REVIEW AND SUMMARY FOR CHAPTER 17

In this chapter we again considered the method of work and energy and the method of impulse and momentum. In the first part of the chapter we studied the method of work and energy and its application to the analysis of the motion of rigid bodies and systems of rigid bodies.

Principle of work and energy for a rigid body

In Sec. 17.2, we first expressed the principle of work and energy for a rigid body in the form

$$T_1 + U_{1\rightarrow 2} = T_2 \tag{17.1}$$

where T_1 and T_2 represent the initial and final values of the kinetic energy of the rigid body and $U_{1\rightarrow 2}$ represents the work of the *external forces* acting on the rigid body.

Work of a force or a couple

In Sec. 17.3, we recalled the expression found in Chap. 13 for the work of a force **F** applied at a point A, namely

$$U_{1\rightarrow 2} = \int_{s_1}^{s_2} (F \cos \alpha)\, ds \tag{17.3$'$}$$

where F was the magnitude of the force, α the angle it formed with the direction of motion of A, and s the variable of integration measuring the distance traveled by A along its path. We also derived the expression for the *work of a couple of moment* **M** applied to a rigid body during a rotation in θ of the rigid body:

$$U_{1\rightarrow 2} = \int_{\theta_1}^{\theta_2} M\, d\theta \tag{17.5}$$

Kinetic energy in plane motion

We then derived an expression for the kinetic energy of a rigid body in plane motion [Sec. 17.4]. We wrote

$$T = \tfrac{1}{2} m\bar{v}^2 + \tfrac{1}{2}\bar{I}\omega^2 \tag{17.9}$$

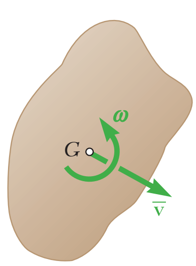


Fig. 17.13

where $\bar{v}$ is the velocity of the mass center G of the body, $\boldsymbol{\omega}$ is the angular velocity of the body, and $\bar{I}$ is its moment of inertia about an axis through G perpendicular to the plane of reference (Fig. 17.13) [Sample Prob. 17.3]. We noted that the kinetic energy of a rigid body in plane motion can be separated into two parts: (1) the kinetic energy $\frac{1}{2}m\bar{v}^2$ associated with the motion of the mass center G of the body, and (2) the kinetic energy $\frac{1}{2}\bar{I}\omega^2$ associated with the rotation of the body about G.

Kinetic energy in rotation

For a rigid body rotating about a fixed axis through O with an angular velocity $\boldsymbol{\omega}$, we had

$$T = \tfrac{1}{2} I_O \omega^2 \qquad (17.10)$$

where I_O was the moment of inertia of the body about the fixed axis. We noted that the result obtained is not limited to the rotation of plane slabs or of bodies symmetrical with respect to the reference plane, but is valid regardless of the shape of the body or of the location of the axis of rotation.

Systems of rigid bodies

Equation (17.1) can be applied to the motion of systems of rigid bodies [Sec. 17.5] as long as all the forces acting on the various bodies involved—internal as well as external to the system—are included in the computation of $U_{1\rightarrow 2}$. However, in the case of systems consisting of pin-connected members, or blocks and pulleys connected by inextensible cords, or meshed gears, the points of application of the internal forces move through equal distances and the work of these forces cancels out [Sample Probs. 17.1 and 17.2].

Conservation of energy

When a rigid body, or a system of rigid bodies, moves under the action of conservative forces, the principle of work and energy can be expressed in the form

$$T_1 + V_1 = T_2 + V_2 \qquad (17.12)$$

which is referred to as the *principle of conservation of energy* [Sec. 17.6]. This principle can be used to solve problems involving conservative forces such as the force of gravity or the force exerted by a spring [Sample Probs. 17.4 and 17.5]. However, when a reaction is to be determined, the principle of conservation of energy must be supplemented by the application of d'Alembert's principle [Sample Prob. 17.4].

Power

In Sec. 17.7, we extended the concept of power to a rotating body subjected to a couple, writing

$$\text{Power} = \frac{dU}{dt} = \frac{M\,d\theta}{dt} = M\omega \qquad (17.13)$$

where M is the magnitude of the couple and ω the angular velocity of the body.

The middle part of the chapter was devoted to the method of impulse and momentum and its application to the solution of various types of problems involving the plane motion of rigid slabs and rigid bodies symmetrical with respect to the reference plane.

Principle of impulse and momentum for a rigid body

We first recalled the *principle of impulse and momentum* as it was derived in Sec. 14.9 for a system of particles and applied it to the *motion of a rigid body* [Sec. 17.8]. We wrote

$$\textbf{Syst Momenta}_1 + \textbf{Syst Ext Imp}_{1\rightarrow 2} = \textbf{Syst Momenta}_2 \qquad (17.14)$$

Next we showed that for a rigid slab or a rigid body symmetrical with respect to the reference plane, the system of the momenta of the particles forming the body is equivalent to a vector $m\bar{\mathbf{v}}$ attached at the mass center G of the body and a couple $\bar{I}\boldsymbol{\omega}$ (Fig. 17.14). The

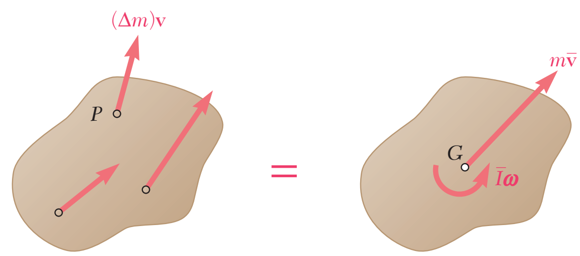


Fig. 17.14

vector $m\bar{\mathbf{v}}$ is associated with the translation of the body with G and represents the *linear momentum* of the body, while the couple $\bar{I}\boldsymbol{\omega}$ corresponds to the rotation of the body about G and represents the *angular momentum* of the body about an axis through G.

Equation (17.14) can be expressed graphically as shown in Fig. 17.15 by drawing three diagrams representing respectively the system of the initial momenta of the body, the impulses of the external forces acting on the body, and the system of the final momenta of the body.

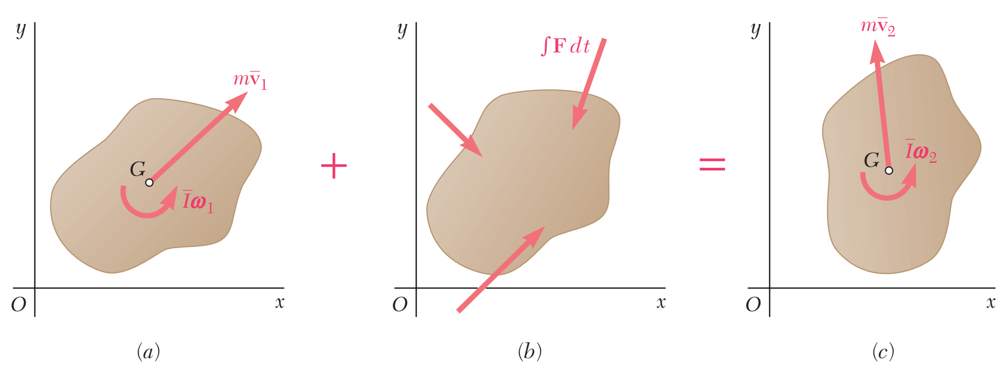


Fig. 17.15

Summing and equating respectively the x *components*, the y *components*, and the *moments about any given point* of the vectors shown in that figure, we obtain three equations of motion which can be solved for the desired unknowns [Sample Probs. 17.6 and 17.7].

In problems dealing with several connected rigid bodies [Sec. 17.9], each body can be considered separately [Sample Prob. 17.6], or, if no more than three unknowns are involved, the principle of impulse and momentum can be applied to the entire system, considering the impulses of the external forces only [Sample Prob. 17.8].

Conservation of angular momentum

When the lines of action of all the external forces acting on a system of rigid bodies pass through a given point O, the angular momentum of the system about O is conserved [Sec. 17.10]. It was suggested that problems involving conservation of angular momentum be solved by the general method described above [Sample Prob. 17.8].

Impulsive motion

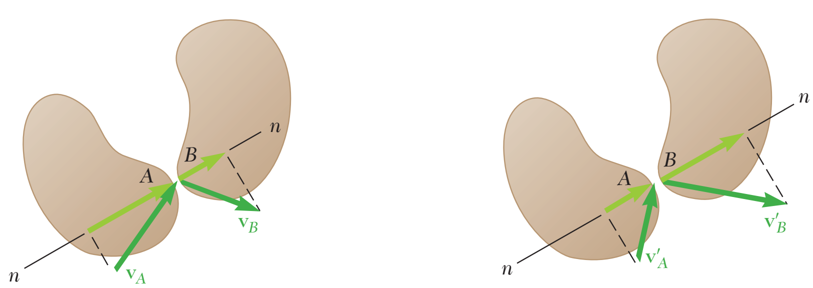
The last part of the chapter was devoted to the *impulsive motion* and the *eccentric impact* of rigid bodies. In Sec. 17.11, we recalled that the method of impulse and momentum is the only practicable method for the solution of problems involving impulsive motion and that the computation of impulses in such problems is particularly simple [Sample Prob. 17.9].

Eccentric impact

In Sec. 17.12, we recalled that the eccentric impact of two rigid bodies is defined as an impact in which the mass centers of the colliding bodies are *not* located on the line of impact. It was shown that in such a situation a relation similar to that derived in Chap. 13 for the central impact of two particles and involving the coefficient of restitution e still holds, but that *the velocities of points A and B where contact occurs during the impact should be used.* We have

$$(v'_B)_n - (v'_A)_n = e[(v_A)_n - (v_B)_n] \qquad (17.19)$$

where $(v_A)_n$ and $(v_B)_n$ are the components along the line of impact of the velocities of A and B before the impact, and $(v'_A)_n$ and $(v'_B)_n$ are their components after the impact (Fig. 17.16). Equation



(*a*) Before impact (*b*) After impact

Fig. 17.16

(17.19) is applicable not only when the colliding bodies move freely after the impact but also when the bodies are partially constrained in their motion. It should be used in conjunction with one or several other equations obtained by applying the principle of impulse and momentum [Sample Prob. 17.10]. We also considered problems where the method of impulse and momentum and the method of work and energy can be combined [Sample Prob. 17.11].