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REVIEW AND SUMMARY FOR CHAPTER 11

Position coordinate of a particle in rectilinear motion

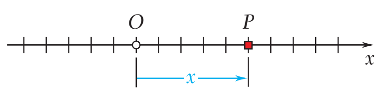

Fig. 11.27

In the first half of the chapter, we analyzed the *rectilinear motion of a particle,* that is, the motion of a particle along a straight line. To define the position P of the particle on that line, we chose a fixed origin O and a positive direction (Fig. 11.27). The distance x from O to P, with the appropriate sign, completely defines the position of the particle on the line and is called the *position coordinate* of the particle [Sec. 11.2].

Velocity and acceleration in rectilinear motion

The *velocity* v of the particle was shown to be equal to the time derivative of the position coordinate x,

$$v = \frac{dx}{dt} \tag{11.1}$$

and the *acceleration* a was obtained by differentiating v with respect to t,

$$a = \frac{dv}{dt} \tag{11.2}$$

or

$$a = \frac{d^2x}{dt^2} \tag{11.3}$$

We also noted that a could be expressed as

$$a = v\frac{dv}{dx} \tag{11.4}$$

We observed that the velocity v and the acceleration a were represented by algebraic numbers which can be positive or negative. A positive value for v indicates that the particle moves in the positive direction, and a negative value that it moves in the negative direction. A positive value for a, however, may mean that the particle is truly accelerated (i.e., moves faster) in the positive direction, or that it is decelerated (i.e., moves more slowly) in the negative direction. A negative value for a is subject to a similar interpretation [Sample Prob. 11.1].

Determination of the velocity and acceleration by integration

In most problems, the conditions of motion of a particle are defined by the type of acceleration that the particle possesses and by the initial conditions [Sec. 11.3]. The velocity and position of the particle can then be obtained by integrating two of the equations (11.1) to (11.4). Which of these equations should be selected depends upon the type of acceleration involved [Sample Probs. 11.2 and 11.3].

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Uniform rectilinear motion

Two types of motion are frequently encountered: the *uniform rectilinear motion* [Sec. 11.4], in which the velocity v of the particle is constant and

$$x = x_0 + vt \tag{11.5}$$

Uniformly accelerated rectilinear motion

and the *uniformly accelerated rectilinear motion* [Sec. 11.5], in which the acceleration a of the particle is constant and we have

$$v = v_0 + at \tag{11.6}$$

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2 \tag{11.7}$$

$$v^2 = v_0^2 + 2a(x - x_0) \tag{11.8}$$

Relative motion of two particles

When two particles A and B move along the same straight line, we may wish to consider the *relative motion* of B with respect to

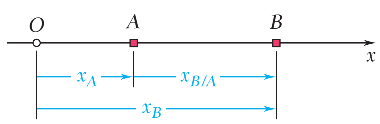

Fig. 11.28

A [Sec. 11.6]. Denoting by $x_{B/A}$ the *relative position coordinate* of B with respect to A (Fig. 11.28), we had

$$x_B = x_A + x_{B/A} \tag{11.9}$$

Differentiating Eq. (11.9) twice with respect to t, we obtained successively

$$v_B = v_A + v_{B/A} \tag{11.10}$$

$$a_B = a_A + a_{B/A} \tag{11.11}$$

where $v_{B/A}$ and $a_{B/A}$ represent, respectively, the *relative velocity* and the *relative acceleration* of B with respect to A.

Blocks connected by inextensible cords

When several blocks are *connected by inextensible cords,* it is possible to write a *linear relation* between their position coordinates. Similar relations can then be written between their velocities and between their accelerations and can be used to analyze their motion [Sample Prob. 11.5].

Graphical solutions

It is sometimes convenient to use a *graphical solution* for problems involving the rectilinear motion of a particle [Secs. 11.7 and 11.8]. The graphical solution most commonly used involves the x–t, v–t, and a–t curves [Sec. 11.7; Sample Prob. 11.6]. It was shown that, at any given time t,

$$v = \text{slope of } x\text{–}t \text{ curve}$$
$$a = \text{slope of } v\text{–}t \text{ curve}$$

while, over any given time interval from t_1 to t_2,

$$v_2 - v_1 = \text{area under } a\text{–}t \text{ curve}$$
$$x_2 - x_1 = \text{area under } v\text{–}t \text{ curve}$$

Position vector and velocity in curvilinear motion

In the second half of the chapter, we analyzed the *curvilinear motion of a particle,* that is, the motion of a particle along a curved path. The position P of the particle at a given time [Sec. 11.9] was

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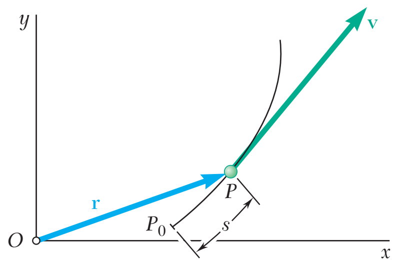

Fig. 11.29

defined by the *position vector* **r** joining the origin O of the coordinates and point P (Fig. 11.29). The *velocity* **v** of the particle was defined by the relation

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} \tag{11.15}$$

and was found to be a *vector tangent to the path of the particle* and of magnitude v (called the *speed* of the particle) equal to the time derivative of the length s of the arc described by the particle:

$$v = \frac{ds}{dt} \tag{11.16}$$

Acceleration in curvilinear motion

The *acceleration* **a** of the particle was defined by the relation

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} \tag{11.18}$$

and we noted that, in general, *the acceleration is not tangent to the path of the particle.*

Derivative of a vector function

Before proceeding to the consideration of the components of velocity and acceleration, we reviewed the formal definition of the derivative of a vector function and established a few rules governing the differentiation of sums and products of vector functions. We then showed that the rate of change of a vector is the same with respect to a fixed frame and with respect to a frame in translation [Sec. 11.10].

Rectangular components of velocity and acceleration

Denoting by x, y, and z the rectangular coordinates of a particle P, we found that the rectangular components of the velocity and acceleration of P equal, respectively, the first and second derivatives with respect to t of the corresponding coordinates:

$$v_x = \dot{x} \qquad v_y = \dot{y} \qquad v_z = \dot{z} \tag{11.29}$$
$$a_x = \ddot{x} \qquad a_y = \ddot{y} \qquad a_z = \ddot{z} \tag{11.30}$$

Component motions

When the component a_x of the acceleration depends only upon t, x, and/or v_x, and when similarly a_y depends only upon t, y, and/or v_y, and a_z upon t, z, and/or v_z, Eqs. (11.30) can be integrated independently. The analysis of the given curvilinear motion can thus be reduced to the analysis of three independent rectilinear component motions [Sec. 11.11]. This approach is particularly effective in the study of the motion of projectiles [Sample Probs. 11.7 and 11.8].

Relative motion of two particles

For two particles A and B moving in space (Fig. 11.30), we considered the relative motion of B with respect to A, or more precisely, with respect to a moving frame attached to A and in translation with A [Sec. 11.12]. Denoting by $\mathbf{r}_{B/A}$ the *relative position vector* of B with respect to A (Fig. 11.30), we had

$$\mathbf{r}_B = \mathbf{r}_A + \mathbf{r}_{B/A} \tag{11.31}$$

Denoting by $\mathbf{v}_{B/A}$ and $\mathbf{a}_{B/A}$, respectively, the *relative velocity* and the *relative acceleration* of B with respect to A, we also showed that

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A} \tag{11.33}$$

and

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A} \tag{11.34}$$

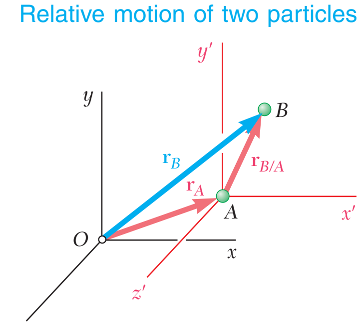

Fig. 11.30

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Tangential and normal components

It is sometimes convenient to resolve the velocity and acceleration of a particle P into components other than the rectangular x, y, and z components. For a particle P moving along a path contained in a plane, we attached to P unit vectors $\mathbf{e}_t$ tangent to the path and $\mathbf{e}_n$ normal to the path and directed toward the center of curvature of the path [Sec. 11.13]. We then expressed the velocity and acceleration of the particle in terms of tangential and normal components. We wrote

$$\mathbf{v} = v\mathbf{e}_t \qquad (11.36)$$

and

$$\mathbf{a} = \frac{dv}{dt}\mathbf{e}_t + \frac{v^2}{\rho}\mathbf{e}_n \qquad (11.39)$$

where v is the speed of the particle and ρ the radius of curvature of its path [Sample Probs. 11.10 and 11.11]. We observed that while the velocity $\mathbf{v}$ is directed along the tangent to the path, the acceleration $\mathbf{a}$ consists of a component $\mathbf{a}_t$ directed along the tangent to the path and a component $\mathbf{a}_n$ directed toward the center of curvature of the path (Fig. 11.31).

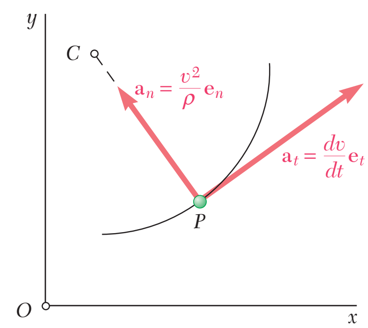

Fig. 11.31

Motion along a space curve

For a particle P moving along a space curve, we defined the plane which most closely fits the curve in the neighborhood of P as the *osculating plane.* This plane contains the unit vectors $\mathbf{e}_t$ and $\mathbf{e}_n$ which define, respectively, the tangent and principal normal to the curve. The unit vector $\mathbf{e}_b$ which is perpendicular to the osculating plane defines the *binormal.*

Radial and transverse components

When the position of a particle P moving in a plane is defined by its polar coordinates r and θ, it is convenient to use radial and transverse components directed, respectively, along the position vector $\mathbf{r}$ of the particle and in the direction obtained by rotating $\mathbf{r}$ through 90° counterclockwise [Sec. 11.14]. We attached to P unit vectors $\mathbf{e}_r$ and $\mathbf{e}_\theta$ directed, respectively, in the radial and transverse directions (Fig. 11.32). We then expressed the velocity and acceleration of the particle in terms of radial and transverse components

$$\mathbf{v} = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta \qquad (11.43)$$

$$\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta \qquad (11.44)$$

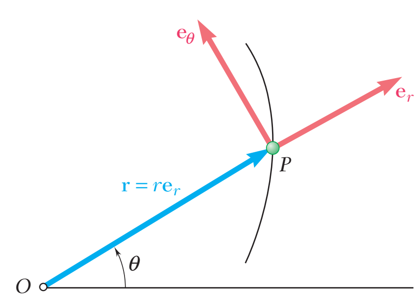

Fig. 11.32

where dots are used to indicate differentiation with respect to time. The scalar components of the velocity and acceleration in the radial and transverse directions are therefore

$$v_r = \dot{r} \qquad v_\theta = r\dot{\theta} \qquad (11.45)$$

$$a_r = \ddot{r} - r\dot{\theta}^2 \qquad a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} \qquad (11.46)$$

It is important to note that a_r is *not* equal to the time derivative of v_r, and that a_θ is *not* equal to the time derivative of v_θ [Sample Prob. 11.12].

The chapter ended with a discussion of the use of cylindrical coordinates to define the position and motion of a particle in space.