

SUMMARY AND CONCLUSIONS

24.6

Slide 24.44 Quick Quiz

This chapter introduces the wide world of option valuation and some of its more important implications for corporate finance. In it, we saw that:

1. The put-call parity (PCP) condition tells us that among a call option, a put option, a risk-free investment like a T-bill, and an underlying asset such as shares of stock, we can replicate any one using the other three.
2. The Black-Scholes Option Pricing Model (OPM) lets us explicitly value call options given values for the five relevant inputs, which are the price of the underlying asset, the strike price, the time to expiration, the risk-free rate, and the standard deviation of the return on the underlying asset.
3. The effect of changing the inputs into the Black-Scholes OPM varies. Some have positive effects, some negative. The magnitude also varies; relatively small changes in the risk-free rate don't have much of an effect, but changes in the standard deviation can have a very large effect. These various effects are known as the "greeks" because of the Greek (and quasi-Greek) letters used to identify them.
4. The equity in a leveraged corporation can be viewed as a call option on the assets of the firm. This gives the stockholders a strong incentive to increase the volatility of the return on the firm's assets, even if that means accepting projects with lower NPVs.

Chapter Review and Self-Test Problems

- 24.1 Put-Call Parity** A share of stock sells for \$40. The continuously compounded risk-free rate is 8 percent per year. A call option with one month to expiration and a strike price of \$45 sells for \$1. What's the value of a put option with the same expiration and strike?
- 24.2 Black-Scholes** A share of stock sells for \$40. The continuously compounded risk-free rate is 4 percent. The standard deviation of the return on the stock is 80 percent. What is the value of a put option with a strike of \$45 and a three-month expiration?

Answers to Chapter Review and Self-Test Problems

- 24.1** The PCP condition says that:

$$S + P = E \times e^{-Rt} + C$$

Filling in the relevant numbers and rearranging to solve for P , the put price, we get:

$$\begin{aligned} P &= \$45 \times e^{-.08(1/12)} + 1 - 40 \\ &= \$5.70 \end{aligned}$$

- 24.2** We will do this one the long way and then check our answer using an options calculator. We will calculate the value of a call option and then convert it to a put using PCP. We first need d_1 and d_2 :

$$\begin{aligned} d_1 &= [\ln(S/E) + (R + \sigma^2/2) \times t] / (\sigma \times \sqrt{t}) \\ &= [\ln(40/45) + (.04 + .8^2/2) \times 1/4] / (.8 \times \sqrt{1/4}) \end{aligned}$$



$$\begin{aligned}
 &= -.07 \\
 d_2 &= d_1 - \sigma \times \sqrt{t} \\
 &= -.47
 \end{aligned}$$

Referring to Table 24.3, the values of $N(d_1)$ and $N(d_2)$ are .4721 and .3192, respectively. Notice that in both cases we average two values. Plugging all the numbers in:

$$\begin{aligned}
 C &= S \times N(d_1) - E \times e^{-Rt} \times N(d_2) \\
 &= \$40 \times .4721 - \$45 \times e^{-.04(1/4)} \times .3192 \\
 &= \$4.66
 \end{aligned}$$

Converting to a put as in our previous question:

$$\begin{aligned}
 P &= \$45 \times e^{-.04(1/4)} + 4.66 - 40 \\
 &= \$9.21
 \end{aligned}$$

Using the options calculator at www.numa.com, we get \$9.21, so our “by hand” approach was pretty accurate in this case.

Concepts Review and Critical Thinking Questions

- Options and Expiration Dates** What is the impact of lengthening the time to expiration on an option’s value? Explain.
- Options and Stock Price Volatility** What is the impact of an increase in the volatility of the underlying stock’s return on an option’s value? Explain.
- Options and Interest Rates** How do interest rates affect option prices? Explain.
- Protective Puts** The protective put strategy we discussed in the chapter is sometimes referred to as “stock price insurance.” Why?
- Intrinsic Value** What is the intrinsic value of a call option? Of a put option? How do we interpret this value?
- Time Value** What is the time value of a call option? Of a put option? What happens to the time value of a call option as the maturity increases? What about a put option?
- Option Valuation and NPV** You are CEO of Titan Industries and have just been awarded a large number of employee stock options. The company has two mutually exclusive projects available. The first project has a large NPV and will reduce the total risk of the company. The second project has a small NPV and will increase the total risk of the company. You have decided to accept the first project when you remember your employee stock options. How might this affect your decision?
- Put-Call Parity** You find a put and a call with the same exercise price and maturity. What do you know about the relative prices of the put and call? Prove your answer and provide an intuitive explanation.
- Put-Call Parity** A put and a call have the same maturity and strike price. If they have the same price, which one is in the money? Prove your answer and provide an intuitive explanation.
- Put-Call Parity** One thing put-call parity tells us is that given any three of a stock, a call, a put, and a T-bill, the fourth can be synthesized or replicated using

the other three. For example, how can we replicate a share of stock using a call, a put, and a T-bill?

Questions and Problems

- Continuous Compounding** If you have \$1,000 today, how much will it be worth in five years at 7 percent per year compounded continuously?
- Continuous Compounding** If you need \$10,000 in three years, how much will you need to deposit today if you can earn 10 percent per year compounded continuously?
- Put-Call Parity** A stock is currently selling for \$54 per share. A call option with an exercise price of \$55 sells for \$3.10 and expires in three months. If the risk-free rate of interest is 2.6 percent per year, compounded continuously, what is the price of a put option with the same exercise price?
- Put-Call Parity** A put option that expires in six months with an exercise price of \$65 sells for \$2.05. The stock is currently priced at \$67, and the risk-free rate is 3.6 percent per year, compounded continuously. What is the price of a call option with the same exercise price?
- Put-Call Parity** A put option and a call option with an exercise price of \$80 and five months to expiration sell for \$2.05 and \$4.80, respectively. If the risk-free rate is 4.8 percent per year, compounded continuously, what is the current stock price?
- Put-Call Parity** A put option and call option with an exercise price of \$65 expire in two months and sell for \$2.50 and \$0.90, respectively. If the stock is currently priced at \$63.20, what is the annual continuously compounded rate of interest?
- Put-Call Parity** A put option with a maturity of five months sells for \$6.33. A call with the same expiration sells for \$9.30. If the exercise price is \$75 and the stock is currently priced at \$77.20, what is the annual continuously compounded interest rate?
- Black-Scholes** What are the prices of a call option and a put option with the following characteristics?

Stock price = \$32
 Exercise price = \$30
 Risk-free rate = 5% per year, compounded continuously
 Maturity = 3 months
 Standard deviation = 54% per year

- Black-Scholes** What are the prices of a call option and a put option with the following characteristics?

Stock price = \$98
 Exercise price = \$105
 Risk-free rate = 4% per year, compounded continuously
 Maturity = 9 months
 Standard deviation = 62% per year

- Delta** What are the deltas of a call option and a put option with the following characteristics? What does the delta of the option tell you?

Basic

(Questions 1–14)

Basic*(continued)*

Stock price = \$64
 Exercise price = \$60
 Risk-free rate = 5% per year, compounded continuously
 Maturity = 9 months
 Standard deviation = 56% per year

11. **Black-Scholes and Asset Value** You own a lot in Key West, Florida, that is currently unused. Similar lots have recently sold for \$1.5 million. Over the past five years, the price of land in the area has increased 12 percent per year, with an annual standard deviation of 25 percent. A buyer has recently approached you and wants an option to buy the land in the next 12 months for \$1.75 million. The risk-free rate of interest is 5 percent per year, compounded continuously. How much should you charge for the option?
12. **Black-Scholes and Asset Value** In the previous problem, suppose you wanted the option to sell the land to the buyer in one year. Assuming all the facts are the same, describe the transaction that would occur today. What is the price of the transaction today?
13. **Time Value of Options** You are given the following information concerning options on a particular stock:

Stock price = \$72
 Exercise price = \$75
 Risk-free rate = 6% per year, compounded continuously
 Maturity = 6 months
 Standard deviation = 48% per year

- a. What is the intrinsic value of the call option? Of the put option?
 - b. What is the time value of the call option? Of the put option?
 - c. Does the call or the put have the larger time value component? Would you expect this to be true in general?
14. **Put-Call Parity** A call option with an exercise price of \$90 and four months to expiration has a price of \$9.02. The stock is currently priced at \$94.30, and the risk-free rate is 5 percent per year, compounded continuously. What is the price of a put option with the same exercise price?
 15. **Black-Scholes** A call option matures in six months. The underlying stock price is \$85, and the stock's return has a standard deviation of 20 percent per year. The risk-free rate is 4 percent per year, compounded continuously. If the exercise price is \$0, what is the price of the call option?
 16. **Black-Scholes** A call option has an exercise price of \$75 and matures in six months. The current stock price is \$80, and the risk-free rate is 5 percent per year, compounded continuously. What is the price of the call if the standard deviation of the stock is 0 percent per year?
 17. **Black-Scholes** A stock is currently priced at \$35. A call option with an expiration of one year has an exercise price of \$50. The risk-free rate is 12 percent per year, compounded continuously, and the standard deviation of the stock's return is infinitely large. What is the price of the call option?
 18. **Equity as an Option** Sunburn Sunscreen has a zero coupon bond issue outstanding with a \$10,000 face value that matures in one year. The current market value of the firm's assets is \$11,000. The standard deviation of the return on the

Intermediate*(Questions 15–22)*

firm's assets is 42 percent per year, and the annual risk-free rate is 5 percent per year, compounded continuously. Based on the Black-Scholes model, what is the market value of the firm's equity and debt?

Intermediate
(continued)

- 19. Equity as an Option and NPV** Suppose the firm in the previous problem is considering two mutually exclusive investments. Project A has a NPV of \$700, and Project B has an NPV of \$1,000. As the result of taking Project A, the standard deviation of the return on the firm's assets will increase to 55 percent per year. If Project B is taken, the standard deviation will fall to 34 percent per year.
- What is the value of the firm's equity and debt if Project A is undertaken? If Project B is undertaken?
 - Which project would the stockholders prefer? Can you reconcile your answer with the NPV rule?
 - Suppose the stockholders and bondholders are in fact the same group of investors. Would this affect your answer to *b*?
 - What does this problem suggest to you about stockholder incentives?
- 20. Equity as an Option** Frostbite Thermalwear has a zero coupon bond issue outstanding with a face value of \$20,000 that matures in one year. The current market value of the firm's assets is \$20,000. The standard deviation of the return on the firm's assets is 53 percent per year, and the annual risk-free rate is 5 percent per year, compounded continuously. Based on the Black-Scholes model, what is the market value of the firm's equity and debt? What is the firm's continuously compounded cost of debt?
- 21. Mergers and Equity as an Option** Suppose Sunburn Sunscreen and Frostbite Thermalwear in the previous problems have decided to merge. Since the two companies have seasonal sales, the combined firm's return on assets will have a standard deviation of 34 percent per year.
- What is the combined value of equity in the two existing companies? Value of debt?
 - What is the value of the new firm's equity? Value of debt?
 - What was the gain or loss for shareholders? For bondholders?
 - What happened to shareholder value here?
- 22. Equity as an Option and NPV** A company has a single zero coupon bond outstanding which matures in 10 years with a face value of \$25 million. The current value of the company's assets is \$22 million, and the standard deviation of the return on the firm's assets is 42 percent per year. The risk-free rate is 6 percent per year, compounded continuously.
- What is the current market value of the company's equity?
 - What is the current market value of the company's debt?
 - What is the company's continuously compounded cost of debt?
 - The company has a new project available. The project has an NPV of \$500,000. If the company undertakes the project, what will be the new market value of equity?
 - Assuming the company undertakes the new project and does not borrow any additional funds, what is the new continuously compounded cost of debt? What is happening here?
- 23. Debt Valuation and Time to Maturity** Christina Industries has a zero coupon bond issue that matures in two years with a face value of \$25,000. The current value of the company's assets is \$12,400, and the standard deviation of the return on assets is 60 percent per year.

Challenge
(Questions 23–28)

Challenge*(continued)*

- a. Assume the risk-free rate is 5 percent per year, compounded continuously. What is the value of a risk-free bond with the same face value and maturity as the company's bond?
- b. What price would the bondholders have to pay for a put option on the firm's assets with a strike price equal to the face value of the debt?
- c. Using the answers from *a* and *b*, what is the value of the firm's debt? What is the continuously compounded yield on the company's debt?
- d. From an examination of the value of the assets of Christina Industries, and the fact that the debt must be repaid in two years, it seems likely that the company will default on its debt. Management has approached bondholders and proposed a plan whereby the company would repay the same face value of debt, but the repayment would not occur for five years. What is the value of the debt under the proposed plan? What is the new continuously compounded yield on the debt? Explain why this occurs.
- 24. Debt Valuation and Asset Variance** Ozzy Corp. has a zero coupon bond that matures in five years with a face value of \$50,000. The current value of the company's assets is \$48,000, and the standard deviation of its return on assets is 40 percent per year. The risk-free rate is 6 percent per year, compounded continuously.
- a. What is the value of a risk-free bond with the same face value and maturity as the current bond?
- b. What is the value of a put option on the firm's assets with a strike price equal to the face value of the debt?
- c. Using the answers from *a* and *b*, what is the value of the firm's debt? What is the continuously compounded yield on the company's debt?
- d. Assume the company can restructure its assets so that the standard deviation of its return on assets increases to 50 percent per year. What happens to the value of the debt? What is the new continuously compounded yield on the debt? Reconcile your answers in *c* and *d*.
- e. What happens to bondholders if the company restructures its assets? What happens to shareholders? How does this create an agency problem?
- 25. Black-Scholes and Dividends** In addition to the five factors discussed in the chapter, dividends also affect the price of an option. The Black-Scholes Option Pricing Model with dividends is:
- $$C = S \times e^{-dt} \times N(d_1) - E \times e^{-Rt} \times N(d_2)$$
- $$d_1 = [\ln(S/E) + (R - d + \sigma^2/2) \times t] / (\sigma \times \sqrt{t})$$
- $$d_2 = d_1 - \sigma \times \sqrt{t}$$
- All of the variables are the same as the Black-Scholes model without dividends except for the variable *d*, which is the continuously compounded dividend yield on the stock.
- a. What effect do you think the dividend yield will have on the price of a call option? Explain.
- b. A stock is currently priced at \$76 per share, the standard deviation of its return is 45 percent per year, and the risk-free rate is 5 percent per year, compounded continuously. What is the price of a call option with a strike price of \$80 and a maturity of 6 months if the stock has a dividend yield of 2 percent per year?
- 26. Put-Call Parity and Dividends** The put-call parity condition is altered when dividends are paid. The dividend-adjusted put-call parity formula is:

$$S \times e^{-dt} + P = E \times e^{-Rt} + C$$

where d is again the continuously compounded dividend yield.

- a. What effect do you think the dividend yield will have on the price of a put option? Explain.
- b. From the previous question, what is the price of a put option with the same strike and time to expiration as the call option?
27. **Put Delta** In the chapter, we noted that the delta for a put option is $N(d_1) - 1$. Is this the same thing as $-N(-d_1)$? (Hint: Yes, but why?)
28. **Black-Scholes Put Pricing Model** Use the Black-Scholes model for pricing a call, put-call parity, and the previous question to show that the Black-Scholes model for directly pricing a put can be written as:

$$P = E \times e^{-Rt} \times N(-d_2) - S \times N(-d_1)$$

29. **Black-Scholes** A stock is currently priced at \$50. The stock will never pay a dividend. The risk-free rate is 12 percent per year, compounded continuously, and the standard deviation of the stock's return is 60 percent. A European call option on the stock has a strike price of \$100 and no expiration date, meaning that it has an infinite life. Based on Black-Scholes, what is the value of the call option? Do you see a paradox here? Do you see a way out of the paradox?
30. **Delta** You purchase one call and sell one put with the same strike price and expiration date. What is the delta of your portfolio? Why?

- 24.1 **Black-Scholes** Go to www.cfo.com and, under CFO.com Toolbox, follow the "Stock Options Calculator" link, then the "Options Calculator (Java)" link. There is a call and a put option on a stock that expires in 30 days. The strike price is \$50 and the current stock price is \$51.20. The standard deviation of the return on the stock is 60 percent per year, and the risk-free rate is 4.8 percent per year, compounded continuously. What is the price of the call and the put? What are the deltas?
- 24.2 **Black-Scholes** Go to www.cboe.com, click on the "Trading Tools" tab, then the "Option Calculator" link. A stock is currently priced at \$93 per share, and its return has a standard deviation of 48 percent per year. Options are available with an exercise price of \$90, and the risk-free rate is 5.2 percent per year, compounded continuously. What is the price of the call and the put that expire next month? What are the deltas? How do your answers change for an exercise price of \$95?
- 24.3 **Implied Standard Deviation** Go to www.numa.com and look under the section titled "Options" and follow the calculator link. You purchased a call option for \$10.50 that matures in 51 days. The strike price is \$100, and the underlying stock has a price of \$102. If the risk-free rate is 4.8 percent, compounded continuously, what is the implied return standard deviation of the stock? Using this implied standard deviation, what is the price of a put option with the same characteristics?
- 24.4 **Black-Scholes with Dividends** Recalculate the first two problems assuming a dividend yield of 2 percent per year. How does this change your answers? Can you explain why dividends have the effect they do?

Challenge

(continued)

What's On
the Web?