C H A P T E R

4

Equilibrium of Rigid Bodies

On many nstruction projects, large cranes such as those shown are used. The sum of the moments of the loads and the counterweights about the base of a crane must be adjusted so that the couple of the reaction at the base does not cause failure of the tower.

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EQUILIBRIUM OF RIGID BODIES

4.1. INTRODUCTION

We saw in the preceding chapter that the external forces acting on a rigid body can be reduced to a force-couple system at some arbitrary point O. When the force and the couple are both equal to zero, the external forces form a system equivalent to zero, and the rigid body is said to be in *equilibrium.*

The necessary and sufficient conditions for the equilibrium of a rigid body, therefore, can be obtained by setting $\mathbf{R}$ and $\mathbf{M}_O^R$ equal to zero in the relations (3.52) of Sec. 3.17:

$$\Sigma \mathbf{F} = 0 \qquad \Sigma \mathbf{M}_O = \Sigma(\mathbf{r} \times \mathbf{F}) = 0 \tag{4.1}$$

Resolving each force and each moment into its rectangular components, we can express the necessary and sufficient conditions for the equilibrium of a rigid body with the following six scalar equations:

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma F_z = 0 \tag{4.2}$$

$$\Sigma M_x = 0 \qquad \Sigma M_y = 0 \qquad \Sigma M_z = 0 \tag{4.3}$$

The equations obtained can be used to determine unknown forces applied to the rigid body or unknown reactions exerted on it by its supports. We note that Eqs. (4.2) express the fact that the components of the external forces in the x, y, and z directions are balanced; Eqs. (4.3) express the fact that the moments of the external forces about the x, y, and z axes are balanced. Therefore, for a rigid body in equilibrium, the system of the external forces will impart no translational or rotational motion to the body considered.

In order to write the equations of equilibrium for a rigid body, it is essential to first identify all of the forces acting on that body and then to draw the corresponding *free-body diagram.* In this chapter we first consider the equilibrium of *two-dimensional structures* subjected to forces contained in their planes and learn how to draw their free-body diagrams. In addition to the forces *applied* to a structure, the *reactions* exerted on the structure by its supports will be considered. A specific reaction will be associated with each type of support. You will learn how to determine whether the structure is properly supported, so that you can know in advance whether the equations of equilibrium can be solved for the unknown forces and reactions.

Later in the chapter, the equilibrium of three-dimensional structures will be considered, and the same kind of analysis will be given to these structures and their supports.

4.2. FREE-BODY DIAGRAM

In solving a problem concerning the equilibrium of a rigid body, it is essential to consider *all* of the forces acting on the body; it is equally important to exclude any force which is not directly applied to the body. Omitting a force or adding an extraneous one would destroy the conditions of equilibrium. Therefore, the first step in the solution of the problem should be to draw a *free-body diagram* of the rigid body under consideration. Free-body diagrams have already been used on many occasions in Chap. 2. However, in view of their importance to the solution of equilibrium problems, we summarize here the various steps which must be followed in drawing a free-body diagram.

1. A clear decision should be made regarding the choice of the free body to be used. This body is then detached from the ground and is separated from all other bodies. The contour of the body thus isolated is sketched.

2. All external forces should be indicated on the free-body diagram. These forces represent the actions exerted *on* the free body *by* the ground and *by* the bodies which have been detached; they should be applied at the various points where the free body was supported by the ground or was connected to the other bodies. The *weight* of the free body should also be included among the external forces, since it represents the attraction exerted by the earth on the various particles forming the free body. As will be seen in Chap. 5, the weight should be applied at the center of gravity of the body. When the free body is made of several parts, the forces the various parts exert on each other should *not* be included among the external forces. These forces are internal forces as far as the free body is concerned.

3. The magnitudes and directions of the *known external forces* should be clearly marked on the free-body diagram. When indicating the directions of these forces, it must be remembered that the forces shown on the free-body diagram must be those which are exerted *on,* and not *by,* the free body. Known external forces generally include the *weight* of the free body and *forces applied* for a given purpose.

4. *Unknown external forces* usually consist of the *reactions,* through which the ground and other bodies oppose a possible motion of the free body. The reactions constrain the free body to remain in the same position, and, for that reason, are sometimes called *constraining forces.* Reactions are exerted at the points where the free body is *supported by* or *connected to* other bodies and should be clearly indicated. Reactions are discussed in detail in Secs. 4.3 and 4.8.

5. The free-body diagram should also include dimensions, since these may be needed in the computation of moments of forces. Any other detail, however, should be omitted.

Photo 4.1 A free-body diagram of the tractor shown would include all of the external forces acting on the tractor: the weight of the tractor, the weight of the load in ther bucket, and the forces exerted by the ground on the tires.

Photo 4.2 In chap. 6, we will discuss how to determine the internal forces in structures made of several connected pieces, such as the forces in the members that support the bucket of the tractor of Photo 4.1.

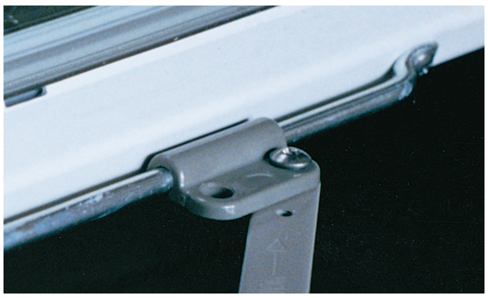


Photo 4.3 As the link of the awning window opening mechanism is extended, the force it exerts on the slider results in a normal force being applied to the rod, which causes the window to open.

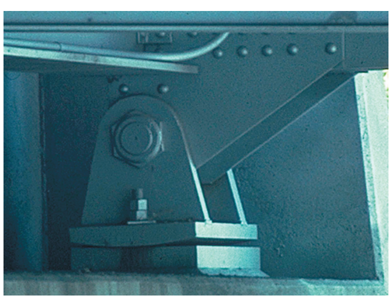


Photo 4.4 The abutment-mounted rocker bearing shown is used to support the roadway of a bridge.

Photo 4.5 Shown is the rocker expansion bearing of a plate girder bridge. The convex surface of the rocker allows the support of the girder to move horizontally.

EQUILIBRIUM IN TWO DIMENSIONS

4.3. REACTIONS AT SUPPORTS AND CONNECTIONS FOR A TWO-DIMENSIONAL STRUCTURE

In the first part of this chapter, the equilibrium of a two-dimensional structure is considered; that is, it is assumed that the structure being analyzed and the forces applied to it are contained in the same plane. Clearly, the reactions needed to maintain the structure in the same position will also be contained in this plane.

The reactions exerted on a two-dimensional structure can be divided into three groups corresponding to three types of *supports,* or *connections:*

1. *Reactions Equivalent to a Force with Known Line of Action.* Supports and connections causing reactions of this type include *rollers, rockers, frictionless surfaces, short links and cables, collars on frictionless rods,* and *frictionless pins in slots.* Each of these supports and connections can prevent motion in one direction only. They are shown in Fig. 4.1, together with the reactions they produce. Each of these reactions involves *one unknown,* namely, the magnitude of the reaction; this magnitude should be denoted by an appropriate letter. The line of action of the reaction is known and should be indicated clearly in the free-body diagram. The sense of the reaction must be as shown in Fig. 4.1 for the cases of a frictionless surface (toward the free body) or a cable (away from the free body). The reaction can be directed either way in the case of double-track rollers, links, collars on rods, and pins in slots. Single-track rollers and rockers are generally assumed to be reversible, and thus the corresponding reactions can also be directed either way.

2. *Reactions Equivalent to a Force of Unknown Direction and Magnitude.* Supports and connections causing reactions of this type include *frictionless pins in fitted holes, hinges,* and *rough surfaces.* They can prevent translation of the free body in all directions, but they cannot prevent the body from rotating about the connection. Reactions of this group involve *two unknowns* and are usually represented by their x and y components. In the case of a rough surface, the component normal to the surface must be directed away from the surface.

3. *Reactions Equivalent to a Force and a Couple.* These reactions are caused by *fixed supports,* which oppose any motion of the free body and thus constrain it completely. Fixed supports actually produce forces over the entire surface of contact; these forces, however, form a system which can be reduced to a force and a couple. Reactions of this group involve *three unknowns,* consisting usually of the two components of the force and the moment of the couple.

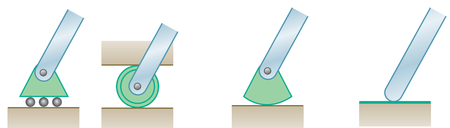
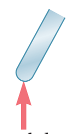
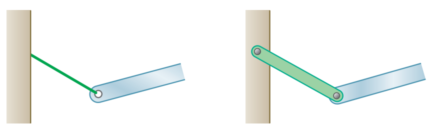
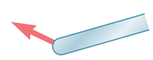
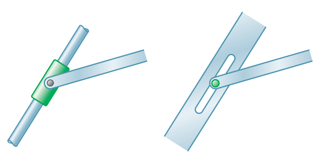
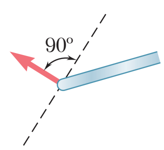
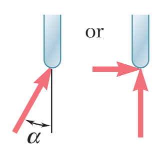
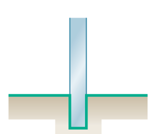
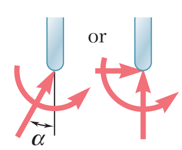
Support or Connection	Reaction	Number of Unknowns
 Rollers · Rocker · Frictionless surface	 Force with known line of action	1
 Short cable · Short link	 Force with known line of action	1
 Collar on frictionless rod · Frictionless pin in slot	 90° Force with known line of action	1
 Frictionless pin or hinge · Rough surface	 or α Force of unknown direction	2
 Fixed support	 or α Force and couple	3

Fig. 4.1 Reactions at supports and connections.

When the sense of an unknown force or couple is not readily apparent, no attempt should be made to determine it. Instead, the sense of the force or couple should be arbitrarily assumed; the sign of the answer obtained will indicate whether the assumption is correct or not.

4.4. EQUILIBRIUM OF A RIGID BODY IN TWO DIMENSIONS

The conditions stated in Sec. 4.1 for the equilibrium of a rigid body become considerably simpler for the case of a two-dimensional structure. Choosing the x and y axes to be in the plane of the structure, we have

$$F_z = 0 \qquad M_x = M_y = 0 \qquad M_z = M_O$$

for each of the forces applied to the structure. Thus, the six equations of equilibrium derived in Sec. 4.1 reduce to

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma M_O = 0 \tag{4.4}$$

and to three trivial identities, $0 = 0$. Since $\Sigma M_O = 0$ must be satisfied regardless of the choice of the origin O, we can write the equations of equilibrium for a two-dimensional structure in the more general form

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma M_A = 0 \tag{4.5}$$

where A is any point in the plane of the structure. The three equations obtained can be solved for no more than *three unknowns.*

We saw in the preceding section that unknown forces include reactions and that the number of unknowns corresponding to a given reaction depends upon the type of support or connection causing that reaction. Referring to Sec. 4.3, we observe that the equilibrium equations (4.5) can be used to determine the reactions associated with two rollers and one cable, one fixed support, or one roller and one pin in a fitted hole, etc.

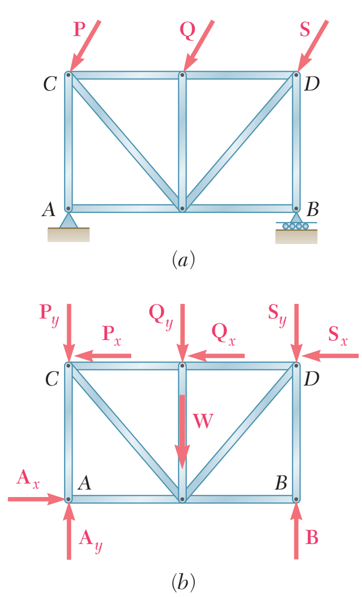


Fig. 4.2

Consider Fig. 4.2*a*, in which the truss shown is subjected to the given forces **P**, **Q**, and **S**. The truss is held in place by a pin at A and a roller at B. The pin prevents point A from moving by exerting on the truss a force which can be resolved into the components $\mathbf{A}_x$ and $\mathbf{A}_y$; the roller keeps the truss from rotating about A by exerting the vertical force **B**. The free-body diagram of the truss is shown in Fig. 4.2*b*; it includes the reactions $\mathbf{A}_x$, $\mathbf{A}_y$, and **B** as well as the applied forces **P**, **Q**, **S** and the weight **W** of the truss. Expressing that the sum of the moments about A of all of the forces shown in Fig. 4.2*b* is zero, we write the equation $\Sigma M_A = 0$, which can be used to determine the magnitude B since it does not contain A_x or A_y. Next, expressing that the sum of the x components and the sum of the y components of the forces are zero, we write the equations $\Sigma F_x = 0$ and $\Sigma F_y = 0$, from which we can obtain the components A_x and A_y, respectively.

An additional equation could be obtained by expressing that the sum of the moments of the external forces about a point other than A is zero. We could write, for instance, $\Sigma M_B = 0$. Such a statement, however, does not contain any new information, since it has already been established that the system of the forces shown in Fig. 4.2*b* is equivalent to zero. The additional equation *is not independent* and cannot be used to determine a fourth unknown. It will be useful, however, for checking the solution obtained from the original three equations of equilibrium.

While the three equations of equilibrium cannot be *augmented* by additional equations, any of them can be *replaced* by another equation. Therefore, an alternative system of equations of equilibrium is

$$\Sigma F_x = 0 \qquad \Sigma M_A = 0 \qquad \Sigma M_B = 0 \tag{4.6}$$

where the second point about which the moments are summed (in this case, point B) cannot lie on the line parallel to the y axis that passes through point A (Fig. 4.2*b*). These equations are sufficient conditions for the equilibrium of the truss. The first two equations indicate that the external forces must reduce to a single vertical force at A. Since the third equation requires that the moment of this force be zero about a point B which is not on its line of action, the force must be zero, and the rigid body is in equilibrium.

A third possible set of equations of equilibrium is

$$\Sigma M_A = 0 \qquad \Sigma M_B = 0 \qquad \Sigma M_C = 0 \tag{4.7}$$

where the points A, B, and C do not lie in a straight line (Fig. 4.2*b*). The first equation requires that the external forces reduce to a single force at A; the second equation requires that this force pass through B; and the third equation requires that it pass through C. Since the points A, B, C do not lie in a straight line, the force must be zero, and the rigid body is in equilibrium.

The equation $\Sigma M_A = 0$, which expresses that the sum of the moments of the forces about pin A is zero, possesses a more definite physical meaning than either of the other two equations (4.7). These two equations express a similar idea of balance, but with respect to points about which the rigid body is not actually hinged. They are, however, as useful as the first equation, and our choice of equilibrium equations should not be unduly influenced by the physical meaning of these equations. Indeed, it will be desirable in practice to choose equations of equilibrium containing only one unknown, since this eliminates the necessity of solving simultaneous equations. Equations containing only one unknown can be obtained by summing moments about the point of intersection of the lines of action of two unknown forces or, if these forces are parallel, by summing components in a direction perpendicular to their common direction. For example, in Fig. 4.3, in which the truss shown is held by rollers at A and B and a short link at D, the reactions at A and B can be eliminated by summing x components. The reactions at A and D will be eliminated by summing moments about C, and the reactions at B and D by summing moments about D. The equations obtained are

$$\Sigma F_x = 0 \qquad \Sigma M_C = 0 \qquad \Sigma M_D = 0$$

Each of these equations contains only one unknown.

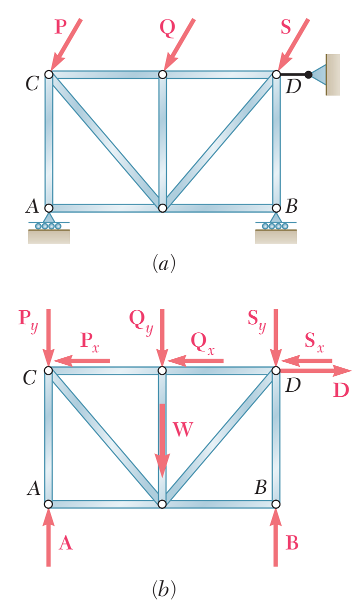


Fig. 4.3

4.5. STATICALLY INDETERMINATE REACTIONS. PARTIAL CONSTRAINTS

In the two examples considered in the preceding section (Figs. 4.2 and 4.3), the types of supports used were such that the rigid body could not possibly move under the given loads or under any other loading conditions. In such cases, the rigid body is said to be *completely constrained.* We also recall that the reactions corresponding to these supports involved *three unknowns* and could be determined by solving the three equations of equilibrium. When such a situation exists, the reactions are said to be *statically determinate.*

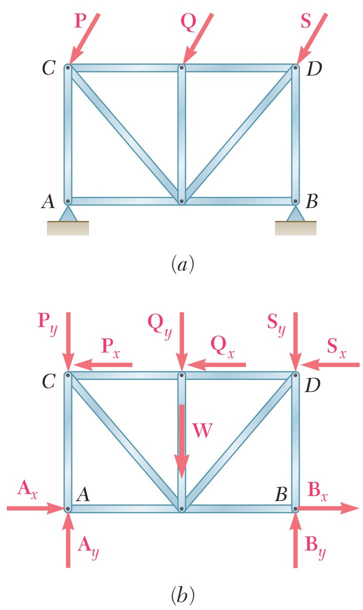


Fig. 4.4 Statically indeterminate reactions.

Consider Fig. 4.4*a*, in which the truss shown is held by pins at A and B. These supports provide more constraints than are necessary to keep the truss from moving under the given loads or under any other loading conditions. We also note from the free-body diagram of Fig. 4.4*b* that the corresponding reactions involve *four unknowns.* Since, as was pointed out in Sec. 4.4, only three independent equilibrium equations are available, there are *more unknowns than equations;* thus, all of the unknowns cannot be determined. While the equations $\Sigma M_A = 0$ and $\Sigma M_B = 0$ yield the vertical components B_y and A_y, respectively, the equation $\Sigma F_x = 0$ gives only the sum $A_x + B_x$ of the horizontal components of the reactions at A and B. The components A_x and B_x are said to be *statically indeterminate.* They could be determined by considering the deformations produced in the truss by the given loading, but this method is beyond the scope of statics and belongs to the study of mechanics of materials.

The supports used to hold the truss shown in Fig. 4.5*a* consist of rollers at A and B. Clearly, the constraints provided by these supports are not sufficient to keep the truss from moving. While any vertical motion is prevented, the truss is free to move horizontally. The truss is said to be *partially constrained.*† Turning our attention to Fig. 4.5*b*, we note that the reactions at A and B involve only *two unknowns.* Since three equations of equilibrium must still be satisfied, there are *fewer unknowns than equations,* and, in general, one of the equilibrium equations will not be satisfied. While the equations $\Sigma M_A = 0$ and $\Sigma M_B = 0$ can be satisfied by a proper choice of reactions at A and B, the equation $\Sigma F_x = 0$ will not be satisfied unless the sum of the horizontal components of the applied forces happens to be zero. We thus observe that the equilibrium of the truss of Fig. 4.5 cannot be maintained under general loading conditions.

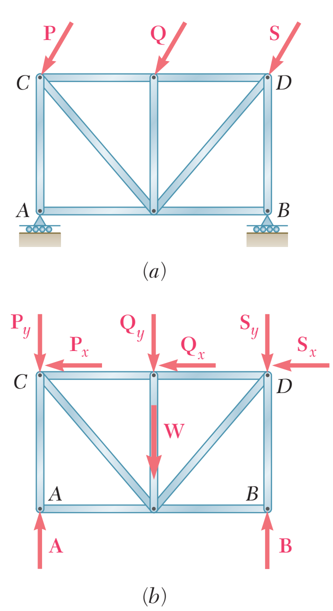


Fig. 4.5 Partial constraints.

It appears from the above that if a rigid body is to be completely constrained and if the reactions at its supports are to be statically determinate, *there must be as many unknowns as there are equations of equilibrium.* When this condition is *not* satisfied, we can be certain either that the rigid body is not completely constrained or that the reactions at its supports are not statically determinate; it is also possible that the rigid body is not completely constrained *and* that the reactions are statically indeterminate.

We should note, however, that, while *necessary,* the above condition is *not sufficient.* In other words, the fact that the number of un-

†Partially constrained bodies are often referred to as *unstable.* However, to avoid confusion between this type of instability, due to insufficient constraints, and the type of instability considered in Chap. 10, which relates to the behavior of a rigid body when its equilibrium is disturbed, we will restrict the use of the words *stable* and *unstable* to the latter case.

knowns is equal to the number of equations is no guarantee that the body is completely constrained or that the reactions at its supports are statically determinate. Consider Fig. 4.6*a*, in which the truss shown is held by rollers at *A*, *B*, and *E*. While there are three unknown reactions, **A**, **B**, and **E** (Fig. 4.6*b*), the equation $\Sigma F_x = 0$ will not be satisfied unless the sum of the horizontal components of the applied forces happens to be zero. Although there are a sufficient number of constraints, these constraints are not properly arranged, and the truss is free to move horizontally. We say that the truss is *improperly constrained.* Since only two equilibrium equations are left for determining three unknowns, the reactions will be statically indeterminate. Thus, improper constraints also produce static indeterminacy.

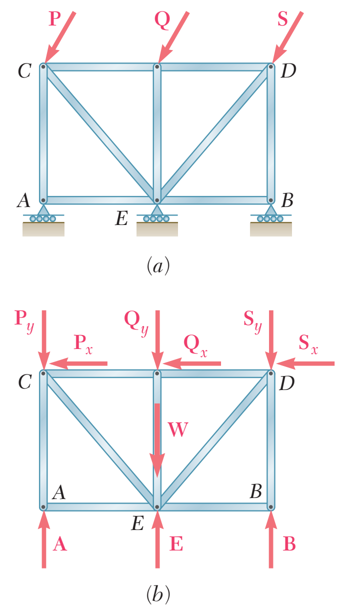


Fig. 4.6 Improper constraints.

Another example of improper constraints—and of static indeterminacy—is provided by the truss shown in Fig. 4.7. This truss is held by a pin at *A* and by rollers at *B* and *C*, which altogether involve four unknowns. Since only three independent equilibrium equations are available, the reactions at the supports are statically indeterminate. On the other hand, we note that the equation $\Sigma M_A = 0$ cannot be satisfied under general loading conditions, since the lines of action of the reactions **B** and **C** pass through *A*. We conclude that the truss can rotate about *A* and that it is improperly constrained.†

The examples of Figs. 4.6 and 4.7 lead us to conclude that *a rigid body is improperly constrained whenever the supports,* even though they may provide a sufficient number of reactions, *are arranged in such a way that the reactions must be either concurrent or parallel.*‡

In summary, to be sure that a two-dimensional rigid body is completely constrained and that the reactions at its supports are statically determinate, we should verify that the reactions involve three—and only three—unknowns and that the supports are arranged in such a way that they do not require the reactions to be either concurrent or parallel.

Supports involving statically indeterminate reactions should be used with care in the *design* of structures and only with a full knowledge of the problems they may cause. On the other hand, the *analysis* of structures possessing statically indeterminate reactions often can be partially carried out by the methods of statics. In the case of the truss of Fig. 4.4, for example, the vertical components of the reactions at *A* and *B* were obtained from the equilibrium equations.

For obvious reasons, supports producing partial or improper constraints should be avoided in the design of stationary structures. However, a partially or improperly constrained structure will not necessarily collapse; under particular loading conditions, equilibrium can be maintained. For example, the trusses of Figs. 4.5 and 4.6 will be in equilibrium if the applied forces **P**, **Q**, and **S** are vertical. Besides, structures which are designed to move *should* be only partially constrained. A railroad car, for instance, would be of little use if it were completely constrained by having its brakes applied permanently.

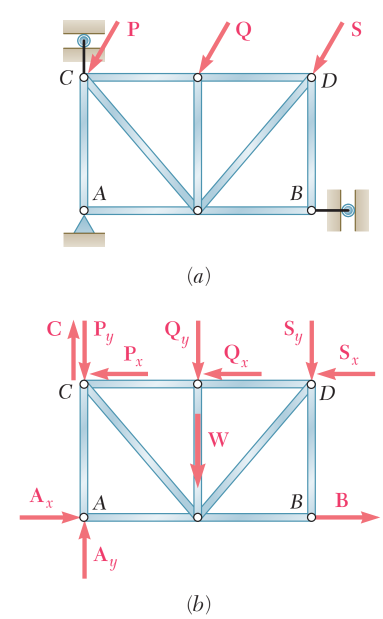
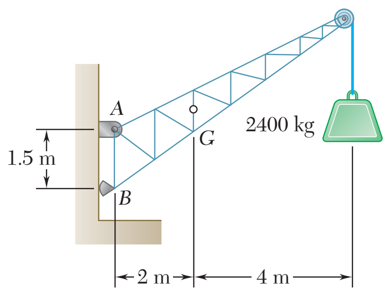


Fig. 4.7 Improper constraints.

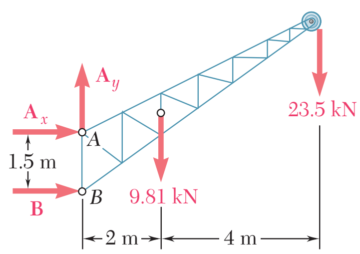
†Rotation of the truss about *A* requires some "play" in the supports at *B* and *C*. In practice such play will always exist. In addition, we note that if the play is kept small, the displacements of the rollers *B* and *C* and, thus, the distances from *A* to the lines of action of the reactions **B** and **C** will also be small. The equation $\Sigma M_A = 0$ then requires that **B** and **C** be very large, a situation which can result in the failure of the supports at *B* and *C*.

‡Because this situation arises from an inadequate arrangement or *geometry* of the supports, it is often referred to as *geometric instability.*



SAMPLE PROBLEM 4.1

A fixed crane has a mass of 1000 kg and is used to lift a 2400-kg crate. It is held in place by a pin at A and a rocker at B. The center of gravity of the crane is located at G. Determine the components of the reactions at A and B.



SOLUTION

Free-Body Diagram. A free-body diagram of the crane is drawn. By multiplying the masses of the crane and of the crate by $g = 9.81\ \mathrm{m/s^2}$, we obtain the corresponding weights, that is, 9810 N or 9.81 kN, and 23 500 N or 23.5 kN. The reaction at pin A is a force of unknown direction; it is represented by its components $\mathbf{A}_x$ and $\mathbf{A}_y$. The reaction at the rocker B is perpendicular to the rocker surface; thus, it is horizontal. We assume that $\mathbf{A}_x$, $\mathbf{A}_y$, and $\mathbf{B}$ act in the directions shown.

Determination of *B*. We express that the sum of the moments of all external forces about point A is zero. The equation obtained will contain neither A_x nor A_y, since the moments of $\mathbf{A}_x$ and $\mathbf{A}_y$ about A are zero. Multiplying the magnitude of each force by its perpendicular distance from A, we write

$$+\circlearrowleft \Sigma M_A = 0: \qquad +B(1.5\ \mathrm{m}) - (9.81\ \mathrm{kN})(2\ \mathrm{m}) - (23.5\ \mathrm{kN})(6\ \mathrm{m}) = 0$$

$$B = +107.1\ \mathrm{kN} \qquad\qquad \mathbf{B} = 107.1\ \mathrm{kN} \rightarrow \quad ◀$$

Since the result is positive, the reaction is directed as assumed.

Determination of $\mathbf{A}_x$. The magnitude of $\mathbf{A}_x$ is determined by expressing that the sum of the horizontal components of all external forces is zero.

$$\overset{+}{\rightarrow} \Sigma F_x = 0: \qquad A_x + B = 0$$

$$A_x + 107.1\ \mathrm{kN} = 0$$

$$A_x = -107.1\ \mathrm{kN} \qquad\qquad \mathbf{A}_x = 107.1\ \mathrm{kN} \leftarrow \quad ◀$$

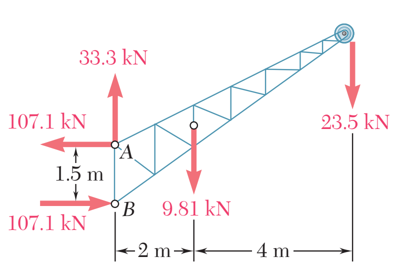
Since the result is negative, the sense of $\mathbf{A}_x$ is opposite to that assumed originally.

Determination of $\mathbf{A}_y$. The sum of the vertical components must also equal zero.

$$+\uparrow \Sigma F_y = 0: \qquad A_y - 9.81\ \mathrm{kN} - 23.5\ \mathrm{kN} = 0$$

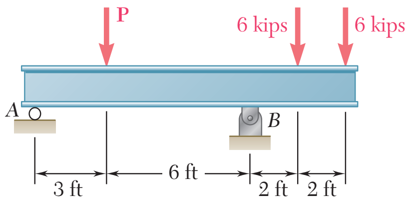
$$A_y = +33.3\ \mathrm{kN} \qquad\qquad \mathbf{A}_y = 33.3\ \mathrm{kN} \uparrow \quad ◀$$

Adding vectorially the components $\mathbf{A}_x$ and $\mathbf{A}_y$, we find that the reaction at A is 112.2 kN ⦪17.3°.



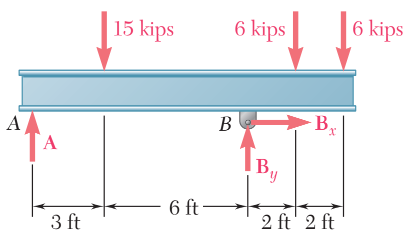
Check. The values obtained for the reactions can be checked by recalling that the sum of the moments of all of the external forces about any point must be zero. For example, considering point B, we write

$$+\circlearrowleft \Sigma M_B = -(9.81\ \mathrm{kN})(2\ \mathrm{m}) - (23.5\ \mathrm{kN})(6\ \mathrm{m}) + (107.1\ \mathrm{kN})(1.5\ \mathrm{m}) = 0$$



SAMPLE PROBLEM 4.2

Three loads are applied to a beam as shown. The beam is supported by a roller at A and by a pin at B. Neglecting the weight of the beam, determine the reactions at A and B when $P = 15$ kips.



SOLUTION

Free-Body Diagram. A free-body diagram of the beam is drawn. The reaction at A is vertical and is denoted by $\mathbf{A}$. The reaction at B is represented by components $\mathbf{B}_x$ and $\mathbf{B}_y$. Each component is assumed to act in the direction shown.

Equilibrium Equations. We write the following three equilibrium equations and solve for the reactions indicated:

$$\xrightarrow{+} \Sigma F_x = 0: \qquad B_x = 0 \qquad \mathbf{B}_x = 0 \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_A = 0:$$
$$-(15 \text{ kips})(3 \text{ ft}) + B_y(9 \text{ ft}) - (6 \text{ kips})(11 \text{ ft}) - (6 \text{ kips})(13 \text{ ft}) = 0$$
$$B_y = +21.0 \text{ kips} \qquad \mathbf{B}_y = 21.0 \text{ kips} \uparrow \blacktriangleleft$$

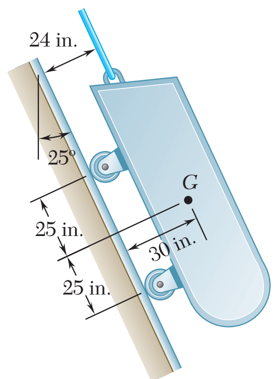
$$+\circlearrowleft \Sigma M_B = 0:$$
$$-A(9 \text{ ft}) + (15 \text{ kips})(6 \text{ ft}) - (6 \text{ kips})(2 \text{ ft}) - (6 \text{ kips})(4 \text{ ft}) = 0$$
$$A = +6.00 \text{ kips} \qquad \mathbf{A} = 6.00 \text{ kips} \uparrow \blacktriangleleft$$

Check. The results are checked by adding the vertical components of all of the external forces:

$$+\uparrow \Sigma F_y = +6.00 \text{ kips} - 15 \text{ kips} + 21.0 \text{ kips} - 6 \text{ kips} - 6 \text{ kips} = 0$$

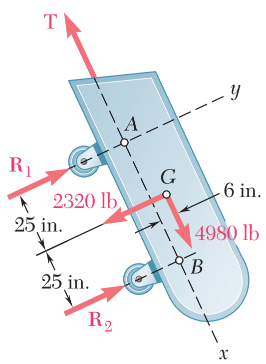
Remark. In this problem the reactions at both A and B are vertical; however, these reactions are vertical for different reasons. At A, the beam is supported by a roller; hence the reaction cannot have a horizontal component. At B, the horizontal component of the reaction is zero because it must satisfy the equilibrium equation $\Sigma F_x = 0$ and because none of the other forces acting on the beam has a horizontal component.

We could have noticed at first glance that the reaction at B was vertical and dispensed with the horizontal component $\mathbf{B}_x$. This, however, is a bad practice. In following it, we would run the risk of forgetting the component $\mathbf{B}_x$ when the loading conditions require such a component (i.e., when a horizontal load is included). Also, the component $\mathbf{B}_x$ was found to be zero by using and solving an equilibrium equation, $\Sigma F_x = 0$. By setting $\mathbf{B}_x$ equal to zero immediately, we might not realize that we actually made use of this equation and thus might lose track of the number of equations available for solving the problem.



SAMPLE PROBLEM 4.3

A loading car is at rest on a track forming an angle of 25° with the vertical. The gross weight of the car and its load is 5500 lb, and it is applied at a point 30 in. from the track, halfway between the two axles. The car is held by a cable attached 24 in. from the track. Determine the tension in the cable and the reaction at each pair of wheels.



SOLUTION

Free-Body Diagram. A free-body diagram of the car is drawn. The reaction at each wheel is perpendicular to the track, and the tension force **T** is parallel to the track. For convenience, we choose the x axis parallel to the track and the y axis perpendicular to the track. The 5500-lb weight is then resolved into x and y components.

$$W_x = +(5500 \text{ lb}) \cos 25° = +4980 \text{ lb}$$
$$W_y = -(5500 \text{ lb}) \sin 25° = -2320 \text{ lb}$$

Equilibrium Equations. We take moments about A to eliminate **T** and $\mathbf{R}_1$ from the computation.

$$+\circlearrowleft \Sigma M_A = 0: \quad -(2320 \text{ lb})(25 \text{ in.}) - (4980 \text{ lb})(6 \text{ in.}) + R_2(50 \text{ in.}) = 0$$
$$R_2 = +1758 \text{ lb} \qquad \mathbf{R}_2 = 1758 \text{ lb} \nearrow \blacktriangleleft$$

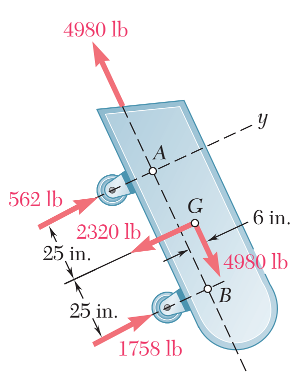
Now, taking moments about B to eliminate **T** and $\mathbf{R}_2$ from the computation, we write

$$+\circlearrowleft \Sigma M_B = 0: \quad (2320 \text{ lb})(25 \text{ in.}) - (4980 \text{ lb})(6 \text{ in.}) - R_1(50 \text{ in.}) = 0$$
$$R_1 = +562 \text{ lb} \qquad \mathbf{R}_1 = +562 \text{ lb} \nearrow \blacktriangleleft$$

The value of T is found by writing

$$\searrow + \Sigma F_x = 0: \quad +4980 \text{ lb} - T = 0$$
$$T = +4980 \text{ lb} \qquad \mathbf{T} = 4980 \text{ lb} \nwarrow \blacktriangleleft$$

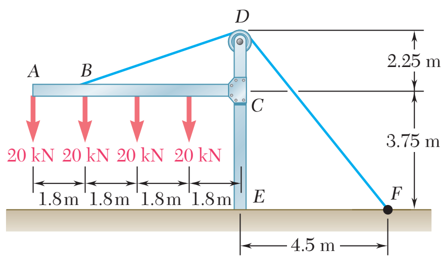
The computed values of the reactions are shown in the adjacent sketch.



Check. The computations are verified by writing

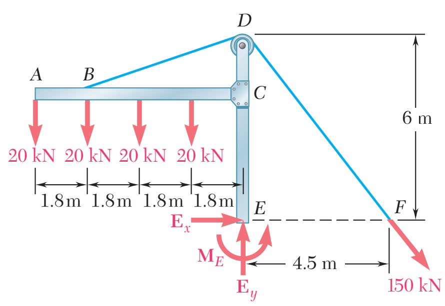
$$\nearrow + \Sigma F_y = +562 \text{ lb} + 1758 \text{ lb} - 2320 \text{ lb} = 0$$

The solution could also have been checked by computing moments about any point other than A or B.



SAMPLE PROBLEM 4.4

The frame shown supports part of the roof of a small building. Knowing that the tension in the cable is 150 kN, determine the reaction at the fixed end E.



SOLUTION

Free-Body Diagram. A free-body diagram of the frame and of the cable BDF is drawn. The reaction at the fixed end E is represented by the force components $\mathbf{E}_x$ and $\mathbf{E}_y$ and the couple $\mathbf{M}_E$. The other forces acting on the free body are the four 20-kN loads and the 150-kN force exerted at end F of the cable.

Equilibrium Equations. Noting that $DF = \sqrt{(4.5 \text{ m})^2 + (6 \text{ m})^2} = 7.5 \text{ m}$, we write

$\xrightarrow{+}\Sigma F_x = 0$: $\quad E_x + \dfrac{4.5}{7.5}(150 \text{ kN}) = 0$

$E_x = -90.0 \text{ kN} \qquad \mathbf{E}_x = 90.0 \text{ kN} \leftarrow$ ◀

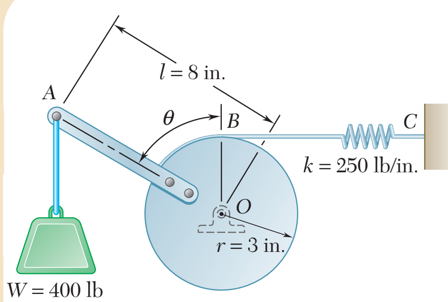
$+\uparrow\Sigma F_y = 0$: $\quad E_y - 4(20 \text{ kN}) - \dfrac{6}{7.5}(150 \text{ kN}) = 0$

$E_y = +200 \text{ kN} \qquad \mathbf{E}_y = 200 \text{ kN} \uparrow$ ◀

$+\circlearrowleft\Sigma M_E = 0$: $\quad (20 \text{ kN})(7.2 \text{ m}) + (20 \text{ kN})(5.4 \text{ m}) + (20 \text{ kN})(3.6 \text{ m})$

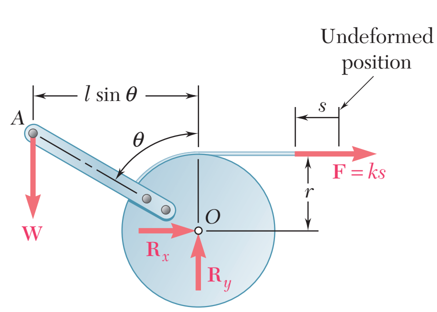
$+ (20 \text{ kN})(1.8 \text{ m}) - \dfrac{6}{7.5}(150 \text{ kN})(4.5 \text{ m}) + M_E = 0$

$M_E = +180.0 \text{ kN} \cdot \text{m} \qquad \mathbf{M}_E = 180.0 \text{ kN} \cdot \text{m} \circlearrowleft$ ◀



SAMPLE PROBLEM 4.5

A 400-lb weight is attached at A to the lever shown. The constant of the spring BC is $k = 250$ lb/in., and the spring is unstretched when $\theta = 0$. Determine the position of equilibrium.



SOLUTION

Free-Body Diagram. We draw a free-body diagram of the lever and cylinder. Denoting by s the deflection of the spring from its undeformed position, and noting that $s = r\theta$, we have $F = ks = kr\theta$.

Equilibrium Equation. Summing the moments of $\mathbf{W}$ and $\mathbf{F}$ about O, we write

$+\circlearrowleft\Sigma M_O = 0$: $\quad Wl \sin\theta - r(kr\theta) = 0 \qquad \sin\theta = \dfrac{kr^2}{Wl}\theta$

Substituting the given data, we obtain

$$\sin\theta = \frac{(250 \text{ lb/in.})(3 \text{ in.})^2}{(400 \text{ lb})(8 \text{ in.})}\theta \qquad \sin\theta = 0.703\theta$$

Solving numerically, we find $\qquad \theta = 0 \qquad \theta = 80.3°$ ◀

SOLVING PROBLEMS ON YOUR OWN

You saw that the external forces acting on a rigid body in equilibrium form a system equivalent to zero. To solve an equilibrium problem your first task is to draw a neat, reasonably large *free-body diagram* on which you will show all external forces and relevant dimensions. Both known and unknown forces must be included.

For a two-dimensional rigid body, the reactions at the supports can involve one, two, or three unknowns depending on the type of support (Fig. 4.1). For the successful solution of a problem, a correct free-body diagram is essential. Never proceed with the solution of a problem until you are sure that your free-body diagram includes all loads, all reactions, and the weight of the body (if appropriate).

As you construct your free-body diagrams, it will be necessary to assign directions to the unknown reactions. We suggest you always assume these forces act in a positive direction, so that positive answers always imply forces acting in a positive direction, while negative answers always imply forces acting in a negative direction. Similarly, we recommend you always assume the unknown force in a rod or cable is tensile, so that a positive result always means a tensile reaction. While a negative or comprehensive reaction is possible for a rod, a negative answer for a cable is impossible and, therefore, implies that there is an error in your solution.

1. You can write three equilibrium equations and solve them for *three unknowns.* The three equations might be

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma M_O = 0$$

However, there are usually several sets of equations that you can write, such as

$$\Sigma F_x = 0 \qquad \Sigma M_A = 0 \qquad \Sigma M_B = 0$$

where point B is chosen in such a way that the line AB is not parallel to the y axis, or

$$\Sigma M_A = 0 \qquad \Sigma M_B = 0 \qquad \Sigma M_C = 0$$

where the points A, B, and C do not lie in a straight line.

2. To simplify your solution, it may be helpful to use one of the following solution techniques if applicable.

a. By summing moments about the point of intersection of the lines of action of two unknown forces, you will obtain an equation in a single unknown.

b. By summing components in a direction perpendicular to two unknown parallel forces, you will obtain an equation in a single unknown.

In some of the following problems you will be asked to determine the *allowable range of values* of the applied load for a given set of constraints, such as the maximum reaction at a support or the maximum force in one or more cables or rods. For problems of this type, you first assume a *maximum* loading situation (for example, the maximum allowed force in a rod), and then apply the equations of equilibrium to determine the corresponding unknown reactions and applied load. If the reactions satisfy the constraints, then the applied load is either the maximum or minimum value of the allowable range. However, if the solution violates a constraint (for example, the force in a cable is compressive), the initial assumption is wrong and another loading condition must be assumed (for the previous example, you would assume the force in the cable is zero, the *minimum* allowed reaction). The solution process is then repeated for another possible *maximum* loading to complete the determination of the allowable range of values of the applied load.

As in Chap. 2, we strongly recommend you always write the equations of equilibrium in the same form that we have used in the preceding sample problems. That is, both the known and unknown quantities are placed on the left side of the equation, and their sum is set equal to zero.

Problems

4.1 The boom on a 4300-kg truck is used to unload a pallet of shingles of mass 1600 kg. Determine the reaction at each of the two (*a*) rear wheels *B*, (*b*) front wheels *C*.

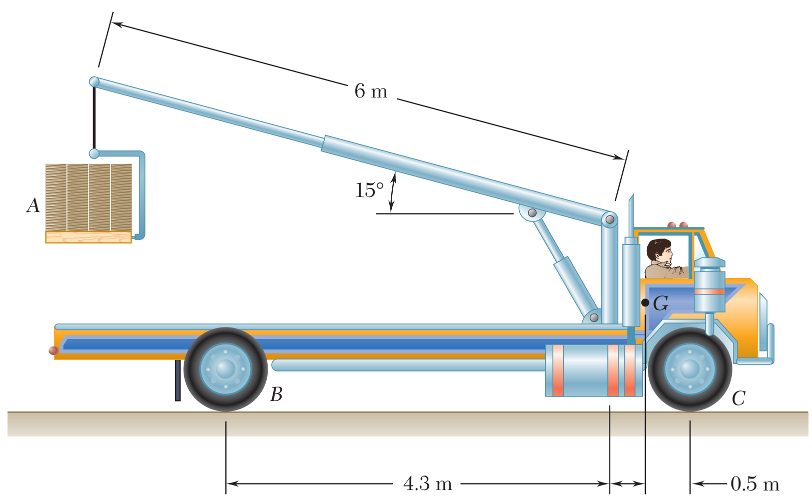


Fig. P4.1

4.2 Two children are standing on a diving board of mass 65 kg. Knowing that the masses of the children at *C* and *D* are 28 kg and 40 kg, respectively, determine (*a*) the reaction at *A*, (*b*) the reaction at *B*.

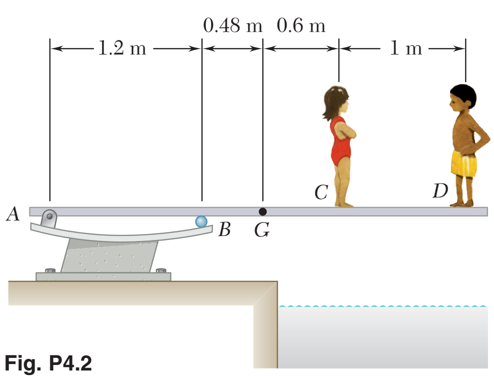


Fig. P4.2

4.3 Two crates, each weighing 250 lb, are placed as shown in the bed of a 3000-lb pickup truck. Determine the reactions at each of the two (*a*) rear wheels *A*, (*b*) front wheels *B*.

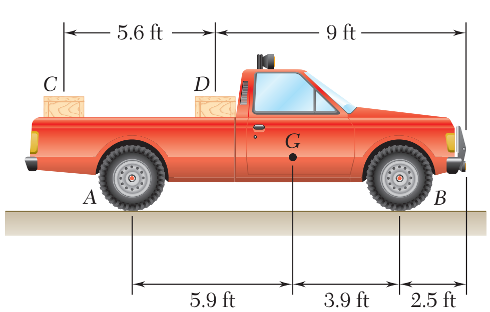


Fig. P4.3

4.4 Solve Prob. 4.3 assuming that crate *D* is removed and that the position of crate *C* is unchanged.

4.5 A T-shaped bracket supports the four loads shown. Determine the reactions at *A* and *B* if (*a*) $a = 100$ mm, (*b*) $a = 70$ mm.

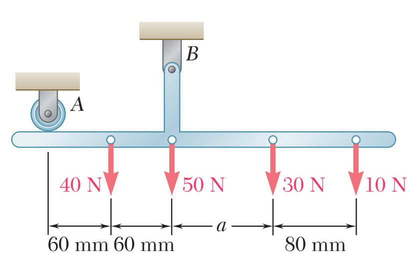


Fig. P4.5

4.6 For the bracket and loading of Prob. 4.5, determine the smallest distance *a* if the bracket is not to move.

4.7 A hand truck is used to move two barrels, each weighing 80 lb. Neglecting the weight of the hand truck, determine (*a*) the vertical force **P** which should be applied to the handle to maintain equilibrium when $\alpha = 35°$, (*b*) the corresponding reaction at each of the two wheels.

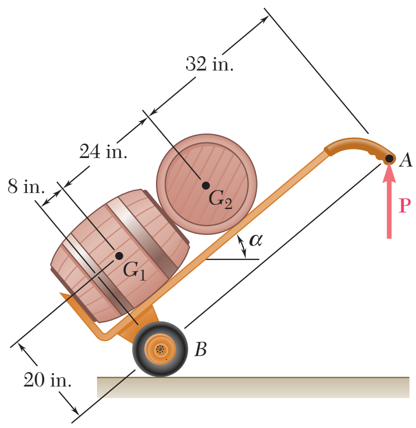


Fig. ***P4.7***

4.8 Solve Prob. 4.7 when $\alpha = 40°$.

4.9 Four boxes are placed on a uniform 14-kg wooden plank which rests on two sawhorses. Knowing that the masses of boxes *B* and *D* are 4.5 kg and 45 kg, respectively, determine the range of values of the mass of box *A* so that the plank remains in equilibrium when box *C* is removed.

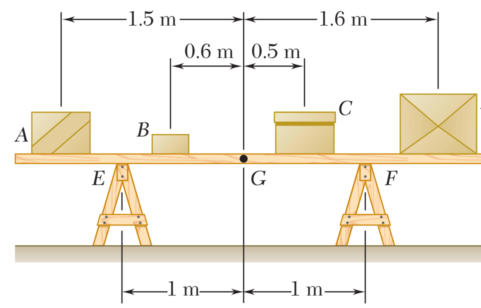


Fig. P4.9

4.10 A control rod is attached to a crank at *A* and cords are attached at *B* and *C*. For the given force in the rod, determine the range of values of the tension in the cord at *C* knowing that the cords must remain taut and that the maximum allowed tension in a cord is 180 N.

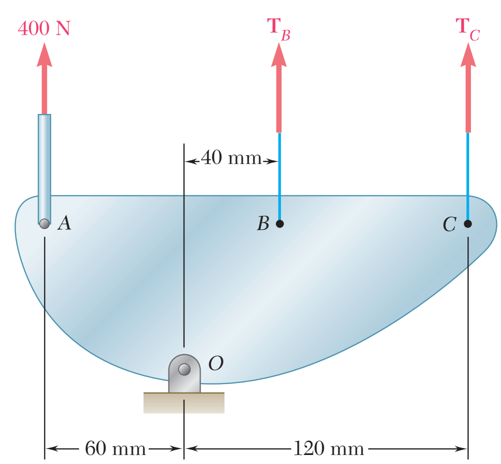


Fig. P4.10

4.11 The maximum allowable value of each of the reactions is 360 N. Neglecting the weight of the beam, determine the range of values of the distance *d* for which the beam is safe.

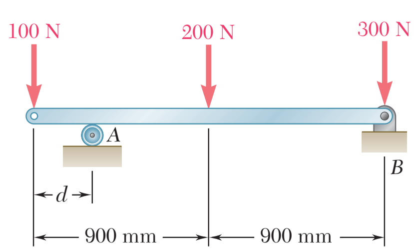


Fig. ***P4.11***

4.12 Solve Prob. 4.11 assuming that the 100-N load is replaced by a 160-N load.

4.13 For the beam of Sample Prob. 4.2, determine the range of values of *P* for which the beam will be safe knowing that the maximum allowable value of each of the reactions is 45 kips and that the reaction at *A* must be directed upward.

4.14 For the beam and loading shown, determine the range of values of the distance *a* for which the reaction at *B* does not exceed 50 lb downward or 100 lb upward.

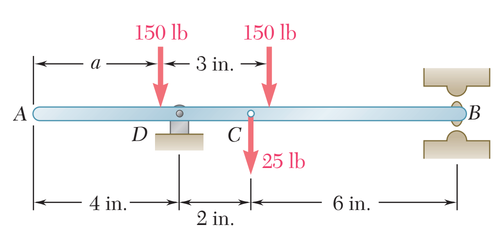


Fig. P4.14

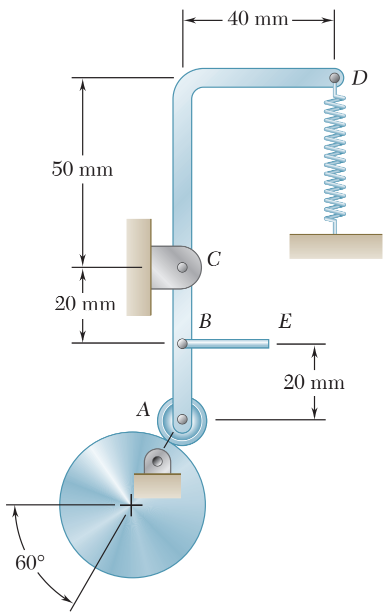


Fig. P4.15

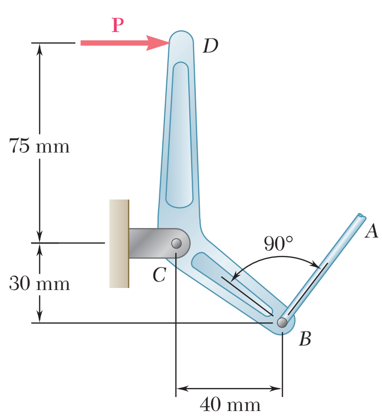


Fig. *P4.19* and *P4.20*

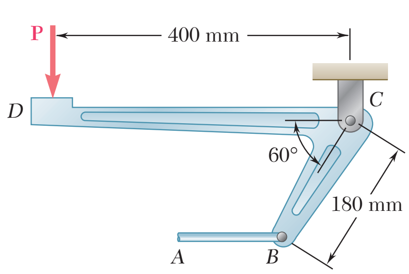


Fig. P4.21 and P4.22

4.15 A follower *ABCD* is held against a circular cam by a stretched spring, which exerts a force of 21 N for the position shown. Knowing that the tension in rod *BE* is 14 N, determine (*a*) the force exerted on the roller at *A*, (*b*) the reaction at bearing *C*.

4.16 A 6-m-long pole *AB* is placed in a hole and is guyed by three cables. Knowing that the tensions in cables *BD* and *BE* are 442 N and 322 N, respectively, determine (*a*) the tension in cable *CD*, (*b*) the reaction at *A*.

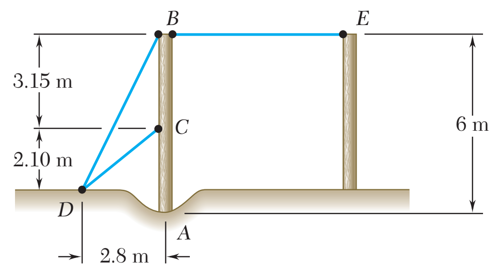


Fig. P4.16

4.17 Determine the reactions at *A* and *C* when (*a*) $\alpha = 0$, (*b*) $\alpha = 30°$.

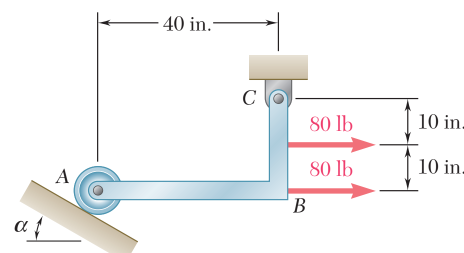


Fig. P4.17

4.18 Determine the reactions at *A* and *B* when (*a*) $h = 0$, (*b*) $h = 8$ in.

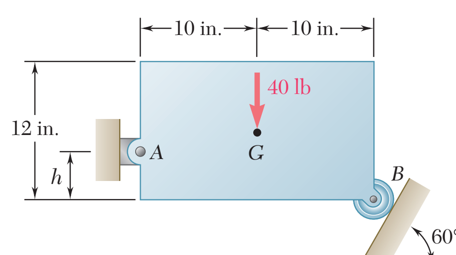


Fig. P4.18

4.19 The lever *BCD* is hinged at *C* and is attached to a control rod at *B*. If $P = 200$ N, determine (*a*) the tension in rod *AB*, (*b*) the reaction at *C*.

4.20 The lever *BCD* is hinged at *C* and is attached to a control rod at *B*. Determine the maximum force **P** which can be safely applied at *D* if the maximum allowable value of the reaction at *C* is 500 N.

4.21 The required tension in cable *AB* is 800 N. Determine (*a*) the vertical force **P** which must be applied to the pedal, (*b*) the corresponding reaction at *C*.

4.22 Determine the maximum tension which can be developed in cable *AB* if the maximum allowable value of the reaction at *C* is 1000 N.

4.23 and 4.24 A steel rod is bent to form a mounting bracket. For each of the mounting brackets and loadings shown, determine the reactions at *A* and *B*.

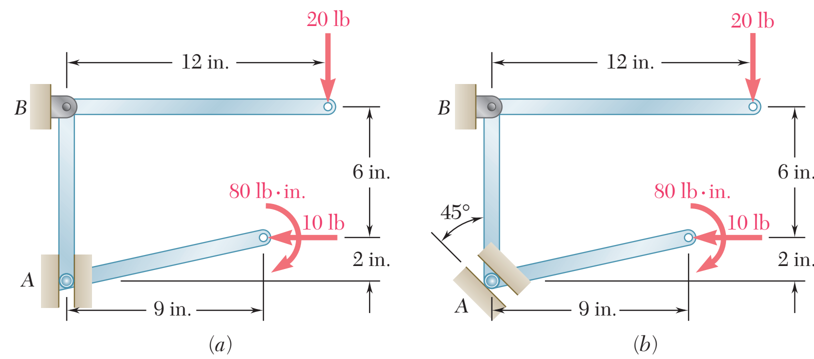


Fig. P4.23

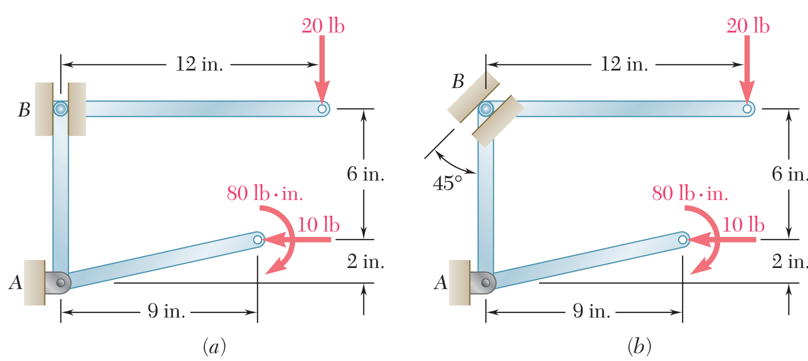


Fig. P4.24

4.25 A sign is hung by two chains from mast *AB*. The mast is hinged at *A* and is supported by cable *BD*. Knowing that the tensions in chains *DE* and *FH* are 50 lb and 30 lb, respectively, and that $d = 1.3$ ft, determine (*a*) the tension in cable *BC*, (*b*) the reaction at *A*.

4.26 A sign is hung by two chains from mast *AB*. The mast is hinged at *A* and is supported by cable *BD*. Knowing that the tensions in chains *DE* and *FH* are 30 lb and 20 lb, respectively, and that $d = 1.54$ ft, determine (*a*) the tension in cable *BC*, (*b*) the reaction at *A*.

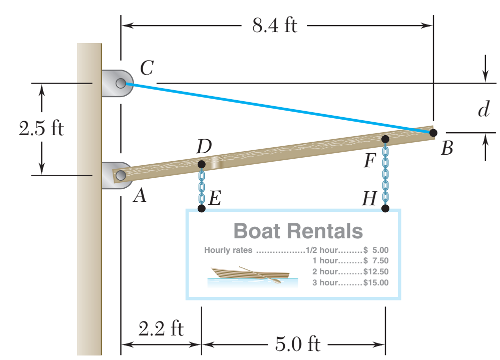


Fig. *P4.25* and *P4.26*

4.27 For the frame and loading shown, determine the reactions at *A* and *E* when (*a*) $\alpha = 30°$, (*b*) $\alpha = 45°$.

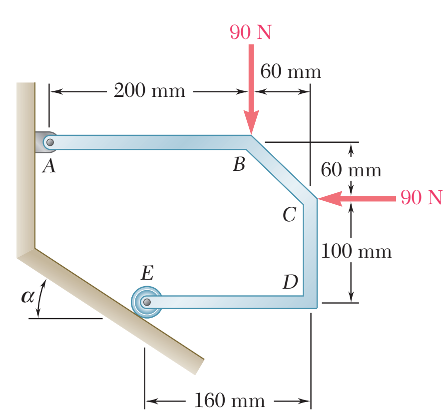


Fig. P4.27

4.28 A lever AB is hinged at C and is attached to a control cable at A. If the lever is subjected to a 300-N vertical force at B, determine (*a*) the tension in the cable, (*b*) the reaction at C.

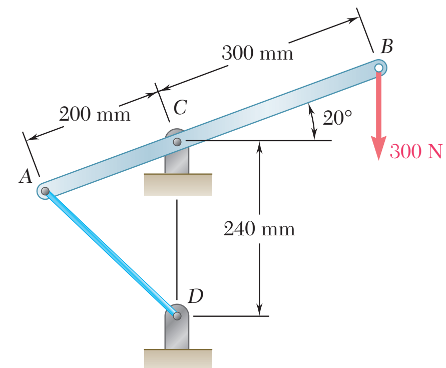


Fig. P4.28

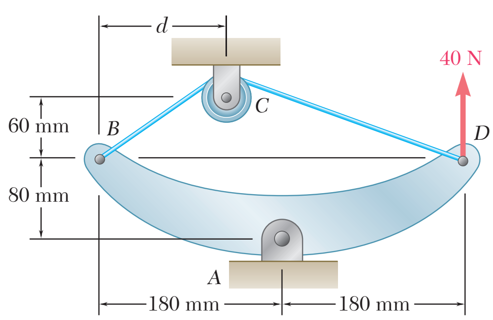


Fig. P4.29 and P4.30

4.29 Neglecting friction and the radius of the pulley, determine the tension in cable BCD and the reaction at support A when $d = 80$ mm.

4.30 Neglecting friction and the radius of the pulley, determine the tension in cable BCD and the reaction at support A when $d = 144$ mm.

4.31 Neglecting friction, determine the tension in cable ABD and the reaction at support C.

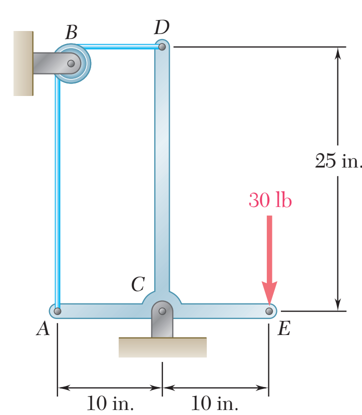


Fig. P4.31

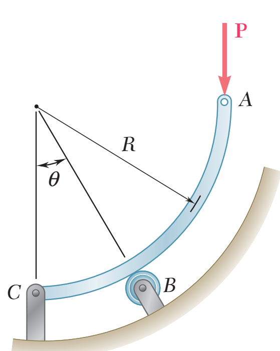


Fig. *P4.32* and P4.33

4.32 Rod ABC is bent in the shape of a circular arc of radius R. Knowing that $\theta = 35°$, determine the reaction (*a*) at B, (*b*) at C.

4.33 Rod ABC is bent in the shape of a circular arc of radius R. Knowing that $\theta = 50°$, determine the reaction (*a*) at B, (*b*) at C.

4.34 Neglecting friction and the radius of the pulley, determine (*a*) the tension in cable *ADB*, (*b*) the reaction at *C*.

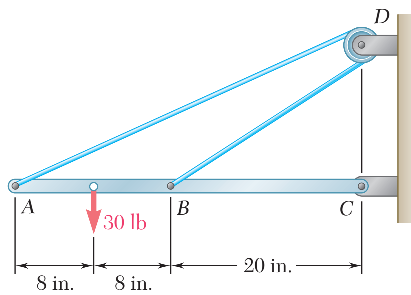


Fig. P4.34

4.35 Neglecting friction, determine the tension in cable *ABD* and the reaction at *C* when $\theta = 60°$.

4.36 Neglecting friction, determine the tension in cable *ABD* and the reaction at *C* when $\theta = 30°$.

4.37 Determine the tension in each cable and the reaction at *D*.

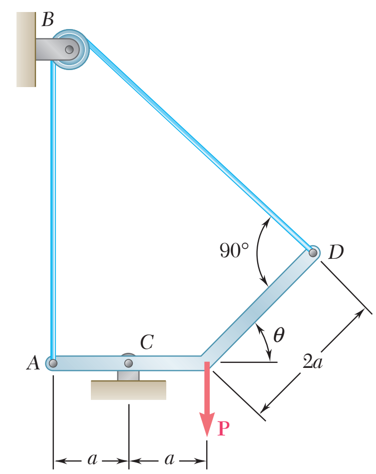


Fig. P4.35 and *P4.36*

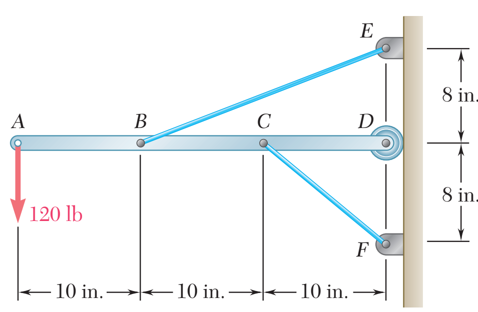


Fig. P4.37

4.38 Rod *ABCD* is bent in the shape of a circular arc of radius 80 mm and rests against frictionless surfaces at *A* and *D*. Knowing that the collar at *B* can move freely on the rod and that $\theta = 45°$, determine (*a*) the tension in cord *OB*, (*b*) the reactions at *A* and *D*.

4.39 Rod *ABCD* is bent in the shape of a circular arc of radius 80 mm and rests against frictionless surfaces at *A* and *D*. Knowing that the collar at *B* can move freely on the rod, determine (*a*) the value of θ for which the tension in cord *OB* is as small as possible, (*b*) the corresponding value of the tension, (*c*) the reactions at *A* and *D*.

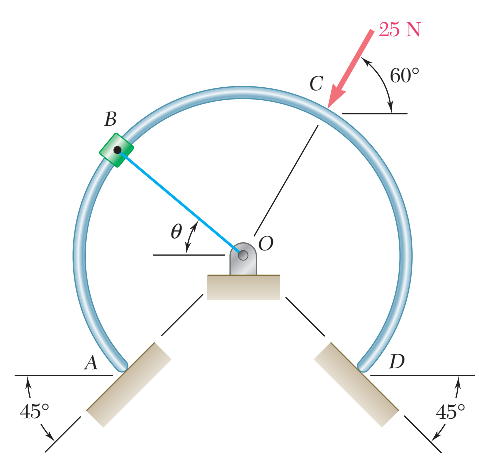


Fig. P4.38 and P4.39

4.40 Bar AC supports two 100-lb loads as shown. Rollers A and C rest against frictionless surfaces and a cable BD is attached at B. Determine (*a*) the tension in cable BD, (*b*) the reaction at A, (*c*) the reaction at C.

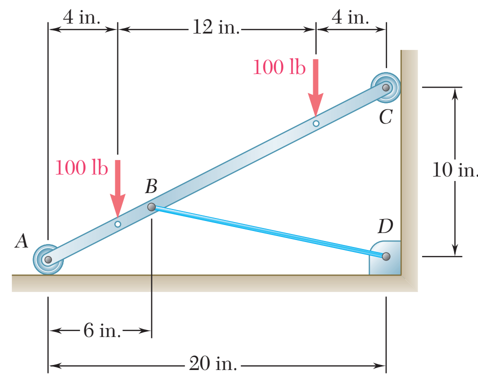


Fig. *P4.40*

4.41 A parabolic slot has been cut in plate AD, and the plate has been placed so that the slot fits two fixed, frictionless pins B and C. The equation of the slot is $y = x^2/100$, where x and y are expressed in mm. Knowing that the input force $P = 4$ N, determine (*a*) the force each pin exerts on the plate, (*b*) the output force $\mathbf{Q}$.

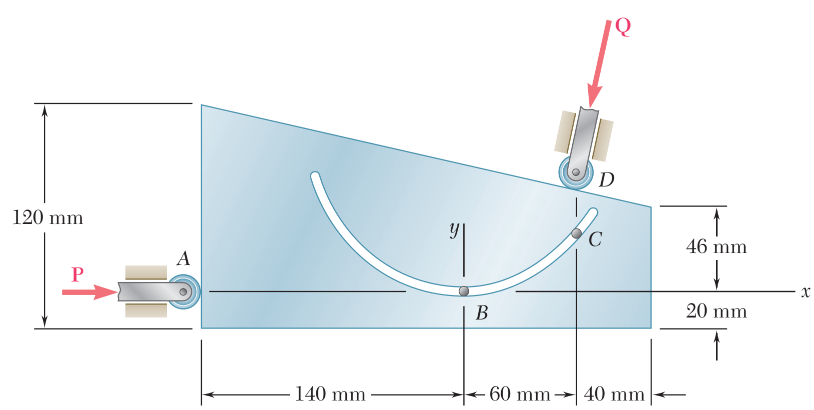


Fig. P4.41 and P4.42

4.42 A parabolic slot has been cut in plate AD, and the plate has been placed so that the slot fits two fixed, frictionless pins B and C. The equation of the slot is $y = x^2/100$, where x and y are expressed in mm. Knowing that the maximum allowable force exerted on the roller at D is 8.5 N, determine (*a*) the corresponding magnitude of the input force $\mathbf{P}$, (*b*) the force each pin exerts on the plate.

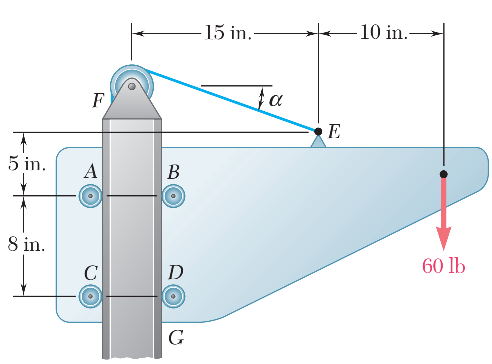


Fig. P4.43

4.43 A movable bracket is held at rest by a cable attached at E and by frictionless rollers. Knowing that the width of post FG is slightly less than the distance between the rollers, determine the force exerted on the post by each roller when $\alpha = 20°$.

4.44 Solve Prob. 4.43 when $\alpha = 30°$.

4.45 A 20-lb weight can be supported in the three different ways shown. Knowing that the pulleys have a 4-in. radius, determine the reaction at A in each case.

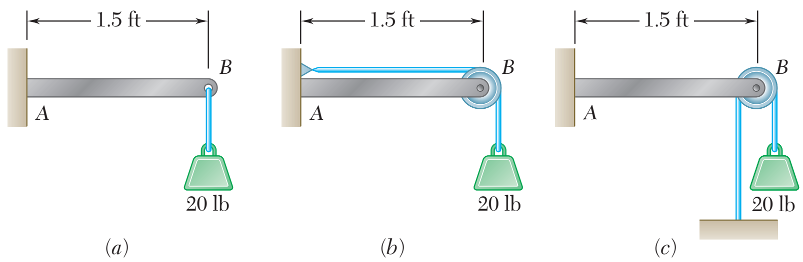


Fig. P4.45

4.46 A belt passes over two 50-mm-diameter pulleys which are mounted on a bracket as shown. Knowing that $M = 0$ and $T_i = T_o = 24$ N, determine the reaction at C.

4.47 A belt passes over two 50-mm-diameter pulleys which are mounted on a bracket as shown. Knowing that $M = 0.40$ N · m and that T_i and T_o are equal to 32 N and 16 N, respectively, determine the reaction at C.

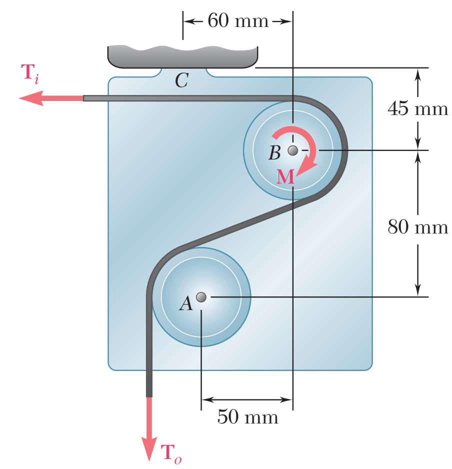


Fig. P4.46 and P4.47

4.48 A 350-lb utility pole is used to support at C the end of an electric wire. The tension in the wire is 120 lb, and the wire forms an angle of 15° with the horizontal at C. Determine the largest and smallest allowable tensions in the guy cable BD if the magnitude of the couple at A may not exceed 200 lb · ft.

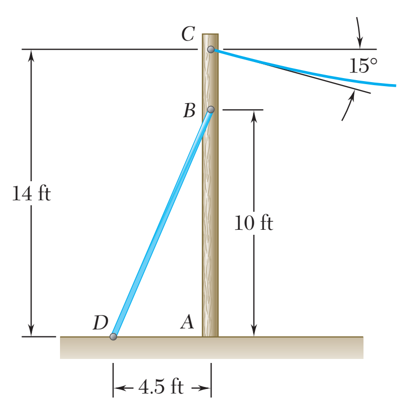


Fig. P4.48

4.49 In a laboratory experiment, students hang the masses shown from a beam of negligible mass. (a) Determine the reaction at the fixed support A knowing that end D of the beam does not touch support E. (b) Determine the reaction at the fixed support A knowing that the adjustable support E exerts an upward force of 6 N on the beam.

4.50 In a laboratory experiment, students hang the masses shown from a beam of negligible mass. Determine the range of values of the force exerted on the beam by the adjustable support E for which the magnitude of the couple at A does not exceed 2.5 N · m.

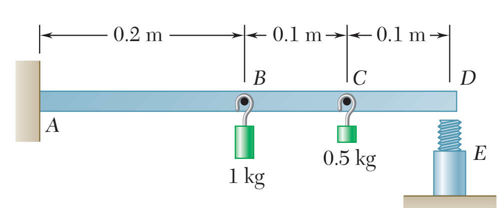


Fig. P4.49 and P4.50

4.51 Knowing that the tension in wire BD is 300 lb, determine the reaction at fixed support C for the frame shown.

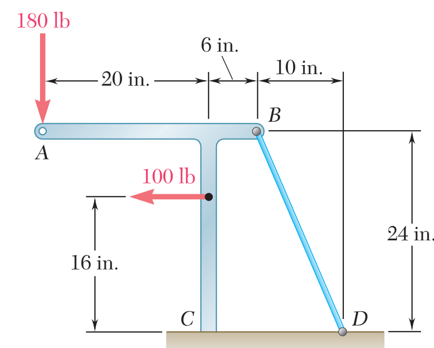


Fig. *P4.51* and *P4.52*

4.52 Determine the range of allowable values of the tension in wire BD if the magnitude of the couple at the fixed support C is not to exceed 75 lb · ft.

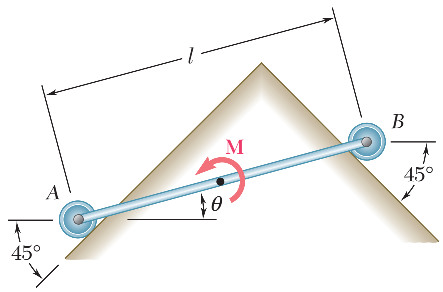


Fig. P4.53

4.53 Uniform rod AB of length l and weight W lies in a vertical plane and is acted upon by a couple **M**. The ends of the rod are connected to small rollers which rest against frictionless surfaces. (a) Express the angle θ corresponding to equilibrium in terms of M, W, and l. (b) Determine the value of θ corresponding to equilibrium when $M = 1.5$ lb · ft, $W = 4$ lb, and $l = 2$ ft.

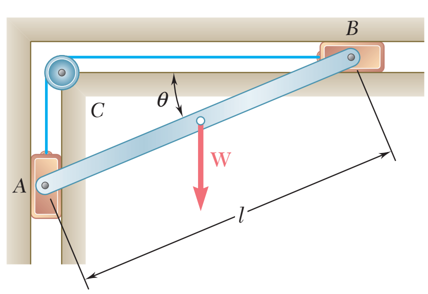


Fig. P4.54

4.54 A slender rod AB, of weight W, is attached to blocks A and B, which move freely in the guides shown. The blocks are connected by an elastic cord which passes over a pulley at C. (a) Express the tension in the cord in terms of W and θ. (b) Determine the value of θ for which the tension in the cord is equal to $3W$.

4.55 A thin, uniform ring of mass m and radius R is attached by a frictionless pin to a collar at A and rests against a small roller at B. The ring lies in a vertical plane, and the collar can move freely on a horizontal rod and is acted upon by a horizontal force **P**. (a) Express the angle θ corresponding to equilibrium in terms of m and P. (b) Determine the value of θ corresponding to equilibrium when $m = 500$ g and $P = 5$ N.

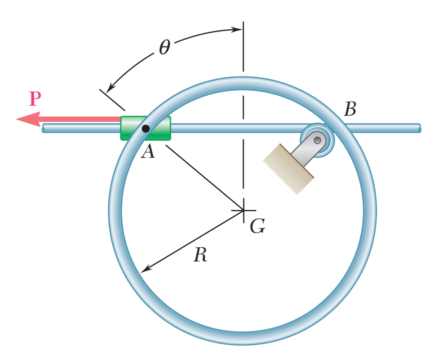


Fig. P4.55

4.56 Rod AB is acted upon by a couple **M** and two forces, each of magnitude P. (a) Derive an equation in θ, P, M, and l which must be satisfied when the rod is in equilibrium. (b) Determine the value of θ corresponding to equilibrium when $M = 150$ lb · in., $P = 20$ lb, and $l = 6$ in.

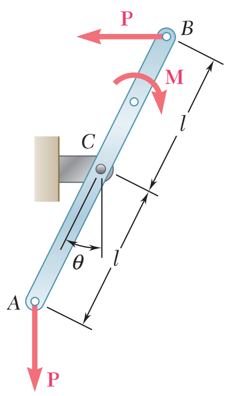


Fig. *P4.56*

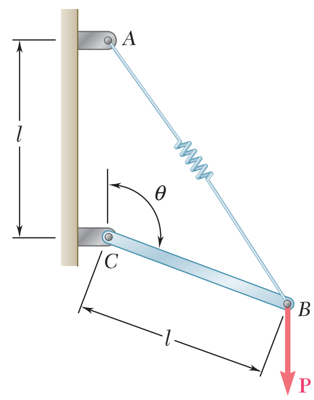


Fig. P4.57

4.57 A vertical load **P** is applied at end B of rod BC. The constant of the spring is k, and the spring is unstretched when $\theta = 90°$. (a) Neglecting the weight of the rod, express the angle θ corresponding to equilibrium in terms of P, k, and l. (b) Determine the value of θ corresponding to equilibrium when $P = \frac{1}{4}kl$.

4.58 Solve Sample Prob. 4.5 assuming that the spring is unstretched when $\theta = 90°$.

4.59 A collar B of weight W can move freely along the vertical rod shown. The constant of the spring is k, and the spring is unstretched when $\theta = 0$. (a) Derive an equation in θ, W, k, and l which must be satisfied when the collar is in equilibrium. (b) Knowing that $W = 3$ lb, $l = 6$ in., and $k = 8$ lb/ft, determine the value of θ corresponding to equilibrium.

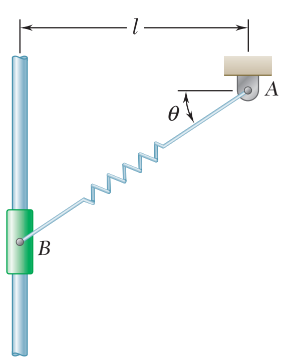


Fig. P4.59

4.60 A slender rod AB, of mass m, is attached to blocks A and B which move freely in the guides shown. The constant of the spring is k, and the spring is unstretched when $\theta = 0$. (a) Neglecting the mass of the blocks, derive an equation in m, g, k, l, and θ which must be satisfied when the rod is in equilibrium. (b) Determine the value of θ when $m = 2$ kg, $l = 750$ mm, and $k = 30$ N/m.

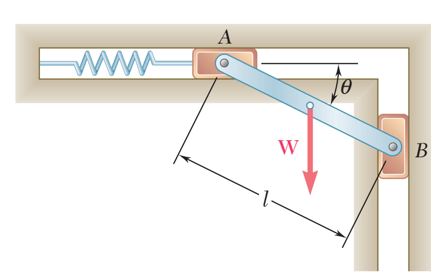


Fig. *P4.60*

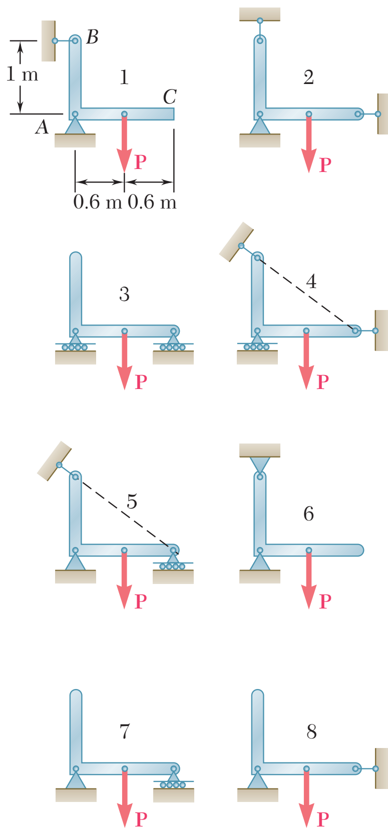


Fig. P4.61

4.61 The bracket *ABC* can be supported in the eight different ways shown. All connections consist of smooth pins, rollers, or short links. In each case, determine whether (*a*) the plate is completely, partially, or improperly constrained, (*b*) the reactions are statically determinate or indeterminate, (*c*) the equilibrium of the plate is maintained in the position shown. Also, wherever possible, compute the reactions assuming that the magnitude of the force **P** is 100 N.

4.62 Eight identical 20 × 30-in. rectangular plates, each weighing 50 lb, are held in a vertical plane as shown. All connections consist of frictionless pins, rollers, or short links. For each case, answer the questions listed in Prob. 4.61, and, wherever possible, compute the reactions.

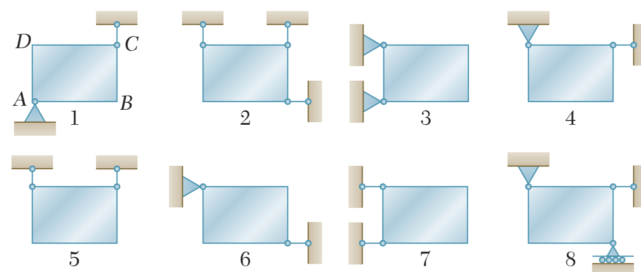


Fig. *P4.62*

4.6. EQUILIBRIUM OF A TWO-FORCE BODY

A particular case of equilibrium which is of considerable interest is that of a rigid body subjected to two forces. Such a body is commonly called a *two-force body.* It will be shown that *if a two-force body is in equilibrium, the two forces must have the same magnitude, the same line of action, and opposite sense.*

Consider a corner plate subjected to two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ acting at *A* and *B*, respectively (Fig. 4.8*a*). If the plate is to be in equilibrium, the sum of the moments of $\mathbf{F}_1$ and $\mathbf{F}_2$ about any point must be zero. First, we sum moments about *A*. Since the moment of $\mathbf{F}_1$ is obviously zero, the moment of $\mathbf{F}_2$ must also be zero and the line of action of $\mathbf{F}_2$ must pass through *A* (Fig. 4.8*b*). Summing moments about *B*, we prove similarly that the line of action of $\mathbf{F}_1$ must pass through *B* (Fig. 4.8*c*). Therefore, both forces have the same line of

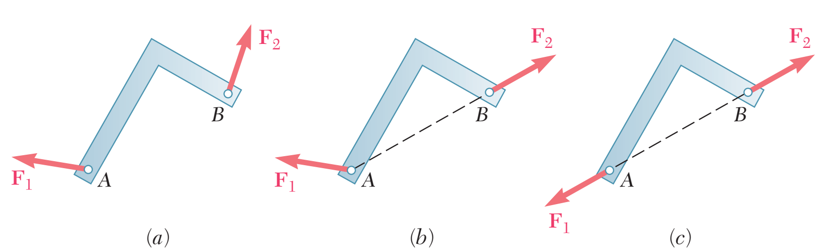


Fig. 4.8

action (line AB). From either of the equations $\Sigma F_x = 0$ and $\Sigma F_y = 0$ it is seen that they must also have the same magnitude but opposite sense.

If several forces act at two points A and B, the forces acting at A can be replaced by their resultant $\mathbf{F}_1$ and those acting at B can be replaced by their resultant $\mathbf{F}_2$. Thus a two-force body can be more generally defined as *a rigid body subjected to forces acting at only two points.* The resultants $\mathbf{F}_1$ and $\mathbf{F}_2$ then must have the same line of action, the same magnitude, and opposite sense (Fig. 4.8).

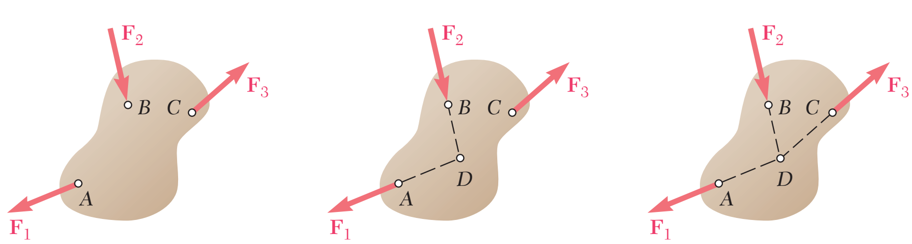
In the study of structures, frames, and machines, you will see how the recognition of two-force bodies simplifies the solution of certain problems.

4.7. EQUILIBRIUM OF A THREE-FORCE BODY

Another case of equilibrium that is of great interest is that of a *three-force body,* that is, a rigid body subjected to three forces or, more generally, *a rigid body subjected to forces acting at only three points.* Consider a rigid body subjected to a system of forces which can be reduced to three forces $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ acting at A, B, and C, respectively (Fig. 4.9*a*). It will be shown that if the body is in equilibrium, *the lines of action of the three forces must be either concurrent or parallel.*

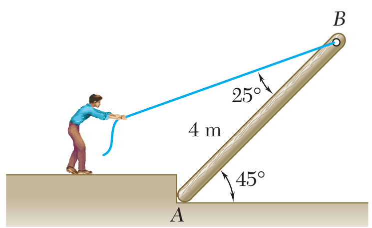
Since the rigid body is in equilibrium, the sum of the moments of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ about any point must be zero. Assuming that the lines of action of $\mathbf{F}_1$ and $\mathbf{F}_2$ intersect and denoting their point of intersection by D, we sum moments about D (Fig. 4.9*b*). Since the moments of $\mathbf{F}_1$ and $\mathbf{F}_2$ about D are zero, the moment of $\mathbf{F}_3$ about D must also be zero, and the line of action of $\mathbf{F}_3$ must pass through D (Fig. 4.9*c*). Therefore, the three lines of action are concurrent. The only exception occurs when none of the lines intersect; the lines of action are then parallel.

Although problems concerning three-force bodies can be solved by the general methods of Secs. 4.3 to 4.5, the property just established can be used to solve them either graphically or mathematically from simple trigonometric or geometric relations.



(*a*) (*b*) (*c*)

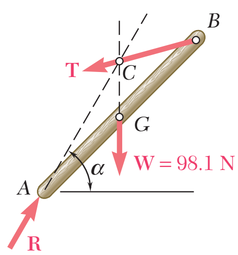
Fig. 4.9



SAMPLE PROBLEM 4.6

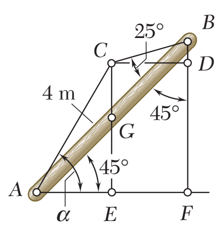
A man raises a 10-kg joist, of length 4 m, by pulling on a rope. Find the tension T in the rope and the reaction at A.

SOLUTION



Free-Body Diagram. The joist is a three-force body, since it is acted upon by three forces: its weight **W**, the force **T** exerted by the rope, and the reaction **R** of the ground at A. We note that

$$W = mg = (10 \text{ kg})(9.81 \text{ m/s}^2) = 98.1 \text{ N}$$



Three-Force Body. Since the joist is a three-force body, the forces acting on it must be concurrent. The reaction **R**, therefore, will pass through the point of intersection C of the lines of action of the weight **W** and the tension force **T**. This fact will be used to determine the angle α that **R** forms with the horizontal.

Drawing the vertical BF through B and the horizontal CD through C, we note that

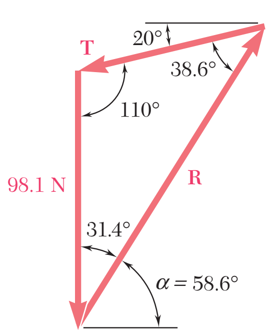
$$\begin{aligned}
AF &= BF = (AB)\cos 45° = (4 \text{ m})\cos 45° = 2.828 \text{ m} \\
CD &= EF = AE = \tfrac{1}{2}(AF) = 1.414 \text{ m} \\
BD &= (CD)\cot(45° + 25°) = (1.414 \text{ m})\tan 20° = 0.515 \text{ m} \\
CE &= DF = BF - BD = 2.828 \text{ m} - 0.515 \text{ m} = 2.313 \text{ m}
\end{aligned}$$

We write

$$\tan\alpha = \frac{CE}{AE} = \frac{2.313 \text{ m}}{1.414 \text{ m}} = 1.636$$

$$\alpha = 58.6° \quad \blacktriangleleft$$

We now know the direction of all the forces acting on the joist.



Force Triangle. A force triangle is drawn as shown, and its interior angles are computed from the known directions of the forces. Using the law of sines, we write

$$\frac{T}{\sin 31.4°} = \frac{R}{\sin 110°} = \frac{98.1 \text{ N}}{\sin 38.6°}$$

$$T = 81.9 \text{ N} \quad \blacktriangleleft$$
$$\mathbf{R} = 147.8 \text{ N} \measuredangle 58.6° \quad \blacktriangleleft$$

SOLVING PROBLEMS ON YOUR OWN

The preceding sections covered two particular cases of equilibrium of a rigid body.

1. A two-force body is a body subjected to forces at only two points. The resultants of the forces acting at each of these points must have the *same magnitude, the same line of action, and opposite sense.* This property will allow you to simplify the solutions of some problems by replacing the two unknown components of a reaction by a single force of unknown magnitude but of *known direction*.

2. A three-force body is subjected to forces at only three points. The resultants of the forces acting at each of these points must be *concurrent or parallel.* To solve a problem involving a three-force body with concurrent forces, draw your free-body diagram showing that the lines of action of these three forces pass through the same point. The use of geometry will then allow you to complete the solution using a force triangle [Sample Prob. 4.6].

Although the principle noted above for the solution of problems involving three-force bodies is easily understood, it can be difficult to determine the needed geometric constructions. If you encounter difficulty, first draw a reasonably large free-body diagram and then seek a relation between known or easily calculated lengths and a dimension that involves an unknown. This was done in Sample Prob. 4.6, where the easily calculated dimensions AE and CE were used to determine the angle α.

Problems

4.63 Horizontal and vertical links are hinged to a wheel, and forces are applied to the links as shown. Knowing that $a = 3.0$ in., determine the value of P and the reaction at A.

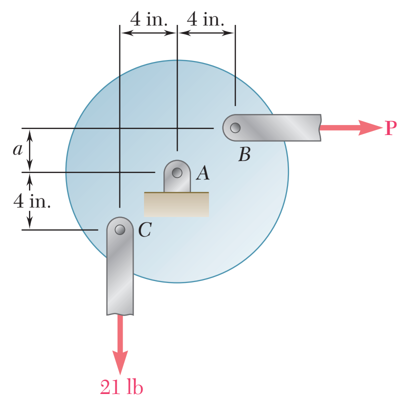


Fig. P4.63 and P4.64

4.64 Horizontal and vertical links are hinged to a wheel, and forces are applied to the links as shown. Determine the range of values of the distance a for which the magnitude of the reaction at A does not exceed 42 lb.

4.65 Using the method of Sec. 4.7, solve Prob. 4.21.

4.66 Using the method of Sec. 4.7, solve Prob. 4.22.

4.67 To remove a nail, a small block of wood is placed under a crowbar, and a horizontal force **P** is applied as shown. Knowing that $l = 3.5$ in. and $P = 30$ lb, determine the vertical force exerted on the nail and the reaction at B.

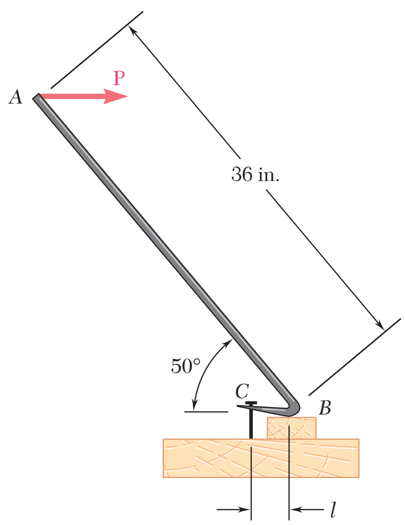


Fig. *P4.67* and *P4.68*

4.68 To remove a nail, a small block of wood is placed under a crowbar, and a horizontal force **P** is applied as shown. Knowing that the maximum vertical force needed to extract the nail is 600 lb and that the horizontal force **P** is not to exceed 65 lb, determine the largest acceptable value of distance l.

4.69 For the frame and loading shown, determine the reactions at C and D.

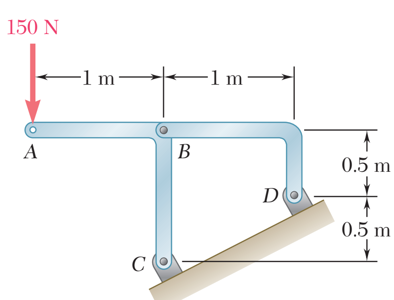


Fig. P4.69

4.70 For the frame and loading shown, determine the reactions at A and C.

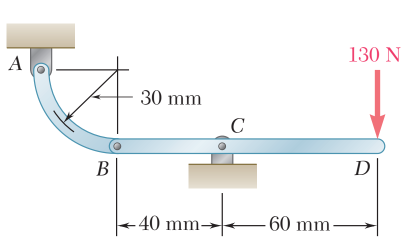


Fig. P4.70

4.71 To remove the lid from a 5-gallon pail, the tool shown is used to apply an upward and radially outward force to the bottom inside rim of the lid. Assuming that the rim rests against the tool at *A* and that a 100-N force is applied as indicated to the handle, determine the force acting on the rim.

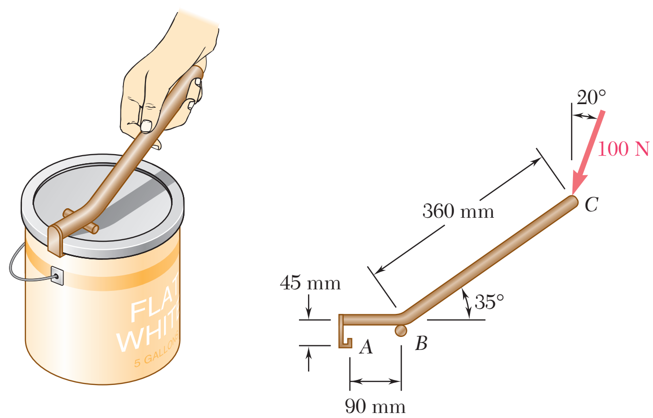


Fig. P4.71

4.72 To remove the lid from a 5-gallon pail, the tool shown is used to apply an upward and radially outward force to the bottom inside rim of the lid. Assuming that the top and the rim of the lid rest against the tool at *A* and *B*, respectively, and that a 60-N force is applied as indicated to the handle, determine the force acting on the rim.

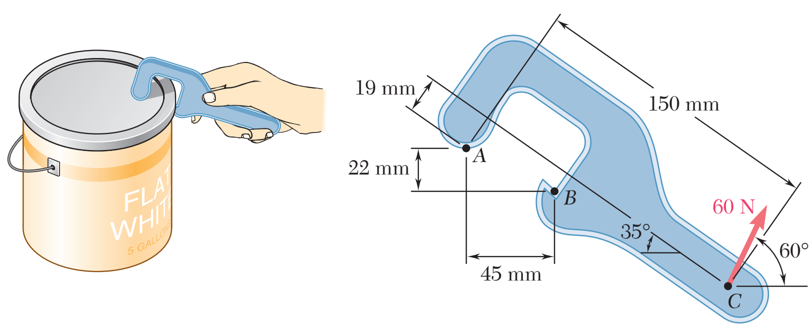


Fig. P4.72

4.73 A 200-lb crate is attached to the trolley-beam system shown. Knowing that $a = 1.5$ ft, determine (*a*) the tension in cable *CD*, (*b*) the reaction at *B*.

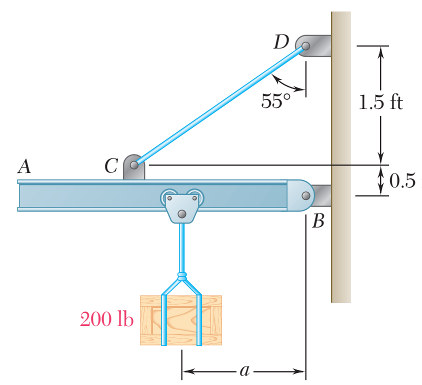


Fig. P4.73

4.74 Solve Prob. 4.73 assuming that $a = 3$ ft.

4.75 A 20-kg roller, of diameter 200 mm, which is to be used on a tile floor, is resting directly on the subflooring as shown. Knowing that the thickness of each tile is 8 mm, determine the force **P** required to move the roller onto the tiles if the roller is pushed to the left.

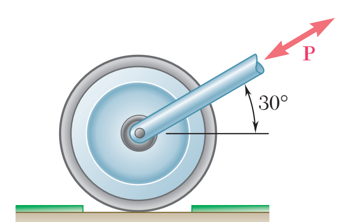


Fig. *P4.75* and P4.76

4.76 A 20-kg roller, of diameter 200 mm, which is to be used on a tile floor, is resting directly on the subflooring as shown. Knowing that the thickness of each tile is 8 mm, determine the force **P** required to move the roller onto the tiles if the roller is pulled to the right.

4.77 A small hoist is mounted on the back of a pickup truck and is used to lift a 120-kg crate. Determine (*a*) the force exerted on the hoist by the hydraulic cylinder *BC*, (*b*) the reaction at *A*.

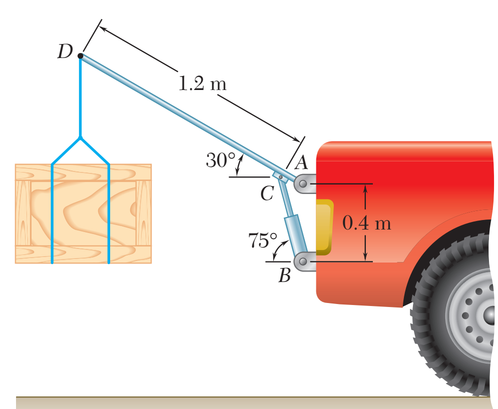


Fig. P4.77

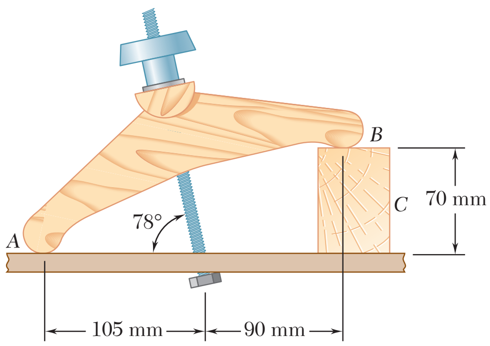


Fig. ***P4.78***

4.78 The clamp shown is used to hold the rough workpiece C. Knowing that the maximum allowable compressive force on the workpiece is 200 N and neglecting the effect of friction at A, determine the corresponding (a) reaction at B, (b) reaction at A, (c) tension in the bolt.

4.79 A modified peavey is used to lift a 0.2-m-diameter log of mass 36 kg. Knowing that $\theta = 45°$ and that the force exerted at C by the worker is perpendicular to the handle of the peavey, determine (a) the force exerted at C, (b) the reaction at A.

****4.80*** A modified peavey is used to lift a 0.2-m-diameter log of mass 36 kg. Knowing that $\theta = 60°$ and that the force exerted at C by the worker is perpendicular to the handle of the peavey, determine (a) the force exerted at C, (b) the reaction at A.

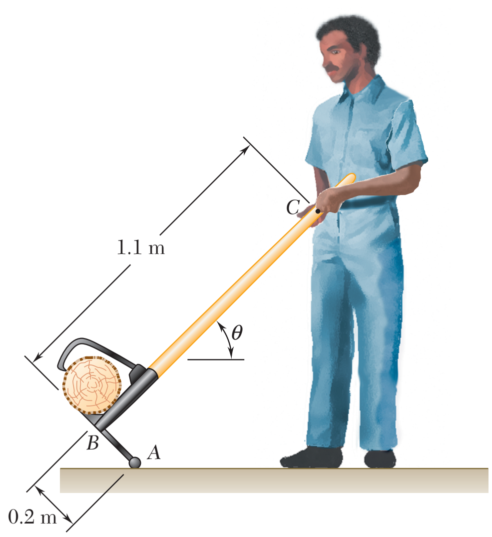


Fig. P4.79 and ***P4.80***

4.81 Member ABC is supported by a pin and bracket at B and by an inextensible cord attached at A and C and passing over a frictionless pulley at D. The tension may be assumed to be the same in portion AD and CD of the cord. For the loading shown and neglecting the size of the pulley, determine the tension in the cord and the reaction at B.

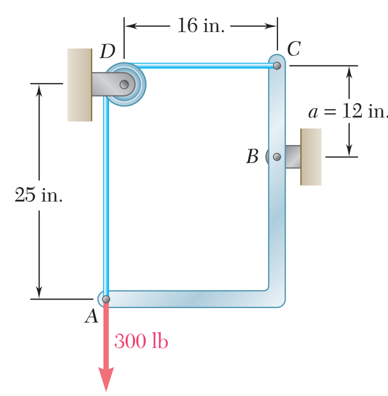


Fig. P4.81

4.82 Member $ABCD$ is supported by a pin and bracket at C and by an inextensible cord attached at A and D and passing over frictionless pulleys at B and E. Neglecting the size of the pulleys, determine the tension in the cord and the reaction at C.

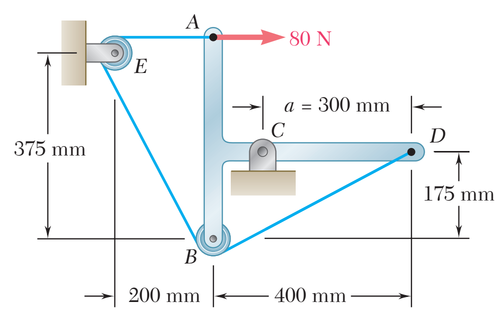


Fig. P4.82

4.83 Using the method of Sec. 4.7, solve Prob. 4.18.

4.84 Using the method of Sec. 4.7, solve Prob. 4.28.

4.85 Knowing that $\theta = 35°$, determine the reaction (a) at B, (b) at C.

4.86 Knowing that $\theta = 50°$, determine the reaction (a) at B, (b) at C.

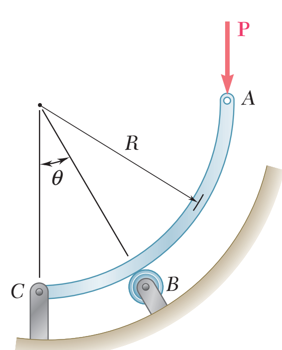


Fig. P4.85 and P4.86

4.87 A slender rod of length L and weight W is held in equilibrium as shown, with one end against a frictionless wall and the other end attached to a cord of length S. Derive an expression for the distance h in terms of L and S. Show that this position of equilibrium does not exist if $S > 2L$.

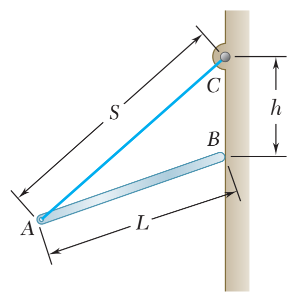


Fig. P4.87 and P4.88

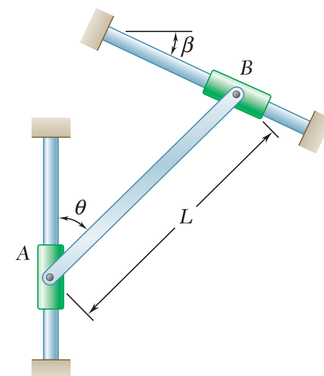


Fig. *P4.89* and P4.90

4.88 A slender rod of length $L = 200$ mm is held in equilibrium as shown, with one end against a frictionless wall and the other end attached to a cord of length $S = 300$ mm. Knowing that the mass of the rod is 1.5 kg, determine (*a*) the distance h, (*b*) the tension in the cord, (*c*) the reaction at B.

4.89 A slender rod of length L and weight W is attached to collars which can slide freely along the guides shown. Knowing that the rod is in equilibrium, derive an expression for the angle θ in terms of the angle β.

4.90 A 10-kg slender rod of length L is attached to collars which can slide freely along the guides shown. Knowing that the rod is in equilibrium and that $\beta = 25°$, determine (*a*) the angle θ that the rod forms with the vertical, (*b*) the reactions at A and B.

4.91 A uniform slender rod of mass 5 g and length 250 mm is balanced on a glass of inner diameter 70 mm. Neglecting friction, determine the angle θ corresponding to equilibrium.

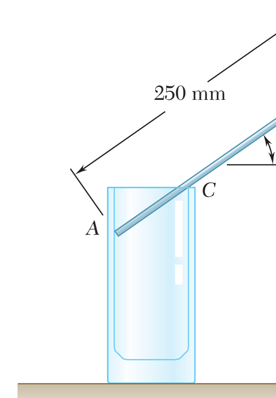


Fig. P4.91

4.92 Rod AB is bent into the shape of a circular arc and is lodged between two pegs D and E. It supports a load **P** at end B. Neglecting friction and the weight of the rod, determine the distance c corresponding to equilibrium when $a = 1$ in. and $R = 5$ in.

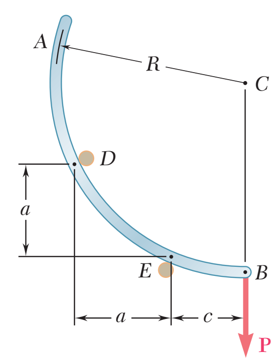


Fig. *P4.92*

4.93 A uniform rod AB of weight W and length $2L$ rests inside a hemispherical bowl of radius R as shown. Neglecting friction, determine the angle θ corresponding to equilibrium.

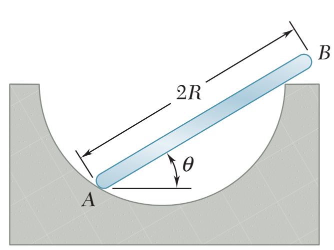


Fig. *P4.93*

4.94 A uniform slender rod of mass m and length $4r$ rests on the surface shown and is held in the given equilibrium position by the force **P**. Neglecting the effect of friction at A and C, (*a*) determine the angle θ, (*b*) derive an expression for P in terms of m.

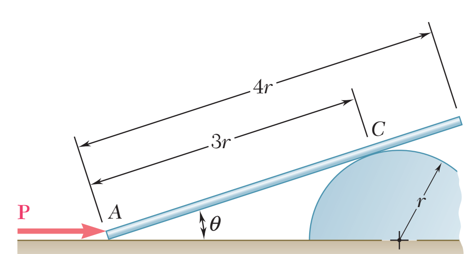


Fig. P4.94

4.95 A uniform slender rod of length $2L$ and mass m rests against a roller at D and is held in the equilibrium position shown by a cord of length a. Knowing that $L = 200$ mm, determine (*a*) the angle θ, (*b*) the length a.

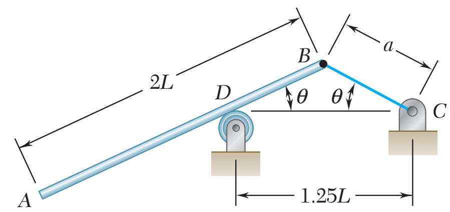


Fig. P4.95

EQUILIBRIUM IN THREE DIMENSIONS

4.8. EQUILIBRIUM OF A RIGID BODY IN THREE DIMENSIONS

We saw in Sec. 4.1 that six scalar equations are required to express the conditions for the equilibrium of a rigid body in the general three-dimensional case:

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma F_z = 0 \tag{4.2}$$
$$\Sigma M_x = 0 \qquad \Sigma M_y = 0 \qquad \Sigma M_z = 0 \tag{4.3}$$

These equations can be solved for no more than *six unknowns,* which generally will represent reactions at supports or connections.

In most problems the scalar equations (4.2) and (4.3) will be more conveniently obtained if we first express in vector form the conditions for the equilibrium of the rigid body considered. We write

$$\Sigma \mathbf{F} = 0 \qquad \Sigma \mathbf{M}_O = \Sigma(\mathbf{r} \times \mathbf{F}) = 0 \tag{4.1}$$

and express the forces $\mathbf{F}$ and position vectors $\mathbf{r}$ in terms of scalar components and unit vectors. Next, we compute all vector products, either by direct calculation or by means of determinants (see Sec. 3.8). We observe that as many as three unknown reaction components may be eliminated from these computations through a judicious choice of the point O. By equating to zero the coefficients of the unit vectors in each of the two relations (4.1), we obtain the desired scalar equations.†

4.9. REACTIONS AT SUPPORTS AND CONNECTIONS FOR A THREE-DIMENSIONAL STRUCTURE

The reactions on a three-dimensional structure range from the single force of known direction exerted by a frictionless surface to the force-couple system exerted by a fixed support. Consequently, in problems involving the equilibrium of a three-dimensional structure, there can be between one and six unknowns associated with the reaction at each

†In some problems, it will be found convenient to eliminate the reactions at two points A and B from the solution by writing the equilibrium equation $\Sigma M_{AB} = 0$, which involves the determination of the moments of the forces about the axis AB joining points A and B (see Sample Prob. 4.10).

support or connection. Various types of supports and connections are shown in Fig. 4.10 with their corresponding reactions. A simple way of determining the type of reaction corresponding to a given support or connection and the number of unknowns involved is to find which of the six fundamental motions (translation in the x, y, and z directions, rotation about the x, y, and z axes) are allowed and which motions are prevented.

Ball supports, frictionless surfaces, and cables, for example, prevent translation in one direction only and thus exert a single force whose line of action is known; each of these supports involves one unknown, namely, the magnitude of the reaction. Rollers on rough surfaces and wheels on rails prevent translation in two directions; the corresponding reactions consist of two unknown force components. Rough surfaces in direct contact and ball-and-socket supports prevent translation in three directions; these supports involve three unknown force components.

Some supports and connections can prevent rotation as well as translation; the corresponding reactions include couples as well as forces. For example, the reaction at a fixed support, which prevents any motion (rotation as well as translation), consists of three unknown forces and three unknown couples. A universal joint, which is designed to allow rotation about two axes, will exert a reaction consisting of three unknown force components and one unknown couple.

Photo 4.6 Universal joints, easily seen on the drive shafts of rear-wheel-drive cars and trucks, allow rotational motion to be transferred between two non-collinear shafts.

Other supports and connections are primarily intended to prevent translation; their design, however, is such that they also prevent some rotations. The corresponding reactions consist essentially of force components but *may* also include couples. One group of supports of this type includes hinges and bearings designed to support radial loads only (for example, journal bearings, roller bearings). The corresponding reactions consist of two force components but may also include two couples. Another group includes pin-and-bracket supports, hinges, and bearings designed to support an axial thrust as well as a radial load (for example, ball bearings). The corresponding reactions consist of three force components but may include two couples. However, these supports will not exert any appreciable couples under normal conditions of use. Therefore, *only* force components should be included in their analysis *unless* it is found that couples are necessary to maintain the equilibrium of the rigid body, or unless the support is known to have been specifically designed to exert a couple (see Probs. 4.128 through 4.131).

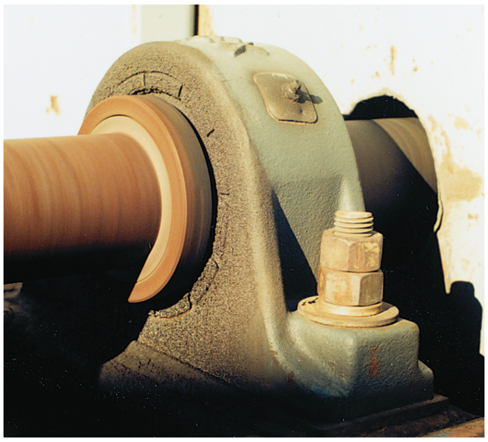


Photo 4.7 The pillow block bearing shown supports the shaft of a fan used to ventilate a foundry.

If the reactions involve more than six unknowns, there are more unknowns than equations, and some of the reactions are *statically indeterminate*. If the reactions involve fewer than six unknowns, there are more equations than unknowns, and some of the equations of equilibrium cannot be satisfied under general loading conditions; the rigid body is only *partially constrained.* Under the particular loading conditions corresponding to a given problem, however, the extra equations often reduce to trivial identities, such as $0 = 0$, and can be disregarded; although only partially constrained, the rigid body remains in equilibrium (see Sample Probs. 4.7 and 4.8). Even with six or more unknowns, it is possible that some equations of equilibrium will not be satisfied. This can occur when the reactions associated with the given supports either are parallel or intersect the same line; the rigid body is then *improperly constrained.*

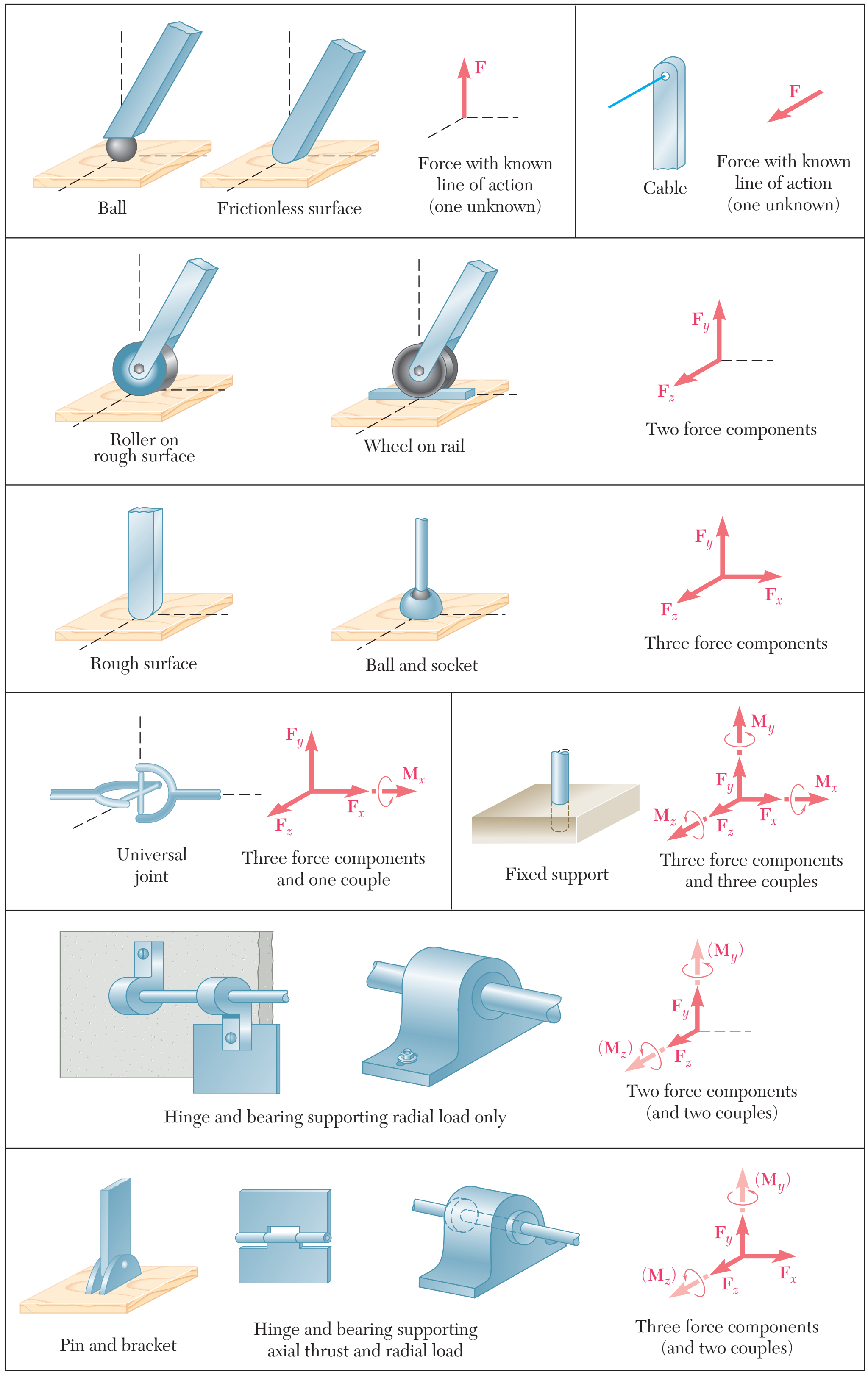
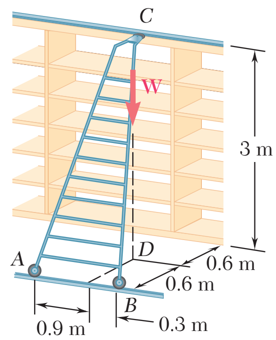
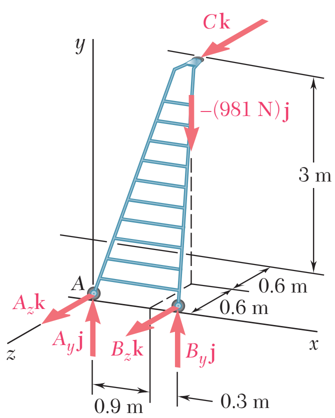


Fig. 4.10 Reactions at supports and connections.



SAMPLE PROBLEM 4.7

A 20-kg ladder used to reach high shelves in a storeroom is supported by two flanged wheels A and B mounted on a rail and by an unflanged wheel C resting against a rail fixed to the wall. An 80-kg man stands on the ladder and leans to the right. The line of action of the combined weight **W** of the man and ladder intersects the floor at point D. Determine the reactions at A, B, and C.



SOLUTION

Free-Body Diagram. A free-body diagram of the ladder is drawn. The forces involved are the combined weight of the man and ladder.

$$\mathbf{W} = -mg\mathbf{j} = -(80\text{ kg} + 20\text{ kg})(9.81\text{ m/s}^2)\mathbf{j} = -(981\text{ N})\mathbf{j}$$

and five unknown reaction components, two at each flanged wheel and one at the unflanged wheel. The ladder is thus only partially constrained; it is free to roll along the rails. It is, however, in equilibrium under the given load since the equation $\Sigma F_x = 0$ is satisfied.

Equilibrium Equations. We express that the forces acting on the ladder form a system equivalent to zero:

$$\Sigma\mathbf{F} = 0{:}\qquad A_y\mathbf{j} + A_z\mathbf{k} + B_y\mathbf{j} + B_z\mathbf{k} - (981\text{ N})\mathbf{j} + C\mathbf{k} = 0$$
$$(A_y + B_y - 981\text{ N})\mathbf{j} + (A_z + B_z + C)\mathbf{k} = 0$$

$$\Sigma\mathbf{M}_A = \Sigma(\mathbf{r}\times\mathbf{F}) = 0{:}\ 1.2\mathbf{i}\times(B_y\mathbf{j} + B_z\mathbf{k}) + (0.9\mathbf{i} - 0.6\mathbf{k})\times(-981\mathbf{j})$$
$$+ (0.6\mathbf{i} + 3\mathbf{j} - 1.2\mathbf{k})\times C\mathbf{k} = 0$$

Computing the vector product, we have†

$$1.2B_y\mathbf{k} - 1.2B_z\mathbf{j} - 882.9\mathbf{k} - 588.6\mathbf{i} - 0.6C\mathbf{j} + 3C\mathbf{i} = 0$$
$$(3C - 588.6)\mathbf{i} - (1.2B_z + 0.6C)\mathbf{j} + (1.2B_y - 882.9)\mathbf{k} = 0 \qquad (2)$$

Setting the coefficients of **i**, **j**, **k** equal to zero in Eq. (2), we obtain the following three scalar equations, which express that the sum of the moments about each coordinate axis must be zero:

$$3C - 588.6 = 0 \qquad C = +196.2\text{ N}$$
$$1.2B_z + 0.6C = 0 \qquad B_z = -98.1\text{ N}$$
$$1.2B_y - 882.9 = 0 \qquad B_y = +736\text{ N}$$

The reactions at B and C are therefore

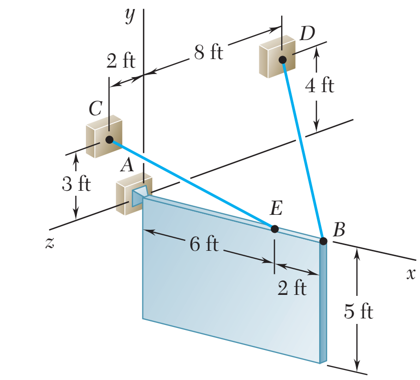
$$\mathbf{B} = +(736\text{ N})\mathbf{j} - (98.1\text{ N})\mathbf{k} \qquad \mathbf{C} = +(196.2\text{ N})\mathbf{k} \blacktriangleleft$$

Setting the coefficients of **j** and **k** equal to zero in Eq. (1), we obtain two scalar equations expressing that the sums of the components in the y and z directions are zero. Substituting for B_y, B_z, and C the values obtained above, we write

$$A_y + B_y - 981 = 0 \qquad A_y + 736 - 981 = 0 \qquad A_y = +245\text{ N}$$
$$A_z + B_z + C = 0 \qquad A_z - 98.1 + 196.2 = 0 \qquad A_z = -98.1\text{ N}$$

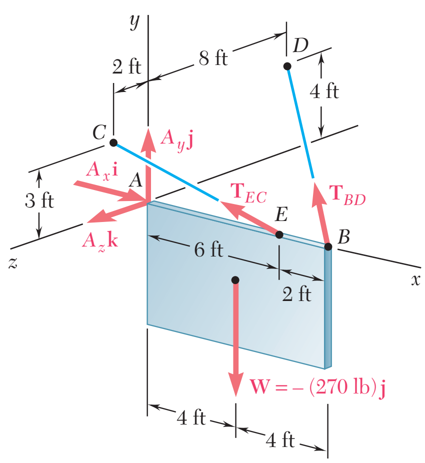
We conclude that the reaction at A is $\quad \mathbf{A} = +(245\text{ N})\mathbf{j} - (98.1\text{ N})\mathbf{k} \blacktriangleleft$

†The moments in this sample problem and in Sample Probs. 4.8 and 4.9 can also be expressed in the form of determinants (see Sample Prob. 3.10).



SAMPLE PROBLEM 4.8

A 5 × 8-ft sign of uniform density weighs 270 lb and is supported by a ball-and-socket joint at A and by two cables. Determine the tension in each cable and the reaction at A.



SOLUTION

Free-Body Diagram. A free-body diagram of the sign is drawn. The forces acting on the free body are the weight $\mathbf{W} = -(270 \text{ lb})\mathbf{j}$ and the reactions at A, B, and E. The reaction at A is a force of unknown direction and is represented by three unknown components. Since the directions of the forces exerted by the cables are known, these forces involve only one unknown each, namely, the magnitudes T_{BD} and T_{EC}. Since there are only five unknowns, the sign is partially constrained. It can rotate freely about the x axis; it is, however, in equilibrium under the given loading, since the equation $\Sigma M_x = 0$ is satisfied.

The components of the forces $\mathbf{T}_{BD}$ and $\mathbf{T}_{EC}$ can be expressed in terms of the unknown magnitudes T_{BD} and T_{EC} by writing

$$\overrightarrow{BD} = -(8 \text{ ft})\mathbf{i} + (4 \text{ ft})\mathbf{j} - (8 \text{ ft})\mathbf{k} \qquad BD = 12 \text{ ft}$$
$$\overrightarrow{EC} = -(6 \text{ ft})\mathbf{i} + (3 \text{ ft})\mathbf{j} + (2 \text{ ft})\mathbf{k} \qquad EC = 7 \text{ ft}$$
$$\mathbf{T}_{BD} = T_{BD}\left(\frac{\overrightarrow{BD}}{BD}\right) = T_{BD}(-\tfrac{2}{3}\mathbf{i} + \tfrac{1}{3}\mathbf{j} - \tfrac{2}{3}\mathbf{k})$$
$$\mathbf{T}_{EC} = T_{EC}\left(\frac{\overrightarrow{EC}}{EC}\right) = T_{EC}(-\tfrac{6}{7}\mathbf{i} + \tfrac{3}{7}\mathbf{j} + \tfrac{2}{7}\mathbf{k})$$

Equilibrium Equations. We express that the forces acting on the sign form a system equivalent to zero:

$$\Sigma\mathbf{F} = 0: \qquad A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k} + \mathbf{T}_{BD} + \mathbf{T}_{EC} - (270 \text{ lb})\mathbf{j} = 0$$
$$(A_x - \tfrac{2}{3}T_{BD} - \tfrac{6}{7}T_{EC})\mathbf{i} + (A_y + \tfrac{1}{3}T_{BD} + \tfrac{3}{7}T_{EC} - 270 \text{ lb})\mathbf{j} + (A_z - \tfrac{2}{3}T_{BD} + \tfrac{2}{7}T_{EC})\mathbf{k} = 0 \qquad (1)$$

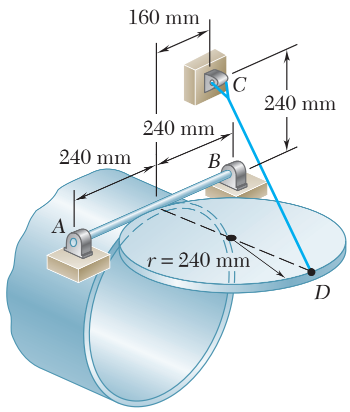
$$\Sigma\mathbf{M}_A = \Sigma(\mathbf{r} \times \mathbf{F}) = 0:$$
$$(8 \text{ ft})\mathbf{i} \times T_{BD}(-\tfrac{2}{3}\mathbf{i} + \tfrac{1}{3}\mathbf{j} - \tfrac{2}{3}\mathbf{k}) + (6 \text{ ft})\mathbf{i} \times T_{EC}(-\tfrac{6}{7}\mathbf{i} + \tfrac{3}{7}\mathbf{j} + \tfrac{2}{7}\mathbf{k}) + (4 \text{ ft})\mathbf{i} \times (-270 \text{ lb})\mathbf{j} = 0$$
$$(2.667T_{BD} + 2.571T_{EC} - 1080 \text{ lb})\mathbf{k} + (5.333T_{BD} - 1.714T_{EC})\mathbf{j} = 0 \qquad (2)$$

Setting the coefficients of $\mathbf{j}$ and $\mathbf{k}$ equal to zero in Eq. (2), we obtain two scalar equations which can be solved for T_{BD} and T_{EC}:

$$T_{BD} = 101.3 \text{ lb} \qquad T_{EC} = 315 \text{ lb} \quad ◀$$

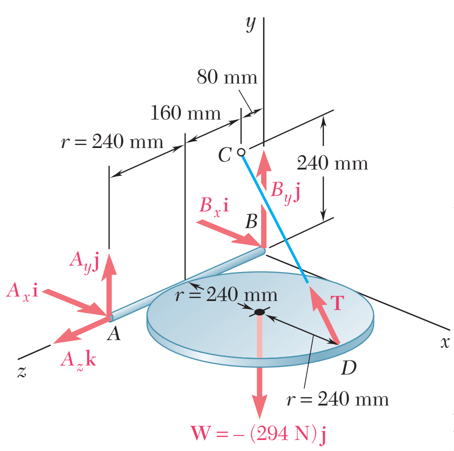
Setting the coefficients of $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ equal to zero in Eq. (1), we obtain three more equations, which yield the components of $\mathbf{A}$. We have

$$\mathbf{A} = +(338 \text{ lb})\mathbf{i} + (101.2 \text{ lb})\mathbf{j} - (22.5 \text{ lb})\mathbf{k} \quad ◀$$



SAMPLE PROBLEM 4.9

A uniform pipe cover of radius $r = 240$ mm and mass 30 kg is held in a horizontal position by the cable CD. Assuming that the bearing at B does not exert any axial thrust, determine the tension in the cable and the reactions at A and B.



SOLUTION

Free-Body Diagram. A free-body diagram is drawn with the coordinate axes shown. The forces acting on the free body are the weight of the cover.

$$\mathbf{W} = -mg\mathbf{j} = -(30\text{ kg})(9.81\text{ m/s}^2)\mathbf{j} = -(294\text{ N})\mathbf{j}$$

and reactions involving six unknowns, namely, the magnitude of the force $\mathbf{T}$ exerted by the cable, three force components at hinge A, and two at hinge B. The components of $\mathbf{T}$ are expressed in terms of the unknown magnitude T by resolving the vector $\overrightarrow{DC}$ into rectangular components and writing

$$\overrightarrow{DC} = -(480\text{ mm})\mathbf{i} + (240\text{ mm})\mathbf{j} - (160\text{ mm})\mathbf{k} \qquad DC = 560\text{ mm}$$

$$\mathbf{T} = T\frac{\overrightarrow{DC}}{DC} = -\tfrac{6}{7}T\mathbf{i} + \tfrac{3}{7}T\mathbf{j} - \tfrac{2}{7}T\mathbf{k}$$

Equilibrium Equations. We express that the forces acting on the pipe cover form a system equivalent to zero:

$$\Sigma\mathbf{F} = 0: \qquad A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k} + B_x\mathbf{i} + B_y\mathbf{j} + \mathbf{T} - (294\text{ N})\mathbf{j} = 0$$

$$(A_x + B_x - \tfrac{6}{7}T)\mathbf{i} + (A_y + B_y + \tfrac{3}{7}T - 294\text{ N})\mathbf{j} + (A_z - \tfrac{2}{7}T)\mathbf{k} = 0 \qquad (1)$$

$$\Sigma\mathbf{M}_B = \Sigma(\mathbf{r} \times \mathbf{F}) = 0:$$

$$2r\mathbf{k} \times (A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k})$$
$$+ (2r\mathbf{i} + r\mathbf{k}) \times (-\tfrac{6}{7}T\mathbf{i} + \tfrac{3}{7}T\mathbf{j} - \tfrac{2}{7}T\mathbf{k})$$
$$+ (r\mathbf{i} + r\mathbf{k}) \times (-294\text{ N})\mathbf{j} = 0$$

$$(-2A_y - \tfrac{3}{7}T + 294\text{ N})r\mathbf{i} + (2A_x - \tfrac{2}{7}T)r\mathbf{j} + (\tfrac{6}{7}T - 294\text{ N})r\mathbf{k} = 0 \qquad (2)$$

Setting the coefficients of the unit vectors equal to zero in Eq. (2), we write three scalar equations, which yield

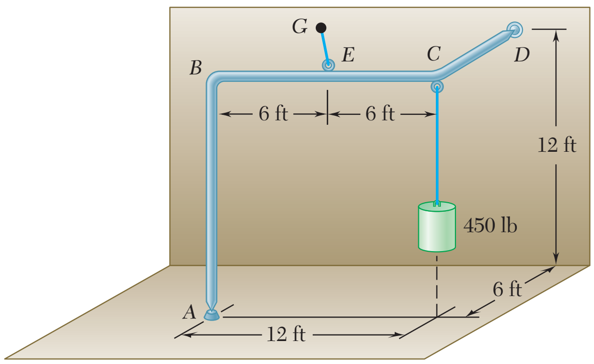
$$A_x = +49.0\text{ N} \qquad A_y = +73.5\text{ N} \qquad T = 343\text{ N} \blacktriangleleft$$

Setting the coefficients of the unit vectors equal to zero in Eq. (1), we obtain three more scalar equations. After substituting the values of T, A_x, and A_y into these equations, we obtain

$$A_z = +98.0\text{ N} \qquad B_x = +245\text{ N} \qquad B_y = +73.5\text{ N}$$

The reactions at A and B are therefore

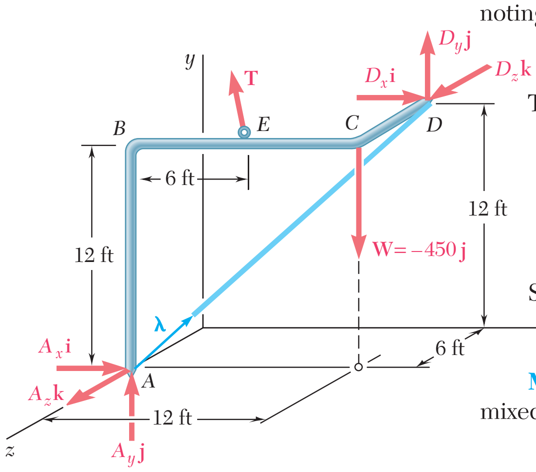
$$\mathbf{A} = +(49.0\text{ N})\mathbf{i} + (73.5\text{ N})\mathbf{j} + (98.0\text{ N})\mathbf{k} \blacktriangleleft$$
$$\mathbf{B} = +(245\text{ N})\mathbf{i} + (73.5\text{ N})\mathbf{j} \blacktriangleleft$$



SAMPLE PROBLEM 4.10

A 450-lb load hangs from the corner C of a rigid piece of pipe $ABCD$ which has been bent as shown. The pipe is supported by the ball-and-socket joints A and D, which are fastened, respectively, to the floor and to a vertical wall, and by a cable attached at the midpoint E of the portion BC of the pipe and at a point G on the wall. Determine (*a*) where G should be located if the tension in the cable is to be minimum, (*b*) the corresponding minimum value of the tension.

SOLUTION



Free-Body Diagram. The free-body diagram of the pipe includes the load $\mathbf{W} = (-450 \text{ lb})\mathbf{j}$, the reactions at A and D, and the force $\mathbf{T}$ exerted by the cable. To eliminate the reactions at A and D from the computations, we express that the sum of the moments of the forces about AD is zero. Denoting by $\boldsymbol{\lambda}$ the unit vector along AD, we write

$$\Sigma M_{AD} = 0: \qquad \boldsymbol{\lambda} \cdot (\overrightarrow{AE} \times \mathbf{T}) + \boldsymbol{\lambda} \cdot (\overrightarrow{AC} \times \mathbf{W}) = 0 \tag{1}$$

The second term in Eq. (1) can be computed as follows:

$$\overrightarrow{AC} \times \mathbf{W} = (12\mathbf{i} + 12\mathbf{j}) \times (-450\mathbf{j}) = -5400\mathbf{k}$$

$$\boldsymbol{\lambda} = \frac{\overrightarrow{AD}}{AD} = \frac{12\mathbf{i} + 12\mathbf{j} - 6\mathbf{k}}{18} = \tfrac{2}{3}\mathbf{i} + \tfrac{2}{3}\mathbf{j} - \tfrac{1}{3}\mathbf{k}$$

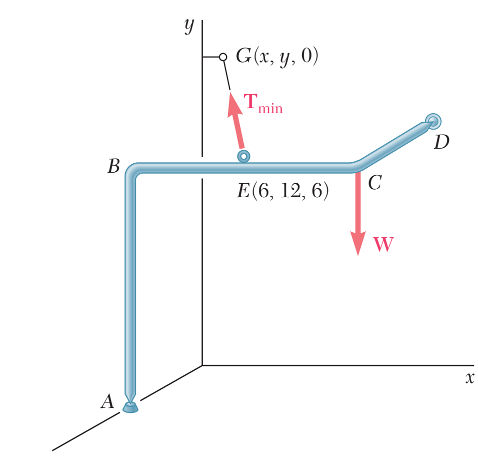
$$\boldsymbol{\lambda} \cdot (\overrightarrow{AC} \times \mathbf{W}) = (\tfrac{2}{3}\mathbf{i} + \tfrac{2}{3}\mathbf{j} - \tfrac{1}{3}\mathbf{k}) \cdot (-5400\mathbf{k}) = +1800$$

Substituting the value obtained into Eq. (1), we write

$$\boldsymbol{\lambda} \cdot (\overrightarrow{AE} \times \mathbf{T}) = -1800 \text{ lb} \cdot \text{ft} \tag{2}$$

Minimum Value of Tension. Recalling the commutative property for mixed triple products, we rewrite Eq. (2) in the form

$$\mathbf{T} \cdot (\boldsymbol{\lambda} \times \overrightarrow{AE}) = -1800 \text{ lb} \cdot \text{ft} \tag{3}$$



which shows that the projection of $\mathbf{T}$ on the vector $\boldsymbol{\lambda} \times \overrightarrow{AE}$ is a constant. It follows that $\mathbf{T}$ is minimum when parallel to the vector

$$\boldsymbol{\lambda} \times \overrightarrow{AE} = (\tfrac{2}{3}\mathbf{i} + \tfrac{2}{3}\mathbf{j} - \tfrac{1}{3}\mathbf{k}) \times (6\mathbf{i} + 12\mathbf{j}) = 4\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$$

Since the corresponding unit vector is $\tfrac{2}{3}\mathbf{i} - \tfrac{1}{3}\mathbf{j} + \tfrac{2}{3}\mathbf{k}$, we write

$$\mathbf{T}_{\text{min}} = T(\tfrac{2}{3}\mathbf{i} - \tfrac{1}{3}\mathbf{j} + \tfrac{2}{3}\mathbf{k}) \tag{4}$$

Substituting for $\mathbf{T}$ and $\boldsymbol{\lambda} \times \overrightarrow{AE}$ in Eq. (3) and computing the dot products, we obtain $6T = -1800$ and, thus, $T = -300$. Carrying this value into (4), we obtain

$$\mathbf{T}_{\text{min}} = -200\mathbf{i} + 100\mathbf{j} - 200\mathbf{k} \qquad T_{\text{min}} = 300 \text{ lb} \blacktriangleleft$$

Location of G. Since the vector $\overrightarrow{EG}$ and the force $\mathbf{T}_{\text{min}}$ have the same direction, their components must be proportional. Denoting the coordinates of G by x, y, 0, we write

$$\frac{x-6}{-200} = \frac{y-12}{+100} = \frac{0-6}{-200} \qquad x = 0 \qquad y = 15 \text{ ft} \blacktriangleleft$$

SOLVING PROBLEMS ON YOUR OWN

The equilibrium of a *three-dimensional body* was considered in the sections you just completed. It is again most important that you draw a complete *free-body diagram* as the first step of your solution.

1. As you draw the free-body diagram, pay particular attention to the reactions at the supports. The number of unknowns at a support can range from one to six (Fig. 4.10). To decide whether an unknown reaction or reaction component exists at a support, ask yourself whether the support prevents motion of the body in a certain direction or about a certain axis.

a. If motion is prevented in a certain direction, include in your free-body diagram an unknown reaction or *reaction component* that acts in the *same direction.*

b. If a support prevents rotation about a certain axis, include in your free-body diagram a *couple* of unknown magnitude that acts about the *same* axis.

2. The external forces acting on a three-dimensional body form a system equivalent to zero. Writing $\Sigma\mathbf{F} = 0$ and $\Sigma\mathbf{M}_A = 0$ about an appropriate point A, and setting the coefficients of $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ in both equations equal to zero will provide you with six scalar equations. In general, these equations will contain six unknowns and can be solved for these unknowns.

3. After completing your free-body diagram, you may want to seek equations involving as few unknowns as possible. The following strategies may help you.

a. By summing moments about a ball-and-socket support or a hinge, you will obtain equations from which three unknown reaction components have been eliminated [Sample Probs. 4.8 and 4.9].

b. If you can draw an axis through the points of application of all but one of the unknown reactions, summing moments about that axis will yield an equation in a single unknown [Sample Prob. 4.10].

4. After drawing your free-body diagram, we encourage you to compare the number of unknowns to the number of nontrivial, scalar equations of equilibrium for the given problem. Doing so will tell you if the body is properly or partially constrained and whether the problem is statically determinate or indeterminate. Further, as we consider more complex problems in later chapters, keeping track of the numbers of unknowns and equations will help you to develop correct solutions.

Problems

4.96 Gears *A* and *B* are attached to a shaft supported by bearings at *C* and *D*. The diameters of gears *A* and *B* are 150 mm and 75 mm, respectively, and the tangential and radial forces acting on the gears are as shown. Knowing that the system rotates at a constant rate, determine the reactions at *C* and *D*. Assume that the bearing at *C* does not exert any axial force, and neglect the weights of the gears and the shaft.

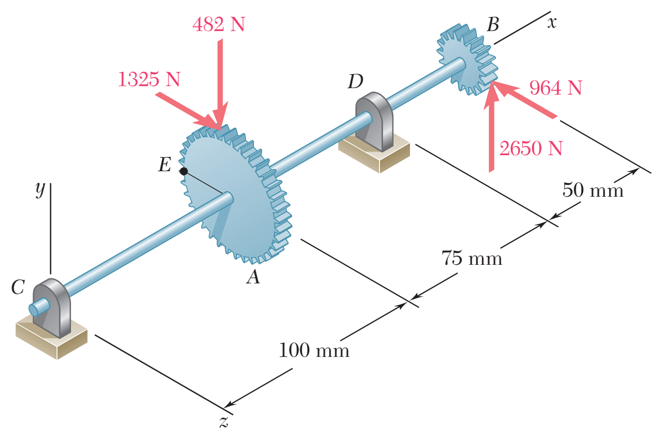


Fig. P4.96

4.97 Solve Prob. 4.96 assuming that for gear *A* the tangential and radial forces are acting at *E*, so that $\mathbf{F}_A = (1325\text{ N})\mathbf{j} + (482\text{ N})\mathbf{k}$.

4.98 Two transmission belts pass over sheaves welded to an axle supported by bearings at *B* and *D*. The sheave at *A* has a radius of 50 mm, and the sheave at *C* has a radius of 40 mm. Knowing that the system rotates with a constant rate, determine (*a*) the tension *T*, (*b*) the reactions at *B* and *D*. Assume that the bearing at *D* does not exert any axial thrust and neglect the weights of the sheaves and the axle.

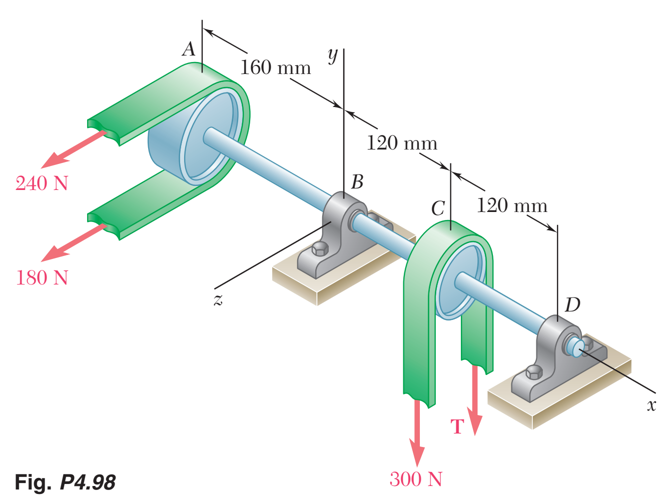


Fig. *P4.98*

4.99 For the portion of a machine shown, the 4-in.-diameter pulley *A* and wheel *B* are fixed to a shaft supported by bearings at *C* and *D*. The spring of constant 2 lb/in. is unstretched when $\theta = 0$, and the bearing at *C* does not exert any axial force. Knowing that $\theta = 180°$ and that the machine is at rest and in equilibrium, determine (*a*) the tension *T*, (*b*) the reactions at *C* and *D*. Neglect the weights of the shaft, pulley, and wheel.

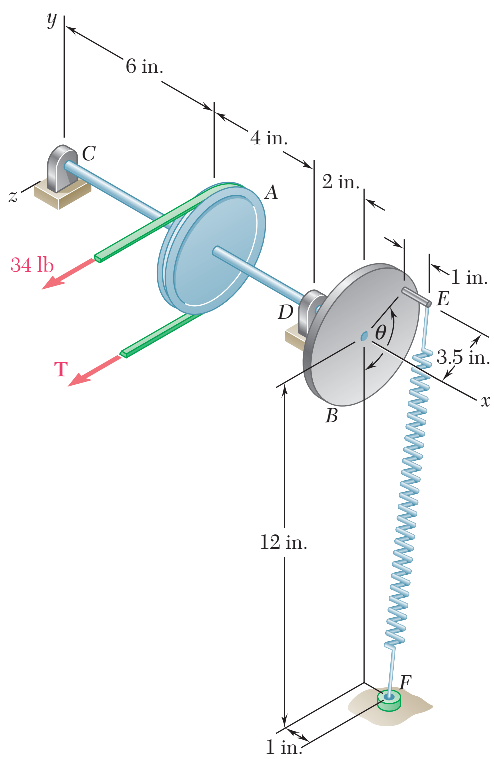


Fig. P4.99

4.100 Solve Prob. 4.99 for $\theta = 90°$.

4.101 A 1.2×2.4-m sheet of plywood having a mass of 17 kg has been temporarily placed among three pipe supports. The lower edge of the sheet rests on small collars *A* and *B* and its upper edge leans against pipe *C*. Neglecting friction at all surfaces, determine the reactions at *A*, *B*, and *C*.

4.102 The 200×200-mm square plate shown has a mass of 25 kg and is supported by three vertical wires. Determine the tension in each wire.

4.103 The 200×200-mm square plate shown has a mass of 25 kg and is supported by three vertical wires. Determine the mass and location of the lightest block which should be placed on the plate if the tensions in the three cables are to be equal.

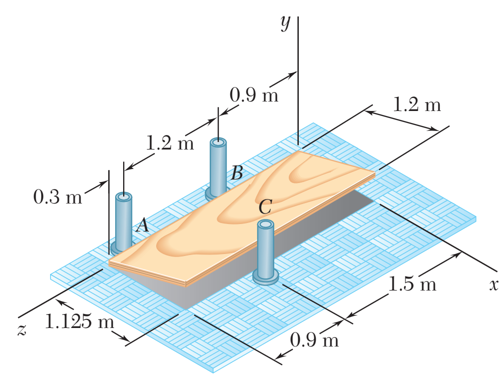


Fig. *P4.101*

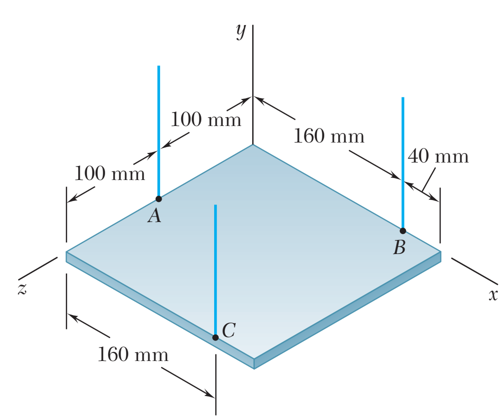


Fig. P4.102 and P4.103

4.104 A camera of mass 240 g is mounted on a small tripod of mass 200 g. Assuming that the mass of the camera is uniformly distributed and that the line of action of the weight of the tripod passes through D, determine (a) the vertical components of the reactions at A, B, and C when $\theta = 0$, (b) the maximum value of θ if the tripod is not to tip over.

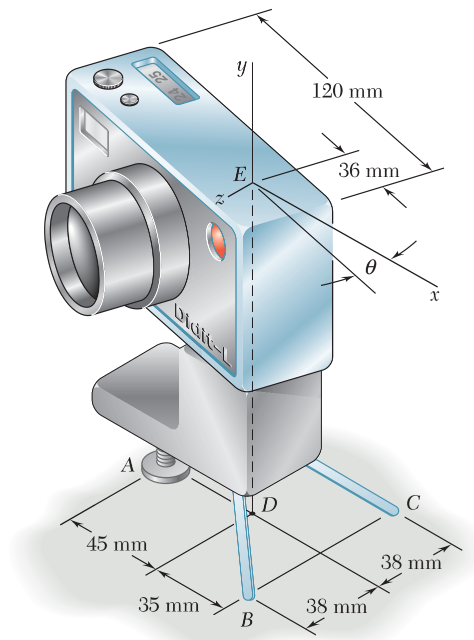


Fig. P4.104

4.105 Two steel pipes AB and BC, each having a weight per unit length of 5 lb/ft, are welded together at B and are supported by three wires. Knowing that $a = 1.25$ ft, determine the tension in each wire.

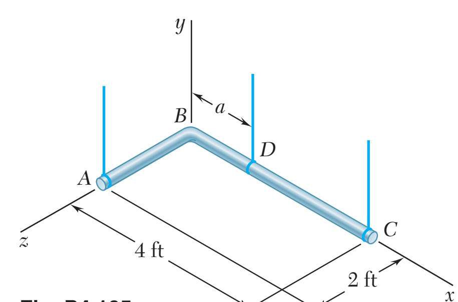


Fig. P4.105

4.106 For the pile assembly of Prob. 4.105, determine (a) the largest permissible value of a if the assembly is not to tip, (b) the corresponding tension in each wire.

4.107 A uniform aluminum rod of weight W is bent into a circular ring of radius R and is supported by three wires as shown. Determine the tension in each wire.

4.108 A uniform aluminum rod of weight W is bent into a circular ring of radius R and is supported by three wires as shown. A small collar of weight W' is then placed on the ring and positioned so that the tensions in the three wires are equal. Determine (a) the position of the collar, (b) the value of W', (c) the tension in the wires.

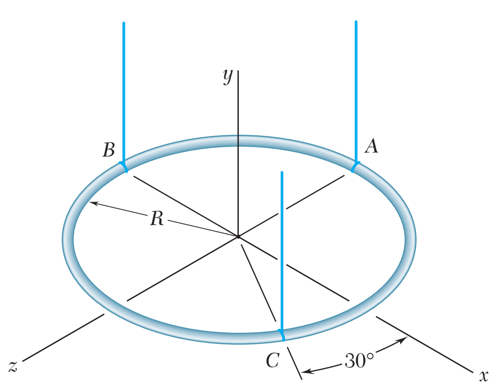


Fig. P4.107 and *P4.108*

4.109 An opening in a floor is covered by a 3 × 4-ft sheet of plywood weighing 12 lb. The sheet is hinged at *A* and *B* and is maintained in a position slightly above the floor by a small block *C*. Determine the vertical component of the reaction (*a*) at *A*, (*b*) at *B*, (*c*) at *C*.

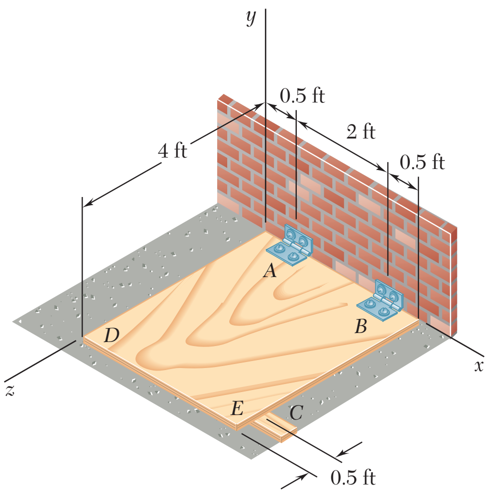


Fig. *P4.109*

4.110 Solve Prob. 4.109 assuming that the small block *C* is moved and placed under edge *DE* at a point 0.5 ft from corner *E*.

4.111 The 10-kg square plate shown is supported by three vertical wires. Determine (*a*) the tension in each wire when *a* = 100 mm, (*b*) the value of *a* for which tensions in the three wires are equal.

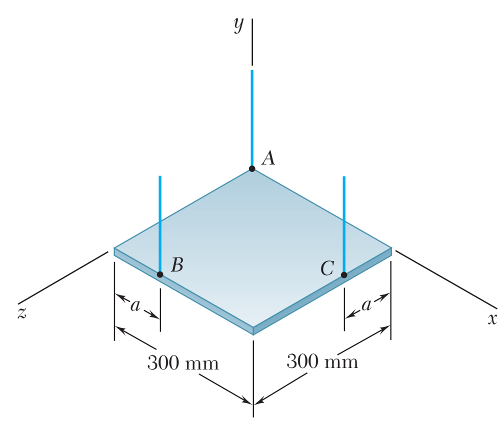


Fig. P4.111

4.112 The 3-m flagpole *AC* forms an angle of 30° with the *z* axis. It is held by a ball-and-socket joint at *C* and by two thin braces *BD* and *BE*. Knowing that the distance *BC* is 0.9 m, determine the tension in each brace and the reaction at *C*.

4.113 A 3-m boom is acted upon by the 4-kN force shown. Determine the tension in each cable and the reaction at the ball-and-socket joint at *A*.

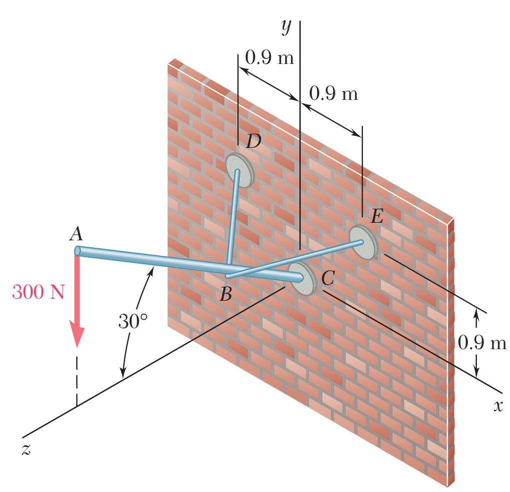


Fig. P4.112

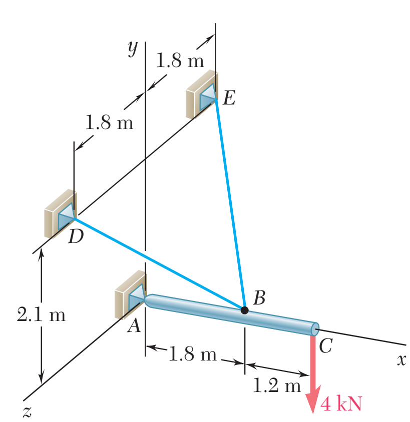


Fig. P4.113

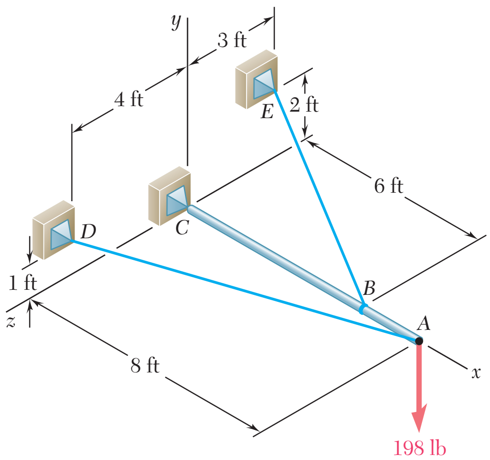


Fig. P4.114

4.114 An 8-ft-long boom is held by a ball-and-socket joint at C and by two cables AD and BE. Determine the tension in each cable and the reaction at C.

4.115 Solve Prob. 4.114 assuming that the given 198-lb load is replaced with two 99-lb loads applied at A and B.

4.116 The 18-ft pole ABC is acted upon by a 210-lb force as shown. The pole is held by a ball-and-socket joint at A and by two cables BD and BE. For $a = 9$ ft, determine the tension in each cable and the reaction at A.

4.117 Solve Prob. 4.116 for $a = 4.5$ ft.

4.118 Two steel pipes $ABCD$ and EBF are welded together at B to form the boom shown. The boom is held by a ball-and-socket joint at D and by two cables EG and $ICFH$; cable $ICFH$ passes around frictionless pulleys at C and F. For the loading shown, determine the tension in each cable and the reaction at D.

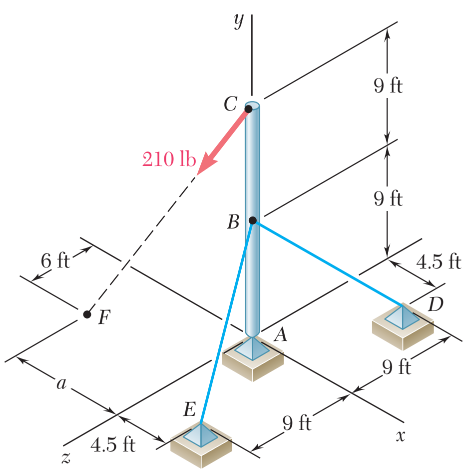


Fig. P4.116

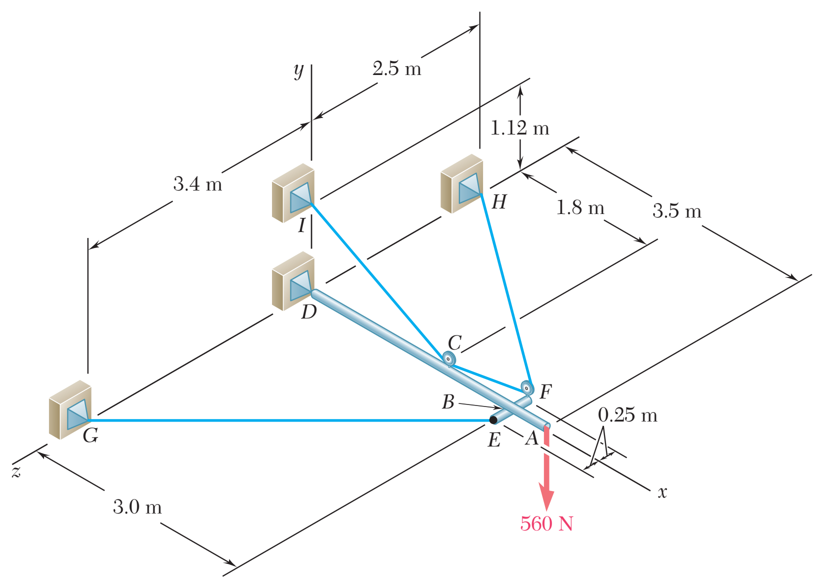


Fig. P4.118

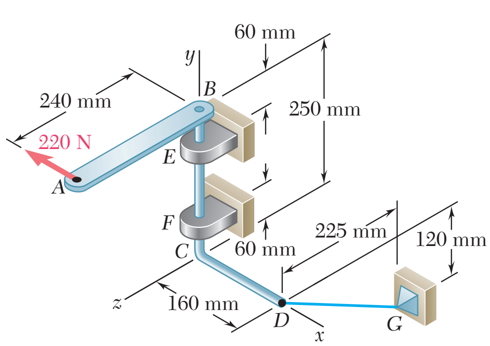


Fig. P4.120

4.119 Solve Prob. 4.118 assuming that the 560-N load is applied at B.

4.120 The lever AB is welded to the bent rod BCD which is supported by bearings at E and F and by cable DG. Knowing that the bearing at E does not exert any axial thrust, determine (a) the tension in cable DG, (b) the reactions at E and F.

4.121 A 30-kg cover for a roof opening is hinged at corners *A* and *B*. The roof forms an angle of 30° with the horizontal, and the cover is maintained in a horizontal position by the brace *CE*. Determine (*a*) the magnitude of the force exerted by the brace, (*b*) the reactions at the hinges. Assume that the hinge at *A* does not exert any axial thrust.

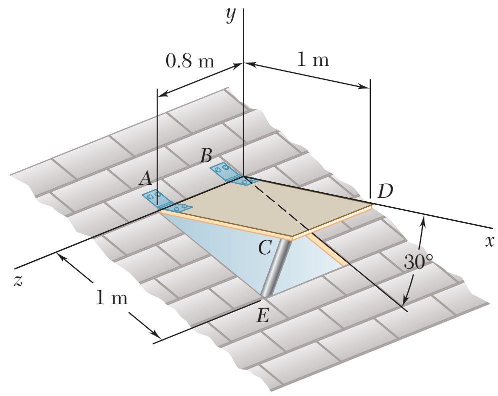


Fig. ***P4.121***

4.122 The rectangular plate shown has a mass of 15 kg and is held in the position shown by hinges *A* and *B* and cable *EF*. Assuming that the hinge at *B* does not exert any axial thrust, determine (*a*) the tension in the cable, (*b*) the reactions at *A* and *B*.

4.123 Solve Prob. 4.122 assuming that cable *EF* is replaced by a cable attached at points *E* and *H*.

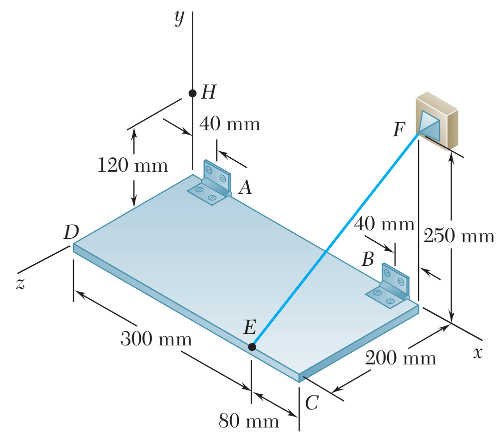


Fig. P4.122

4.124 A small door weighing 16 lb is attached by hinges *A* and *B* to a wall and is held in the horizontal position shown by rope *EFH*. The rope passes around a small, frictionless pulley at *F* and is tied to a fixed cleat at *H*. Assuming that the hinge at *A* does not exert any axial thrust, determine (*a*) the tension in the rope, (*b*) the reactions at *A* and *B*.

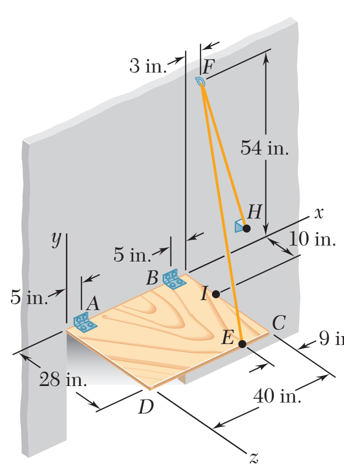


Fig. P4.124

4.125 Solve Prob. 4.124 assuming that the rope is attached to the door at *I*.

4.126 A 285-lb uniform rectangular plate is supported in the position shown by hinges *A* and *B* and by cable *DCE*, which passes over a frictionless hook at *C*. Assuming that the tension is the same in both parts of the cable, determine (*a*) the tension in the cable, (*b*) the reactions at *A* and *B*. Assume that the hinge at *B* does not exert any axial thrust.

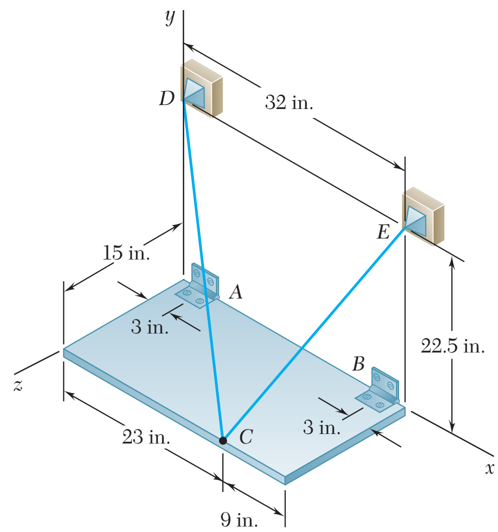


Fig. P4.126

4.127 Solve Prob. 4.126 assuming that cable *DCE* is replaced by a cable attached to point *E* and hook *C*.

4.128 The tensioning mechanism of a belt drive consists of frictionless pulley *A*, mounting plate *B*, and spring *C*. Attached below the mounting plate is slider block *D* which is free to move in the frictionless slot of bracket *E*. Knowing that the pulley and the belt lie in a horizontal plane, with portion *F* of the belt parallel to the *x* axis and portion *G* forming an angle of 30° with the *x* axis, determine (*a*) the force in the spring, (*b*) the reaction at *D*.

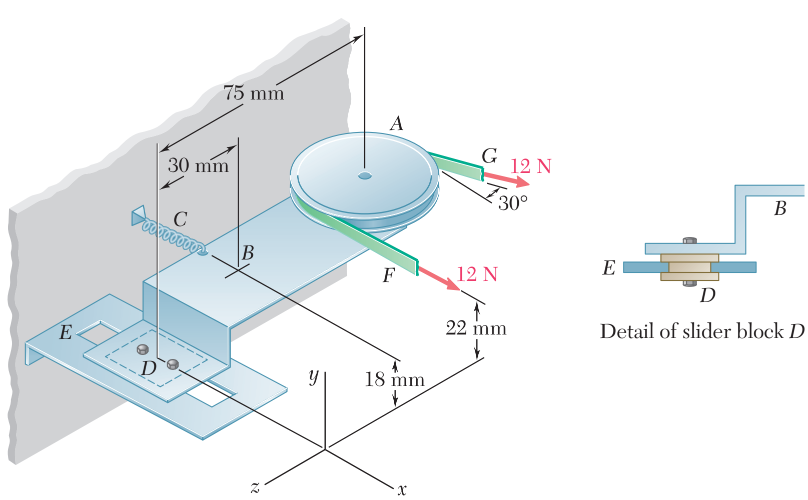


Fig. P4.128

4.129 The assembly shown is welded to collar A which fits on the vertical pin shown. The pin can exert couples about the x and z axes but does not prevent motion about or along the y axis. For the loading shown, determine the tension in each cable and the reaction at A.

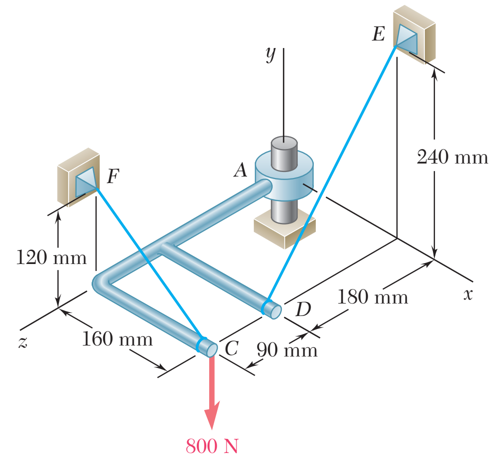


Fig. P4.129

4.130 The lever AB is welded to the bent rod BCD which is supported by bearing E and by cable DG. Assuming that the bearing can exert an axial thrust and couples about axes parallel to the x and z axes, determine (*a*) the tension in cable DG, (*b*) the reaction at E.

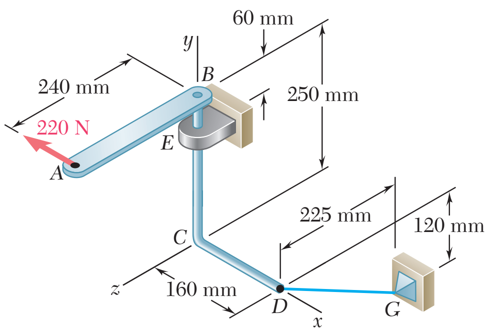


Fig. P4.130

4.131 Solve Prob. 4.124 assuming that the hinge at A is removed and that the hinge at B can exert couples about the y and z axes.

4.132 The frame shown is supported by three cables and a ball-and-socket joint at A. For $\mathbf{P} = 0$, determine the tension in each cable and the reaction at A.

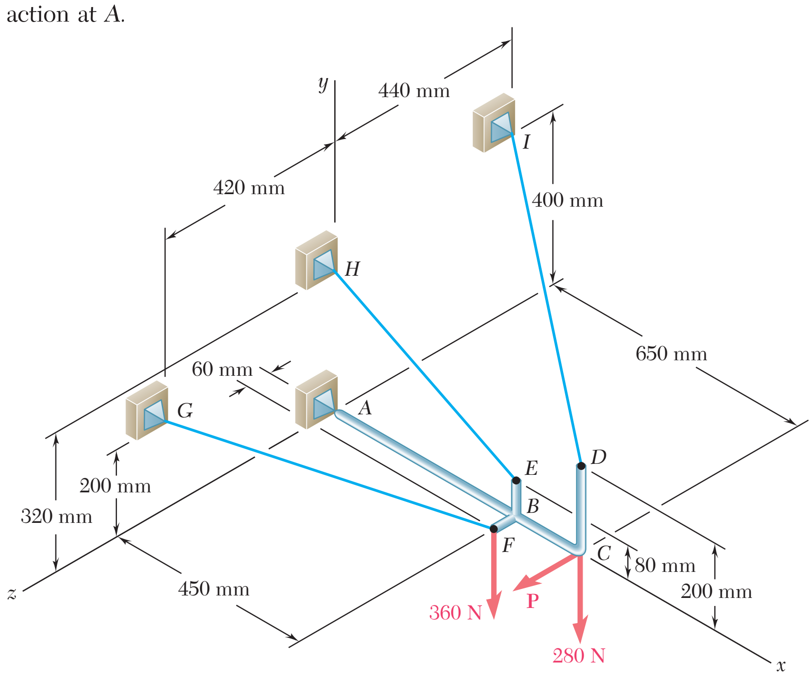


Fig. P4.132 and P4.133

4.133 The frame shown is supported by three cables and a ball-and-socket joint at A. For $P = 50$ N, determine the tension in each cable and the reaction at A.

4.134 The rigid L-shaped member *ABF* is supported by a ball-and-socket joint at *A* and by three cables. For the loading shown, determine the tension in each cable and the reaction at *A*.

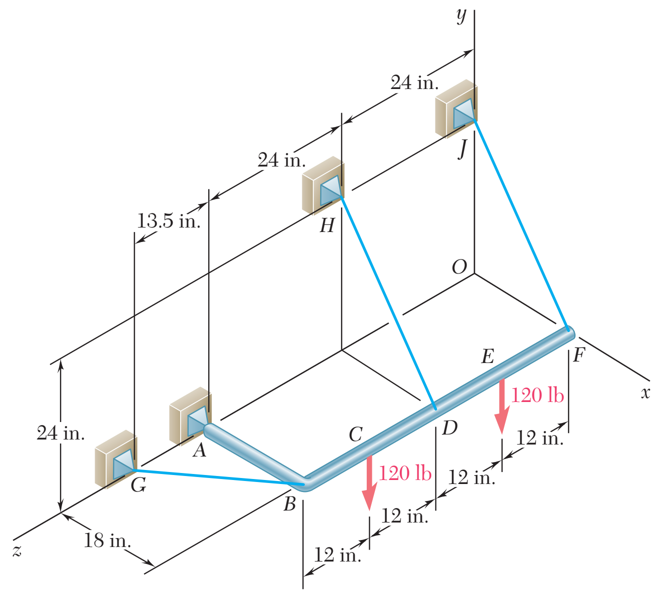


Fig. *P4.134*

4.135 Solve Prob. 4.134 assuming that the load at *C* has been removed.

4.136 In order to clean the clogged drainpipe *AE*, a plumber has disconnected both ends of the pipe and inserted a power snake through the opening at *A*. The cutting head of the snake is connected by a heavy cable to an electric motor which rotates at a constant speed as the plumber forces the cable into the pipe. The forces exerted by the plumber and the motor on the end of the cable can be represented by the wrench $\mathbf{F} = -(60\text{ N})\mathbf{k}$, $\mathbf{M} = -(108\text{ N}\cdot\text{m})\mathbf{k}$. Determine the additional reactions at *B*, *C*, and *D* caused by the cleaning operation. Assume that the reaction at each support consists of two force components perpendicular to the pipe.

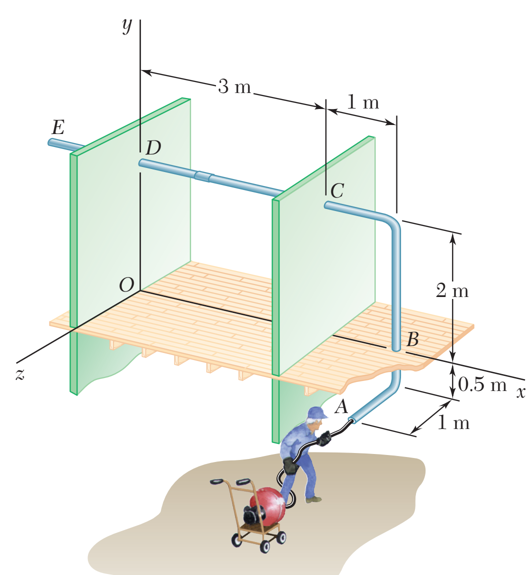


Fig. P4.136

4.137 Solve Prob. 4.136 assuming that the plumber exerts a force $\mathbf{F} = -(60\text{ N})\mathbf{k}$ and that the motor is turned off ($\mathbf{M} = 0$).

4.138 Three rods are welded together to form a "corner" which is supported by three eyebolts. Neglecting friction, determine the reactions at A, B, and C when $P = 240$ N, $a = 120$ mm, $b = 80$ mm, and $c = 100$ mm.

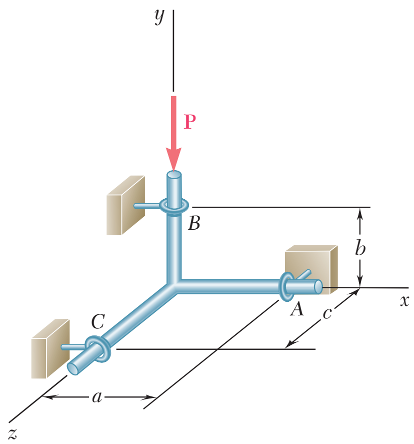


Fig. P4.138

4.139 Solve Prob. 4.138 assuming that the force **P** is removed and is replaced by a couple $\mathbf{M} = +(6\text{ N}\cdot\text{m})\mathbf{j}$ acting at B.

4.140 The uniform 10-lb rod AB is supported by a ball-and-socket joint at A and leans against both the rod CD and the vertical wall. Neglecting the effects of friction, determine (a) the force which rod CD exerts on AB, (b) the reactions at A and B. (*Hint:* The force exerted by CD on AB must be perpendicular to both rods.)

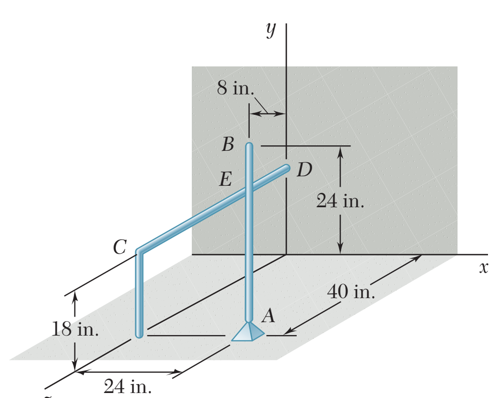


Fig. P4.140

4.141 A 21-in.-long uniform rod AB weighs 6.4 lb and is attached to a ball-and-socket joint at A. The rod rests against an inclined frictionless surface and is held in the position shown by cord BC. Knowing that the cord is 21 in. long, determine (a) the tension in the cord, (b) the reactions at A and B.

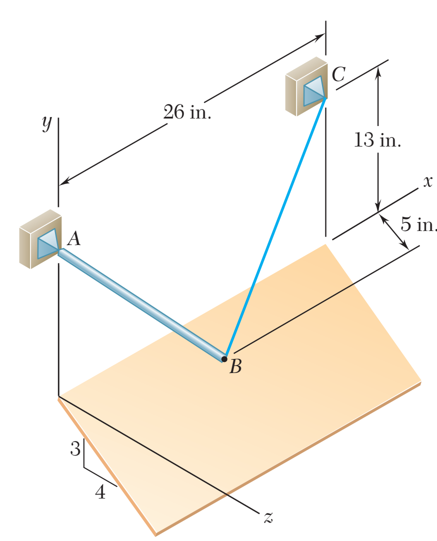


Fig. P4.141

4.142 While being installed, the 56-lb chute *ABCD* is attached to a wall with brackets *E* and *F* and is braced with props *GH* and *IJ*. Assuming that the weight of the chute is uniformly distributed, determine the magnitude of the force exerted on the chute by prop *GH* if prop *IJ* is removed.

4.143 While being installed, the 56-lb chute *ABCD* is attached to a wall with brackets *E* and *F* and is braced with props *GH* and *IJ*. Assuming that the weight of the chute is uniformly distributed, determine the magnitude of the force exerted on the chute by prop *IJ* if prop *GH* is removed.

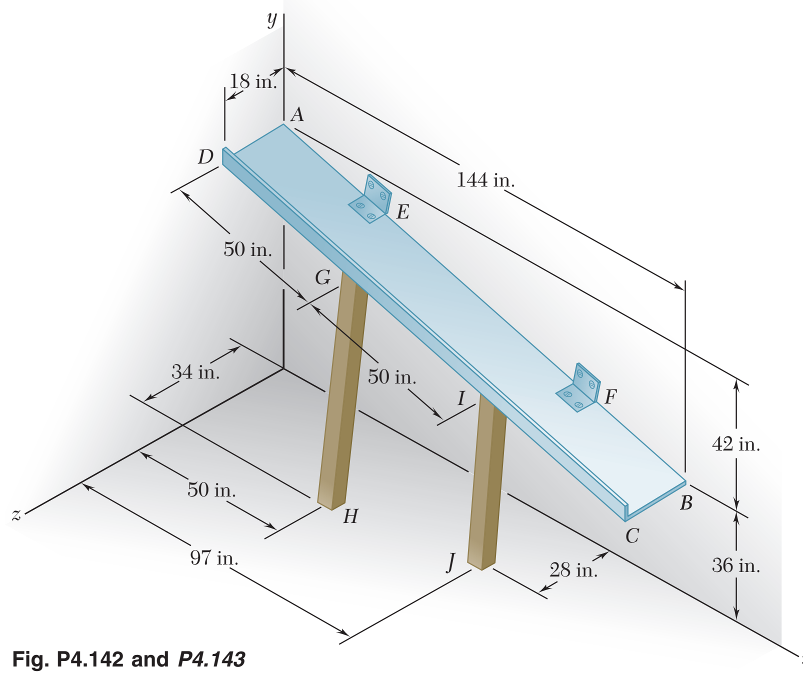


Fig. P4.142 and *P4.143*

4.144 To water seedlings, a gardener joins three lengths of pipe, *AB*, *BC*, and *CD*, fitted with spray nozzles and suspends the assembly using hinged supports at *A* and *D* and cable *EF*. Knowing that the pipe weighs 0.85 lb/ft, determine the tension in the cable.

4.145 Solve Prob. 4.144 assuming that cable *EF* is replaced by a cable connecting *E* and *C*.

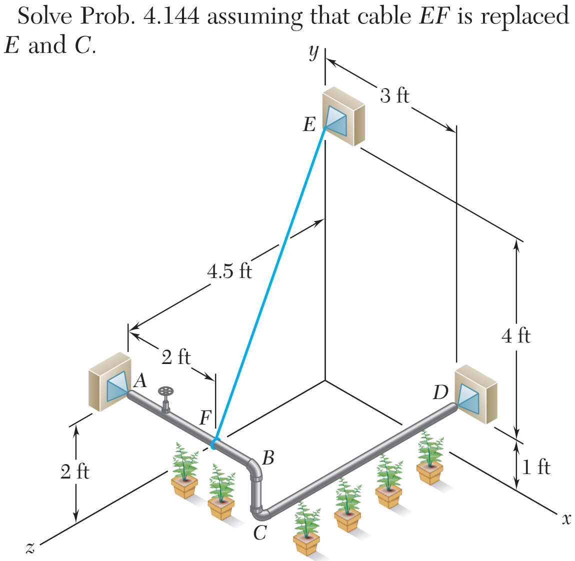


Fig. P4.144

4.146 The bent rod *ABDE* is supported by ball-and-socket joints at *A* and *E* and by the cable *DF.* If a 600-N load is applied at *C* as shown, determine the tension in the cable.

4.147 Solve Prob. 4.146 assuming that cable *DF* is replaced by a cable connecting *B* and *F.*

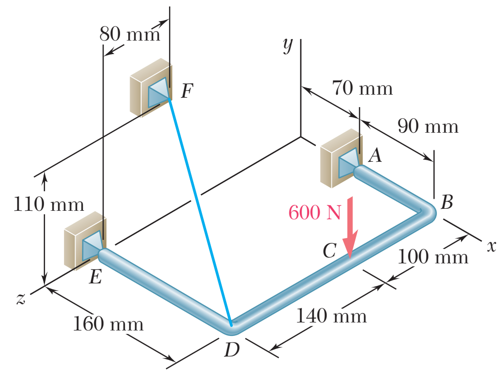


Fig. P4.146

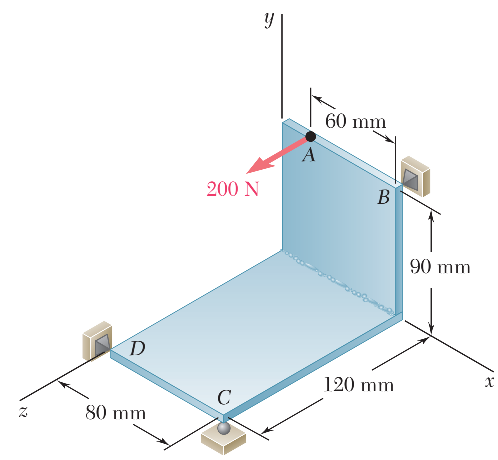


Fig. P4.148

4.148 Two rectangular plates are welded together to form the assembly shown. The assembly is supported by ball-and-socket joints at *B* and *D* and by a ball on a horizontal surface at *C.* For the loading shown, determine the reaction at *C.*

4.149 Two 1 × 2-m plywood panels, each of mass 15 kg, are nailed together as shown. The panels are supported by ball-and-socket joints at *A* and *F* and by the wire *BH.* Determine (*a*) the location of *H* in the *xy* plane if the tension in the wire is to be minimum, (*b*) the corresponding minimum tension.

4.150 Solve Prob. 4.149 subject to the restriction that *H* must lie on the *y* axis.

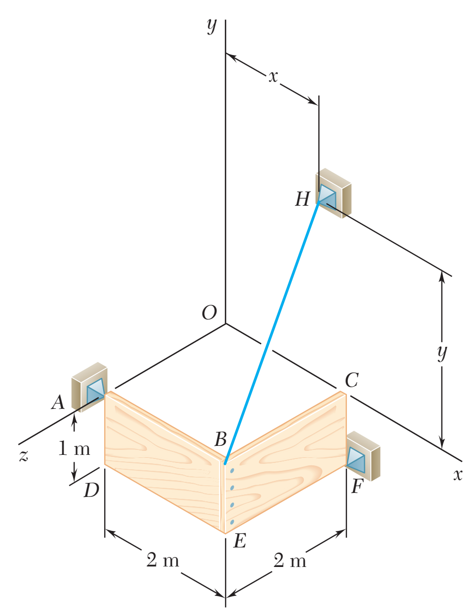


Fig. P4.149

4.151 A uniform 20 × 30-in. steel plate *ABCD* weighs 85 lb and is attached to ball-and-socket joints at *A* and *B.* Knowing that the plate leans against a frictionless vertical wall at *D,* determine (*a*) the location of *D,* (*b*) the reaction at *D.*

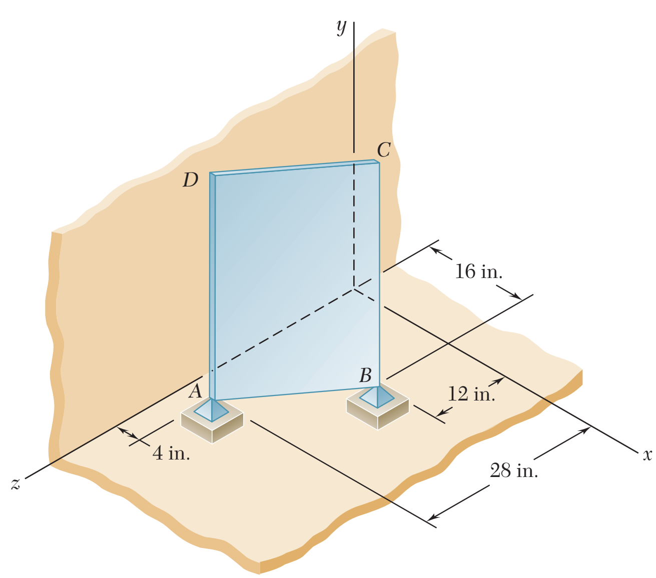


Fig. P4.151

REVIEW AND SUMMARY FOR CHAPTER 4

Equilibrium equations

This chapter was devoted to the study of the *equilibrium of rigid bodies,* that is, to the situation when the external forces acting on a rigid body *form a system equivalent to zero* [Sec. 4.1]. We then have

$$\Sigma \mathbf{F} = 0 \qquad \Sigma \mathbf{M}_O = \Sigma(\mathbf{r} \times \mathbf{F}) = 0 \tag{4.1}$$

Resolving each force and each moment into its rectangular components, we can express the necessary and sufficient conditions for the equilibrium of a rigid body with the following six scalar equations:

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma F_z = 0 \tag{4.2}$$

$$\Sigma M_x = 0 \qquad \Sigma M_y = 0 \qquad \Sigma M_z = 0 \tag{4.3}$$

These equations can be used to determine unknown forces applied to the rigid body or unknown reactions exerted by its supports.

Free-body diagram

When solving a problem involving the equilibrium of a rigid body, it is essential to consider *all* of the forces acting on the body. Therefore, the first step in the solution of the problem should be to draw a *free-body diagram* showing the body under consideration and all of the unknown as well as known forces acting on it [Sec. 4.2].

Equilibrium of a two-dimensional structure

In the first part of the chapter, we considered the *equilibrium of a two-dimensional structure;* that is, we assumed that the structure considered and the forces applied to it were contained in the same plane. We saw that each of the reactions exerted on the structure by its supports could involve one, two, or three unknowns, depending upon the type of support [Sec. 4.3].

In the case of a two-dimensional structure, Eqs. (4.1), or Eqs. (4.2) and (4.3), reduce to *three equilibrium equations,* namely

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma M_A = 0 \tag{4.5}$$

where A is an arbitrary point in the plane of the structure [Sec. 4.4]. These equations can be used to solve for three unknowns. While the three equilibrium equations (4.5) cannot be *augmented* with additional equations, any of them can be *replaced* by another equation. Therefore, we can write alternative sets of equilibrium equations, such as

$$\Sigma F_x = 0 \qquad \Sigma M_A = 0 \qquad \Sigma M_B = 0 \tag{4.6}$$

where point B is chosen in such a way that the line AB is not parallel to the y axis, or

$$\Sigma M_A = 0 \qquad \Sigma M_B = 0 \qquad \Sigma M_C = 0 \tag{4.7}$$

where the points A, B, and C do not lie in a straight line.

Statical indeterminacy

Partial constraints

Improper constraints

Since any set of equilibrium equations can be solved for only three unknowns, the reactions at the supports of a rigid two-dimensional structure cannot be completely determined if they involve *more than three unknowns;* they are said to be *statically indeterminate* [Sec. 4.5]. On the other hand, if the reactions involve *fewer than three unknowns,* equilibrium will not be maintained under general loading conditions; the structure is said to be *partially constrained.* The fact that the reactions involve exactly three unknowns is no guarantee that the equilibrium equations can be solved for all three unknowns. If the supports are arranged in such a way that the reactions are *either concurrent or parallel,* the reactions are statically indeterminate, and the structure is said to be *improperly constrained.*

Two-force body

Three-force body

Two particular cases of equilibrium of a rigid body were given special attention. In Sec. 4.6, a *two-force body* was defined as a rigid body subjected to forces at only two points, and it was shown that the resultants $\mathbf{F}_1$ and $\mathbf{F}_2$ of these forces must have the *same magnitude, the same line of action, and opposite sense* (Fig. 4.11), a property which will simplify the solution of certain problems in later chapters. In Sec. 4.7, a *three-force body* was defined as a rigid body subjected to forces at only three points, and it was shown that the resultants $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ of these forces must be *either concurrent* (Fig. 4.12) *or parallel.* This properly provides us with an alternative approach to the solution of problems involving a three-force body [Sample Prob. 4.6].

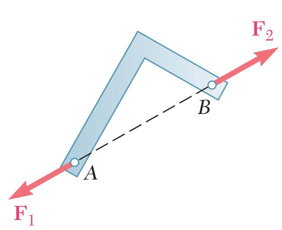


Fig. 4.11

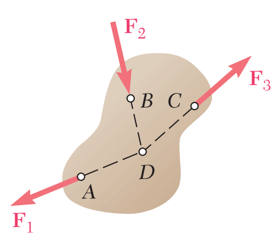


Fig. P4.12

Equilibrium of a three-dimensional body

In the second part of the chapter, we considered the *equilibrium of a three-dimensional body* and saw that each of the reactions exerted on the body by its supports could involve between one and six unknowns, depending upon the type of support [Sec. 4.8].

In the general case of the equilibrium of a three-dimensional body, all of the six scalar equilibrium equations (4.2) and (4.3) listed at the beginning of this review should be used and solved for *six unknowns* [Sec. 4.9]. In most problems, however, these equations will be more conveniently obtained if we first write

$$\Sigma \mathbf{F} = 0 \qquad \Sigma \mathbf{M}_O = \Sigma(\mathbf{r} \times \mathbf{F}) = 0 \tag{4.1}$$

and express the forces $\mathbf{F}$ and position vectors $\mathbf{r}$ in terms of scalar components and unit vectors. The vector products can then be computed either directly or by means of determinants, and the desired scalar equations obtained by equating to zero the coefficients of the unit vectors [Sample Probs. 4.7 through 4.9].

We noted that as many as three unknown reaction components may be eliminated from the computation of $\Sigma\mathbf{M}_O$ in the second of the relations (4.1) through a judicious choice of point O. Also, the reactions at two points A and B can be eliminated from the solution of some problems by writing the equation $\Sigma M_{AB} = 0$, which involves the computation of the moments of the forces about an axis AB joining points A and B [Sample Prob. 4.10].

If the reactions involve more than six unknowns, some of the reactions are *statically indeterminate;* if they involve fewer than six unknowns, the rigid body is only *partially constrained.* Even with six or more unknowns, the rigid body will be *improperly constrained* if the reactions associated with the given supports either are parallel or intersect the same line.

Review Problems

4.152 Beam AD carries the two 40-lb loads shown. The beam is held by a fixed support at D and by the cable BE which is attached to the counter weight W. Determine the reaction at D when (*a*) $W = 100$ lb, (*b*) $W = 90$ lb.

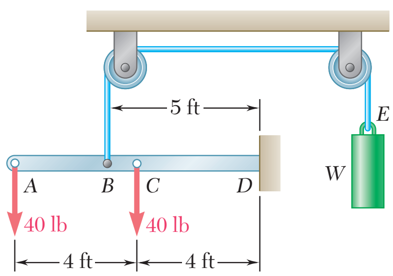


Fig. P4.152 and *P4.153*

4.153 For the beam and loading shown, determine the range of values of W for which the magnitude of the couple at D does not exceed 40 lb · ft.

4.154 Determine the reactions at A and D when $\beta = 30°$.

4.155 Determine the reactions at A and D when $\beta = 60°$.

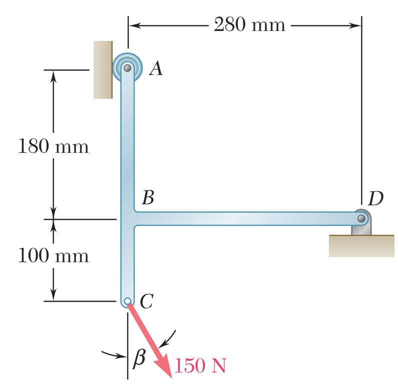


Fig. P4.154 and *P4.155*

4.156 A 2100-lb tractor is used to lift 900 lb of gravel. Determine the reaction at each of the two (*a*) rear wheels A, (*b*) front wheels B.

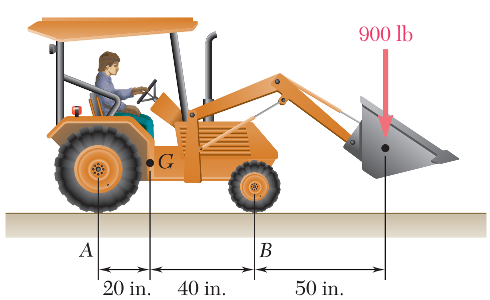


Fig. P4.156

4.157 A tension of 5 lb is maintained in a tape as it passes the support system shown. Knowing that the radius of each pulley is 0.4 in., determine the reaction at C.

4.158 Solve Prob. 4.157 assuming that 0.6-in.-radius pulleys are used.

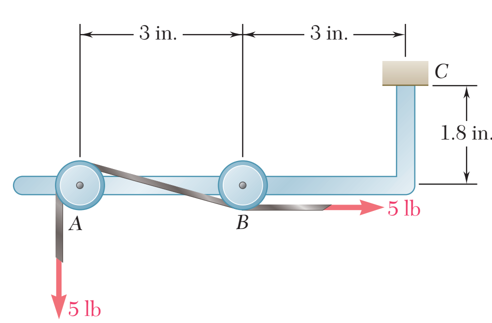


Fig. P4.157

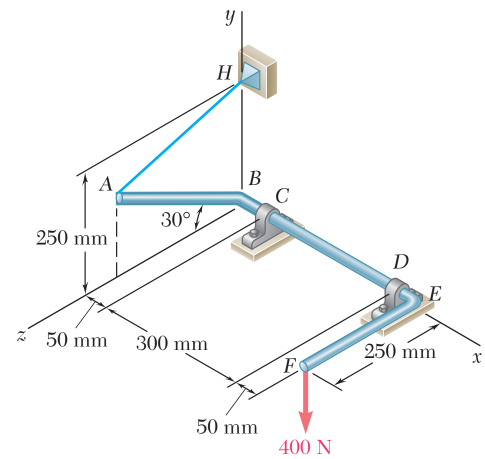


Fig. P4.159

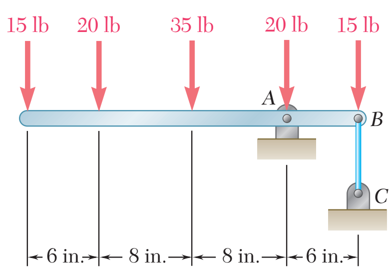


Fig. *P4.160*

4.159 The bent rod *ABEF* is supported by bearings at *C* and *D* and by wire *AH*. Knowing that portion *AB* of the rod is 250 mm long, determine (*a*) the tension in wire *AH*, (*b*) the reactions at *C* and *D*. Assume that the bearing at *D* does not exert any axial thrust.

4.160 For the beam and loading shown, determine (*a*) the reaction at *A*, (*b*) the tension in cable *BC*.

4.161 Frame *ABCD* is supported by a ball-and-socket joint at *A* and by three cables. For $a = 150$ mm, determine the tension in each cable and the reaction at *A*.

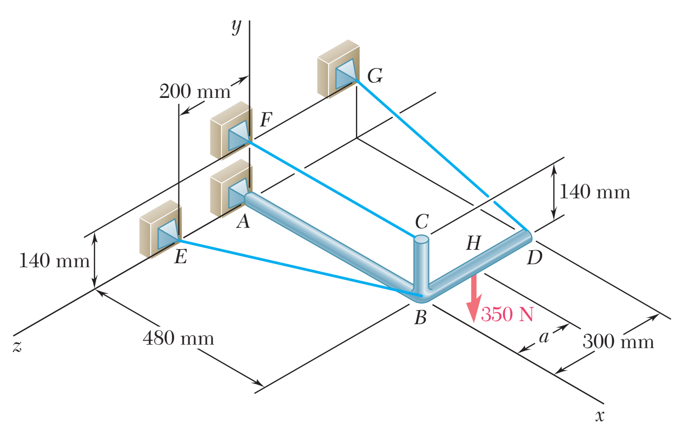


Fig. P4.161 and *P4.162*

4.162 Frame *ABCD* is supported by a ball-and-socket joint at *A* and by three cables. Knowing that the 350-N load is applied at *D* ($a = 300$ mm), determine the tension in each cable and the reaction at *A*.

***4.163** In the problems listed below, the rigid bodies considered were completely constrained and the reactions were statically determinate. For each of these rigid bodies it is possible to create an improper set of constraints by changing a dimension of the body. In each of the following problems determine the value of *a* which results in improper constraints. (*a*) Prob. 4.81, (*b*) Prob. 4.82.

Computer Problems

4.C1 A slender rod AB of weight W is attached to blocks at A and B which can move freely in the guides shown. The constant of the spring is k and the spring is unstretched when the rod is horizontal. Neglecting the weight of the blocks, derive an equation in terms of (θ, W, l, and k which must be satisfied when the rod is in equilibrium. Knowing that $W = 10$ lb and $l = 40$ in., (*a*) calculate and plot the value of the spring constant k as a function of the angle θ for $15° \leq \theta \leq 40°$, (*b*) determine the two values of the angle θ corresponding to equilibrium when $k = 0.7$ lb/in.

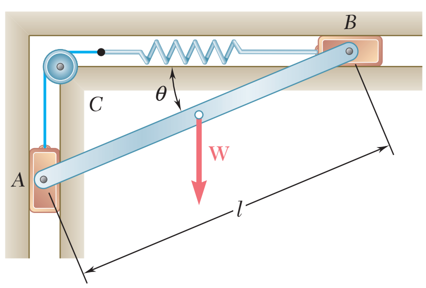


Fig. P4.C1

4.C2 The position of the L-shaped rod shown is controlled by a cable attached at point B. Knowing that the rod supports a load of magnitude $P =$ 200 N, use computational software to calculate and plot the tension T in the cable as a function of θ for values of θ from from 0 to 120°. Determine the maximum tension T_{max} and the corresponding value of θ.

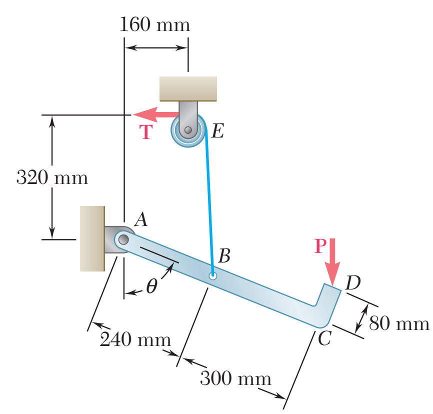


Fig. P4.C2

4.C3 The position of the 20-lb rod AB is controlled by the block shown, which is slowly moved to the left by the force **P**. Neglecting the effect of friction, use computational software to calculate and plot the magnitude P of the force as a function of x for values of x decreasing from 30 in. to 0. Determine the maximum value of P and the corresponding value of x.

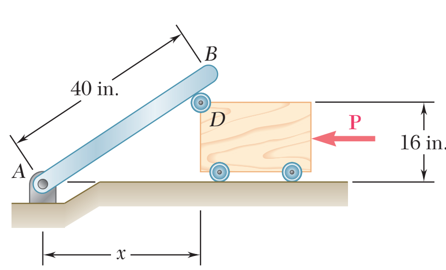


Fig. P4.C3

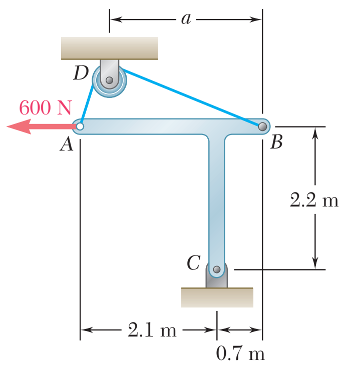


Fig. P4.C4

***4.C4** Member *ABC* is supported by a pin and bracket at *C* and by an inextensible cable of length 3.5 m that is attached at *A* and *B* and passes over a frictionless pulley at *D*. Neglecting the mass of *ABC* and the radius of the pulley, (*a*) plot the tension in the cable as a function of *a* for $0 \le a \le 2.4$ m, (*b*) determine the largest value of *a* for which equilibrium can be maintained.

4.C5 and 4.C6 The constant of spring *AB* is *k*, and the spring is unstretched when $\theta = 0$. Knowing that $R = 200$ mm, $a = 400$ mm, and $k = 1$ kN/m, use computational software to calculate and plot the mass *m* corresponding to equilibrium as a function of θ for values of θ from 0 to 90°. Determine the value of θ corresponding to equilibrium when $m = 2$ kg.

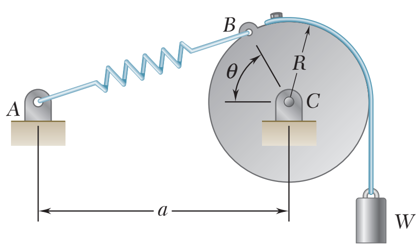


Fig. P4.C5

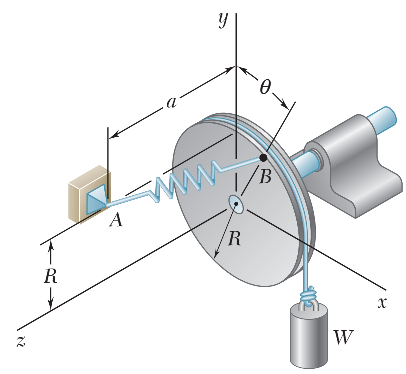


Fig. P4.C6

4.C7 An 8 × 10-in. panel of weight $W = 40$ lb is supported by hinges along edge *AB*. Cable *CDE* is attached to the panel at point *C*, passes over a small pulley at *D*, and supports a cylinder of weight *W*. Neglecting the effect of friction, use computational software to calculate and plot the weight of the cylinder corresponding to equilibrium as a function of θ for values of θ from 0 to 90°. Determine the value of θ corresponding to equilibrium when $W = 20$ lb.

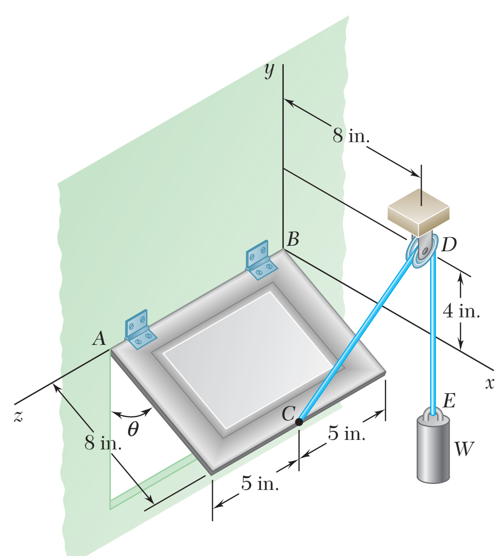


Fig. P4.C7

4.C8 A uniform circular plate of radius 300 mm and mass 26 kg is supported by three vertical wires that are equally spaced around its edge. A small 3-kg block E is placed on the plate at D and is then slowly moved along diameter CD until it reaches C. (a) Plot the tension in wires A and C as functions of a, where a is the distance of the block from D. (b) Determine the value of a for which the tension in wires A and C is minimum.

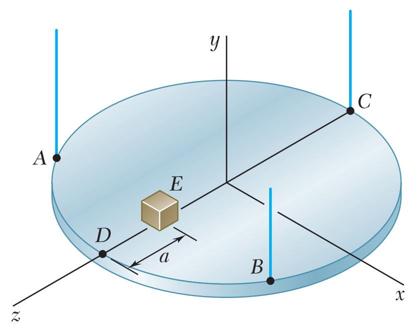


Fig. P4.C8

4.C9 The derrick shown supports a 4000-lb crate. It is held by a ball-and-socket joint at point A and by two cables attached at points D and E. Knowing that the derrick lies in a vertical plane forming an angle ϕ with the xy plane, use computational software to calculate and plot the tension in each cable as a function of ϕ for values of ϕ from 0 to 40°. Determine the value of ϕ for which the tension in cable BE is maximum.

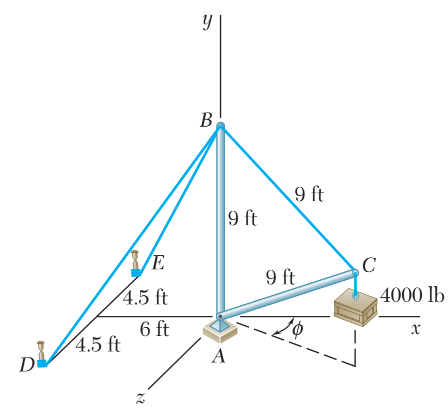


Fig. P4.C9

4.C10 The 140-lb uniform steel plate $ABCD$ is welded to shaft EF and is maintained in the position shown by the couple **M**. Knowing that collars prevent the shaft from sliding in the bearings and that the shaft lies in the yz plane, plot the magnitude M of the couple as a function of θ for $0 \leq \theta \leq 90°$.

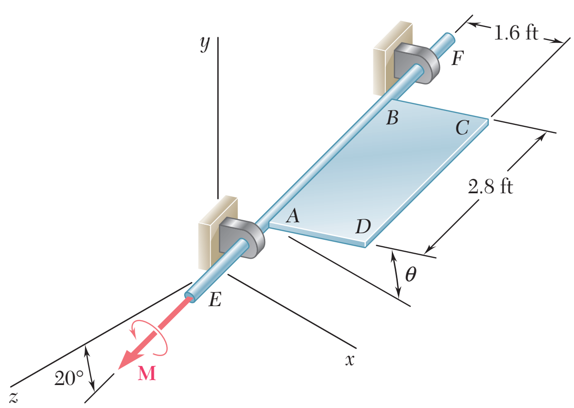


Fig. P4.C10