REVIEW AND SUMMARY FOR CHAPTER 13

This chapter was devoted to the method of work and energy and to the method of impulse and momentum. In the first half of the chapter we studied the method of work and energy and its application to the analysis of the motion of particles.

Work of a force

We first considered a force **F** acting on a particle A and defined the *work of* **F** *corresponding to the small displacement* $d\mathbf{r}$ [Sec. 13.2] as the quantity

$$dU = \mathbf{F} \cdot d\mathbf{r} \tag{13.1}$$

or, recalling the definition of the scalar product of two vectors,

$$dU = F\, ds \cos \alpha \tag{13.1'}$$

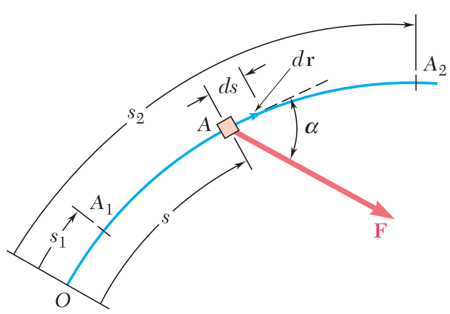

Fig. 13.29

where α is the angle between **F** and $d\mathbf{r}$ (Fig. 13.29). The work of **F** during a finite displacement from A_1 to A_2, denoted by $U_{1\rightarrow 2}$, was obtained by integrating Eq. (13.1) along the path described by the particle:

$$U_{1\rightarrow 2} = \int_{A_1}^{A_2} \mathbf{F} \cdot d\mathbf{r} \tag{13.2}$$

For a force defined by its rectangular components, we wrote

$$U_{1\rightarrow 2} = \int_{A_1}^{A_2} (F_x\, dx + F_y\, dy + F_z\, dz) \tag{13.2''}$$

Work of a weight

The work of the weight **W** of a body as its center of gravity moves from the elevation y_1 to y_2 (Fig. 13.30) was obtained by substituting $F_x = F_z = 0$ and $F_y = -W$ into Eq. (13.2″) and integrating. We found

$$U_{1\rightarrow 2} = -\int_{y_1}^{y_2} W\, dy = Wy_1 - Wy_2 \tag{13.4}$$

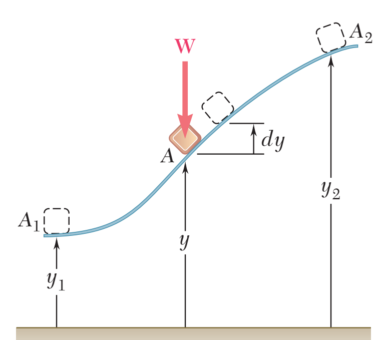

Fig. 13.30

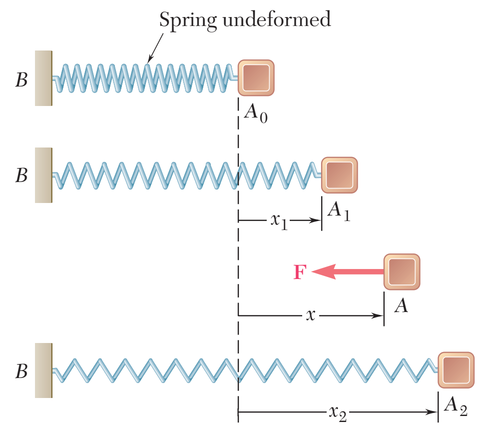

Fig. 13.31

Work of the force exerted by a spring

The work of a force **F** exerted by a spring on a body A during a finite displacement of the body (Fig. 13.31) from $A_1(x = x_1)$ to $A_2(x = x_2)$ was obtained by writing

$$dU = -F\,dx = -kx\,dx$$

$$U_{1\rightarrow 2} = -\int_{x_1}^{x_2} kx\,dx = \tfrac{1}{2}kx_1^2 - \tfrac{1}{2}kx_2^2 \qquad (13.6)$$

The work of **F** is therefore positive *when the spring is returning to its undeformed position.*

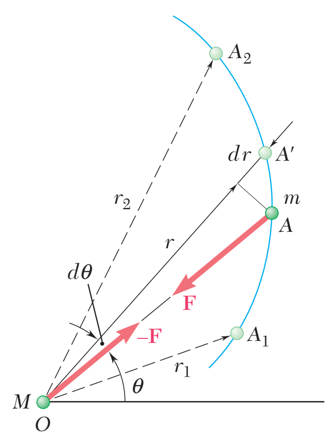

Fig. 13.32

Work of the gravitational force

The *work of the gravitational force* **F** exerted by a particle of mass M located at O on a particle of mass m as the latter moves from A_1 to A_2 (Fig. 13.32) was obtained by recalling from Sec. 12.10 the expression for the magnitude of **F** and writing

$$U_{1\rightarrow 2} = -\int_{r_1}^{r_2} \frac{GMm}{r^2}\,dr = \frac{GMm}{r_2} - \frac{GMm}{r_1} \qquad (13.7)$$

Kinetic energy of a particle

The *kinetic energy of a particle* of mass m moving with a velocity **v** [Sec. 13.3] was defined as the scalar quantity

$$T = \tfrac{1}{2}mv^2 \qquad (13.9)$$

Principle of work and energy

From Newton's second law we derived the *principle of work and energy,* which states that *the kinetic energy of a particle at* A_2 *can be obtained by adding to its kinetic energy at* A_1 *the work done during the displacement from* A_1 to A_2 *by the force* **F** *exerted on the particle:*

$$T_1 + U_{1\rightarrow 2} = T_2 \qquad (13.11)$$

Method of work and energy

The method of work and energy simplifies the solution of many problems dealing with forces, displacements, and velocities, since it does not require the determination of accelerations [Sec. 13.4]. We also note that it involves only scalar quantities and that forces which do no work need not be considered [Sample Probs. 13.1 and 13.3]. However, this method should be supplemented by the direct application of Newton's second law to determine a force normal to the path of the particle [Sample Prob. 13.4].

Power and mechanical efficiency

The power developed by a machine and its mechanical efficiency were discussed in Sec. 13.5. Power was defined as the time rate at which work is done:

$$\text{Power} = \frac{dU}{dt} = \mathbf{F} \cdot \mathbf{v} \qquad (13.12,\ 13.13)$$

where **F** is the force exerted on the particle and **v** the velocity of the particle [Sample Prob. 13.5]. The *mechanical efficiency,* denoted by η, was expressed as

$$\eta = \frac{\text{power output}}{\text{power input}} \qquad (13.15)$$

Conservative force. Potential energy

When the work of a force **F** is independent of the path followed [Secs. 13.6 and 13.7], the force **F** is said to be a *conservative force,* and its work is equal to *minus the change in the potential energy* V associated with **F**:

$$U_{1\rightarrow 2} = V_1 - V_2 \qquad (13.19')$$

The following expressions were obtained for the potential energy associated with each of the forces considered earlier:

Force of gravity (weight): $$V_g = Wy \qquad (13.16)$$

Gravitational force: $$V_g = -\frac{GMm}{r} \qquad (13.17)$$

Elastic force exerted by a spring: $$V_e = \tfrac{1}{2}kx^2 \qquad (13.18)$$

Substituting for $U_{1\rightarrow2}$ from Eq. (13.19′) into Eq. (13.11) and rearranging the terms [Sec. 13.8], we obtained

$$T_1 + V_1 = T_2 + V_2 \tag{13.24}$$

Principle of conservation of energy

This is the *principle of conservation of energy,* which states that when a particle moves under the action of conservative forces, *the sum of its kinetic and potential energies remains constant*. The application of this principle facilitates the solution of problems involving only conservative forces [Sample Probs. 13.6 and 13.7].

Motion under a gravitational force

Recalling from Sec. 12.9 that, when a particle moves under a central force **F**, its angular momentum about the center of force O remains constant, we observed [Sec. 13.9] that, if the central force **F** is also conservative, the principles of conservation of angular momentum and of conservation of energy can be used jointly to analyze the motion of the particle [Sample Prob. 13.8]. Since the gravitational force exerted by the earth on a space vehicle is both central and conservative, this approach was used to study the motion of such vehicles [Sample Prob. 13.9] and was found particularly effective in the case of an *oblique launching*. Considering the initial position P_0 and an arbitrary position P of the vehicle (Fig. 13.33), we wrote

$$(H_O)_0 = H_O: \qquad r_0 m v_0 \sin \phi_0 = r m v \sin \phi \tag{13.25}$$

$$T_0 + V_0 = T + V: \qquad \tfrac{1}{2} m v_0^2 - \frac{GMm}{r_0} = \tfrac{1}{2} m v^2 - \frac{GMm}{r} \tag{13.26}$$

where m was the mass of the vehicle and M the mass of the earth.

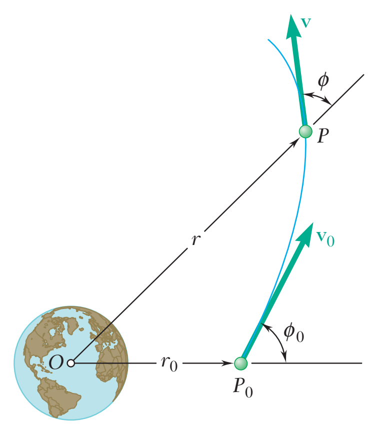

Fig. 13.33

Principle of impulse and momentum for a particle

The second half of the chapter was devoted to the method of impulse and momentum and to its application to the solution of various types of problems involving the motion of particles.

The *linear momentum of a particle* was defined [Sec. 13.10] as the product $m\mathbf{v}$ of the mass m of the particle and its velocity $\mathbf{v}$. From Newton's second law, $\mathbf{F} = m\mathbf{a}$, we derived the relation

$$m\mathbf{v}_1 + \int_{t_1}^{t_2} \mathbf{F}\, dt = m\mathbf{v}_2 \tag{13.28}$$

where $m\mathbf{v}_1$ and $m\mathbf{v}_2$ represent the momentum of the particle at a time t_1 and a time t_2, respectively, and where the integral defines the *linear impulse of the force* **F** during the corresponding time interval. We wrote therefore

$$m\mathbf{v}_1 + \mathbf{Imp}_{1\rightarrow2} = m\mathbf{v}_2 \tag{13.30}$$

which expresses the principle of impulse and momentum for a particle.

When the particle considered is subjected to several forces, the sum of the impulses of these forces should be used; we had

$$m\mathbf{v}_1 + \Sigma\ \mathbf{Imp}_{1\rightarrow 2} = m\mathbf{v}_2 \qquad (13.32)$$

Since Eqs. (13.30) and (13.32) involve *vector quantities*, it is necessary to consider their x and y components separately when applying them to the solution of a given problem [Sample Probs. 13.10 and 13.11].

Impulsive motion

The method of impulse and momentum is particularly effective in the study of the *impulsive motion* of a particle, when very large forces, called *impulsive forces,* are applied for a very short interval of time Δt, since this method involves the impulses $\mathbf{F}\ \Delta t$ of the forces, rather than the forces themselves [Sec. 13.11]. Neglecting the impulse of any nonimpulsive force, we wrote

$$m\mathbf{v}_1 + \Sigma\mathbf{F}\ \Delta t = m\mathbf{v}_2 \qquad (13.35)$$

In the case of the impulsive motion of several particles, we had

$$\Sigma m\mathbf{v}_1 + \Sigma\mathbf{F}\ \Delta t = \Sigma m\mathbf{v}_2 \qquad (13.36)$$

where the second term involves only impulsive, external forces [Sample Prob. 13.12].

In the particular case *when the sum of the impulses of the external forces is zero,* Eq. (13.36) reduces to $\Sigma m\mathbf{v}_1 = \Sigma m\mathbf{v}_2$; that is, *the total momentum of the particles is conserved.*

Direct central impact

In Secs. 13.12 through 13.14, we considered the *central impact* of two colliding bodies. In the case of a *direct central impact* [Sec. 13.13], the two colliding bodies A and B were moving along the *line of impact* with velocities $\mathbf{v}_A$ and $\mathbf{v}_B$, respectively (Fig. 13.34). Two equations could be used to determine their

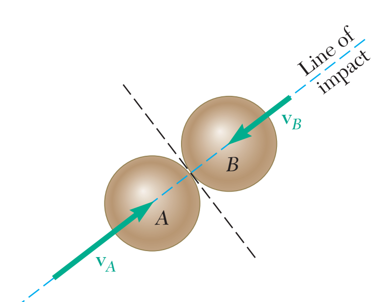

Fig. 13.34

velocities $\mathbf{v}'_A$ and $\mathbf{v}'_B$ after the impact. The first expressed conservation of the total momentum of the two bodies,

$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B \tag{13.37}$$

where a positive sign indicates that the corresponding velocity is directed to the right, while the second related the *relative velocities* of the two bodies before and after the impact,

$$v'_B - v'_A = e(v_A - v_B) \tag{13.43}$$

The constant e is known as the *coefficient of restitution;* its value lies between 0 and 1 and depends in a large measure on the materials involved. When $e = 0$, the impact is said to be *perfectly plastic;* when $e = 1$, it is said to be *perfectly elastic* [Sample Prob. 13.13].

Oblique central impact

In the case of an *oblique central impact* [Sec. 13.14], the velocities of the two colliding bodies before and after the impact were resolved into n components along the line of impact and t components along the common tangent to the surfaces in contact (Fig. 13.35). We observed that the t component of the velocity of

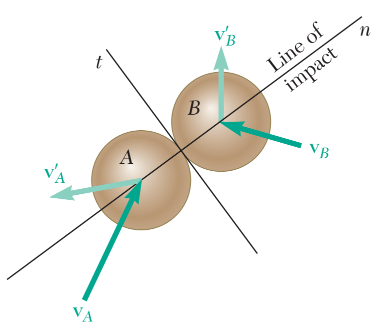

Fig. 13.35

each body remained unchanged, while the n components satisfied equations similar to Eqs. (13.37) and (13.43) [Sample Probs. 13.14 and 13.15]. It was shown that although this method was developed for bodies moving freely before and after the impact, it could be extended to the case when one or both of the colliding bodies is constrained in its motion [Sample Prob. 13.16].

Using the three fundamental methods of kinetic analysis

In Sec. 13.15, we discussed the relative advantages of the three fundamental methods presented in this chapter and the preceding one, namely, Newton's second law, work and energy, and impulse and momentum. We noted that the method of work and energy and the method of impulse and momentum can be combined to solve problems involving a short impact phase during which impulsive forces must be taken into consideration [Sample Prob. 13.17].