REVIEW AND SUMMARY FOR CHAPTER 2

In this chapter we have studied the effect of forces on particles, that is, on bodies of such shape and size that all forces acting on them may be assumed applied at the same point.

Resultant of two forces

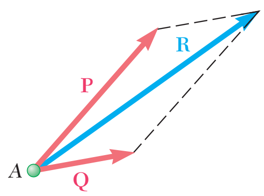

Fig. 2.35

Forces are *vector quantities*; they are characterized by a *point of application*, a *magnitude*, and a *direction*, and they add according to the *parallelogram law* (Fig. 2.35). The magnitude and direction of the resultant **R** of two forces **P** and **Q** can be determined either graphically or by trigonometry, using successively the law of cosines and the law of sines [Sample Prob. 2.1].

Components of a force

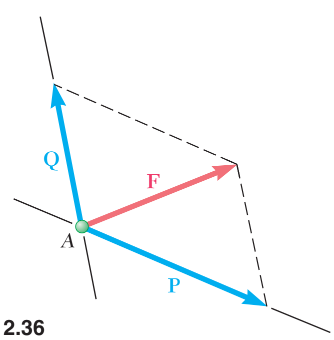

Fig. 2.36

Any given force acting on a particle can be resolved into two or more *components*, that is, it can be replaced by two or more forces which have the same effect on the particle. A force **F** can be resolved into two components **P** and **Q** by drawing a parallelogram which has **F** for its diagonal; the components **P** and **Q** are then represented by the two adjacent sides of the parallelogram (Fig. 2.36) and can be determined either graphically or by trigonometry [Sec. 2.6].

Rectangular components
Unit vectors

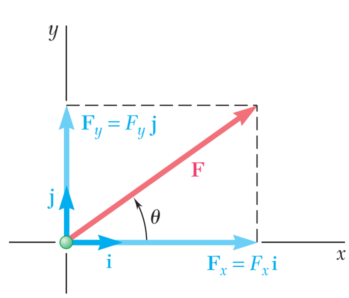

Fig. 2.37

A force **F** is said to have been resolved into two *rectangular components* if its components $\mathbf{F}_x$ and $\mathbf{F}_y$ are perpendicular to each other and are directed along the coordinate axes (Fig. 2.37). Introducing the *unit vectors* **i** and **j** along the x and y axes, respectively, we write [Sec. 2.7]

$$\mathbf{F}_x = F_x\mathbf{i} \qquad \mathbf{F}_y = F_y\mathbf{j} \tag{2.6}$$

and

$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} \tag{2.7}$$

where F_x and F_y are the *scalar components* of **F**. These components, which can be positive or negative, are defined by the relations

$$F_x = F\cos\theta \qquad F_y = F\sin\theta \tag{2.8}$$

When the rectangular components F_x and F_y of a force **F** are given, the angle θ defining the direction of the force can be obtained by writing

$$\tan\theta = \frac{F_y}{F_x} \tag{2.9}$$

The magnitude F of the force can then be obtained by solving one of the equations (2.8) for F or by applying the Pythagorean theorem and writing

$$F = \sqrt{F_x^2 + F_y^2} \tag{2.10}$$

Resultant of several coplanar forces

When *three or more coplanar forces* act on a particle, the rectangular components of their resultant **R** can be obtained by adding algebraically the corresponding components of the given forces [Sec. 2.8]. We have

$$R_x = \Sigma F_x \qquad R_y = \Sigma F_y \tag{2.13}$$

The magnitude and direction of **R** can then be determined from relations similar to Eqs. (2.9) and (2.10) [Sample Prob. 2.3].

Forces in space

A force **F** in *three-dimensional space* can be resolved into rectangular components $\mathbf{F}_x$, $\mathbf{F}_y$, and $\mathbf{F}_z$ [Sec. 2.12]. Denoting by θ_x, θ_y, and θ_z, respectively, the angles that **F** forms with the x, y, and z axes (Fig. 2.38), we have

$$F_x = F\cos\theta_x \qquad F_y = F\cos\theta_y \qquad F_z = F\cos\theta_z \tag{2.19}$$

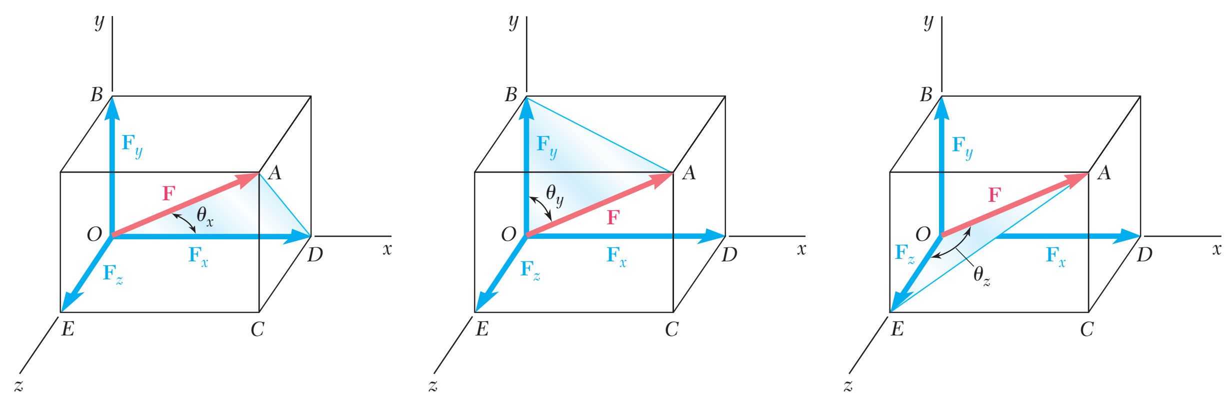

Fig. 2.38

Direction cosines

The cosines of θ_x, θ_y, θ_z are known as the *direction cosines* of the force **F**. Introducing the unit vectors **i**, **j**, **k** along the coordinate axes, we write

$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k} \tag{2.20}$$

or

$$\mathbf{F} = F(\cos\theta_x\mathbf{i} + \cos\theta_y\mathbf{j} + \cos\theta_z\mathbf{k}) \tag{2.21}$$

which shows (Fig. 2.39) that **F** is the product of its magnitude F and the unit vector

$$\boldsymbol{\lambda} = \cos\theta_x\mathbf{i} + \cos\theta_y\mathbf{j} + \cos\theta_z\mathbf{k}$$

Since the magnitude of $\boldsymbol{\lambda}$ is equal to unity, we must have

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1 \tag{2.24}$$

When the rectangular components F_x, F_y, F_z of a force **F** are given, the magnitude F of the force is found by writing

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2} \tag{2.18}$$

and the direction cosines of **F** are obtained from Eqs. (2.19). We have

$$\cos\theta_x = \frac{F_x}{F} \qquad \cos\theta_y = \frac{F_y}{F} \qquad \cos\theta_z = \frac{F_z}{F} \tag{2.25}$$

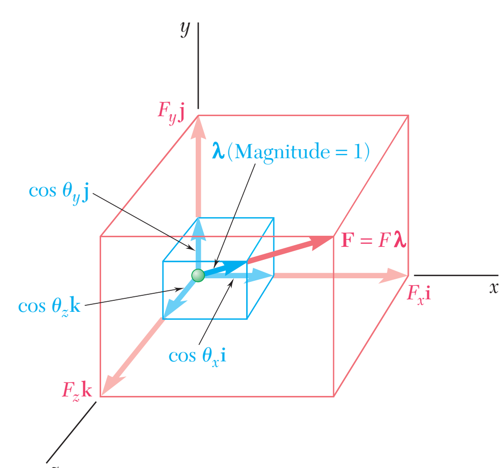

Fig. 2.39

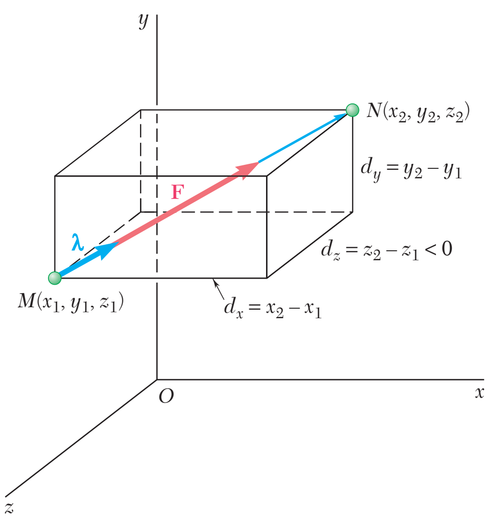

Fig. 2.40

When a force $\mathbf{F}$ is defined in three-dimensional space by its magnitude F and two points M and N on its line of action [Sec. 2.13], its rectangular components can be obtained as follows. We first express the vector $\overrightarrow{MN}$ joining points M and N in terms of its components d_x, d_y, and d_z (Fig. 2.40); we write

$$\overrightarrow{MN} = d_x\mathbf{i} + d_y\mathbf{j} + d_z\mathbf{k} \tag{2.26}$$

We next determine the unit vector $\boldsymbol{\lambda}$ along the line of action of $\mathbf{F}$ by dividing $\overrightarrow{MN}$ by its magnitude $MN = d$:

$$\boldsymbol{\lambda} = \frac{\overrightarrow{MN}}{MN} = \frac{1}{d}(d_x\mathbf{i} + d_y\mathbf{j} + d_z\mathbf{k}) \tag{2.27}$$

Recalling that $\mathbf{F}$ is equal to the product of F and $\boldsymbol{\lambda}$, we have

$$\mathbf{F} = F\boldsymbol{\lambda} = \frac{F}{d}(d_x\mathbf{i} + d_y\mathbf{j} + d_z\mathbf{k}) \tag{2.28}$$

from which it follows [Sample Probs. 2.7 and 2.8] that the scalar components of $\mathbf{F}$ are, respectively,

$$F_x = \frac{Fd_x}{d} \qquad F_y = \frac{Fd_y}{d} \qquad F_z = \frac{Fd_z}{d} \tag{2.29}$$

Resultant of forces in space

When *two or more forces* act on a particle in *three-dimensional space*, the rectangular components of their resultant $\mathbf{R}$ can be obtained by adding algebraically the corresponding components of the given forces [Sec. 2.14]. We have

$$R_x = \Sigma F_x \qquad R_y = \Sigma F_y \qquad R_z = \Sigma F_z \tag{2.31}$$

The magnitude and direction of $\mathbf{R}$ can then be determined from relations similar to Eqs. (2.18) and (2.25) [Sample Prob. 2.8].

Equilibrium of a particle

A particle is said to be in *equilibrium* when the resultant of all the forces acting on it is zero [Sec. 2.9]. The particle will then remain at rest (if originally at rest) or move with constant speed in a straight line (if originally in motion) [Sec. 2.10].

Free-body diagram

To solve a problem involving a particle in equilibrium, one first should draw a *free-body diagram* of the particle showing all the forces acting on it [Sec. 2.11]. If *only three coplanar forces* act on the particle, a *force triangle* may be drawn to express that the particle is in equilibrium. Using graphical methods or trigonometry, this triangle can be solved for no more than two unknowns [Sample Prob. 2.4]. If *more than three coplanar forces* are involved, the equations of equilibrium

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \tag{2.15}$$

should be used. These equations can be solved for no more than two unknowns [Sample Prob. 2.6].

Equilibrium in space

When a particle is in *equilibrium in three-dimensional space* [Sec. 2.15], the three equations of equilibrium

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma F_z = 0 \tag{2.34}$$

should be used. These equations can be solved for no more than three unknowns [Sample Prob. 2.9].