Excerpted from Chapter 14 "Heat-Transfer Equipment - Design and Costs"

EXAMPLE 14-6 Estimation of Heat-Transfer Coefficient and Pressure Drop on the Shell Side of a Shell-and-Tube Exchanger Using the Kern, Bell-Delaware, and Wills-Johnston Methods

A shell-and-tube exchanger with one shell and one tube pass is being used as a cooler. The cooling medium is water with a flow rate of 11 kg/s on the shell side of the exchanger. With an inside diameter of 0.584 m, the shell is packed with a total of 384 tubes in a staggered (triangular) array. The outside diameter of the tubes is 0.019 m with a clearance between tubes of 0.00635 m. Segmental baffles with a 25 percent baffle cut are used on the shell side, and the baffle spacing is set at 0.1524 m. The length of the exchanger is 3.66 m. (Assume a split backing ring, floating heat exchanger.)

The average temperature of the water is 30°C, and the average temperature of the tube walls on the water side is 40°C. Under these conditions, estimate the heat-transfer coefficient for the water and the pressure drop on the shell side, using the Kern, Bell-Delaware, and Wills and Johnston methods.

■ Solution

The procedures for all three methods have been outlined briefly in the shell-and-tube section. Appendix D provides the following data for water:

	30°C	35°C	40°C
Physical property data			
Thermal conductivity k, kJ/s·m·K	0.000616	0.000623	0.000632
Heat capacity C_p, kJ/kg·K	4.179	4.179	4.179
Viscosity μ, Pa·s	0.000803	0.000724	0.000657
Density ρ, kg/m^3	995	995	995
Exchanger configuration			
Shell internal diameter		$D_s = 0.584$ m	
Tube outside diameter		$D_o = 0.019$ m	
Tube pitch (triangular)		$P_T = 0.0254$ m	
Number of tubes		$N_T = 384$	
Baffle spacing		$L_B = 0.1524$ m	
Shell length		$L_s = 3.66$ m	
Bundle-to-shell diametral clearance†		$\Delta_b = 0.035$ m	
Shell-to-baffle diametral clearance†		$\Delta_{sb} = 0.005$ m	
Tube-to-baffle diametral clearance†		$\Delta_{tb} = 0.0008$ m	
Thickness of baffle†		$t_b = 0.005$ m	
Sealing strips per cross-flow row†		$N_{ss}/N_c = 0.2$	

†Items consistent with recommendations by J. Taborek, in *Heat Exchanger Design Handbook,* Hemisphere Publishing, Washington, 1983, Sec. 3.3.5.

Kern Method

Determine the flow area at the shell centerline. The gap between tubes P_D is given as 0.00635 m. The cross-flow area along the centerline of flow in the shell is given by Eq. (14-32).

$$S_s = \frac{D_s P_D L_B}{P_T} = \frac{0.584(0.00635)(0.1524)}{0.0254} = 0.02225 \text{ m}^2$$

Determine D_e from Eq. (14-33).

$$D_e = \frac{4\left(P_T^2 - \pi D_o^2/4\right)}{\pi D_o} = \frac{4[(0.0254)^2 - (\pi/4)(0.019)^2]}{\pi(0.019)} = 0.02423 \text{ m}$$

The mass flow rate G_s is

$$G_s = \frac{\dot{m}_T}{S_s} = \frac{11}{0.02225} = 494.4 \text{ kg/m}^2\text{·s}$$

To obtain the heat-transfer coefficient at an average water-film temperature requires evaluation of the Reynolds and Prandtl numbers.

$$\text{Re} = \frac{D_e G_s}{\mu_f} = \frac{0.02423(494.4)}{0.000724} = 16{,}550$$

$$\text{Pr} = \left(\frac{C_p \mu}{k}\right)_f = \frac{4.179(0.000724)}{0.000623} = 4.86$$

From Eq. (14-30)

$$h_s = 0.36\left(\frac{k}{D_e}\right)\mathrm{Re}^{0.55}\mathrm{Pr}^{0.33}\left(\frac{\mu}{\mu_w}\right)^{0.14}$$

$$= 0.36\left(\frac{0.623}{0.02423}\right)(16{,}550)^{0.55}(4.86)^{0.33}\left(\frac{0.000803}{0.000657}\right)^{0.14}$$

$$= 3369\ \mathrm{W/m^2{\cdot}K}$$

Calculate the pressure drop on the shell side, assuming no effect for any type of fluid leakage. The number of baffles on the shell side is obtained from Eq. (14-36).

$$N_B = \frac{L_s}{L_B + t_b} - 1 = \frac{3.66}{0.1524 + 0.005} - 1 = 22.2 \text{ or } 22$$

For a shell-side Reynolds number of 16,550, Fig. 14-44 provides a value of 0.062 for the friction factor. The pressure drop is obtained from Eq. (14-35) as

$$\Delta p_s = \frac{4fG_s^2 D_s(N_B + 1)}{2\rho D_e(\mu/\mu_w)_s^{0.14}}$$

$$= \frac{4(0.062)(494.4)^2(0.584)(22+1)}{2(995)(0.02423)(0.000803/0.000657)^{0.14}} = 16{,}420\ \mathrm{Pa}$$

Bell-Delaware Method

The first step in this method is to calculate the ideal cross-flow heat-transfer coefficient. Calculate $V_{\max}$ from Eq. (14-39) and obtain S_m from Eq. (14-40) to substitute into Eq. (14-22).

$$S_m = L_B\left[D_s - D_{OTL} + \frac{(D_{OTL} - D_o)(P_T - D_o)}{P_T}\right] \qquad \text{where } D_{OTL} = D_s - \Delta_b = 0.549$$

$$= 0.1524\left[0.035 + \frac{(0.549 - 0.019)(0.0254 - 0.019)}{0.0254}\right] = 0.0255\ \mathrm{m^2}$$

$$V_{\max} = \frac{\dot{m}_T}{\rho S_m} = \frac{11}{995(0.0255)} = 0.4335\ \mathrm{m/s}$$

$$\mathrm{Re} = \frac{\rho V_{\max} D_o}{\mu} = \frac{995(0.4335)(0.019)}{0.000803} = 10{,}205$$

$$\mathrm{Pr} = \frac{C_p \mu}{k} = \frac{4.179(0.000803)}{0.000616} = 5.449$$

The ideal heat-transfer coefficient is given by

$$h_i = \frac{k}{D_o} a\,\mathrm{Re}^m\,\mathrm{Pr}^{0.34} F_1 F_2$$

where constants a and m are obtained from Table 14-1 for a staggered tube array, F_1 from Eq. (14-22*b*), and F_2 from Table 14-2.

$$h_i = \left(\frac{0.616}{0.019}\right)(0.273)(10{,}205)^{0.635}(5.449)^{0.34}\left(\frac{5.449}{4.345}\right)^{0.26}(0.99)$$

$$= 5807\ \mathrm{W/m^2{\cdot}K}$$

The actual shell-side heat-transfer coefficient is obtained from Eq. (14-41). This requires obtaining values for J_C, J_L, and J_B using the appropriate correction factors to account for baffle configuration, leakage, and bypass. Equation (14-42) permits calculation of F_c

$$F_c = \frac{1}{\pi}\left[\pi + \frac{2(D_s - 2L_c)}{D_{OTL}} \sin\left(\cos^{-1}\frac{D_s - 2L_c}{D_{OTL}}\right) - 2\cos^{-1}\frac{D_s - 2L_c}{D_{OTL}}\right]$$

For a baffle cut of 25 percent

$$L_c = 0.25D_s = 0.25(0.584) = 0.146 \text{ m}$$

$$\frac{D_s - 2L_c}{D_{OTL}} = \frac{0.584 - 2(0.146)}{0.549} = 0.5318$$

$$F_c = \frac{1}{\pi}[\pi + 2(0.5318)\sin(\cos^{-1}0.5318) - 2\cos^{-1}0.5318] = 0.6437$$

From Fig. 14-45

$$J_C = 0.55 + 0.72F_c = 0.55 + 0.72(0.6437) = 1.013$$

To obtain J_L, calculate the leakage areas S_{sb} and S_{tb} from Eqs. (14-43*a*) and (14-43*b*), respectively.

$$S_{sb} = D_s\left(\frac{\Delta_{sb}}{2}\right)\left[\pi - \cos^{-1}\left(1 - \frac{2L_c}{D_s}\right)\right]$$

$$= (0.584)\left(\frac{0.005}{2}\right)\left\{\pi - \cos^{-1}\left[1 - \frac{2(0.146)}{0.584}\right]\right\} = 0.003058 \text{ m}^2$$

$$S_{tb} = \pi D_o\left(\frac{\Delta_{tb}}{2}\right) N_T \frac{1 + F_c}{2}$$

$$= \pi(0.019)\left(\frac{0.0008}{2}\right)(384)\left(\frac{1 + 0.6437}{2}\right) = 0.007535 \text{ m}^2$$

The correction factor J_L is obtained from Fig. 14-46, utilizing S_{sb} and S_{tb}.

$$\frac{S_{sb} + S_{tb}}{S_m} = \frac{0.003058 + 0.007535}{0.0255} = 0.4154$$

$$\frac{S_{sb}}{S_{sb} + S_{tb}} = \frac{0.003058}{0.003058 + 0.007535} = 0.2887$$

Figure 14-46 provides a value of 0.56 for J_L.

To obtain the correction factor J_B for bypass in the bundle-shell gap, obtain F_{bp}, the fraction of the cross-flow area available for bypass flow, with Eq. (14-44).

$$F_{bp} = \frac{L_B}{S_m}(D_s - D_{OTL}) = \frac{0.1524}{0.0255}(0.035) = 0.2092$$

Note that $F_{bp} = S_b/S_m$, and Fig. 14-47 can be used to obtain a J_B value of 0.935 when $N_{ss}/N_c = 0.2$. The corrected heat-transfer coefficient from Eq. (14-41) is then

$$h = h_i J_c J_L J_B$$

$$= 5807(1.013)(0.56)(0.935) = 3080 \text{ W/m}^2\cdot\text{K}$$

Evaluation of the pressure drop using the Bell-Delaware method is similar to the process for obtaining the heat-transfer coefficient. The ideal cross-flow pressure drop through one baffle space is obtained with the use of Eq. (14-46).

$$\Delta p_c = (K_a + N_c K_f)\left(\frac{\rho V_{\max}^2}{2}\right) \qquad \text{assume } K_a = 1.5$$

$$N_c = \frac{D_s}{P_{TP}}\left(1 - \frac{2L_c}{D_s}\right) \qquad P_{TP} = 0.866 P_T \text{, for triangular array}$$

$$= \frac{0.584}{0.866(0.0254)}\left[1 - \frac{2(0.146)}{0.584}\right] = 13.27$$

A value of 0.495 for K_f is obtained by using the following relation given in the footnote of Table 14-10:

$$K_f = 0.245 + \frac{0.339 \times 10^4}{\text{Re}} - \frac{0.984 \times 10^7}{\text{Re}^2} + \frac{0.133 \times 10^{11}}{\text{Re}^3} - \frac{0.599 \times 10^{13}}{\text{Re}^4}$$

$$\Delta p_c = [1.5 + 13.27(0.495)](995)\frac{(0.4335)^2}{2} = 754 \text{ Pa}$$

Calculate the window zone pressure loss from Eq. (14-47*b*). First, determine the window flow area S_w from Eq. (14-49).

$$S_w = \frac{D_s^2}{4}\left[\cos^{-1} D_B - D_B\left(1 - D_B^2\right)^{1/2}\right] - \frac{N_T}{8}(1 - F_c)\pi D_o^2$$

where

$$D_B = \frac{D_s - 2L_c}{D_s} = 1 - \frac{2L_c}{D_s} = 1 - \frac{2(0.146)}{0.584} = 0.5$$

$$S_w = \frac{(0.584)^2}{4}\{\cos^{-1} 0.5 - 0.5[1 - (0.5)^2]^{1/2}\} - \left(\frac{384}{8}\right)(1 - 0.6437)\pi(0.019)^2$$

$$= 0.03298 \text{ m}^2$$

Next, calculate the number of effective cross-flow rows in the window zone from Eq. (14-48).

$$N_{cw} = \frac{0.8L_c}{P_{TP}} = \frac{0.8(0.146)}{0.866(0.0254)} = 5.31$$

Now calculate the window zone pressure drop for Re > 100.

$$\Delta p_w = \frac{(2 + 0.6N_{cw})\dot{m}_T^2}{2S_m S_w \rho}$$

$$= [2 + 0.6(5.31)]\frac{(11)^2}{2(0.0255)(0.03298)(995)} = 375 \text{ Pa}$$

Finally, estimate the leakage and bypass correction factors R_B and R_L. To obtain R_B, use the calculated values of F_{bp} and $N_{\text{ss}}/N_c = 0.2$ with Fig. 14-48. This gives a value of 0.82 for R_B. For R_L use the area ratio values of $(S_{sb} + S_{tb})/S_m$ and $S_{sb}/(S_{sb} + S_{tb})$ with Fig. 14-49 to obtain a value of 0.365 for R_L.

The pressure drop across the shell is given by Eq. (14-51).

$$\Delta p_s = [(N_B - 1)\,\Delta p_c R_B + N_B \Delta p_w]R_L + 2\,\Delta p_c R_B\left(1 + \frac{N_{cw}}{N_c}\right)$$

$$= [(22-1)(754)(0.82) + 22(375)](0.365) + 2(754)(0.82)\left(1 + \frac{5.31}{13.27}\right)$$

$$= 7750 + 1731 = 9481 \text{ Pa}$$

Wills and Johnston Method

The heat-transfer coefficient calculated in this method is similar to that used in the Bell-Delaware method except that the value of the Reynolds number is estimated from $\dot{m}_c = F_{cr}\dot{m}_T$. To determine F_{cr} requires evaluating the flow stream resistance coefficients in Fig. 14-50 as defined in Eqs. (14-55*a*) through (14-55*c*), (14-56), (14-58), (14-60), and (14-61).

Calculate the shell-to-baffle resistance coefficient n_s, using Eqs. (14-56) and (14-57).

$$S_s = \pi\left(D_s - \frac{\Delta_{sb}}{2}\right)\left(\frac{\Delta_{sb}}{2}\right) = \pi\left(0.584 - \frac{0.005}{2}\right)\left(\frac{0.005}{2}\right) = 0.004567 \text{ m}^2$$

$$n_s = \frac{0.036(2t_b/\Delta_{sb}) + 2.3(2t_b/\Delta_{sb})^{-0.177}}{2\rho S_s^2}$$

$$= \frac{0.036(2)(0.005)/0.005 + 2.3[2(0.005)/0.005]^{-0.177}}{2(995)(0.004567)^2}$$

$$= 50.75$$

Calculate the tube-to-baffle clearance resistance coefficient n_t from Eqs. (14-58) and (14-59).

$$S_t = N_T\pi\left(D_o + \frac{\Delta_{tb}}{2}\right)\left(\frac{\Delta_{tb}}{2}\right)$$

$$= 384\pi(0.019 + 0.0004)(0.0004) = 0.00936 \text{ m}^2$$

$$n_t = \frac{0.036(2t_b/\Delta_{tb}) + 2.3(2t_b/\Delta_{tb})^{-0.177}}{2\rho S_t^2}$$

$$= \frac{0.036(2)(0.005/0.0008) + 2.3[2(0.005/0.0008)]^{-0.177}}{2(995)(0.00936)^2}$$

$$= 11.02$$

Calculate the window flow resistance coefficient n_w from Eq. (14-60).

$$n_w = \frac{1.9e^{0.6856\,S_w/S_m}}{2\rho S_w^2}$$

where $S_m = 0.0255 \text{ m}^2$ and $S_w = 0.03298 \text{ m}^2$ from the Bell-Delaware calculations.

$$n_w = \frac{1.9\exp[0.6856(0.03298/0.0255)]}{2(995)(0.03298)^2} = 2.13$$

The bypass flow resistance coefficient n_b is calculated from Eqs. (14-61) and (14-62).

$$S_b = (\Delta_b + \Delta_{pp})L_B \qquad \text{assume } \Delta_{pp} \cong 0$$
$$= (0.035 + 0)(0.1524) = 0.00533 \text{ m}^2$$

$$N_{ss} = N_c \frac{N_{ss}}{N_c} = 13.27(0.2) = 2.65 \cong 3$$

$$n_b = \frac{a(D_s - 2L_c)/P_{TP} + N_{ss}}{2\rho S_b^2}$$

Since $N_c = (D_s/P_{TP})(1 - 2L_c/D_s)$, this can be rearranged and simplified to

$$n_b = \frac{aN_c + N_{ss}}{2\rho S_b^2} \qquad \text{where } a = 0.133 \text{ for triangular arrays}$$
$$= \frac{0.133(13.27) + 3}{2(995)(0.00533)^2} = 84.2$$

For a first approximation assume that the fraction F_{cr} of the flow that is in cross-flow over the bundle is 0.5 to initiate a calculation for the flow resistance coefficient n_c. For an F_{cr} of 0.5

$$\text{Re} = \frac{D_o \dot{m}_T F_{cr}}{S_m \mu}$$
$$= \frac{0.019(11)(0.5)}{0.0255(0.000803)} = 5103$$

The flow resistance coefficient n_c is evaluated by using Eq. (14-64) where K_f is obtained from the relation given in Table 14-10 for a triangular tube array with $10^3 < \text{Re} < 10^6$.

$$K_f = 0.245 + \frac{0.339 \times 10^4}{\text{Re}} - \frac{0.984 \times 10^7}{\text{Re}^2} + \frac{0.133 \times 10^{11}}{\text{Re}^3} - \frac{0.599 \times 10^{13}}{\text{Re}^4}$$
$$K_f(\text{Re} = 5103) = 0.6227$$

Calculate n_c, n_{cb}, n_a, and n_p to determine a new value for F_{cr}.

$$n_c = \frac{K_a + N_c K_f}{2\rho S_m^2} \qquad \text{assume } K_a = 1.5$$
$$= \frac{1.5 + 13.27(0.6227)}{2(995)(0.0255)^2} = 7.55$$
$$n_{cb} = \left(n_c^{-1/2} + n_b^{-1/2}\right)^{-2}$$
$$= (7.55^{-1/2} + 84.2^{-1/2})^{-2} = 4.47$$
$$n_a = n_w + n_{cb} = 2.13 + 4.47 = 6.60$$
$$n_p = \left(n_a^{-1/2} + n_s^{-1/2} + n_t^{-1/2}\right)^{-2}$$
$$= [(6.60)^{-1/2} + (50.75)^{-1/2} + (11.02)^{-1/2}]^{-2} = 1.47$$

Now calculate a new F_{cr} with Eq. (14-65).

$$F_{cr} = \frac{(n_p/n_a)^{1/2}}{1 + (n_c/n_b)^{1/2}}$$
$$= \frac{(1.47/6.60)^{1/2}}{1 + (7.55/84.2)^{1/2}} = 0.363$$

Repeat the above calculations beginning with the Reynolds number evaluation to determine a new value for F_{cr} until a convergence value for F_{cr} is obtained. The iteration results are shown below.

	Iteration attempts			
	1	**2**	**3**	**4**
F_{cr} (initial)	0.50	0.363	0.355	0.354
Re	5103	3705	3618	3614
K_f	0.6227	0.6729	0.676	0.676
n_c	7.55	8.06	8.09	8.09
n_{cb}	4.47	4.70	4.72	4.72
n_a	6.60	6.83	6.85	6.85
n_p	1.47	1.47	1.474	1.474
F_{cr} (calc.)	0.363	0.355	0.354	0.354

The iteration establishes F_{cr} at a value of 0.354 and fixes the Reynolds number for this calculation of the heat-transfer coefficient from Eq. (14-22) with constants a and m listed in Table 14-1, and F_1 and F_2 obtained from Eq. (14-22a) and Table 14-2, respectively.

$$h = \frac{k}{D_o} a \mathrm{Re}^m \mathrm{Pr}^{0.34} F_1 F_2$$
$$= \frac{0.616}{0.019}(0.273)(3614)^{0.635}(5.449)^{0.34}\left(\frac{5.449}{4.345}\right)^{0.26}(0.99)$$
$$= 3004 \text{ W/m}^2\cdot\text{K}$$

For the pressure drop calculation determine the various flow fractions.

Equation (14-66) for shell-to-baffle leakage flow:

$$F_s = \left(\frac{n_p}{n_t}\right)^{1/2} = \left(\frac{1.474}{50.75}\right)^{1/2} = 0.1704$$

Equation (14-67) for tube-to-baffle leakage:

$$F_t = \left(\frac{n_p}{n_t}\right)^{1/2} = \left(\frac{1.474}{11.02}\right)^{1/2} = 0.3657$$

Equation (14-68) for bypass flow:

$$F_b = \frac{(n_p/n_a)^{1/2}}{1 + (n_b/n_c)^{1/2}}$$
$$= \frac{(1.474/6.85)^{1/2}}{1 + (84.2/8.09)^{1/2}} = 0.1098$$

Check on the flow fractions that should equal unity.

$$F_s + F_t + F_b + F_{cr} \equiv 1.000$$

$$0.1704 + 0.3657 + 0.1098 + 0.3540 = 0.9999 \qquad \text{good check}$$

Calculate the total pressure drop per baffle on the shell side, using Eq. (14-54*b*).

$$\Delta p = n_p \dot{m}_T^2 = (1.474)(11)^2 = 178.4 \text{ Pa}$$

The total shell-side pressure drop is given by

$$\Delta p_s = (N + 1)\Delta p = (22 + 1)(178.4) = 4103 \text{ Pa}$$

A comparison of the results for the shell-side heat-transfer coefficient and shell-side pressure drop from the three methods as well as from a widely used computer program is shown below:

Method	***h*, W/m^2·K**	**Δ*p*, Pa**
Kern	3,369	16,420
Bell-Delaware	3,080	9,481
Wills-Johnston	3,004	4,103
Computer (CC-Therm)	3,035	4,155

Note that the Kern method provides higher values for the heat-transfer coefficient and pressure drop on the shell side. The Bell-Delaware and Wills-Johnston methods provide similar results for the heat-transfer coefficient.