REVIEW AND SUMMARY FOR CHAPTER 12

This chapter was devoted to Newton's second law and its application to the analysis of the motion of particles.

Newton's second law

Denoting by m the mass of a particle, by $\Sigma\mathbf{F}$ the sum, or resultant, of the forces acting on the particle, and by $\mathbf{a}$ the acceleration of the particle relative to a *newtonian frame of reference* [Sec. 12.2], we wrote

$$\Sigma\mathbf{F} = m\mathbf{a} \tag{12.2}$$

Linear momentum

Introducing the *linear momentum* of a particle, $\mathbf{L} = m\mathbf{v}$ [Sec. 12.3], we saw that Newton's second law can also be written in the form

$$\Sigma\mathbf{F} = \dot{\mathbf{L}} \tag{12.5}$$

which expresses that *the resultant of the forces acting on a particle is equal to the rate of change of the linear momentum of the particle.*

Consistent systems of units

Equation (12.2) holds only if a consistent system of units is used. With SI units, the forces should be expressed in newtons, the masses in kilograms, and the accelerations in $\mathrm{m/s^2}$; with U.S. customary units, the forces should be expressed in pounds, the masses in $\mathrm{lb \cdot s^2/ft}$ (also referred to as *slugs*), and the accelerations in $\mathrm{ft/s^2}$ [Sec. 12.4].

Equations of motion for a particle

To solve a problem involving the motion of a particle, Eq. (12.2) should be replaced by equations containing scalar quantities [Sec. 12.5]. Using *rectangular components* of $\mathbf{F}$ and $\mathbf{a}$, we wrote

$$\Sigma F_x = ma_x \qquad \Sigma F_y = ma_y \qquad \Sigma F_z = ma_z \tag{12.8}$$

Using *tangential and normal components,* we had

$$\Sigma F_t = m\frac{dv}{dt} \qquad \Sigma F_n = m\frac{v^2}{\rho} \tag{12.9$'$}$$

Dynamic equilibrium

We also noted [Sec. 12.6] that the equations of motion of a particle can be replaced by equations similar to the equilibrium equations used in statics if a vector $-m\mathbf{a}$ of magnitude ma but of sense opposite to that of the acceleration is added to the forces applied to the particle; the particle is then said to be in *dynamic equilibrium.* For the sake of uniformity, however, all the Sample Problems were solved by using the equations of motion, first with rectangular components [Sample Probs. 12.1 through 12.4], then with tangential and normal components [Sample Probs. 12.5 and 12.6].

Angular momentum

In the second part of the chapter, we defined the *angular momentum* $\mathbf{H}_O$ of a particle about a point O as the moment about O of the linear momentum $m\mathbf{v}$ of that particle [Sec. 12.7]. We wrote

$$\mathbf{H}_O = \mathbf{r} \times m\mathbf{v} \tag{12.12}$$

and noted that $\mathbf{H}_O$ is a vector perpendicular to the plane containing $\mathbf{r}$ and $m\mathbf{v}$ (Fig. 12.24) and of magnitude

$$H_O = rmv \sin \phi \tag{12.13}$$

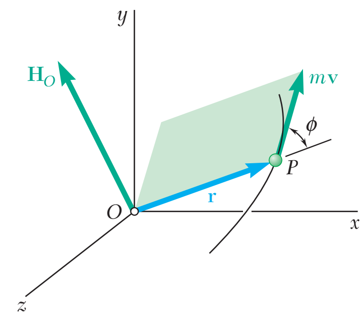

Fig. 12.24

Resolving the vectors $\mathbf{r}$ and $m\mathbf{v}$ into rectangular components, we expressed the angular momentum $\mathbf{H}_O$ in the determinant form

$$\mathbf{H}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & y & z \\ mv_x & mv_y & mv_z \end{vmatrix} \tag{12.14}$$

In the case of a particle moving in the xy plane, we have $z = v_z = 0$. The angular momentum is perpendicular to the xy plane and is completely defined by its magnitude. We wrote

$$H_O = H_z = m(xv_y - yv_x) \tag{12.16}$$

Rate of change of angular momentum

Computing the rate of change $\dot{\mathbf{H}}_O$ of the angular momentum $\mathbf{H}_O$, and applying Newton's second law, we wrote the equation

$$\Sigma \mathbf{M}_O = \dot{\mathbf{H}}_O \tag{12.19}$$

which states that *the sum of the moments about O of the forces acting on a particle is equal to the rate of change of the angular momentum of the particle about O.*

Radial and transverse components

In many problems involving the plane motion of a particle, it is found convenient to use *radial and transverse components* [Sec. 12.8, Sample Prob. 12.7] and to write the equations

$$\Sigma F_r = m(\ddot{r} - r\dot{\theta}^2) \tag{12.21}$$
$$\Sigma F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta}) \tag{12.22}$$

Motion under a central force

When the only force acting on a particle P is a force $\mathbf{F}$ directed toward or away from a fixed point O, the particle is said to be moving *under a central force* [Sec. 12.9]. Since $\Sigma \mathbf{M}_O = 0$ at any given instant, it follows from Eq. (12.19) that $\dot{\mathbf{H}}_O = 0$ for all values of t and, thus, that

$$\mathbf{H}_O = \text{constant} \tag{12.23}$$

We concluded that *the angular momentum of a particle moving under a central force is constant, both in magnitude and direction,* and that the particle moves in a plane perpendicular to the vector $\mathbf{H}_O$.

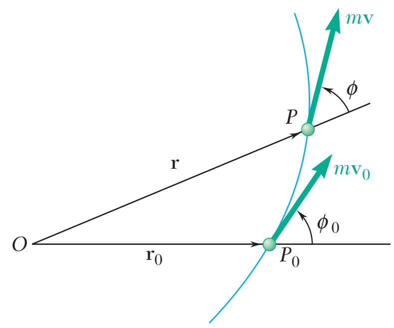

Fig. 12.25

Recalling Eq. (12.13), we wrote the relation

$$r m v \sin \phi = r_0 m v_0 \sin \phi_0 \tag{12.25}$$

for the motion of any particle under a central force (Fig. 12.25). Using polar coordinates and recalling Eq. (12.18), we also had

$$r^2 \dot{\theta} = h \tag{12.27}$$

where h is a constant representing the angular momentum per unit mass, H_O/m, of the particle. We observed (Fig. 12.26) that the infinitesimal area dA swept by the radius vector OP as it rotates through $d\theta$ is equal to $\frac{1}{2}r^2\, d\theta$ and, thus, that the left-hand member of Eq. (12.27) represents twice the *areal velocity dA/dt* of the particle. Therefore, *the areal velocity of a particle moving under a central force is constant.*

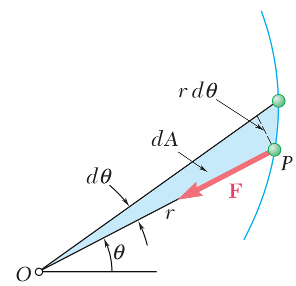

Fig. 12.26

Newton's law of universal gravitation

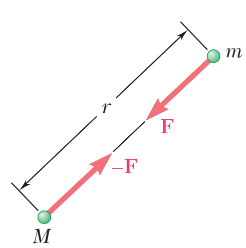

Fig. 12.27

An important application of the motion under a central force is provided by the orbital motion of bodies under gravitational attraction [Sec. 12.10]. According to *Newton's law of universal gravitation,* two particles at a distance r from each other and of masses M and m, respectively, attract each other with equal and opposite forces $\mathbf{F}$ and $-\mathbf{F}$ directed along the line joining the particles (Fig. 12.27). The common magnitude F of the two forces is

$$F = G\frac{Mm}{r^2} \tag{12.28}$$

where G is the *constant of gravitation.* In the case of a body of mass m subjected to the gravitational attraction of the earth, the product GM, where M is the mass of the earth, can be expressed as

$$GM = gR^2 \tag{12.30}$$

where $g = 9.81\ \text{m/s}^2 = 32.2\ \text{ft/s}^2$ and R is the radius of the earth.

Orbital motion

It was shown in Sec. 12.11 that a particle moving under a central force describes a trajectory defined by the differential equation

$$\frac{d^2u}{d\theta^2} + u = \frac{F}{mh^2u^2} \tag{12.37}$$

where $F > 0$ corresponds to an attractive force and $u = 1/r$. In the case of a particle moving under a force of gravitational attraction [Sec. 12.12], we substituted for F the expression given in Eq. (12.28). Measuring θ from the axis OA joining the focus O to the point A of the trajectory closest to O (Fig. 12.28), we found that the solution to Eq. (12.37) was

$$\frac{1}{r} = u = \frac{GM}{h^2} + C\cos\theta \qquad (12.39)$$

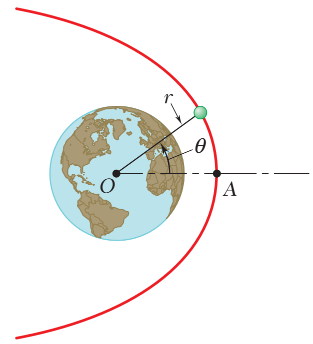

Fig. 12.28

This is the equation of a conic of eccentricity $\varepsilon = Ch^2/GM$. The conic is an *ellipse* if $\varepsilon < 1$, a *parabola* if $\varepsilon = 1$, and a hyperbola if $\varepsilon > 1$. The constants C and h can be determined from the initial conditions; if the particle is projected from point A ($\theta = 0$, $r = r_0$) with an initial velocity $\mathbf{v}_0$ perpendicular to OA, we have $h = r_0 v_0$ [Sample Prob. 12.9].

Escape velocity

It was also shown that the values of the initial velocity corresponding, respectively, to a parabolic and a circular trajectory were

$$v_{esc} = \sqrt{\frac{2GM}{r_0}} \qquad (12.43)$$

$$v_{circ} = \sqrt{\frac{GM}{r_0}} \qquad (12.44)$$

and that the first of these values, called the *escape velocity*, is the smallest value of v_0 for which the particle will not return to its starting point.

Periodic time

The *periodic time* τ of a planet or satellite was defined as the time required by that body to describe its orbit. It was shown that

$$\tau = \frac{2\pi ab}{h} \qquad (12.45)$$

where $h = r_0 v_0$ and where a and b represent the semimajor and semiminor axes of the orbit. It was further shown that these semiaxes are respectively equal to the arithmetic and geometric means of the maximum and minimum values of the radius vector r.

Kepler's laws

The last section of the chapter [Sec. 12.13] presented *Kepler's laws of planetary motion* and showed that these empirical laws, obtained from early astronomical observations, confirm Newton's laws of motion as well as his law of gravitation.