus are explicitly noted. Exercises that develop results used in the text are clearly identified with the right pointing hand symbol \rightarrow . Answers or outlined solutions to all odd-

numbered exercises are provided at the back of the text. The solutions include proofs in which most of the steps are clearly spelled out.

REVIEW QUESTIONS A set of review questions is provided at the end of each chapter. These questions are designed to help students focus their study on the most important concepts and techniques of that chapter. To answer these questions students need to write long answers, rather than just perform calculations or give short replies.

SUPPLEMENTARY EXERCISE SETS Each chapter is followed by a rich and varied set of supplementary exercises. These exercises are generally more difficult than those in the exercise sets following the sections. The supplementary exercises reinforce the concepts of the chapter and integrate different topics more effectively.

COMPUTER PROJECTS Each chapter is followed by a set of computer projects. The computer projects tie together what students may have learned in computing and in discrete mathematics. Computer projects that are more difficult than average, from both a mathematical and a programming point of view, are marked with a star, and those that are extremely challenging are marked with two stars.

COMPUTATIONS AND EXPLORATIONS A set of computations and explorations is included at the conclusion of each chapter. These exercises are designed to be completed using existing software tools, such as programs that students or instructors have written or mathematical computation packages such as MapleTM or MathematicaTM. Many of these exercises give students the opportunity to uncover new facts and ideas through computation.

WRITING PROJECTS Each chapter is followed by a set of writing projects. To do these projects students need to consult the mathematical literature. Some of these projects are historical in nature and may involve looking up original sources. Others are designed to serve as gateways to new topics and ideas. All are designed to expose students to ideas not covered in depth in the text. These projects tie mathematical concepts together with the writing process and help expose students to possible areas for future study.

APPENDIXES There are three appendixes to the text. The first introduces axioms for real numbers and the positive integers, and illustrates how facts are proved directly from these axioms. The second covers exponential and logarithmic functions, reviewing some basic material used heavily in the course. The third specifies the pseudocode used to describe algorithms in this text.

SUGGESTED READINGS A list of suggested readings for the overall book and for each chapter is provided after the appendixes. These suggested readings include books at or below the level of this text, more difficult books, expository articles, and articles in which discoveries in discrete mathematics were originally published. Some of these publications are classics, published many years ago, while others have been published in the last few years.

Acknowledgments

I would like to thank the many instructors and students at a variety of schools who have used this book and provided me with their valuable feedback and helpful suggestions. Their input has made this a much better book than it would have been otherwise. I especially want to thank Jerrold Grossman, Jean-Claude Evard, and Georgia Mederer for their technical reviews of the seventh edition and their “eagle eyes,” which have helped ensure the accuracy of this book. I also appreciate the help provided by all those who have submitted comments via the website.

I thank the reviewers of this seventh and the six previous editions. These reviewers have provided much helpful criticism and encouragement to me. I hope this edition lives up to their high expectations.

Reviewers for the Seventh Edition

Philip Barry <i>University of Minnesota, Minneapolis</i>	Kim Factor <i>Marquette University</i>	Darrell Minor <i>Columbus State Community College</i>
Miklos Bona <i>University of Florida</i>	Margaret Fleck <i>University of Illinois, Champaign</i>	Keith Olson <i>Utah Valley University</i>
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I would also like to thank the original editor, Wayne Yuhasz, whose insights and skills helped ensure the book's success, as well as all the many other previous editors of this book.

I want to express my appreciation to the staff of RPK Editorial Services for their valuable work on this edition, including Rose Kernan, who served as both the developmental editor and the production editor, and the other members of the RPK team, Fred Dahl, Martha McMaster, Erin Wagner, Harlan James, and Shelly Gerger-Knechtl. I thank Paul Mailhot of PreTeX, Inc., the compositor, for the tremendous amount of work he devoted to producing this edition, and for his intimate knowledge of LaTeX. Thanks also to Danny Meldung of Photo Affairs, Inc., who was resourceful obtaining images for the new biographical footnotes.

The accuracy and quality of this new edition owe much to Jerry Grossman and Jean-Claude Evard, who checked the entire manuscript for technical accuracy and Georgia Mederer, who

checked the accuracy of the answers at the end of the book and the *Instructor's Resource Guide*. As usual, I cannot thank Jerry Grossman enough for all his work authoring this essential ancillary.

I would also express my appreciation the Science, Engineering, and Mathematics (SEM) Division of McGraw-Hill Higher Education for their valuable support for this new edition and the associated media content. In particular, thanks go to Kurt Strand: President, SEM, McGraw-Hill Higher Education, Marty Lange: Editor-in-Chief, SEM, Michael Lange: Editorial Director, Raghothaman Srinivasan: Global Publisher, Bill Stenquist: Executive Editor, Curt Reynolds: Executive Marketing Manager, Robin A. Reed: Project Manager, Sandy Ludovissey: Buyer, Lorraine Buczek: In-house Developmental Editor, Brenda Rowles: Design Coordinator, Carrie K. Burger: Lead Photo Research Coordinator, and Tammy Juran: Media Project Manager.

Kenneth H. Rosen

Changes in the Global Edition

This adapted edition caters to the curriculum requisites of most of the international institutes and universities. It includes a new chapter on Algebraic Structures and Coding Theory which deals with basic properties of **semigroups**, **monoids**, **groups** and **rings**. Since coding theory is also taught in the course on Discrete Structures in some places, especially in mathematics departments, a section on this is added including **group codes**, **Hamming codes**, and **polynomial rings and codes**.

The section on **cardinality** is elaborated and theory related to **lattices** is introduced. A note about **exponential generating functions**; and some **additional methods of solving recurrence relations** have been added. Additional material is given in the website and at appropriate places, the link information is shown. In keeping with the style of the previous edition, key terms and results, exercises, review questions, supplementary exercises, computer projects, computations and explorations, writing projects are given in the new chapter. Exercises are added at the end of added sections. For odd-numbered exercises, solutions have been given.

In a nutshell, the new additions incorporated in this Global Edition are the following:

- Chapter 1 on The Foundations: Logic and Proofs includes a new section on normal forms and expanded coverage of Resolution Principle with its application to Prolog.
- Chapter 2 on Basic Structures: Sets, Functions, Sequences, Sums, and Matrices includes added material on Cardinality of Sets.
- Chapter 8 on Advanced Counting Techniques includes a subsection on Exponential Generating Functions and additional methods of solving recurrence relations by substitution.
- A new chapter on Algebraic Structures and Coding Theory is added as Chapter 12.
- Two chapters are provided at the Global Edition website www.mhhe.com/rosenGE. They are: Boolean Algebra and Modeling Computation.