



VALUATION OF FUTURE CASH FLOWS

CHAPTER 5 Introduction to Valuation: The Time Value of Money One of the most important questions in finance is: What is the value today of a cash flow to be received at a later date? The answer depends on the time value of money, the subject of this chapter.

CHAPTER 6 Discounted Cash Flow Valuation This chapter expands on the basic results from Chapter 5 to discuss valuation of multiple future cash flows. We consider a number of related topics, including loan valuation, calculation of loan payments, and determination of rates of return.

CHAPTER 7 Interest Rates and Bond Valuation Bonds are a very important type of financial instrument. This chapter shows how the valuation techniques of Chapter 6 can be used to determine bond prices. We describe essential features of bonds and how their prices are reported in the financial press. Interest rates and their influence on bond prices are also examined.

CHAPTER 8 Stock Valuation The final chapter of Part Three considers the determinants of the value of a share of stock. Important features of common and preferred stock, such as shareholder rights, are discussed, and stock price quotes are examined.





Introduction to Valuation: The Time Value of Money

On December 2, 1982, General Motors Acceptance Corporation (GMAC), a subsidiary of General Motors, offered some securities for sale to the public. Under the terms of the deal, GMAC promised to repay the owner of one of these securities \$10,000 on December 1, 2012, but investors would receive nothing until then. Investors paid GMAC \$500 for each of these securities, so they gave up \$500 on December 2, 1982, for the promise of a \$10,000 payment 30 years later. Such a security, for which you pay some amount today in exchange for a promised lump sum to be received at a future date, is about the simplest possible type.

Is giving up \$500 in exchange for \$10,000 in 30 years a good deal? On the plus side, you get back \$20 for every \$1 you put up. That probably sounds good, but, on the down side, you have to wait 30 years to get it. What you need to know is how to analyze this trade-off; this chapter gives you the tools you need.

One of the basic problems faced by the financial manager is how to determine the value today of cash flows expected in the future. For example, the jackpot in a PowerBall™ lottery drawing was \$110 million. Does this mean the winning ticket was worth \$110 million? The answer is no because the jackpot was actually going to pay out over a 20-year period at a rate of \$5.5 million per year. How much was the ticket worth then? The answer depends on the time value of money, the subject of this chapter.

In the most general sense, the phrase *time value of money* refers to the fact that a dollar in hand today is worth more than a dollar promised at some time in the future. On a practical level, one reason for this is that you could earn interest while you waited; so a dollar today would grow to more than a dollar later. The trade-off between money now and money later thus depends on, among other things, the rate you can earn by investing. Our goal in this chapter is to explicitly evaluate this trade-off between dollars today and dollars at some future time.

A thorough understanding of the material in this chapter is critical to understanding material in subsequent chapters, so you should study it with particular care. We will present a number of examples in this chapter. In many problems, your answer

Slide 5.1 Key Concepts and Skills

Slide 5.2 Chapter Outline

Lecture Tip: Students sometimes find time value terminology confusing. IM 5.1 suggests an analogy utilizing the concept of cash flow “equivalence” as a way of introducing the basic time value concepts.

may differ from ours slightly. This can happen because of rounding and is not a cause for concern.

5.1

future value (FV)

The amount an investment is worth after one or more periods.

Self-Test Problem 5.1

Critical Thinking

Questions 2–4, 6, 7

Problems 1, 2, 12, 18, 19

Lecture Tip: Because of the importance of problem solving in this chapter, we break the “Quick Quiz” into four parts and integrate them throughout the chapter.

Concept Q Answer 5.1a

Slide 5.3 Basic Definitions

Concept Q Answer 5.1b

compounding

The process of accumulating interest on an investment over time to earn more interest.

interest on interest

Interest earned on the reinvestment of previous interest payments.

compound interest

Interest earned on both the initial principal and the interest reinvested from prior periods.

FUTURE VALUE AND COMPOUNDING

The first thing we will study is future value. **Future value (FV)** refers to the amount of money an investment will grow to over some period of time at some given interest rate. Put another way, future value is the cash value of an investment at some time in the future. We start out by considering the simplest case, a single-period investment.

Investing for a Single Period

Suppose you invest \$100 in a savings account that pays 10 percent interest per year. How much will you have in one year? You will have \$110. This \$110 is equal to your original *principal* of \$100 plus \$10 in interest that you earn. We say that \$110 is the future value of \$100 invested for one year at 10 percent, and we simply mean that \$100 today is worth \$110 in one year, given that 10 percent is the interest rate.

In general, if you invest for one period at an interest rate of r , your investment will grow to $(1 + r)$ per dollar invested. In our example, r is 10 percent, so your investment grows to $1 + .10 = 1.1$ dollars per dollar invested. You invested \$100 in this case, so you ended up with $\$100 \times 1.10 = \110 .

Investing for More Than One Period

Going back to our \$100 investment, what will you have after two years, assuming the interest rate doesn't change? If you leave the entire \$110 in the bank, you will earn $\$110 \times .10 = \11 in interest during the second year, so you will have a total of $\$110 + 11 = \121 . This \$121 is the future value of \$100 in two years at 10 percent. Another way of looking at it is that one year from now you are effectively investing \$110 at 10 percent for a year. This is a single-period problem, so you'll end up with \$1.10 for every dollar invested, or $\$110 \times 1.1 = \121 total.

This \$121 has four parts. The first part is the \$100 original principal. The second part is the \$10 in interest you earned in the first year, and the third part is another \$10 you earn in the second year, for a total of \$120. The last \$1 you end up with (the fourth part) is interest you earn in the second year on the interest paid in the first year: $\$10 \times .10 = \1 .

This process of leaving your money and any accumulated interest in an investment for more than one period, thereby *reinvesting* the interest, is called **compounding**. Compounding the interest means earning **interest on interest**, so we call the result **compound interest**. With **simple interest**, the interest is not reinvested, so interest is earned each period only on the original principal.

EXAMPLE 5.1

Interest on Interest

Suppose you locate a two-year investment that pays 14 percent per year. If you invest \$325, how much will you have at the end of the two years? How much of this is simple interest? How much is compound interest?

At the end of the first year, you will have $\$325 \times (1 + .14) = \370.50 . If you reinvest this entire amount, and thereby compound the interest, you will have $\$370.50 \times 1.14 = \422.37 at the end of the second year. The total interest you earn is thus $\$422.37 - 325 = \97.37 .

Your \$325 original principal earns $\$325 \times .14 = \45.50 in interest each year, for a two-year total of \$91 in simple interest. The remaining $\$97.37 - 91 = \6.37 results from compounding. You can check this by noting that the interest earned in the first year is \$45.50. The interest on interest earned in the second year thus amounts to $\$45.50 \times .14 = \6.37 , as we calculated.

We now take a closer look at how we calculated the \$121 future value. We multiplied \$110 by 1.1 to get \$121. The \$110, however, was \$100 also multiplied by 1.1. In other words:

$$\begin{aligned} \$121 &= \$110 \times 1.1 \\ &= (\$100 \times 1.1) \times 1.1 \\ &= \$100 \times (1.1 \times 1.1) \\ &= \$100 \times 1.1^2 \\ &= \$100 \times 1.21 \end{aligned}$$

At the risk of belaboring the obvious, let's ask: How much would our \$100 grow to after three years? Once again, in two years, we'll be investing \$121 for one period at 10 percent. We'll end up with \$1.10 for every dollar we invest, or $\$121 \times 1.1 = \133.10 total. This \$133.10 is thus:

$$\begin{aligned} \$133.10 &= \$121 \times 1.1 \\ &= (\$110 \times 1.1) \times 1.1 \\ &= (\$100 \times 1.1) \times 1.1 \times 1.1 \\ &= \$100 \times (1.1 \times 1.1 \times 1.1) \\ &= \$100 \times 1.1^3 \\ &= \$100 \times 1.331 \end{aligned}$$

You're probably noticing a pattern to these calculations, so we can now go ahead and state the general result. As our examples suggest, the future value of \$1 invested for t periods at a rate of r per period is:

$$\text{Future value} = \$1 \times (1 + r)^t \quad [5.1]$$

The expression $(1 + r)^t$ is sometimes called the *future value interest factor* (or just *future value factor*) for \$1 invested at r percent for t periods and can be abbreviated as $FVIF(r, t)$.

In our example, what would your \$100 be worth after five years? We can first compute the relevant future value factor as:

$$(1 + r)^t = (1 + .10)^5 = 1.1^5 = 1.6105$$

Your \$100 will thus grow to:

$$\$100 \times 1.6105 = \$161.05$$

The growth of your \$100 each year is illustrated in Table 5.1. As shown, the interest earned in each year is equal to the beginning amount multiplied by the interest rate of 10 percent.

In Table 5.1, notice the total interest you earn is \$61.05. Over the five-year span of this investment, the simple interest is $\$100 \times .10 = \10 per year, so you accumulate \$50 this way. The other \$11.05 is from compounding.

simple interest

Interest earned only on the original principal amount invested.

For a discussion of time value concepts (and lots more) see

www.financeprofessor.com.

Slide 5.4 Future Values

Concept Q Answer 5.1c

Slide 5.5 Future Values: General Formula

Slide 5.6 Effects of Compounding

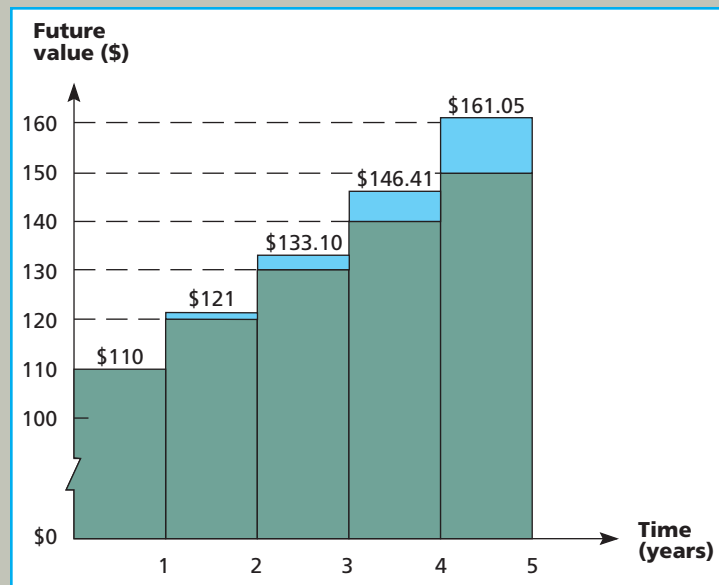
TABLE 5.1

Future Value of \$100 at 10 Percent

Year	Beginning Amount	Simple Interest	Compound Interest	Total Interest Earned	Ending Amount
1	\$100.00	\$10	\$ 0.00	\$10.00	\$110.00
2	110.00	10	1.00	11.00	121.00
3	121.00	10	2.10	12.10	133.10
4	133.10	10	3.31	13.31	146.41
5	146.41	10	4.64	14.64	161.05
		Total \$50 simple interest	Total \$11.05 compound interest	Total \$61.05 interest	

FIGURE 5.1

Future Value, Simple Interest, and Compound Interest



Growth of \$100 original amount at 10% per year. Blue shaded area represents the portion of the total that results from compounding of interest.

A brief introduction to key financial concepts is available at www.teachmefinance.com.

Figure 5.1 illustrates the growth of the compound interest in Table 5.1. Notice how the simple interest is constant each year, but the amount of compound interest you earn gets bigger every year. The amount of the compound interest keeps increasing because more and more interest builds up and there is thus more to compound.

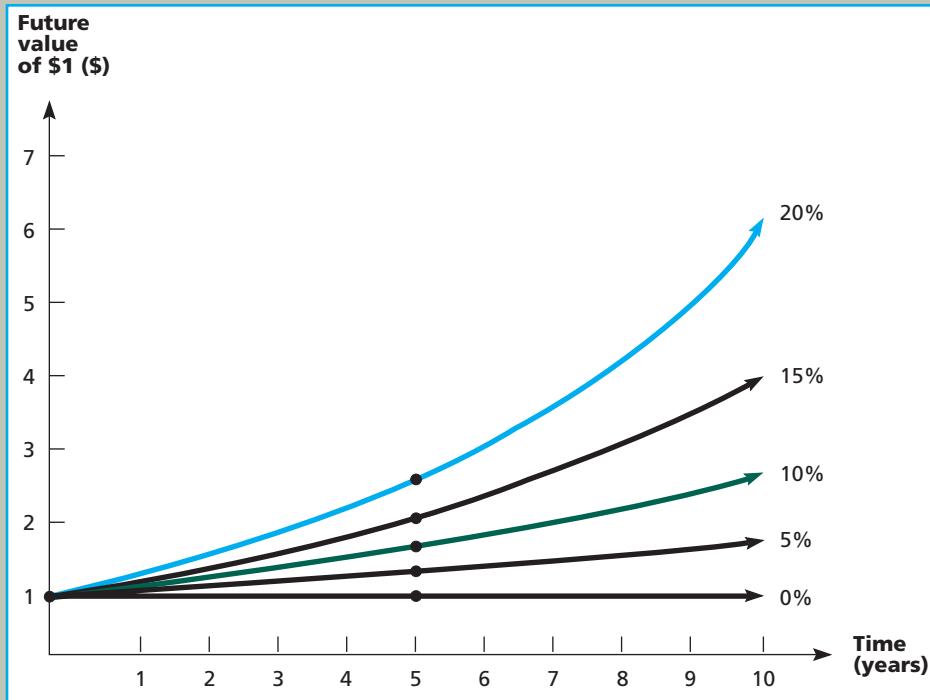
Future values depend critically on the assumed interest rate, particularly for long-lived investments. Figure 5.2 illustrates this relationship by plotting the growth of \$1 for different rates and lengths of time. Notice the future value of \$1 after 10 years is about \$6.20 at a 20 percent rate, but it is only about \$2.60 at 10 percent. In this case, doubling the interest rate more than doubles the future value.

To solve future value problems, we need to come up with the relevant future value factors. There are several different ways of doing this. In our example, we could have multiplied 1.1 by itself five times. This would work just fine, but it would get to be very tedious for, say, a 30-year investment.

Lecture Tip: Use slide 5.6 to emphasize the importance of compounding. See IM 5.1 for why.

Future Value of \$1 for Different Periods and Rates

FIGURE 5.2



Number of Periods	Interest Rate			
	5%	10%	15%	20%
1	1.0500	1.1000	1.1500	1.2000
2	1.1025	1.2100	1.3225	1.4400
3	1.1576	1.3310	1.5209	1.7280
4	1.2155	1.4641	1.7490	2.0736
5	1.2763	1.6105	2.0114	2.4883

TABLE 5.2

Future Value Interest Factors

Fortunately, there are several easier ways to get future value factors. Most calculators have a key labeled “y^x.” You can usually just enter 1.1, press this key, enter 5, and press the “=” key to get the answer. This is an easy way to calculate future value factors because it’s quick and accurate.

Alternatively, you can use a table that contains future value factors for some common interest rates and time periods. Table 5.2 contains some of these factors. Table A.1 in the appendix at the end of the book contains a much larger set. To use the table, find the column that corresponds to 10 percent. Then, look down the rows until you come to five periods. You should find the factor that we calculated, 1.6105.

Tables such as 5.2 are not as common as they once were because they predate inexpensive calculators and are only available for a relatively small number of rates. Interest rates are often quoted to three or four decimal places, so the tables needed to deal

Slide 5.7 Calculator Keys

Slide 5.8 Future Values: Example 2

Slide 5.9 Future Values: Example 3

with these accurately would be quite large. As a result, the real world has moved away from using them. We will emphasize the use of a calculator in this chapter.

These tables still serve a useful purpose. To make sure you are doing the calculations correctly, pick a factor from the table and then calculate it yourself to see that you get the same answer. There are plenty of numbers to choose from.

EXAMPLE 5.2

Compound Interest

You've located an investment that pays 12 percent. That rate sounds good to you, so you invest \$400. How much will you have in three years? How much will you have in seven years? At the end of seven years, how much interest will you have earned? How much of that interest results from compounding?

Based on our discussion, we can calculate the future value factor for 12 percent and three years as:

$$(1 + r)^t = 1.12^3 = 1.4049$$

Your \$400 thus grows to:

$$\$400 \times 1.4049 = \$561.97$$

After seven years, you will have:

$$\$400 \times 1.12^7 = \$400 \times 2.2107 = \$884.27$$

Thus, you will more than double your money over seven years.

Because you invested \$400, the interest in the \$884.27 future value is $\$884.27 - 400 = \484.27 . At 12 percent, your \$400 investment earns $\$400 \times .12 = \48 in simple interest every year. Over seven years, the simple interest thus totals $7 \times \$48 = \336 . The other $\$484.27 - 336 = \148.27 is from compounding.

The effect of compounding is not great over short time periods, but it really starts to add up as the horizon grows. To take an extreme case, suppose one of your more frugal ancestors had invested \$5 for you at a 6 percent interest rate 200 years ago. How much would you have today? The future value factor is a substantial $1.06^{200} = 115,125.90$ (you won't find this one in a table), so you would have $\$5 \times 115,125.91 = \$575,629.52$ today. Notice that the simple interest is just $\$5 \times .06 = \0.30 per year. After 200 years, this amounts to \$60. The rest is from reinvesting. Such is the power of compound interest!

EXAMPLE 5.3

How Much for That Island?

To further illustrate the effect of compounding for long horizons, consider the case of Peter Minuit and the American Indians. In 1626, Minuit bought all of Manhattan Island for about \$24 in goods and trinkets. This sounds cheap, but the Indians may have gotten the better end of the deal. To see why, suppose the Indians had sold the goods and invested the \$24 at 10 percent. How much would it be worth today?

Roughly 375 years have passed since the transaction. At 10 percent, \$24 will grow by quite a bit over that time. How much? The future value factor is approximately:

$$(1 + r)^t = 1.1^{375} \approx 3,000,000,000,000$$

That is, 3 followed by 15 zeroes. The future value is thus on the order of $\$24 \times 3$ quadrillion or about \$72 *quadrillion* (give or take a few hundreds of trillions).

Real-World Tip:
Additional real-world examples of the effects of compound interest appear in IM 5.1.

Well, \$72 quadrillion is a lot of money. How much? If you had it, you could buy the United States. All of it. Cash. With money left over to buy Canada, Mexico, and the rest of the world, for that matter.

This example is something of an exaggeration, of course. In 1626, it would not have been easy to locate an investment that would pay 10 percent every year without fail for the next 375 years.

CALCULATOR HINTS



Using a Financial Calculator

Although there are the various ways of calculating future values we have described so far, many of you will decide that a financial calculator is the way to go. If you are planning on using one, you should read this extended hint; otherwise, skip it.

A financial calculator is simply an ordinary calculator with a few extra features. In particular, it knows some of the most commonly used financial formulas, so it can directly compute things like future values.

Financial calculators have the advantage that they handle a lot of the computation, but that is really all. In other words, you still have to understand the problem; the calculator just does some of the arithmetic. In fact, there is an old joke (somewhat modified) that goes like this: Anyone can make a mistake on a time value of money problem, but to really screw one up takes a financial calculator! We therefore have two goals for this section. First, we'll discuss how to compute future values. After that, we'll show you how to avoid the most common mistakes people make when they start using financial calculators.

How to Calculate Future Values with a Financial Calculator Examining a typical financial calculator, you will find five keys of particular interest. They usually look like this:



For now, we need to focus on four of these. The keys labeled **PV** and **FV** are just what you would guess, present value and future value. The key labeled **N** refers to the number of periods, which is what we have been calling t . Finally, **%i** stands for the interest rate, which we have called r .¹

If we have the financial calculator set up right (see our next section), then calculating a future value is very simple. Take a look back at our question involving the future value of \$100 at 10 percent for five years. We have seen that the answer is \$161.05. The exact keystrokes will differ depending on what type of calculator you use, but here is basically all you do:

1. Enter -100 . Press the **PV** key. (The negative sign is explained below.)
2. Enter 10. Press the **%i** key. (Notice that we entered 10, not .10; see below.)
3. Enter 5. Press the **N** key.

¹The reason financial calculators use N and %i is that the most common use for these calculators is determining loan payments. In this context, N is the number of payments and %i is the interest rate on the loan. But, as we will see, there are many other uses of financial calculators that don't involve loan payments and interest rates.

Now we have entered all of the relevant information. To solve for the future value, we need to ask the calculator what the FV is. Depending on your calculator, you either press the button labeled “CPT” (for compute) and then press **FV**, or else you just press **FV**. Either way, you should get 161.05. If you don’t (and you probably won’t if this is the first time you have used a financial calculator!), we will offer some help in our next section.

Before we explain the kinds of problems that you are likely to run into, we want to establish a standard format for showing you how to use a financial calculator. Using the example we just looked at, in the future, we will illustrate such problems like this:

Enter	5	10	–100	
	N	%i	PMT	PV
Solve for				FV
				161.05

Here is an important tip: Appendix D in the back of the book contains some more detailed instructions for the most common types of financial calculators. See if yours is included, and, if it is, follow the instructions there if you need help. Of course, if all else fails, you can read the manual that came with the calculator.

How to Get the Wrong Answer Using a Financial Calculator There are a couple of common (and frustrating) problems that cause a lot of trouble with financial calculators. In this section, we provide some important *dos* and *don’ts*. If you just can’t seem to get a problem to work out, you should refer back to this section.

There are two categories we examine, three things you need to do only once and three things you need to do every time you work a problem. The things you need to do just once deal with the following calculator settings:

1. *Make sure your calculator is set to display a large number of decimal places.* Most financial calculators only display two decimal places; this causes problems because we frequently work with numbers—like interest rates—that are very small.
2. *Make sure your calculator is set to assume only one payment per period or per year.* Most financial calculators assume monthly payments (12 per year) unless you say otherwise.
3. *Make sure your calculator is in “end” mode.* This is usually the default, but you can accidentally change to “begin” mode.

If you don’t know how to set these three things, see Appendix D or your calculator’s operating manual. There are also three things you need to do *every time you work a problem*:

1. *Before you start, completely clear out the calculator.* This is very important. Failure to do this is the number one reason for wrong answers; you simply must get in the habit of clearing the calculator every time you start a problem. How you do this depends on the calculator (see Appendix D), but you must do more than just clear the display. For example, on a Texas Instruments BA II Plus you must press **2nd** then **CLR TVM** for *clear time value of money*. There is a similar command on your calculator. Learn it!

Note that turning the calculator off and back on won’t do it. Most financial calculators remember everything you enter, even after you turn them off. In other words, they remember all your mistakes unless you explicitly clear them out. Also, if you are in the middle of a problem and make a mistake, *clear it out and start over*. Better to be safe than sorry.

2. *Put a negative sign on cash outflows.* Most financial calculators require you to put a negative sign on cash outflows and a positive sign on cash inflows. As a practical matter, this usually just means that you should enter the present value amount with a negative sign (because normally the present value represents the amount you give up today in exchange for cash inflows later). By the same token, when you solve for a present value, you shouldn't be surprised to see a negative sign.
3. *Enter the rate correctly.* Financial calculators assume that rates are quoted in percent, so if the rate is .08 (or 8 percent), you should enter 8, not .08.

If you follow these guidelines (especially the one about clearing out the calculator), you should have no problem using a financial calculator to work almost all of the problems in this and the next few chapters. We'll provide some additional examples and guidance where appropriate.

A Note on Compound Growth

If you are considering depositing money in an interest-bearing account, then the interest rate on that account is just the rate at which your money grows, assuming you don't remove any of it. If that rate is 10 percent, then each year you simply have 10 percent more money than you had the year before. In this case, the interest rate is just an example of a compound growth rate.

The way we calculated future values is actually quite general and lets you answer some other types of questions related to growth. For example, your company currently has 10,000 employees. You've estimated that the number of employees grows by 3 percent per year. How many employees will there be in five years? Here, we start with 10,000 people instead of dollars, and we don't think of the growth rate as an interest rate, but the calculation is exactly the same:

$$10,000 \times 1.03^5 = 10,000 \times 1.1593 = 11,593 \text{ employees}$$

There will be about 1,593 net new hires over the coming five years.

To give another example, according to Value Line (a leading supplier of business information for investors), Wal-Mart's 2000 sales were about \$200 billion. Suppose sales are projected to increase at a rate of 15 percent per year. What will Wal-Mart's sales be in the year 2005 if this is correct? Verify for yourself that the answer is about 402.3 billion, just over twice as large.

Slide 5.10 Future Value as a General Growth Formula

Slide 5.11 Quick Quiz—Part I

Dividend Growth

The TICO Corporation currently pays a cash dividend of \$5 per share. You believe the dividend will be increased by 4 percent each year indefinitely. How big will the dividend be in eight years?

Here we have a cash dividend growing because it is being increased by management, but, once again, the calculation is the same:

$$\text{Future value} = \$5 \times 1.04^8 = \$5 \times 1.3686 = \$6.84$$

The dividend will grow by \$1.84 over that period. Dividend growth is a subject we will return to in a later chapter.

EXAMPLE 5.4

Lecture Tip: Some caveats on the interpretation of compound values appear in IM 5.1.

CONCEPT QUESTIONS

- 5.1a** What do we mean by the future value of an investment?
5.1b What does it mean to compound interest? How does compound interest differ from simple interest?
5.1c In general, what is the future value of \$1 invested at r per period for t periods?

5.2

Self-Test Problem 5.2

Critical Thinking Questions 1, 5, 9, 10

Problems 3, 10, 11, 14, 17

Slide 5.12 Present Values

Slide 5.13 Present Value—One Period Example

present value (PV)

The current value of future cash flows discounted at the appropriate discount rate.

Concept Q Answers 5.2a, 5.2b

discount

Calculate the present value of some future amount.

PRESENT VALUE AND DISCOUNTING

When we discuss future value, we are thinking of questions like, What will my \$2,000 investment grow to if it earns a 6.5 percent return every year for the next six years? The answer to this question is what we call the future value of \$2,000 invested at 6.5 percent for six years (verify that the answer is about \$2,918).

There is another type of question that comes up even more often in financial management that is obviously related to future value. Suppose you need to have \$10,000 in 10 years, and you can earn 6.5 percent on your money. How much do you have to invest today to reach your goal? You can verify that the answer is \$5,327.26. How do we know this? Read on.

The Single-Period Case

We've seen that the future value of \$1 invested for one year at 10 percent is \$1.10. We now ask a slightly different question: How much do we have to invest today at 10 percent to get \$1 in one year? In other words, we know the future value here is \$1, but what is the **present value (PV)**? The answer isn't too hard to figure out. Whatever we invest today will be 1.1 times bigger at the end of the year. Because we need \$1 at the end of the year:

$$\text{Present value} \times 1.1 = \$1$$

Or, solving for the present value:

$$\text{Present value} = \$1/1.1 = \$.909$$

In this case, the present value is the answer to the following question: What amount, invested today, will grow to \$1 in one year if the interest rate is 10 percent? Present value is thus just the reverse of future value. Instead of compounding the money forward into the future, we **discount** it back to the present.

EXAMPLE 5.5

Single-Period PV

Suppose you need \$400 to buy textbooks next year. You can earn 7 percent on your money. How much do you have to put up today?

We need to know the PV of \$400 in one year at 7 percent. Proceeding as in the previous example:

$$\text{Present value} \times 1.07 = \$400$$

We can now solve for the present value:

$$\text{Present value} = \$400 \times (1/1.07) = \$373.83$$

Thus, \$373.83 is the present value. Again, this just means that investing this amount for one year at 7 percent will result in your having a future value of \$400.

From our examples, the present value of \$1 to be received in one period is generally given as:

$$PV = \$1 \times [1/(1 + r)] = \$1/(1 + r)$$

We next examine how to get the present value of an amount to be paid in two or more periods into the future.

Present Values for Multiple Periods

Suppose you need to have \$1,000 in two years. If you can earn 7 percent, how much do you have to invest to make sure that you have the \$1,000 when you need it? In other words, what is the present value of \$1,000 in two years if the relevant rate is 7 percent?

Based on your knowledge of future values, you know the amount invested must grow to \$1,000 over the two years. In other words, it must be the case that:

$$\begin{aligned} \$1,000 &= PV \times 1.07 \times 1.07 \\ &= PV \times 1.07^2 \\ &= PV \times 1.1449 \end{aligned}$$

Given this, we can solve for the present value:

$$\text{Present value} = \$1,000/1.1449 = \$873.44$$

Therefore, \$873.44 is the amount you must invest in order to achieve your goal.

Lecture Tip: See IM 5.2 for an additional tool for illustrating present values.

Slide 5.14 Present Values—Example 2

Slide 5.15 Present Values—Example 3

Saving Up

You would like to buy a new automobile. You have \$50,000 or so, but the car costs \$68,500. If you can earn 9 percent, how much do you have to invest today to buy the car in two years? Do you have enough? Assume the price will stay the same.

What we need to know is the present value of \$68,500 to be paid in two years, assuming a 9 percent rate. Based on our discussion, this is:

$$PV = \$68,500/1.09^2 = \$68,500/1.1881 = \$57,655.08$$

You're still about \$7,655 short, even if you're willing to wait two years.

EXAMPLE 5.6

As you have probably recognized by now, calculating present values is quite similar to calculating future values, and the general result looks much the same. The present value of \$1 to be received t periods into the future at a discount rate of r is:

$$PV = \$1 \times [1/(1 + r)^t] = \$1/(1 + r)^t \quad [5.2]$$

The quantity in brackets, $1/(1 + r)^t$, goes by several different names. Because it's used to discount a future cash flow, it is often called a *discount factor*. With this name, it is not surprising that the rate used in the calculation is often called the **discount rate**. We will tend to call it this in talking about present values. The quantity in brackets is also called the *present value interest factor* (or just *present value factor*) for \$1 at r percent for t periods and is sometimes abbreviated as $PVIF(r, t)$. Finally, calculating the present value of a future cash flow to determine its worth today is commonly called **discounted cash flow (DCF) valuation**.

To illustrate, suppose you need \$1,000 in three years. You can earn 15 percent on your money. How much do you have to invest today? To find out, we have to determine

Concept Q Answer 5.2c

discount rate

The rate used to calculate the present value of future cash flows.

discounted cash flow (DCF) valuation

Calculating the present value of a future cash flow to determine its value today.

the present value of \$1,000 in three years at 15 percent. We do this by discounting \$1,000 back three periods at 15 percent. With these numbers, the discount factor is:

$$1/(1 + .15)^3 = 1/1.5209 = .6575$$

The amount you must invest is thus:

$$\$1,000 \times .6575 = \$657.50$$

We say that \$657.50 is the present or discounted value of \$1,000 to be received in three years at 15 percent.

There are tables for present value factors just as there are tables for future value factors, and you use them in the same way (if you use them at all). Table 5.3 contains a small set. A much larger set can be found in Table A.2 in the book's appendix.

In Table 5.3, the discount factor we just calculated (.6575) can be found by looking down the column labeled "15%" until you come to the third row.

Lecture Tip: IM 5.2 provides a real-world example of multiperiod discounting utilizing data from the art world.

Lecture Tip: IM 5.2 provides a discussion to accompany slides 5.16 and 5.17.

CALCULATOR HINTS



You solve present value problems on a financial calculator just like you do future value problems. For the example we just examined (the present value of \$1,000 to be received in three years at 15 percent), you would do the following:

Enter	3	15		1,000
	N	%i	PMT	PV
Solve for				-657.50

Notice that the answer has a negative sign; as we discussed above, that's because it represents an outflow today in exchange for the \$1,000 inflow later.

EXAMPLE 5.7

Deceptive Advertising?

Recently, some businesses have been saying things like "Come try our product. If you do, we'll give you \$100 just for coming by!" If you read the fine print, what you find out is that they will give you a savings certificate that will pay you \$100 in 25 years or so. If the going interest rate on such certificates is 10 percent per year, how much are they really giving you today?

What you're actually getting is the present value of \$100 to be paid in 25 years. If the discount rate is 10 percent per year, then the discount factor is:

$$1/1.1^{25} = 1/10.8347 = .0923$$

This tells you that a dollar in 25 years is worth a little more than nine cents today, assuming a 10 percent discount rate. Given this, the promotion is actually paying you about $.0923 \times \$100 = \9.23 . Maybe this is enough to draw customers, but it's not \$100.

Slide 5.16 Present Value—Important Relationship I

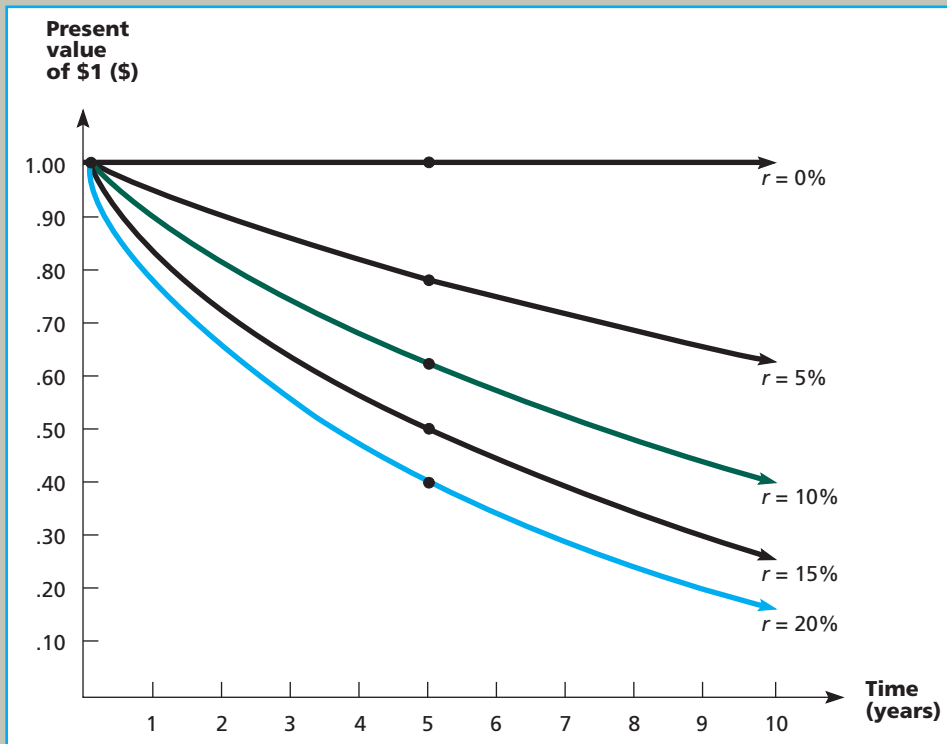
As the length of time until payment grows, present values decline. As Example 5.7 illustrates, present values tend to become small as the time horizon grows. If you look out far enough, they will always get close to zero. Also, for a given length of time, the

Number of Periods	Interest Rate			
	5%	10%	15%	20%
1	.9524	.9091	.8696	.8333
2	.9070	.8264	.7561	.6944
3	.8638	.7513	.6575	.5787
4	.8227	.6830	.5718	.4823
5	.7835	.6209	.4972	.4019

TABLE 5.3
Present Value Interest Factors

Present Value of \$1 for Different Periods and Rates

FIGURE 5.3



higher the discount rate is, the lower is the present value. Put another way, present values and discount rates are inversely related. Increasing the discount rate decreases the PV and vice versa.

The relationship between time, discount rates, and present values is illustrated in Figure 5.3. Notice that by the time we get to 10 years, the present values are all substantially smaller than the future amounts.

Slide 5.17 Present Value—Important Relationship II

Concept Q Answer 5.2d

Slide 5.18 Quick Quiz—Part II

CONCEPT QUESTIONS

- 5.2a** What do we mean by the present value of an investment?
- 5.2b** The process of discounting a future amount back to the present is the opposite of doing what?
- 5.2c** What do we mean by discounted cash flow, or DCF, valuation?
- 5.2d** In general, what is the present value of \$1 to be received in t periods, assuming a discount rate of r per period?

5.3

Self-Test Problems 5.3
and 5.4

Critical Thinking
Question 8

Problems 4–9, 13, 15,
16, 20

Slide 5.19 The Basic PV
Equation—Refresher

Concept Q Answer 5.3a

EXAMPLE 5.8

Lecture Tip: The underlying *economic* rationale for time value analysis is discussed in IM 5.3.

Lecture Tip: See IM 5.3 for a tip on helping students find any of the variables in the basic TVM equation.

MORE ON PRESENT AND FUTURE VALUES

If you look back at the expressions we came up with for present and future values, you will see there is a very simple relationship between the two. We explore this relationship and some related issues in this section.

Present versus Future Value

What we called the present value factor is just the reciprocal of (that is, 1 divided by) the future value factor:

$$\text{Future value factor} = (1 + r)^t$$

$$\text{Present value factor} = 1/(1 + r)^t$$

In fact, the easy way to calculate a present value factor on many calculators is to first calculate the future value factor and then press the “1/x” key to flip it over.

If we let FV_t stand for the future value after t periods, then the relationship between future value and present value can be written very simply as one of the following:

$$PV \times (1 + r)^t = FV_t$$

$$PV = FV_t / (1 + r)^t = FV_t \times [1/(1 + r)^t] \quad [5.3]$$

This last result we will call the *basic present value equation*. We will use it throughout the text. There are a number of variations that come up, but this simple equation underlies many of the most important ideas in corporate finance.

Evaluating Investments

To give you an idea of how we will be using present and future values, consider the following simple investment. Your company proposes to buy an asset for \$335. This investment is very safe. You would sell off the asset in three years for \$400. You know you could invest the \$335 elsewhere at 10 percent with very little risk. What do you think of the proposed investment?

This is not a good investment. Why not? Because you can invest the \$335 elsewhere at 10 percent. If you do, after three years it will grow to:

$$\begin{aligned} \$335 \times (1 + r)^t &= \$335 \times 1.1^3 \\ &= \$335 \times 1.331 \\ &= \$445.89 \end{aligned}$$

Because the proposed investment only pays out \$400, it is not as good as other alternatives we have. Another way of seeing the same thing is to notice that the present value of \$400 in three years at 10 percent is:

$$\$400 \times [1/(1 + r)^3] = \$400/1.1^3 = \$400/1.331 = \$300.53$$

This tells us that we only have to invest about \$300 to get \$400 in three years, not \$335. We will return to this type of analysis later on.

Determining the Discount Rate

It will turn out that we will frequently need to determine what discount rate is implicit in an investment. We can do this by looking at the basic present value equation:

$$PV = FV_t / (1 + r)^t$$

There are only four parts to this equation: the present value (PV), the future value (FV_t), the discount rate (r), and the life of the investment (t). Given any three of these, we can always find the fourth.

Finding r for a Single-Period Investment

You are considering a one-year investment. If you put up \$1,250, you will get back \$1,350. What rate is this investment paying?

First, in this single-period case, the answer is fairly obvious. You are getting a total of \$100 in addition to your \$1,250. The implicit rate on this investment is thus $\$100/\$1,250 = 8$ percent.

More formally, from the basic present value equation, the present value (the amount you must put up today) is \$1,250. The future value (what the present value grows to) is \$1,350. The time involved is one period, so we have:

$$\begin{aligned} \$1,250 &= \$1,350 / (1 + r)^1 \\ 1 + r &= \$1,350 / \$1,250 = 1.08 \\ r &= 8\% \end{aligned}$$

In this simple case, of course, there was no need to go through this calculation, but, as we describe next, it gets a little harder when there is more than one period.

To illustrate what happens with multiple periods, let's say that we are offered an investment that costs us \$100 and will double our money in eight years. To compare this to other investments, we would like to know what discount rate is implicit in these numbers. This discount rate is called the *rate of return*, or sometimes just *return*, on the investment. In this case, we have a present value of \$100, a future value of \$200 (double our money), and an eight-year life. To calculate the return, we can write the basic present value equation as:

$$\begin{aligned} PV &= FV_t / (1 + r)^t \\ \$100 &= \$200 / (1 + r)^8 \end{aligned}$$

It could also be written as:

$$(1 + r)^8 = \$200 / \$100 = 2$$

We now need to solve for r . There are three ways we could do it:

1. Use a financial calculator.
2. Solve the equation for $1 + r$ by taking the eighth root of both sides. Because this is the same thing as raising both sides to the power of $1/8$ or .125, this is actually easy

For a downloadable, Windows-based financial calculator, go to www.calculator.org.

Slide 5.20 Discount Rate

Slide 5.21 Discount Rate—Example 1

EXAMPLE 5.9

Slide 5.22 Discount Rate—Example 2

Slide 5.23 Discount Rate—Example 3

to do with the “y^x” key on a calculator. Just enter 2, then press “y^x,” enter .125, and press the “=” key. The eighth root should be about 1.09, which implies that r is 9 percent.

Lecture Tip: More generally, r is equal to $(FV/PV)^{1/t} - 1$.

- Use a future value table. The future value factor after eight years is equal to 2. If you look across the row corresponding to eight periods in Table A.1, you will see that a future value factor of 2 corresponds to the 9 percent column, again implying that the return here is 9 percent.

Concept Q Answer 5.3b

Actually, in this particular example, there is a useful “back of the envelope” means of solving for r —the Rule of 72. For reasonable rates of return, the time it takes to double your money is given approximately by $72/r\%$. In our example, this means that $72/r\% = 8$ years, implying that r is 9 percent, as we calculated. This rule is fairly accurate for discount rates in the 5 percent to 20 percent range.

EXAMPLE 5.10

Big Mac

In 1998, when Mark McGwire was chasing baseball’s single-season home run record, there was much speculation as to what might be the value of the baseball he hit to break the record (in 1999, the record-setting 70th home run ball sold for \$3 million). One “expert” on such collectibles said, “No matter what it’s worth today, I’m sure it will double in value over the next 10 years.”

So, would the record-breaking home run ball have been a good investment? By the Rule of 72, you already know that since the expert was predicting that the ball would double in value in 10 years, he was predicting that it would earn about $72/10 = 7.2\%$ per year, which is only so-so. Of course, thanks to Barry Bonds, it will probably do much worse!

At one time at least, a rule of thumb in the rarified world of fine art collecting was “your money back in 5 years, double your money in 10 years.” Given this, let’s see how one investment stacked up. In 1976, British Rail purchased the Renoir portrait *La Promenade* for \$1 million as an investment for its pension fund (the goal was to diversify the fund’s holdings more broadly). In 1989, it sold the portrait for nearly \$15 million. Relative to the rule of thumb, how did British Rail do? Did they make money, or did they get railroaded?

The rule of thumb has us doubling our money in 10 years, so, from the Rule of 72, we have that 7.2 percent per year was the norm. We will assume that British Rail bought the painting on January 1, 1976, and sold it at the end of 1989, for a total of 14 years. The present value is \$1 million, and the future value is \$15 million. We need to solve for the unknown rate, r , as follows:

$$\begin{aligned} \$1 \text{ million} &= \$15 \text{ million}/(1 + r)^{14} \\ (1 + r)^{14} &= 15 \end{aligned}$$

Solving for r , we get that British Rail earned about 21.34 percent per year, or almost three times the 7.2 percent rule of thumb. Not bad.

Can’t afford a Renoir? Well, a Schwinn Deluxe Tornado boy’s bicycle sold for \$49.95 when it was new in 1959, and it was a beauty. Assuming it was still in like-new condition in 2001, it was worth about 12 times as much. At what rate did its value grow? Verify for yourself that the answer is about 6.1 percent per year, assuming a 42-year period.

A Mickey Mantle bobbing-head doll was a better investment. It sold for \$2.98 in 1962, but by 2000, it was worth about \$700 (in perfect condition). See if you agree that this collectible gained, on average, 15.45 percent per year.

Why does the Rule of 72 work? See www.datachimp.com.



A slightly more extreme example involves money bequeathed by Benjamin Franklin, who died on April 17, 1790. In his will, he gave 1,000 pounds sterling to Massachusetts and the city of Boston. He gave a like amount to Pennsylvania and the city of Philadelphia. The money had been paid to Franklin when he held political office, but he believed that politicians should not be paid for their service (it appears that this view is not widely shared by modern-day politicians).

Franklin originally specified that the money should be paid out 100 years after his death and used to train young people. Later, however, after some legal wrangling, it was agreed that the money would be paid out in 1990, 200 years after Franklin's death. By that time, the Pennsylvania bequest had grown to about \$2 million; the Massachusetts bequest had grown to \$4.5 million. The money was used to fund the Franklin Institutes in Boston and Philadelphia. Assuming that 1,000 pounds sterling was equivalent to \$1,000, what rate of return did the two states earn (the dollar did not become the official U.S. currency until 1792)?

For Pennsylvania, the future value is \$2 million and the present value is \$1,000. There are 200 years involved, so we need to solve for r in the following:

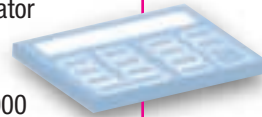
$$\begin{aligned} \$1,000 &= \$2 \text{ million}/(1 + r)^{200} \\ (1 + r)^{200} &= 2,000 \end{aligned}$$

Solving for r , we see that the Pennsylvania money grew at about 3.87 percent per year. The Massachusetts money did better; verify that the rate of return in this case was 4.3 percent. Small differences in returns can add up!

CALCULATOR HINTS

We can illustrate how to calculate unknown rates using a financial calculator using these numbers. For Pennsylvania, you would do the following:

Enter	200		-1,000	2,000,000
	N	%i	PV	FV
Solve for		3.87		



As in our previous examples, notice the minus sign on the present value, representing Franklin's outlay made many years ago. What do you change to work the problem for Massachusetts?

Saving for College

You estimate that you will need about \$80,000 to send your child to college in eight years. You have about \$35,000 now. If you can earn 20 percent per year, will you make it? At what rate will you just reach your goal?

If you can earn 20 percent, the future value of your \$35,000 in eight years will be:

$$FV = \$35,000 \times 1.20^8 = \$35,000 \times 4.2998 = \$150,493.59$$

So, you will make it easily. The minimum rate is the unknown r in the following:

$$\begin{aligned} FV &= \$35,000 \times (1 + r)^8 = \$80,000 \\ (1 + r)^8 &= \$80,000/\$35,000 = 2.2857 \end{aligned}$$

EXAMPLE 5.11

Therefore, the future value factor is 2.2857. Looking at the row in Table A.1 that corresponds to eight periods, we see that our future value factor is roughly halfway between the ones shown for 10 percent (2.1436) and 12 percent (2.4760), so you will just reach your goal if you earn approximately 11 percent. To get the exact answer, we could use a financial calculator or we could solve for r :

$$\begin{aligned}(1 + r)^8 &= \$80,000/35,000 = 2.2857 \\ 1 + r &= 2.2857^{(1/8)} = 2.2857^{.125} = 1.1089 \\ r &= 10.89\%\end{aligned}$$

EXAMPLE 5.12**Only 18,262.5 Days to Retirement**

You would like to retire in 50 years as a millionaire. If you have \$10,000 today, what rate of return do you need to earn to achieve your goal?

The future value is \$1,000,000. The present value is \$10,000, and there are 50 years until payment. We need to calculate the unknown discount rate in the following:

$$\begin{aligned}\$10,000 &= \$1,000,000/(1 + r)^{50} \\ (1 + r)^{50} &= 100\end{aligned}$$

The future value factor is thus 100. You can verify that the implicit rate is about 9.65 percent.

How much do you need at retirement? Check out the "Money/Retirement" link at www.about.com.

Not taking the time value of money into account when computing growth rates or rates of return often leads to some misleading numbers in the real world. For example, in 1997, Nissan announced plans to restore 56 vintage Datsun 240Zs and sell them to consumers. The price tag of a restored Z? About \$25,000, which was at least 609 percent greater than the cost of a 240Z when it sold new 27 years earlier. As expected, many viewed the restored Zs as potential investments because they were virtual carbon copies of the classic original.

If history is any guide, we can get a rough idea of how well you might expect such an investment to perform. According to the numbers quoted above, a Z that originally sold 27 years earlier for about \$3,526 would sell for about \$25,000 in 1997. See if you don't agree that this represents a return of 7.52 percent per year, far less than the gaudy 609 percent difference in the values when the time value of money is ignored.

If classic cars don't capture your fancy, how about classic maps? A few years ago, the first map of America, printed in Rome in 1507, was valued at about \$135,000, 69 percent more than the \$80,000 it was worth 10 years earlier. Your return on investment if you were the proud owner of the map over those 10 years? Verify that it's about 5.4 percent per year, far worse than the 69 percent reported increase in price.

Whether it's maps or cars, it's easy to be misled when returns are quoted without considering the time value of money. However, it's not just the uninitiated who are guilty of this slight form of deception. The title of a feature article in a leading business magazine predicted the Dow-Jones Industrial Average would soar to a 70 percent gain over the coming five years. Do you think it meant a 70 percent return per year on your money? Think again!

Slide 5.24 Quick Quiz—
Part III

Finding the Number of Periods

Slide 5.25 Finding the
Number of Periods

Suppose we are interested in purchasing an asset that costs \$50,000. We currently have \$25,000. If we can earn 12 percent on this \$25,000, how long until we have the \$50,000? Finding the answer involves solving for the last variable in the basic present

value equation, the number of periods. You already know how to get an approximate answer to this particular problem. Notice that we need to double our money. From the Rule of 72, this will take about $72/12 = 6$ years at 12 percent.

To come up with the exact answer, we can again manipulate the basic present value equation. The present value is \$25,000, and the future value is \$50,000. With a 12 percent discount rate, the basic equation takes one of the following forms:

$$\$25,000 = \$50,000/1.12^t$$

$$\$50,000/\$25,000 = 1.12^t = 2$$

We thus have a future value factor of 2 for a 12 percent rate. We now need to solve for t . If you look down the column in Table A.1 that corresponds to 12 percent, you will see that a future value factor of 1.9738 occurs at six periods. It will thus take about six years, as we calculated. To get the exact answer, we have to explicitly solve for t (or use a financial calculator). If you do this, you will see that the answer is 6.1163 years, so our approximation was quite close in this case.

Slide 5.26 Number of Periods—Example 1

Lecture Tip: In general, $t = \ln(FV/PV)/\ln(1 + r)$

Slide 5.27 Number of Periods—Example 2

Slide 5.28 Number of Periods—Example 2 Continued

CALCULATOR HINTS

If you do use a financial calculator, here are the relevant entries:

Enter	12	-25,000	50,000
	N	%i	PMT
		PV	FV
Solve for	6.1163		



Waiting for Godot

You've been saving up to buy the Godot Company. The total cost will be \$10 million. You currently have about \$2.3 million. If you can earn 5 percent on your money, how long will you have to wait? At 16 percent, how long must you wait?

At 5 percent, you'll have to wait a long time. From the basic present value equation:

$$\$2.3 \text{ million} = \$10 \text{ million}/1.05^t$$

$$1.05^t = 4.35$$

$$t = 30 \text{ years}$$

At 16 percent, things are a little better. Verify for yourself that it will take about 10 years.

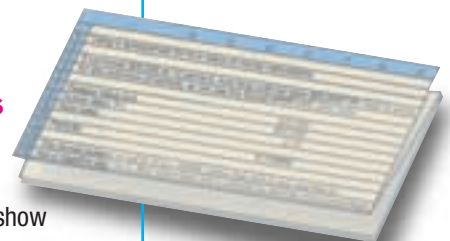
EXAMPLE 5.13

Slide 5.29 Quick Quiz—Part IV

SPREADSHEET STRATEGIES

Using a Spreadsheet for Time Value of Money Calculations

More and more, businesspeople from many different areas (and not just finance and accounting) rely on spreadsheets to do all the different types of calculations that come up in the real world. As a result, in this section, we will show you how to use a spreadsheet to handle the various time value of money problems we presented in this chapter. We will use Microsoft Excel™, but the commands are similar for



Slide 5.30 Spreadsheet Example

other types of software. We assume you are already familiar with basic spreadsheet operations.

As we have seen, you can solve for any one of the following four potential unknowns: future value, present value, the discount rate, or the number of periods. With a spreadsheet, there is a separate formula for each. In Excel, these are as follows:

To Find	Enter This Formula
Future value	= FV (rate,nper,pmt,pv)
Present value	= PV (rate,nper,pmt,fv)
Discount rate	= RATE (nper,pmt,pv,fv)
Number of periods	= NPER (rate,pmt,pv,fv)

In these formulas, pv and fv are present and future value, nper is the number of periods, and rate is the discount, or interest, rate.

There are two things that are a little tricky here. First, unlike a financial calculator, the spreadsheet requires that the rate be entered as a decimal. Second, as with most financial calculators, you have to put a negative sign on either the present value or the future value to solve for the rate or the number of periods. For the same reason, if you solve for a present value, the answer will have a negative sign unless you input a negative future value. The same is true when you compute a future value.

To illustrate how you might use these formulas, we will go back to an example in the chapter. If you invest \$25,000 at 12 percent per year, how long until you have \$50,000? You might set up a spreadsheet like this:

	A	B	C	D	E	F	G	H
1								
2	Using a spreadsheet for time value of money calculations							
3								
4	If we invest \$25,000 at 12 percent, how long until we have \$50,000? We need to solve							
5	for the unknown number of periods, so we use the formula NPER(rate, pmt, pv, fv).							
6								
7	Present value (pv):	\$25,000						
8	Future value (fv):	\$50,000						
9	Rate (rate):	0.12						
10								
11	Periods:	6.1162554						
12								
13	The formula entered in cell B11 is =NPER(B9,0,-B7,B8); notice that pmt is zero and that pv							
14	has a negative sign on it. Also notice that rate is entered as a decimal, not a percentage.							

Learn more about using Excel for time value and other calculations at www.studyfinance.com.



U.S. EE Savings Bonds are a familiar investment for many. A U.S. EE Savings Bond is much like the GMAC Security we described at the start of the chapter. You purchase them for half of their \$100 face value. In other words, you pay \$50 today and get \$100 at some point in the future when the bond “matures.” You receive no interest in between. For EE bonds sold after May 1, 1997, the interest rate is adjusted every six months, so the length of time until your \$50 grows to \$100 depends on future interest rates. However, at worst, the bonds are guaranteed to be worth \$100 at the end of 17 years, so this is the longest you would ever have to wait. If you do have to wait the full 17 years, what rate do you earn?

Because this investment is doubling in value in 17 years, the Rule of 72 tells you the answer right away: $72/17 = 4.24\%$. Remember, this is the minimum guaranteed return. You might do better, and we will return to EE bonds in a later chapter. For now, this

Work the Web

How important is the time value of money? A recent search on one web engine returned over 31,000 hits! It is important to understand the calculations behind the time value of money, but the advent of financial calculators and spreadsheets has eliminated the need for tedious calculations. In fact, many web sites offer time value of money calculators. The following is one example from Cigna's web site, www.cigna.com. You have \$10,000 today and will invest it at 10.5 percent for 30 years. How much will it be worth at that time? With the Cigna calculator, you simply enter the values and hit Calculate:

WWW



Slide 5.31 Work the Web—Example

Present Value/Future Value Calculator

Fill in any 3 of the 4 boxes in the form.

Present Value Amount: \$

Number of Years Invested:

Growth Rate: %

Future Value Amount: \$

The results look like this:

Results	
Present Value Amount	\$10,000
Number of Years Invested	30 years
Growth Rate	10.5%
Future Value Amount	\$199,926

Who said time value of money calculations are hard?

example finishes our introduction to basic time value concepts. Table 5.4 summarizes present and future value calculations for future reference. As our nearby *Work the Web* box shows, online calculators are widely available to handle these calculations, but it is still important to know what is really going on.

CONCEPT QUESTIONS

- 5.3a** What is the basic present value equation?
5.3b What is the Rule of 72?

TABLE 5.4

Summary of Time Value Calculations

Slide 5.32

I. Symbols:

PV = Present value, what future cash flows are worth today

FV_t = Future value, what cash flows are worth in the future*r* = Interest rate, rate of return, or discount rate per period—typically, but not always, one year*t* = Number of periods—typically, but not always, the number of years

C = Cash amount

II. Future value of C invested at *r* percent for *t* periods:

$$FV_t = C \times (1 + r)^t$$

The term $(1 + r)^t$ is called the *future value factor*.**III. Present value of C to be received in *t* periods at *r* percent per period:**

$$PV = C/(1 + r)^t$$

The term $1/(1 + r)^t$ is called the *present value factor*.**IV. The basic present value equation giving the relationship between present and future value is:**

$$PV = FV_t/(1 + r)^t$$

5.4**SUMMARY AND CONCLUSIONS**

This chapter has introduced you to the basic principles of present value and discounted cash flow valuation. In it, we explained a number of things about the time value of money, including:

1. For a given rate of return, the value at some point in the future of an investment made today can be determined by calculating the future value of that investment.
2. The current worth of a future cash flow or series of cash flows can be determined for a given rate of return by calculating the present value of the cash flow(s) involved.
3. The relationship between present value (PV) and future value (FV) for a given rate *r* and time *t* is given by the basic present value equation:

$$PV = FV_t/(1 + r)^t$$

As we have shown, it is possible to find any one of the four components (PV, FV, *r*, or *t*) given the other three.

The principles developed in this chapter will figure prominently in the chapters to come. The reason for this is that most investments, whether they involve real assets or financial assets, can be analyzed using the discounted cash flow (DCF) approach. As a result, the DCF approach is broadly applicable and widely used in practice. Before going on, therefore, you might want to do some of the problems that follow.

Chapter Review and Self-Test Problems

- 5.1 Calculating Future Values** Assume you deposit \$10,000 today in an account that pays 6 percent interest. How much will you have in five years?

- 5.2 Calculating Present Values** Suppose you have just celebrated your 19th birthday. A rich uncle has set up a trust fund for you that will pay you \$150,000 when you turn 30. If the relevant discount rate is 9 percent, how much is this fund worth today?
- 5.3 Calculating Rates of Return** You've been offered an investment that will double your money in 10 years. What rate of return are you being offered? Check your answer using the Rule of 72.
- 5.4 Calculating the Number of Periods** You've been offered an investment that will pay you 9 percent per year. If you invest \$15,000, how long until you have \$30,000? How long until you have \$45,000?

Answers to Chapter Review and Self-Test Problems

- 5.1** We need to calculate the future value of \$10,000 at 6 percent for five years. The future value factor is:

$$1.06^5 = 1.3382$$

The future value is thus $\$10,000 \times 1.3382 = \$13,382.26$.

- 5.2** We need the present value of \$150,000 to be paid in 11 years at 9 percent. The discount factor is:

$$1/1.09^{11} = 1/2.5804 = .3875$$

The present value is thus about **\$58,130**.

- 5.3** Suppose you invest, say, \$1,000. You will have \$2,000 in 10 years with this investment. So, \$1,000 is the amount you have today, or the present value, and \$2,000 is the amount you will have in 10 years, or the future value. From the basic present value equation, we have:

$$\begin{aligned} \$2,000 &= \$1,000 \times (1 + r)^{10} \\ 2 &= (1 + r)^{10} \end{aligned}$$

From here, we need to solve for r , the unknown rate. As shown in the chapter, there are several different ways to do this. We will take the 10th root of 2 (by raising 2 to the power of $1/10$):

$$\begin{aligned} 2^{(1/10)} &= 1 + r \\ 1.0718 &= 1 + r \\ r &= 7.18\% \end{aligned}$$

Using the Rule of 72, we have $72/t = r\%$, or $72/10 = 7.2\%$, so our answer looks good (remember that the Rule of 72 is only an approximation).

- 5.4** The basic equation is:

$$\begin{aligned} \$30,000 &= \$15,000 \times (1 + .09)^t \\ 2 &= (1 + .09)^t \end{aligned}$$

If we solve for t , we get that $t = 8.04$ years. Using the Rule of 72, we get $72/9 = 8$ years, so, once again, our answer looks good. To get \$45,000, verify for yourself that you will have to wait **12.75** years.

Concepts Review and Critical Thinking Questions

- Present Value** The basic present value equation has four parts. What are they?
- Compounding** What is compounding? What is discounting?
- Compounding and Period** As you increase the length of time involved, what happens to future values? What happens to present values?
- Compounding and Interest Rates** What happens to a future value if you increase the rate r ? What happens to a present value?
- Ethical Considerations** Take a look back at Example 5.7. Is it deceptive advertising? Is it unethical to advertise a future value like this without a disclaimer?

To answer the next five questions, refer to the GMAC security we discussed to open the chapter.

- Time Value of Money** Why would GMAC be willing to accept such a small amount today (\$500) in exchange for a promise to repay 20 times that amount (\$10,000) in the future?
- Call Provisions** GMAC has the right to buy back the securities anytime it wishes by paying \$10,000 (this is a term of this particular deal). What impact does this feature have on the desirability of this security as an investment?
- Time Value of Money** Would you be willing to pay \$500 today in exchange for \$10,000 in 30 years? What would be the key considerations in answering yes or no? Would your answer depend on who is making the promise to repay?
- Investment Comparison** Suppose that when GMAC offered the security for \$500, the U.S. Treasury had offered an essentially identical security. Do you think it would have had a higher or lower price? Why?
- Length of Investment** The GMAC security is actively bought and sold on the New York Stock Exchange. If you looked in *The Wall Street Journal* today, do you think the price would exceed the \$500 original price? Why? If you looked in the year 2008, do you think the price would be higher or lower than today's price? Why?

Questions and Problems

Basic

(Questions 1–15)

\$644.47

\$67,410
\$36,964
\$128,670
\$258,619

- Simple Interest versus Compound Interest** First Tappan Bank pays 5 percent simple interest on its savings account balances, whereas First Mullineaux Bank pays 5 percent interest compounded annually. If you made a \$5,000 deposit in each bank, how much more money would you earn from your First Mullineaux Bank account at the end of 10 years?
- Calculating Future Values** For each of the following, compute the future value:

Present Value	Years	Interest Rate	Future Value
\$ 2,250	30	12%	
9,310	16	9	
76,355	3	19	
183,796	7	5	

- Calculating Present Values** For each of the following, compute the present value:

Present Value	Years	Interest Rate	Future Value
	5	4%	\$ 15,451
	8	12	51,557
	19	22	886,073
	15	20	550,164

Basic

(continued)

- \$12,700
- \$20,823
- \$20,260
- \$35,709

4. **Calculating Interest Rates** Solve for the unknown interest rate in each of the following:

Present Value	Years	Interest Rate	Future Value
\$ 265	3		\$ 307
360	9		761
39,000	15		136,771
46,523	30		255,810

- 5.03%
- 8.67%
- 8.72%
- 5.85%

5. **Calculating the Number of Periods** Solve for the unknown number of years in each of the following:

Present Value	Years	Interest Rate	Future Value
\$ 625		4%	\$ 1,284
810		9	4,341
18,400		23	402,662
21,500		34	173,439

- 18.36 years
- 19.48 years
- 14.91 years
- 7.13 years

6. **Calculating Interest Rates** Assume the total cost of a college education will be \$200,000 when your child enters college in 18 years. You presently have \$27,000 to invest. What annual rate of interest must you earn on your investment to cover the cost of your child's college education?

11.77%

7. **Calculating the Number of Periods** At 6 percent interest, how long does it take to double your money? To quadruple it?

- 11.90 years
- 23.79 years

8. **Calculating Interest Rates** You are offered an investment that requires you to put up \$12,000 today in exchange for \$40,000 15 years from now. What is the annual rate of return on this investment?

8.36%

9. **Calculating the Number of Periods** You're trying to save to buy a new \$120,000 Ferrari. You have \$40,000 today that can be invested at your bank. The bank pays 5.5 percent annual interest on its accounts. How long will it be before you have enough to buy the car?

20.52 years

10. **Calculating Present Values** Imprudential, Inc., has an unfunded pension liability of \$650 million that must be paid in 20 years. To assess the value of the firm's stock, financial analysts want to discount this liability back to the present. If the relevant discount rate is 8.5 percent, what is the present value of this liability?

\$127.15 million

11. **Calculating Present Values** You have just received notification that you have won the \$1 million first prize in the Centennial Lottery. However, the prize will be awarded on your 100th birthday (assuming you're around to collect), 80 years from now. What is the present value of your windfall if the appropriate discount rate is 13 percent?

\$56.72

12. **Calculating Future Values** Your coin collection contains fifty 1952 silver dollars. If your parents purchased them for their face value when they were new,

\$4,547.83

Basic

(continued)

8.55%
\$22,096,171.20

\$7.00

-7.53%

Intermediate

(Questions 16–20)

10.50%
11.97%
7.62%

\$42,262.14

\$96,654.57
\$40,827.94

\$41,365.28

23.93 years

S&P Problems**STANDARD
& POOR'S**

- how much will your collection be worth when you retire in 2067, assuming they appreciate at a 4 percent annual rate?
13. **Calculating Interest Rates and Future Values** In 1895, the first U.S. Open Golf Championship was held. The winner's prize money was \$150. In 2001, the winner's check was \$900,000. What was the percentage increase in the winner's check over this period? If the winner's prize increases at the same rate, what will it be in 2040?
14. **Calculating Present Values** In 2001, a mechanized toy robot from the television series *Lost in Space* sold for \$750. This represented a 13.86 percent annual return. For this to be true, what must the robot have sold for new in 1965?
15. **Calculating Rates of Return** Although appealing to more refined tastes, art as a collectible has not always performed so profitably. During 1995, Christie's auctioned the William de Kooning painting *Untitled*. The highest bid of \$2.2 million was rejected by the owner, who had purchased the painting at the height of the art market in 1989 for \$3.52 million. Had the seller accepted the bid, what would his annual rate of return have been?
16. **Calculating Rates of Return** Referring to the GMAC security we discussed at the very beginning of the chapter:
- Based upon the \$500 price, what rate was GMAC paying to borrow money?
 - Suppose that, on December 1, 2002, this security's price was \$4,800. If an investor had purchased it for \$500 at the offering and sold it on this day, what annual rate of return would she have earned?
 - If an investor had purchased the security at market on December 1, 2002, and held it until it matured, what annual rate of return would she have earned?
17. **Calculating Present Values** Suppose you are still committed to owning a \$120,000 Ferrari (see Question 9). If you believe your mutual fund can achieve an 11 percent annual rate of return and you want to buy the car in 10 years on the day you turn 30, how much must you invest today?
18. **Calculating Future Values** You have just made your first \$2,000 contribution to your individual retirement account. Assuming you earn a 9 percent rate of return and make no additional contributions, what will your account be worth when you retire in 45 years? What if you wait 10 years before contributing? (Does this suggest an investment strategy?)
19. **Calculating Future Values** You are scheduled to receive \$30,000 in two years. When you receive it, you will invest it for six more years at 5.5 percent per year. How much will you have in eight years?
20. **Calculating the Number of Periods** You expect to receive \$10,000 at graduation in two years. You plan on investing it at 12 percent until you have \$120,000. How long will you wait from now?

1. **Calculating Future Values** Find the monthly adjusted prices for Tyco International LTD (TYC). If the stock appreciates 11 percent per year, what stock price do you expect to see in five years? In 10 years? Ignore dividends in your calculations.

2. **Calculating Interest Rates** Find the monthly adjusted prices for Redhook Ale Brewery Inc. (HOOK). What is the average annual return over the past four years?
3. **Calculating the Number of Periods** Find the monthly adjusted stock prices for Nucor Corp. (NUE). You find an analyst who projects the stock price will increase 12 percent per year for the foreseeable future. Based on the most recent monthly stock price, if the projection holds true, when will the stock price reach \$150? When will it reach \$200?
- 5.1 **Calculating Future Values** Go to www.dinkytown.net and follow the “Savings Calculator” link. If you currently have \$10,000 and invest this money at 9 percent, how much will you have in 30 years? Assume you will not make any additional contributions. How much will you have if you can earn 11 percent?
- 5.2 **Calculating the Number of Periods** Go to www.dinkytown.net and follow the “Cool Million” link. You want to be a millionaire. You can earn 11.5 percent per year. Using your current age, at what age will you become a millionaire if you have \$25,000 to invest, assuming you make no other deposits (ignore inflation)?
- 5.3 **Calculating the Number of Periods** Cigna has a financial calculator available at www.cigna.com. To get to the calculator, follow the “Calculator & Tools” link, then the “Present/Future Value Calculator” link. You want to buy a Lamborghini Diablo VTTT. The current market price of the car is \$330,000 and you have \$33,000. If you can earn an 11 percent return, how many years until you can buy this car (assuming the price stays the same)?
- 5.4 **Calculating Rates of Return** Use the Cigna financial calculator to solve the following problem. You still want to buy the Lamborghini VTTT, but you have \$50,000 to deposit and want to buy the car in 15 years. What interest rate do you have to earn to accomplish this (assuming the price stays the same)?
- 5.5 **Future Values and Taxes** Taxes can greatly affect the future value of your investment. The Financial Calculators web site at www.fincalc.com has a financial calculator that adjusts your return for taxes. Follow the “Projected Savings” link on this page to find this calculator. Suppose you have \$50,000 to invest today. If you can earn a 12 percent return and no additional annual savings, how much will you have in 20 years? (Enter 0 percent as the tax rate.) Now, assume that your marginal tax rate is 27.5 percent. How much will you have at this tax rate?

What's On the Web?

Spreadsheet Templates 5-1, 5-2, 5-3, 5-4, 5-5

