Problems

12.94 A particle of mass m is projected from point A with an initial velocity $\mathbf{v}_0$ perpendicular to OA and moves under a central force $\mathbf{F}$ along an elliptic path defined by the equation $r = r_0/(2 - \cos\theta)$. Using Eq. (12.37), show that $\mathbf{F}$ is inversely proportional to the square of the distance r from the particle to the center of force O.
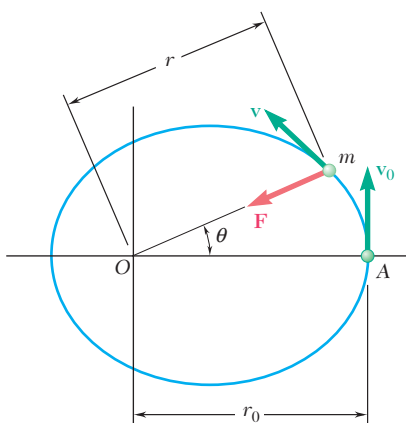

Fig. P12.94

12.95 A particle of mass m describes the path defined by the equation $r = r_0/(6\cos\theta - 5)$ under a central force $\mathbf{F}$ directed away from the center of force O. Using Eq. (12.37), show that $\mathbf{F}$ is inversely proportional to the square of the distance r from the particle to O.
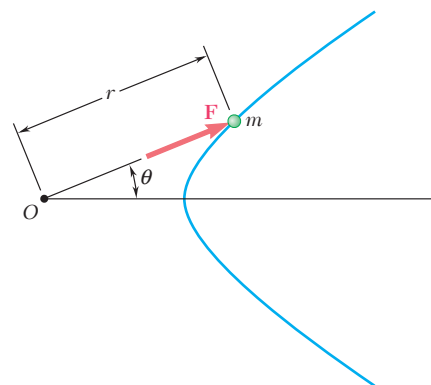
12.96 A particle of mass m describes the parabola $y = x^2/4r_0$ under a central force $\mathbf{F}$ directed toward the center of force C. Using Eq. (12.37) and Eq. (12.39′) with $\varepsilon = 1$, show that $\mathbf{F}$ is inversely proportional to the square of the distance r from the particle to the center of force and that the angular momentum per unit mass $h = \sqrt{2GMr_0}$.
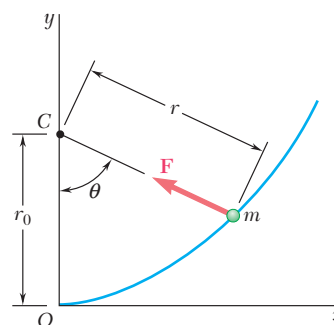
12.97 For the particle of Prob. 12.74, and using Eq. (12.37), show that the central force $\mathbf{F}$ is proportional to the distance r from the particle to the center of force O.

12.98 It was observed that during the Galileo spacecraft's first flyby of the earth, its maximum altitude was 600 mi above the surface of the earth. Assuming that the trajectory of the spacecraft was parabolic, determine the maximum velocity of Galileo during its first flyby of the earth.
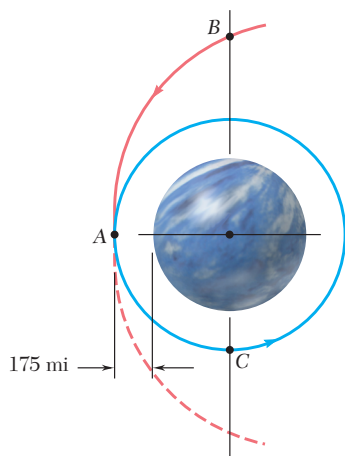

Fig. P12.95

Fig. P12.96

Fig. P12.99

12.99 As a space probe approaching the planet Venus on a parabolic trajectory reaches point A closest to the planet, its velocity is decreased to insert it into a circular orbit. Knowing that the mass and the radius of Venus are 334×10^{21} lb · s²/ft and 3761 mi, respectively, determine (*a*) the velocity of the probe as it approaches A, (*b*) the decrease in velocity required to insert it into a circular orbit.

12.100 It was observed that as the Galileo spacecraft reached the point of its trajectory closest to Io, a moon of the planet Jupiter, it was at a distance of 2820 km from the center of Io and had a velocity of 15 km/s. Knowing that the mass of Io is 0.01496 times the mass of the earth, determine the eccentricity of the trajectory of the spacecraft as it approached Io.

12.101 It was observed that during its second flyby of the earth, the Galileo spacecraft had a velocity of 14.1 km/s as it reached its minimum altitude of 303 km above the surface of the earth. Determine the eccentricity of the trajectory of the spacecraft during this portion of its flight.

12.102 A satellite describes an elliptic orbit about a planet of mass M. Denoting by r_0 and r_1, respectively, the minimum and maximum values of the distance r from the satellite to the center of the planet, derive the relation

$$\frac{1}{r_0} + \frac{1}{r_1} = \frac{2GM}{h^2}$$

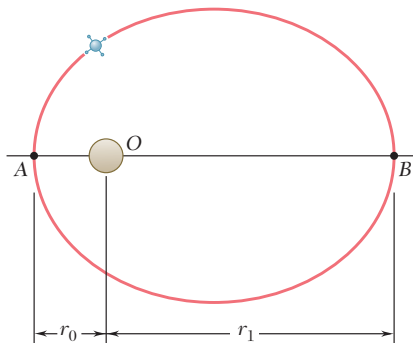where h is the angular momentum per unit mass of the satellite.

Fig. P12.102

12.103 The Chandra X-ray observatory, launched in 1999, achieved an elliptical orbit of minimum altitude 6200 mi and maximum altitude 86,900 mi above the surface of the earth. Assuming that the observatory was transferred to this orbit from a circular orbit of altitude 6200 mi at point A, determine (*a*) the increase in speed required at A, (*b*) the speed of the observatory at B.
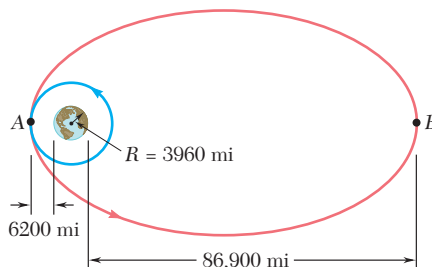

Fig. P12.103

12.104 A satellite describes a circular orbit at an altitude of 19 110 km above the surface of the earth. Determine (a) the increase in speed required at point A for the satellite to achieve the escape velocity and enter a parabolic orbit, (b) the decrease in speed required at point A for the satellite to enter an elliptic orbit of minimum altitude 6370 km, (c) the eccentricity ε of the elliptic orbit.

12.105 As it describes an elliptic orbit about the sun, a spacecraft reaches a maximum distance of 325×10^6 km from the center of the sun at point A (called the *aphelion*) and a minimum distance of 148×10^6 km at point B (called the *perihelion*). To place the spacecraft in a smaller elliptic orbit with aphelion A' and perihelion B', where A' and B' are located 264.7×10^6 km and 137.6×10^6 km, respectively, from the center of the sun, the speed of the spacecraft is first reduced as it passes through A and then is further reduced as it passes through B'. Knowing that the mass of the sun is 332.8×10^3 times the mass of the earth, determine (a) the speed of the spacecraft at A, (b) the amounts by which the speed of the spacecraft should be reduced at A and B' to insert it into the desired elliptic orbit.
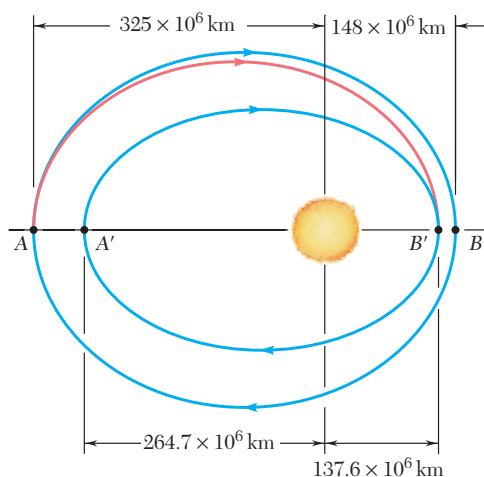

Fig. *P12.104*

Fig. *P12.105*

12.106 A space probe is in a circular orbit about the planet Mars. The orbit must have a radius of 2500 mi and be located in a specified plane which is different from the plane of the approach trajectory. As the probe reaches A, the point of its original trajectory closest to Mars, it is inserted into a first elliptic transfer orbit by reducing its speed by Δv_A. This orbit brings it to point B with a much reduced velocity. There the probe is inserted into a second transfer orbit located in the specified plane by changing the direction of its velocity and further reducing its speed by Δv_B. Finally, as the probe reaches point C, it is inserted into the desired circular orbit by reducing its speed by Δv_C. Knowing that the mass of Mars is 0.1074 times the mass of the earth, that $r_A = 5625$ mi and $r_B = 112{,}500$ mi, and that the probe approaches A on a parabolic trajectory, determine by how much the speed of the probe should be reduced (a) at A, (b) at B, (c) at C.
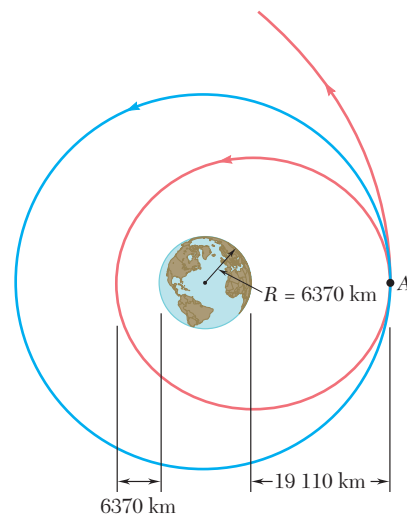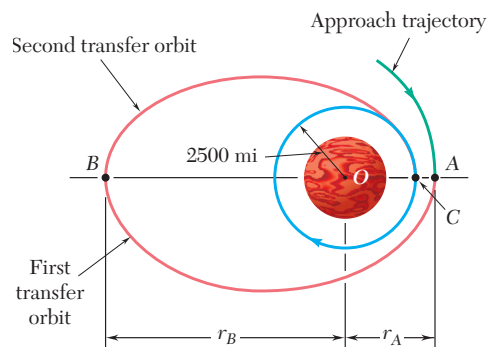

Fig. P12.106

12.107 For the probe of Prob. 12.106, it is known that $r_A = 5625$ mi and that the speed of the probe is reduced by 1300 ft/s as it passes through A. Determine (a) the distance from the center of Mars to point B, (b) the amounts by which the speed of the probe should be reduced at B and C, respectively.
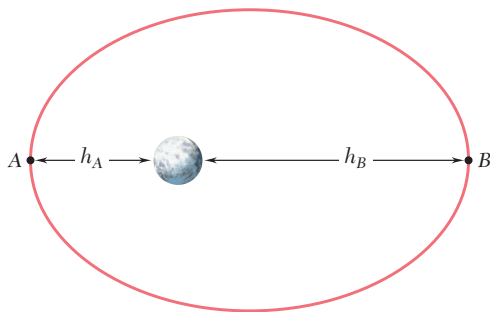
Fig. P12.109

12.108 Determine the time needed for the space probe of Prob. 12.106 to travel from A to B on its first transfer orbit.

12.109 The Clementine spacecraft described an elliptic orbit of minimum altitude $h_A = 250$ mi and a maximum altitude of $h_B = 1840$ mi above the surface of the moon. Knowing that the radius of the moon is 1080 mi and the mass of the moon is 0.01230 times the mass of the earth, determine the periodic time of the spacecraft.

12.110 A spacecraft and a satellite are at diametrically opposite positions in the same circular orbit of altitude 500 km above the earth. As it passes through point A, the spacecraft fires its engine for a short interval of time to increase its speed and enter an elliptic orbit. Knowing that the spacecraft returns to A at the same time the satellite reaches A after completing one and a half orbits, determine (a) the increase in speed required, (b) the periodic time for the elliptic orbit.
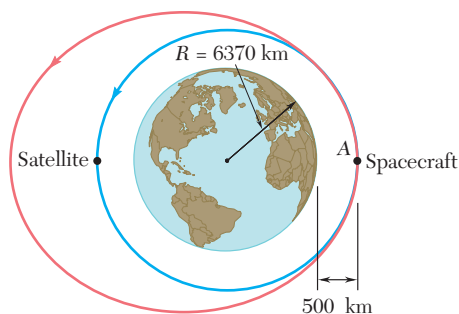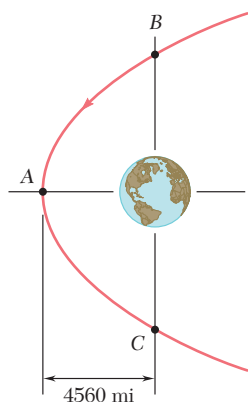

Fig. P12.110

Fig. P12.112

12.111 Based on observations made during the 1996 sighting of the comet Hyakutake, it was concluded that the trajectory of the comet is a highly elongated ellipse for which the eccentricity is approximately $\varepsilon = 0.999887$. Knowing that for the 1996 sighting the minimum distance between the comet and the sun was $0.230R_E$, where R_E is the mean distance from the sun to the earth, determine the periodic time of the comet.

12.112 It was observed that during its first flyby of the earth, the Galileo spacecraft had a velocity of 6.48 mi/s as it reached its minimum distance of 4560 mi from the center of the earth. Assuming that the trajectory of the spacecraft was parabolic, determine the time needed for the spacecraft to travel from B to C on its trajectory.

12.113 Determine the time needed for the space probe of Prob. 12.99 to travel from B to C.

12.114 A space probe is describing a circular orbit of radius nR with a speed v_0 about a planet of radius R and center O. As the probe passes through point A, its speed is reduced from v_0 to βv_0, where $\beta < 1$, to place the probe on a crash trajectory. Express in terms of n and β the angle AOB, where B denotes the point of impact of the probe on the planet.
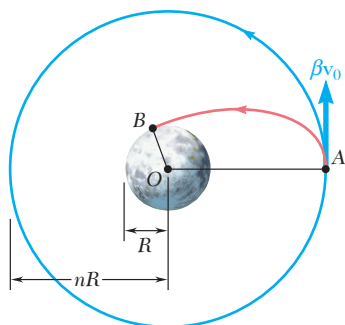

Fig. P12.114

12.115 Prior to the Apollo missions to the moon, several Lunar Orbiter spacecraft were used to photograph the lunar surface to obtain information regarding possible landing sites. At the conclusion of each mission, the trajectory of the spacecraft was adjusted so that the spacecraft would crash on the moon to further study the characteristics of the lunar surface. Shown is the elliptic orbit of Lunar Orbiter 2. Knowing that the mass of the moon is 0.01230 times the mass of the earth, determine the amount by which the speed of the orbiter should be reduced at point B so that it impacts the lunar surface at point C. (*Hint.* Point B is the apogee of the elliptic impact trajectory.)
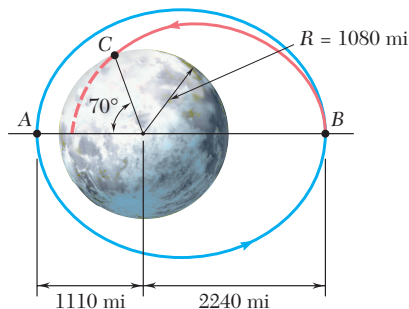

Fig. P12.115

to come

Fig. P12.116

12.116 A long-range ballistic trajectory between points A and B on the earth's surface consists of a portion of an ellipse with the apogee at point C. Knowing that point C is 1500 km above the surface of the earth and the range $R\phi$ of the trajectory is 6000 km, determine (*a*) the velocity of the projectile at C, (*b*) the eccentricity ε of the trajectory.

12.117 A space shuttle is describing a circular orbit at an altitude of 563 km above the surface of the earth. As it passes through point A, it fires its engine for a short interval of time to reduce its speed by 152 m/s and begin its decent toward the earth. Determine the angle AOB so that the altitude of the shuttle at point B is 121 km. (*Hint.* Point A is the apogee of the elliptic decent orbit.)

Fig. P12.117

12.118 A satellite describes an elliptic orbit about a planet. Denoting by r_0 and r_1 the distances corresponding, respectively, to the perigee and apogee of the orbit, show that the curvature of the orbit at each of these two points can be expressed as

$$\frac{1}{\rho} = \frac{1}{2}\left(\frac{1}{r_0} + \frac{1}{r_1}\right)$$

12.119 (*a*) Express the eccentricity ε of the elliptic orbit described by a satellite about a planet in terms of the distances r_0 and r_1 corresponding, respectively, to the perigee and apogee of the orbit. (*b*) Use the result obtained in part *a* and the data given in Prob. 12.111, where $R_\varepsilon = 93.0 \times 10^6$ mi, to determine the appropriate maximum distance from the sun reached by comet Hyakutake.

12.120 Derive Kepler's third law of planetary motion from Eqs. (12.39) and (12.45).

12.121 Show that the angular momentum per unit mass h of a satellite describing an elliptic orbit of semimajor axis a and eccentricity ε about a planet of mass M can be expressed as

$$h = \sqrt{GMa(1 - \varepsilon^2)}$$

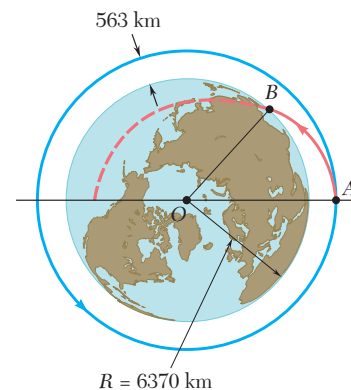

Fig. P12.118 and P12.119