

2. The difference between a firm's available balance and its book balance is the firm's net float. The float reflects the fact that some checks have not cleared and are thus uncollected. The financial manager must always work with collected cash balances and not with the company's book balance. To do otherwise is to use the bank's cash without the bank's knowing it, which raises ethical and legal questions.
3. The firm can make use of a variety of procedures to manage the collection and disbursement of cash in such a way as to speed up the collection of cash and slow down the payments. Some methods to speed up the collection are the use of lockboxes, concentration banking, and wire transfers.
4. Because of seasonal and cyclical activities, to help finance planned expenditures, or as a contingency reserve, firms temporarily hold a cash surplus. The money market offers a variety of possible vehicles for "parking" this idle cash.

Chapter Review and Self-Test Problem

- 20.1 Float Measurement** On a typical day, a firm writes checks totaling \$3,000. These checks clear in seven days. Simultaneously, the firm receives \$1,700. The cash is available in two days on average. Calculate the disbursement, the collection, and the net floats. How do you interpret the answer?

Answer to Chapter Review and Self-Test Problem

- 20.1** The disbursement float is $7 \text{ days} \times \$3,000 = \$21,000$. The collection float is $2 \text{ days} \times (-\$1,700) = -\$3,400$. The net float is $\$21,000 + (-\$3,400) = \$17,600$. In other words, at any given time, the firm typically has uncashed checks outstanding of \$21,000. At the same time, it has uncollected receipts of \$3,400. Thus, the firm's book balance is typically \$17,600 less than its available balance, for a positive \$17,600 net float.

Concepts Review and Critical Thinking Questions

1. **Cash Management** Is it possible for a firm to have too much cash? Why would shareholders care if a firm accumulates large amounts of cash?
2. **Cash Management** What options are available to a firm if it believes it has too much cash? How about too little?
3. **Agency Issues** Are stockholders and creditors likely to agree on how much cash a firm should keep on hand?
4. **Motivations for Holding Cash** Most often, when news breaks about a firm's cash position, it's because the firm is running low, but that wasn't the case for Ford, Chrysler, and General Motors at the end of 1997. At that time, Ford held \$20.8 billion in cash and marketable securities, GM had \$14.5 billion, and Chrysler had \$7.1 billion. Similarly, by the end of 1998, Nissan, the Japanese auto manufacturer, had piled up about 400 billion yen, which amounted to several billion dollars. Thus, each company had substantial cash reserves; in fact, particularly in Ford's case, *enormous* might be more descriptive. Why would firms such as these hold such large quantities of cash?

5. **Cash Management versus Liquidity Management** What is the difference between cash management and liquidity management?
6. **Short-Term Investments** Why is a preferred stock with a dividend tied to short-term interest rates an attractive short-term investment for corporations with excess cash?
7. **Collection and Disbursement Floats** Which would a firm prefer: a net collection float or a net disbursement float? Why?
8. **Float** Suppose a firm has a book balance of \$2 million. At the automatic teller machine (ATM), the cash manager finds out that the bank balance is \$2.5 million. What is the situation here? If this is an ongoing situation, what ethical dilemma arises?
9. **Short-Term Investments** For each of the short-term marketable securities given here, provide an example of the potential disadvantages the investment has for meeting a corporation's cash management goals.
 - a. U.S. Treasury bills
 - b. Ordinary preferred stock
 - c. Negotiable certificates of deposit (NCDs)
 - d. Commercial paper
 - e. Revenue anticipation notes
 - f. Repurchase agreements
10. **Agency Issues** It is sometimes argued that excess cash held by a firm can aggravate agency problems (discussed in Chapter 1) and, more generally, reduce incentives for shareholder wealth maximization. How would you frame the issue here?
11. **Use of Excess Cash** One option a firm usually has with any excess cash is to pay its suppliers more quickly. What are the advantages and disadvantages of this use of excess cash?
12. **Use of Excess Cash** Another option usually available is to reduce the firm's outstanding debt. What are the advantages and disadvantages of this use of excess cash?
13. **Float** An unfortunately common practice goes like this (warning: don't try this at home): Suppose you are out of money in your checking account; however, your local grocery store will, as a convenience to you as a customer, cash a check for you. So, you cash a check for \$200. Of course, this check will bounce unless you do something. To prevent this, you go to the grocery the next day and cash another check for \$200. You take this \$200 and deposit it. You repeat this process every day, and, in doing so, you make sure that no checks bounce. Eventually, manna from heaven arrives (perhaps in the form of money from home), and you are able to cover your outstanding checks.

To make it interesting, suppose you are absolutely certain that no checks will bounce along the way. Assuming this is true, and ignoring any question of legality (what we have described is probably illegal check kiting), is there anything unethical about this? If you say yes, then why? In particular, who is harmed?

Questions and Problems

1. **Calculating Float** In a typical month, the Bungee Jump Corporation receives 100 checks totaling \$90,000. These are delayed six days on average. What is the average daily float? **Basic**
(Questions 1–10)

Basic*(continued)*

2. **Calculating Net Float** Each business day, on average, a company writes checks totaling \$30,000 to pay its suppliers. The usual clearing time for the checks is four days. Meanwhile, the company is receiving payments from its customers each day, in the form of checks, totaling \$50,000. The cash from the payments is available to the firm after two days.
 - a. Calculate the company's disbursement float, collection float, and net float.
 - b. How would your answer to part (a) change if the collected funds were available in one day instead of two?
3. **Costs of Float** Purple Feet Wine, Inc., receives an average of \$6,000 in checks per day. The delay in clearing is typically five days. The current interest rate is .025 percent per day.
 - a. What is the company's float?
 - b. What is the most Purple Feet should be willing to pay today to eliminate its float entirely?
 - c. What is the highest daily fee the company should be willing to pay to eliminate its float entirely?
4. **Float and Weighted Average Delay** Your neighbor goes to the post office once a month and picks up two checks, one for \$20,000 and one for \$4,000. The larger check takes four days to clear after it is deposited; the smaller one takes six days.
 - a. What is the total float for the month?
 - b. What is the average daily float?
 - c. What are the average daily receipts and weighted average delay?
5. **NPV and Collection Time** Your firm has an average receipt size of \$60. A bank has approached you concerning a lockbox service that will decrease your total collection time by three days. You typically receive 12,000 checks per day. The daily interest rate is .018 percent. If the bank charges a fee of \$225 per day, should the lockbox project be accepted? What would the net annual savings be if the service were adopted?
6. **Using Weighted Average Delay** A mail-order firm processes 5,000 checks per month. Of these, 60 percent are for \$50 and 40 percent are for \$70. The \$50 checks are delayed two days on average; the \$70 checks are delayed three days on average.
 - a. What is the average daily collection float? How do you interpret your answer?
 - b. What is the weighted average delay? Use the result to calculate the average daily float.
 - c. How much should the firm be willing to pay to eliminate the float?
 - d. If the interest rate is 8 percent per year, calculate the daily cost of the float.
 - e. How much should the firm be willing to pay to reduce the weighted average float by 1.5 days?
7. **Value of Lockboxes** Paper Submarine Manufacturing is investigating a lockbox system to reduce its collection time. It has determined the following:

Average number of payments per day	300
Average value of payment	\$1,500
Variable lockbox fee (per transaction)	\$.75
Daily interest rate on money market securities	.02%

The total collection time will be reduced by three days if the lockbox system is adopted.

- a. What is the PV of adopting the system?
 b. What is the NPV of adopting the system?
 c. What is the net cash flow per day from adopting? Per check?
8. **Lockboxes and Collections** It takes Cookie Cutter Modular Homes, Inc., about five days to receive and deposit checks from customers. Cookie Cutter's management is considering a lockbox system to reduce the firm's collection times. It is expected that the lockbox system will reduce receipt and deposit times to three days total. Average daily collections are \$140,000, and the required rate of return is 10 percent per year.
- a. What is the reduction in outstanding cash balances as a result of implementing the lockbox system?
 b. What is the dollar return that could be earned on these savings?
 c. What is the maximum monthly charge Cookie Cutter should pay for this lockbox system?
9. **Value of Delay** Pain Free Dentistry, Inc., disburses checks every two weeks that average \$80,000 and take seven days to clear. How much interest can the company earn annually if it delays transfer of funds from an interest-bearing account that pays .02 percent per day for these seven days? Ignore the effects of compounding interest.
10. **NPV and Reducing Float** Puddle of Mudd Corporation has an agreement with Lollipop Bank whereby the bank handles \$6 million in collections a day and requires a \$500,000 compensating balance. Puddle of Mudd is contemplating canceling the agreement and dividing its eastern region so that two other banks will handle its business. Banks A and B will each handle \$3 million of collections a day, and each requires a compensating balance of \$300,000. Puddle of Mudd's financial management expects that collections will be accelerated by one day if the eastern region is divided. Should the company proceed with the new system? What will be the annual net savings? Assume that the T-bill rate is 4 percent annually.
11. **Lockboxes and Collection Time** Bird's Eye Treehouses, Inc., a Kentucky company, has determined that a majority of its customers are located in the Pennsylvania area. It therefore is considering using a lockbox system offered by a bank located in Pittsburgh. The bank has estimated that use of the system will reduce collection time by two days. Based on the following information, should the lockbox system be adopted?

Average number of payments per day	700
Average value of payment	\$1,100
Variable lockbox fee (per transaction)	\$.35
Annual interest rate on money market securities	6.0%

How would your answer change if there were a fixed charge of \$1,000 per year in addition to the variable charge?

12. **Calculating Transactions Required** Bumper Crop, Inc., a large fertilizer distributor based in California, is planning to use a lockbox system to speed up collections from its customers located on the East Coast. A Philadelphia-area bank will provide this service for an annual fee of \$30,000 plus 10 cents per transaction. The estimated reduction in collection and processing time is one day. If the average customer payment in this region is \$6,000, how many customers each

Basic*(continued)***Intermediate***(Questions 11–12)*

Basic*(continued)***Challenge**

(Question 13)

day, on average, are needed to make the system profitable for Bumper Crop? Treasury bills are currently yielding 5 percent per year.

13. **Concentration Banking** Mojo Corporation currently employs a lockbox system with collection centers in San Francisco, St. Louis, Atlanta, and Boston. Each lockbox center, on average, handles \$150,000 in payments every day. Mojo's current policy is to invest these payments in short-term marketable securities daily at the collection center banks. Every two weeks, the investment accounts are swept and the proceeds are wire-transferred to Mojo's headquarters in Dallas to meet the company's payroll. The investment accounts pay .015 percent per day, and wire transfers cost .15 percent of the amount transferred.
- What is Mojo's total net cash flow available from its lockbox system to meet the payroll?
 - Suppose Late Nite Bank, located just outside Dallas, offers to set up a concentration bank system for Mojo. Late Nite will accept each of the lockbox center's daily payments via automated clearinghouse, ACH, transfers (in lieu of wire transfers) and deposit the funds in the same marketable securities investments yielding .015 percent per day. ACH-transferred funds are not available for use for one day. If the ACH transfers cost \$700 each, should Mojo proceed with the concentration bank plan?
 - In part (b), at what cost of ACH transfers would Mojo be indifferent between the two systems?


What's On the Web?

- 20.1 **Commercial Paper** Chevron sells commercial paper to interested institutional investors. Go to the Chevron web site at www.chevron.com, follow the "Investors" link and the "Commercial Paper" link. What is the credit rating for Chevron's commercial paper? Now follow the link for the commercial paper and find the "Offering Memorandum" link. What is the minimum size Chevron will sell? What size do they request for one- to four-day commercial paper?
- 20.2 **Commercial Paper Rates** What are the highest and lowest historical interest rates for commercial paper? Go to www.economagic.com and follow the "Interest Rates" link. Find the highest and lowest interest rates for one-, two-, and three-month nonfinancial commercial paper over the time reported. When did they occur?
- 20.3 **ACH Services** One provider of float reduction services is ePayment Systems. You can find their web site at www.epaymentssystems.com. Follow the "Services" link and then the "Electronic Check Conversion" link. What does an electronic check conversion accomplish? How does the system work? How long does it take for the funds to be deposited in the merchant's account?

Spreadsheet Templates 20-3, 20-5, 20-6, 20-8



20A

DETERMINING THE TARGET CASH BALANCE

target cash balance

A firm's desired cash level as determined by the trade-off between carrying costs and shortage costs.

Based on our general discussion of current assets in the previous chapter, the **target cash balance** involves a trade-off between the opportunity costs of holding too much cash (the carrying costs) and the costs of holding too little (the shortage costs, also

called **adjustment costs**). The nature of these costs depends on the firm's working capital policy.

If the firm has a flexible working capital policy, then it will probably maintain a marketable securities portfolio. In this case, the adjustment, or shortage, costs will be the trading costs associated with buying and selling securities. If the firm has a restrictive working capital policy, it will probably borrow in the short term to meet cash shortages. The costs in this case will be the interest and other expenses associated with arranging a loan.

In our discussion that follows, we will assume that the firm has a flexible policy. Its cash management, then, consists of moving money in and out of marketable securities. This is a very traditional approach to the subject, and it is a nice way of illustrating the costs and benefits of holding cash. Keep in mind, however, that the distinction between cash and money market investments is becoming increasingly blurred.

For example, how do we classify a money market fund with check-writing privileges? Such near-cash arrangements are becoming more and more common. It may be that the prime reason they are not universal is regulation limiting their usage. We will return to this subject of such arrangements at various points in the following discussion.

The Basic Idea

Figure 20A.1 presents the cash management problem for our flexible firm. If a firm tries to keep its cash holdings too low, it will find itself running out of cash more often than is desirable, and thus selling marketable securities (and perhaps later buying marketable securities to replace those sold) more frequently than would be the case if the cash balance were higher. Thus, trading costs will be high when the cash balance is small. These costs will fall as the cash balance becomes larger.

adjustment costs
The costs associated with holding too little cash. Also, shortage costs.

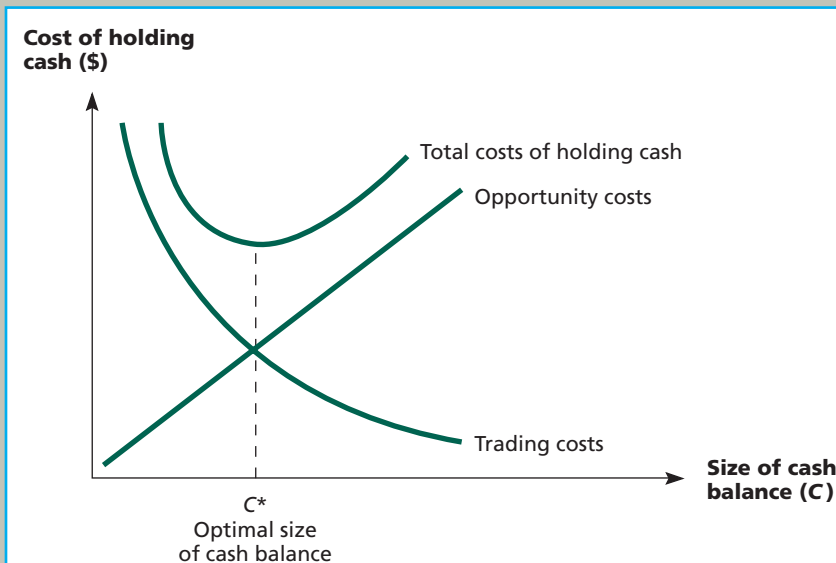


FIGURE 20A.1
Cost of Holding Cash

Trading costs are increased when the firm must sell securities to establish a cash balance. Opportunity costs are increased when there is a cash balance because there is no return on cash.

In contrast, the opportunity costs of holding cash are very low if the firm holds very little cash. These costs increase as the cash holdings rise because the firm is giving up more and more in interest that could have been earned.

In Figure 20A.1, the sum of the costs is given by the total cost curve. As shown, the minimum total cost occurs where the two individual cost curves cross at Point C^* . At this point, the opportunity costs and the trading costs are equal. This point represents the target cash balance, and it is the point the firm should try to find.

Figure 20A.1 is essentially the same as Figure 19.2 in the previous chapter. As we discuss next, however, we can now say more about the optimum investment in cash and the factors that influence it.

The BAT Model

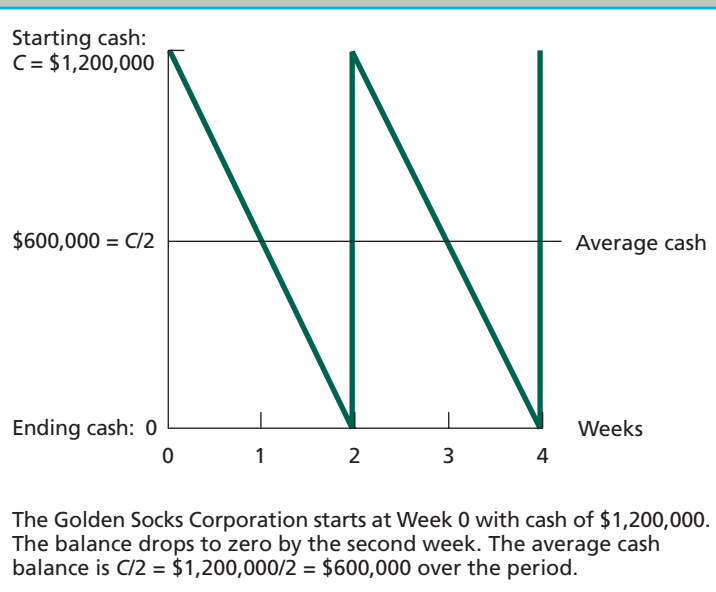
The Baumol-Allais-Tobin (BAT) model is a classic means of analyzing our cash management problem. We will show how this model can be used to actually establish the target cash balance. It is a straightforward model and very useful for illustrating the factors in cash management and, more generally, current asset management.

To develop the BAT model, suppose the Golden Socks Corporation starts off at Week 0 with a cash balance of $C = \$1.2$ million. Each week, outflows exceed inflows by \$600,000. As a result, the cash balance will drop to zero at the end of Week 2. The average cash balance will be the beginning balance (\$1.2 million) plus the ending balance (\$0) divided by 2, or $(\$1.2 \text{ million} + 0)/2 = \$600,000$, over the two-week period. At the end of Week 2, Golden Socks replenishes its cash by depositing another \$1.2 million.

As we have described, the cash management strategy for Golden Socks is very simple and boils down to depositing \$1.2 million every two weeks. This policy is shown in Figure 20A.2. Notice how the cash balance declines by \$600,000 per week. Because the company brings the account up to \$1.2 million, the balance hits zero every two weeks. This results in the sawtooth pattern displayed in Figure 20A.2.

FIGURE 20A.2

Cash Balances for the Golden Socks Corporation



Implicitly, we assume that the net cash outflow is the same every day and that it is known with certainty. These two assumptions make the model easy to handle. We will indicate in the next section what happens when they do not hold.

If C were set higher, say, at \$2.4 million, cash would last four weeks before the firm would have to sell marketable securities, but the firm's average cash balance would increase to \$1.2 million (from \$600,000). If C were set at \$600,000, cash would run out in one week, and the firm would have to replenish cash more frequently, but the average cash balance would fall from \$600,000 to \$300,000.

Because transactions costs (for example, the brokerage costs of selling marketable securities) must be incurred whenever cash is replenished, establishing large initial balances will lower the trading costs connected with cash management. However, the larger the average cash balance, the greater is the opportunity cost (the return that could have been earned on marketable securities).

To determine the optimal strategy, Golden Socks needs to know the following three things:

- F = The fixed cost of making a securities trade to replenish cash.
- T = The total amount of new cash needed for transactions purposes over the relevant planning period, say, one year.
- R = The opportunity cost of holding cash. This is the interest rate on marketable securities.

With this information, Golden Socks can determine the total costs of any particular cash balance policy. It can then determine the optimal cash balance policy.

The Opportunity Costs To determine the opportunity costs of holding cash, we have to find out how much interest is forgone. Golden Socks has, on average, $C/2$ in cash. This amount could be earning interest at rate R . So the total dollar opportunity costs of cash balances are equal to the average cash balance multiplied by the interest rate:

$$\text{Opportunity costs} = (C/2) \times R \quad [20A.1]$$

For example, the opportunity costs of various alternatives are given here assuming that the interest rate is 10 percent:

Initial Cash Balance	Average Cash Balance	Opportunity Cost ($R = .10$)
C	$C/2$	$(C/2) \times R$
\$4,800,000	\$2,400,000	\$240,000
2,400,000	1,200,000	120,000
1,200,000	600,000	60,000
600,000	300,000	30,000
300,000	150,000	15,000

In our original case, in which the initial cash balance is \$1.2 million, the average balance is \$600,000. The interest Golden Socks could have earned on this (at 10 percent) is \$60,000, so this is what the firm gives up with this strategy. Notice that the opportunity costs increase as the initial (and average) cash balance rises.

The Trading Costs To determine the total trading costs for the year, we need to know how many times Golden Socks will have to sell marketable securities during the year. First of all, the total amount of cash disbursed during the year is \$600,000 per week, so

$T = \$600,000 \times 52 \text{ weeks} = \31.2 million . If the initial cash balance is set at $C = \$1.2 \text{ million}$, then Golden Socks will sell \$1.2 million in marketable securities $T/C = \$31.2 \text{ million}/1.2 \text{ million} = 26$ times per year. It costs F dollars each time, so trading costs are given by:

$$\frac{\$31.2 \text{ million}}{\$1.2 \text{ million}} \times F = 26 \times F$$

In general, the total trading costs will be given by:

$$\text{Trading costs} = (T/C) \times F \quad [20A.2]$$

In this example, if F were \$1,000 (an unrealistically large amount), then the trading costs would be \$26,000.

We can calculate the trading costs associated with some different strategies as follows:

Total Amount of Disbursements during Relevant Period	Initial Cash Balance	Trading Costs ($F = \$1,000$)
T	C	$(T/C) \times F$
\$31,200,000	\$4,800,000	\$ 6,500
31,200,000	2,400,000	13,000
31,200,000	1,200,000	26,000
31,200,000	600,000	52,000
31,200,000	300,000	104,000

The Total Cost Now that we have the opportunity costs and the trading costs, we can calculate the total cost by adding them together:

$$\begin{aligned} \text{Total cost} &= \text{Opportunity costs} + \text{Trading costs} \\ &= (C/2) \times R + (T/C) \times F \end{aligned} \quad [20A.3]$$

Using the numbers generated earlier, we have:

Cash Balance	Opportunity Costs	+	Trading Costs	=	Total Cost
\$4,800,000	\$240,000		\$ 6,500		\$246,500
2,400,000	120,000		13,000		133,000
1,200,000	60,000		26,000		86,000
600,000	30,000		52,000		82,000
300,000	15,000		104,000		119,000

Notice how the total cost starts out at almost \$250,000 and declines to about \$82,000 before starting to rise again.

The Solution We can see from the preceding schedule that a \$600,000 cash balance results in the lowest total cost of the possibilities presented: \$82,000. But what about \$700,000 or \$500,000 or other possibilities? It appears that the optimum balance is somewhere between \$300,000 and \$1.2 million. With this in mind, we could easily proceed by trial and error to find the optimum balance. It is not difficult to find it directly, however, so we do this next.

Take a look back at Figure 20A.1. As the figure is drawn, the optimal size of the cash balance, C^* , occurs right where the two lines cross. At this point, the opportunity costs and the trading costs are exactly equal. So, at C^* , we must have that:

$$\begin{aligned}\text{Opportunity costs} &= \text{Trading costs} \\ (C^*/2) \times R &= (T/C^*) \times F\end{aligned}$$

With a little algebra, we can write:

$$C^{*2} = (2T \times F)/R$$

To solve for C^* , we take the square root of both sides to get:

$$C^* = \sqrt{(2T \times F)/R} \quad [20A.4]$$

This is the optimum initial cash balance.

For Golden Socks, we have $T = \$31.2$ million, $F = \$1,000$, and $R = 10\%$. We can now find the optimum cash balance:

$$\begin{aligned}C^* &= \sqrt{(2 \times \$31,200,000 \times 1,000)/.10} \\ &= \sqrt{\$624 \text{ billion}} \\ &= \$789,937\end{aligned}$$

We can verify this answer by calculating the various costs at this balance, as well as a little above and a little below:

Cash Balance	Opportunity Costs	+	Trading Costs	=	Total Cost
\$850,000	\$42,500		\$36,706		\$79,206
800,000	40,000		39,000		79,000
789,937	39,497		39,497		78,994
750,000	37,500		41,600		79,100
700,000	35,000		44,571		79,571

The total cost at the optimum cash level is \$78,994, and it does appear to increase as we move in either direction.

The BAT Model

The Vulcan Corporation has cash outflows of \$100 per day, seven days a week. The interest rate is 5 percent, and the fixed cost of replenishing cash balances is \$10 per transaction. What is the optimal initial cash balance? What is the total cost?

The total cash needed for the year is $365 \text{ days} \times \$100 = \$36,500$. From the BAT model, we have that the optimal initial balance is:

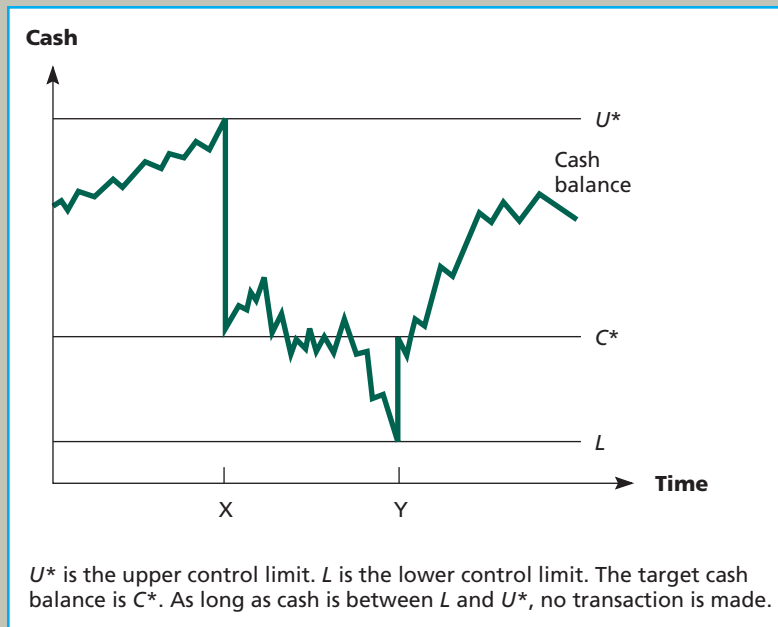
$$\begin{aligned}C^* &= \sqrt{(2T \times F)/R} \\ &= \sqrt{(2 \times \$36,500 \times 10)/.05} \\ &= \sqrt{\$14.6 \text{ million}} \\ &= \$3,821\end{aligned}$$

The average cash balance is $\$3,821/2 = \$1,911$, so the opportunity cost is $\$1,911 \times .05 = \96 . Because Vulcan needs \$100 per day, the \$3,821 balance will last $\$3,821/100 = 38.21$ days. The firm needs to resupply the account $365/38.21 = 9.6$ times per year, so the trading (order) cost is \$96. The total cost is \$192.

EXAMPLE 20A.1

FIGURE 20A.3

The Miller-Orr Model



Conclusion The BAT model is possibly the simplest and most stripped-down sensible model for determining the optimal cash position. Its chief weakness is that it assumes steady, certain cash outflows. We next discuss a more involved model designed to deal with this limitation.

The Miller-Orr Model: A More General Approach

We now describe a cash management system designed to deal with cash inflows and outflows that fluctuate randomly from day to day. With this model, we again concentrate on the cash balance, but, in contrast to the situation with the BAT model, we assume that this balance fluctuates up and down randomly and that the average change is zero.

The Basic Idea Figure 20A.3 shows how the system works. It operates in terms of an upper limit to the amount of cash (U^*) and a lower limit (L), and a target cash balance (C^*). The firm allows its cash balance to wander around between the lower and upper limits. As long as the cash balance is somewhere between U^* and L , nothing happens.

When the cash balance reaches the upper limit (U^*), such as it does at Point X, the firm moves $U^* - C^*$ dollars out of the account and into marketable securities. This action moves the cash balance down to C^* . In the same way, if the cash balance falls to the lower limit (L), as it does at Point Y, the firm will sell $C^* - L$ worth of securities and deposit the cash in the account. This action takes the cash balance up to C^* .

Using the Model To get started, management sets the lower limit (L). This limit essentially defines a safety stock; so, where it is set depends on how much risk of a cash shortfall the firm is willing to tolerate. Alternatively, the minimum might just equal a required compensating balance.

As with the BAT model, the optimal cash balance depends on trading costs and opportunity costs. Once again, the cost per transaction of buying and selling marketable securities, F , is assumed to be fixed. Also, the opportunity cost of holding cash is R , the interest rate per period on marketable securities.

The only extra piece of information needed is σ^2 , the variance of the net cash flow per period. For our purposes, the period can be anything, a day or a week, for example, as long as the interest rate and the variance are based on the same length of time.

Given L , which is set by the firm, Miller and Orr show that the cash balance target, C^* , and the upper limit, U^* , that minimize the total costs of holding cash are:²

$$C^* = L + (3/4 \times F \times \sigma^2/R)^{(1/3)} \quad [20A.5]$$

$$U^* = 3 \times C^* - 2 \times L \quad [20A.6]$$

Also, the average cash balance in the Miller-Orr model is:

$$\text{Average cash balance} = (4 \times C^* - L)/3 \quad [20A.7]$$

The derivation of these expressions is relatively complex, so we will not present it here. Fortunately, as we illustrate next, the results are not difficult to use.

For example, suppose $F = \$10$, the interest rate is 1 percent per month, and the standard deviation of the monthly net cash flows is \$200. The variance of the monthly net cash flows is:

$$\sigma^2 = \$200^2 = \$40,000$$

We assume a minimum cash balance of $L = \$100$. We can calculate the cash balance target, C^* , as:

$$\begin{aligned} C^* &= L + (3/4 \times F \times \sigma^2/R)^{(1/3)} \\ &= \$100 + (3/4 \times 10 \times 40,000/.01)^{(1/3)} \\ &= \$100 + 30,000,000^{(1/3)} \\ &= \$100 + 311 = \$411 \end{aligned}$$

The upper limit, U^* , is thus:

$$\begin{aligned} U^* &= 3 \times C^* - 2 \times L \\ &= 3 \times \$411 - 2 \times 100 \\ &= \$1,033 \end{aligned}$$

Finally, the average cash balance will be:

$$\begin{aligned} \text{Average cash balance} &= (4 \times C^* - L)/3 \\ &= (4 \times \$411 - 100)/3 \\ &= \$515 \end{aligned}$$

Implications of the BAT and Miller-Orr Models

Our two cash management models differ in complexity, but they have some similar implications. In both cases, all other things being equal, we see that:

1. The greater the interest rate, the lower is the target cash balance.
2. The greater the order cost, the higher is the target balance.

²M. H. Miller and D. Orr, "A Model of the Demand for Money by Firms," *Quarterly Journal of Economics*, August 1966.

These implications are both fairly obvious. The advantage of the Miller-Orr model is that it improves our understanding of the problem of cash management by considering the effect of uncertainty as measured by the variation in net cash inflows.

The Miller-Orr model shows that the greater the uncertainty is (the higher σ^2 is), the greater is the difference between the target balance and the minimum balance. Similarly, the greater the uncertainty is, the higher is the upper limit and the higher is the average cash balance. These statements all make intuitive sense. For example, the greater the variability is, the greater is the chance that the balance will drop below the minimum. We thus keep a higher balance to guard against this happening.

Other Factors Influencing the Target Cash Balance

Before moving on, we briefly discuss two additional considerations that affect the target cash balance.

First, in our discussion of cash management, we assume that cash is invested in marketable securities such as Treasury bills. The firm obtains cash by selling these securities. Another alternative is to borrow cash. Borrowing introduces additional considerations to cash management:

1. Borrowing is likely to be more expensive than selling marketable securities because the interest rate is likely to be higher.
2. The need to borrow will depend on management's desire to hold low cash balances. A firm is more likely to have to borrow to cover an unexpected cash outflow the greater its cash flow variability and the lower its investment in marketable securities.

Second, for large firms, the trading costs of buying and selling securities are very small when compared to the opportunity costs of holding cash. For example, suppose a firm has \$1 million in cash that won't be needed for 24 hours. Should the firm invest the money or leave it sitting?

Suppose the firm can invest the money at an annualized rate of 7.57 percent per year. The daily rate in this case is about two basis points (.02 percent or .0002).³ The daily return earned on \$1 million is thus $.0002 \times \$1 \text{ million} = \200 . In many cases, the order cost will be much less than this; so a large firm will buy and sell securities very often before it will leave substantial amounts of cash idle.

CONCEPT QUESTIONS

- 20A.1a** What is a target cash balance?
20A.1b What is the basic trade-off in the BAT model?
20A.1c Describe how the Miller-Orr model works.

Appendix Review and Self-Test Problem

20A.1 The BAT Model Given the following information, calculate the target cash balance using the BAT model:

³A basis point is 1 percent of 1 percent. Also, the annual interest rate is calculated as $(1 + R)^{365} = 1.0757$, implying a daily rate of .02 percent.

Annual interest rate	12%
Fixed order cost	\$100
Total cash needed	\$240,000

What are the opportunity cost of holding cash, the trading cost, and the total cost? What would these be if \$15,000 were held instead? If \$25,000 were held?

Answer to Appendix Review and Self-Test Problem

20A.1 From the BAT model, we know that the target cash balance is:

$$\begin{aligned}
 C^* &= \sqrt{(2T \times F)/R} \\
 &= \sqrt{(2 \times \$240,000 \times 100)/.12} \\
 &= \sqrt{\$400,000,000} \\
 &= \$20,000
 \end{aligned}$$

The average cash balance will be $C^*/2 = \$20,000/2 = \$10,000$. The opportunity cost of holding \$10,000 when the going rate is 12 percent is $\$10,000 \times .12 = \$1,200$. There will be $\$240,000/20,000 = 12$ orders during the year, so the order cost, or trading cost, is also $12 \times \$100 = \$1,200$. The total cost is thus $\$2,400$.

If \$15,000 is held, then the average balance is $\$7,500$. Verify that the opportunity, trading, and total costs in this case are $\$900$, $\$1,600$, and $\$2,500$, respectively. If \$25,000 is held, these numbers are $\$1,500$, $\$960$, and $\$2,460$, respectively.

Questions and Problems

- Changes in Target Cash Balances** Indicate the likely impact of each of the following on a company's target cash balance. Use the letter *I* to denote an increase and *D* to denote a decrease. Briefly explain your reasoning in each case.
 - Commissions charged by brokers decrease.
 - Interest rates paid on money market securities rise.
 - The compensating balance requirement of a bank is raised.
 - The firm's credit rating improves.
 - The cost of borrowing increases.
 - Direct fees for banking services are established.
- Using the BAT Model** Given the following information, calculate the target cash balance using the BAT model:

Annual interest rate	6%
Fixed order cost	\$9
Total cash needed	\$5,000

How do you interpret your answer?

- Opportunity versus Trading Costs** White Whale Corporation has an average daily cash balance of \$300. Total cash needed for the year is \$30,000. The interest rate is 5 percent, and replenishing the cash costs \$6 each time. What are the opportunity cost of holding cash, the trading cost, and the total cost? What do you think of White Whale's strategy?

Basic

(Questions 1–10)

Basic*(continued)*

4. **Costs and the BAT Model** Debit and Credit Bookkeepers needs a total of \$4,000 in cash during the year for transactions and other purposes. Whenever cash runs low, it sells off \$300 in securities and transfers the cash in. The interest rate is 6 percent per year, and selling off securities costs \$25 per sale.
 - a. What is the opportunity cost under the current policy? The trading cost? With no additional calculations, would you say that Debit and Credit keeps too much or too little cash? Explain.
 - b. What is the target cash balance derived using the BAT model?
5. **Determining Optimal Cash Balances** The Bud Wiser Company is currently holding \$600,000 in cash. It projects that over the next year its cash outflows will exceed cash inflows by \$340,000 per month. How much of the current cash holding should be retained and how much should be used to increase the company's holdings of marketable securities? Each time these securities are bought or sold through a broker, the company pays a fee of \$500. The annual interest rate on money market securities is 5.5 percent. After the initial investment of excess cash, how many times during the next 12 months will securities be sold?
6. **Interpreting Miller-Orr** Econoline Crush, Inc., uses a Miller-Orr cash management approach with a lower limit of \$30,000, an upper limit of \$125,000, and a target balance of \$45,000. Explain what each of these points represents and then explain how the system will work.
7. **Using Miller-Orr** Slap Shot Corporation has a fixed cost associated with buying and selling marketable securities of \$100. The interest rate is currently .019 percent per day, and the firm has estimated that the standard deviation of its daily net cash flows is \$50. Management has set a lower limit of \$1,100 on cash holdings. Calculate the target cash balance and upper limit using the Miller-Orr model. Describe how the system will work.
8. **Interpreting Miller-Orr** Based on the Miller-Orr model, describe what will happen to the lower limit, the upper limit, and the spread (the distance between the two) if the variation in net cash flow grows. Give an intuitive explanation for why this happens. What happens if the variance drops to zero?
9. **Using Miller-Orr** The variance of the daily cash flows for the Pele Bicycle Shop is \$1.05 million. The opportunity cost to the firm of holding cash is 7 percent per year. What should be the target cash level and the upper limit if the tolerable lower limit has been established as \$100,000? The fixed cost of buying and selling securities is \$500 per transaction.
10. **Using BAT** Bates Corporation has determined that its target cash balance if it uses the BAT model is \$1,000. The total cash needed for the year is \$16,000, and the order cost is \$5. What interest rate must Bates be using?