GETTING MORE INVOLVED

64. ***Discussion.*** Which of the following equations is not a proportion? Explain.

a) $\frac{1}{2} = \frac{1}{2}$

b) $\frac{x}{x + 2} = \frac{4}{5}$

c) $\frac{x}{4} = \frac{9}{x}$

d) $\frac{8}{x + 2} - 1 = \frac{5}{x + 2}$

65. ***Discussion.*** Find all of the errors in the following solution to an equation.

$$\frac{7}{x} = \frac{8}{x + 3} + 1$$
$$7(x + 3) = 8x + 1$$
$$7x + 3 = 8x$$
$$-x = -3$$
$$x = 3$$

7.8 APPLICATIONS OF RATIONAL EXPRESSIONS

In this section

- Formulas
- Uniform Motion Problems
- Work Problems
- Purchasing Problems

In this section we will study additional applications of rational expressions.

Formulas

Many formulas involve rational expressions. When solving a formula of this type for a certain variable, we usually multiply each side by the LCD to eliminate the denominators.

EXAMPLE 1 **An equation of a line**

The equation for the line through $(-2, 4)$ with slope $3/2$ can be written as

$$\frac{y - 4}{x + 2} = \frac{3}{2}.$$

We studied equations of this type in Chapter 4. Solve this equation for y.

Solution

To isolate y on the left-hand side of the equation, we multiply each side by $x + 2$:

$$\frac{y - 4}{x + 2} = \frac{3}{2} \quad \text{Original equation}$$

$$(x + 2) \cdot \frac{y - 4}{x + 2} = (x + 2) \cdot \frac{3}{2} \quad \text{Multiply by } x + 2.$$

$$y - 4 = \frac{3}{2}x + 3 \quad \text{Simplify.}$$

$$y = \frac{3}{2}x + 7 \quad \text{Add 4 to each side.}$$

Because the original equation is a proportion, we could have used the extremes-means property to solve it for y. ■

helpful hint

When this equation was written in the form

$y - y_1 = m(x - x_1)$

in Chapter 4, we called it the point-slope formula for the equation of a line.

EXAMPLE 2 **Distance, rate, and time**

Solve the formula $\frac{D}{T} = R$ for T.

study tip

As you study from the text, think about the material. Ask yourself questions. If you were the professor, what questions would you ask on the test?

Solution

Because the only denominator is T, we multiply each side by T:

$$\frac{D}{T} = R \quad \text{Original formula}$$

$$T \cdot \frac{D}{T} = T \cdot R \quad \text{Multiply each side by } T.$$

$$D = TR$$

$$\frac{D}{R} = \frac{TR}{R} \quad \text{Divide each side by } R.$$

$$\frac{D}{R} = T \quad \text{Simplify.}$$

The formula solved for T is $T = \frac{D}{R}$.

EXAMPLE 3 **Focal length of a lens**

The formula $\frac{1}{f} = \frac{1}{o} + \frac{1}{i}$ gives the relationship between the focal length f, the object distance o, and the image distance i for a lens. Solve the formula for i.

Solution

The LCD for f, o, and i is foi.

$$\frac{1}{f} = \frac{1}{o} + \frac{1}{i} \quad \text{Original formula}$$

$$foi \cdot \frac{1}{f} = foi \cdot \frac{1}{o} + foi \cdot \frac{1}{i} \quad \text{Multiply each side by the LCD, } foi.$$

$$oi = fi + fo \quad \text{All denominators are eliminated.}$$

$$oi - fi = fo \quad \text{Get all terms involving } i \text{ onto the left side.}$$

$$i(o - f) = fo \quad \text{Factor out } i.$$

$$i = \frac{fo}{o - f} \quad \text{Divide each side by } o - f.$$

EXAMPLE 4 **Finding the value of a variable**

In the formula of Example 1, find x if $y = -3$.

Solution

Substitute $y = -3$ into the formula, then solve for x:

$$\frac{y - 4}{x + 2} = \frac{3}{2} \quad \text{Original formula}$$

$$\frac{-3 - 4}{x + 2} = \frac{3}{2} \quad \text{Replace } y \text{ by } -3.$$

$$\frac{-7}{x + 2} = \frac{3}{2} \quad \text{Simplify.}$$

$$3x + 6 = -14 \quad \text{Extremes-means property}$$

$$3x = -20$$

$$x = -\frac{20}{3}$$

Uniform Motion Problems

In uniform motion problems we use the formula $D = RT$. In some problems in which the time is unknown, we can use the formula $T = \frac{D}{R}$ to get an equation involving rational expressions.

EXAMPLE 5

Driving to Florida

Susan drove 1500 miles to Daytona Beach for spring break. On the way back she averaged 10 miles per hour less, and the drive back took her 5 hours longer. Find Susan's average speed on the way to Daytona Beach.

Solution

If x represents her average speed going there, then $x - 10$ is her average speed for the return trip. See Fig. 7.1. We use the formula $T = \frac{D}{R}$ to make the following table.

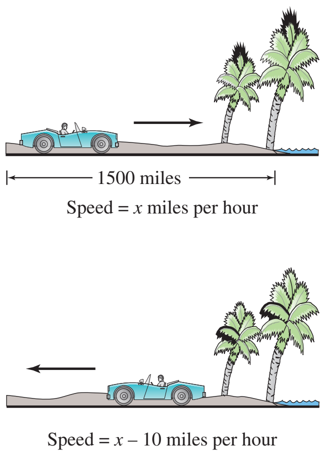


FIGURE 7.1

	D	**R**	**T**	
Going	1500	x	$\frac{1500}{x}$	← Shorter time
Returning	1500	$x - 10$	$\frac{1500}{x - 10}$	← Longer time

Because the difference between the two times is 5 hours, we have

$$\text{longer time} - \text{shorter time} = 5.$$

Using the time expressions from the table, we get the following equation:

$$\frac{1500}{x - 10} - \frac{1500}{x} = 5$$

$$x(x - 10)\frac{1500}{x - 10} - x(x - 10)\frac{1500}{x} = x(x - 10)5 \quad \text{Multiply by } x(x - 10).$$

$$1500x - 1500(x - 10) = 5x^2 - 50x$$

$$15{,}000 = 5x^2 - 50x \quad \text{Simplify.}$$

$$3000 = x^2 - 10x \quad \text{Divide each side by 5.}$$

$$0 = x^2 - 10x - 3000$$

$$(x + 50)(x - 60) = 0 \quad \text{Factor.}$$

$$x + 50 = 0 \quad \text{or} \quad x - 60 = 0$$

$$x = -50 \quad \text{or} \quad x = 60$$

The answer $x = -50$ is a solution to the equation, but it cannot indicate the average speed of the car. Her average speed going to Daytona Beach was 60 mph. ■

helpful hint

Notice that a work rate is the same as a slope from Chapter 4. The only difference is that the work rates here can contain a variable.

Work Problems

If you can complete a job in 3 hours, then you are working at the rate of $\frac{1}{3}$ of the job per hour. If you work for 2 hours at the rate of $\frac{1}{3}$ of the job per hour, then you will complete $\frac{2}{3}$ of the job. The product of the rate and time is the amount of work completed. For problems involving work, we will always assume that the work is done at a constant rate. So if a job takes x hours to complete, then the rate is $\frac{1}{x}$ of the job per hour.

EXAMPLE 6

Shoveling snow

After a heavy snowfall, Brian can shovel all of the driveway in 30 minutes. If his younger brother Allen helps, the job takes only 20 minutes. How long would it take Allen to do the job by himself?

Solution

Let x represent the number of minutes it would take Allen to do the job by himself. Brian's rate for shoveling is $\frac{1}{30}$ of the driveway per minute, and Allen's rate for shoveling is $\frac{1}{x}$ of the driveway per minute. We organize all of the information in a table like the table in Example 5.

	Rate	Time	Work
Brian	$\frac{1 \text{ job}}{30 \text{ min}}$	20 min	$\frac{2}{3}$ job
Allen	$\frac{1 \text{ job}}{x \text{ min}}$	20 min	$\frac{20}{x}$ job

helpful hint

The secret to work problems is remembering that the individual rates or the amounts of work can be added when people work together. If your painting rate is 1/10 of the house per day and your helper's rate is 1/5 of the house per day, then your rate together will be 3/10 of the house per day. In 2 days you will paint 2/10 of the house and your helper will paint 2/5 of the house for a total of 3/5 of the house completed.

If Brian works for 20 min at the rate $\frac{1}{30}$ of the job per minute, then he does $\frac{20}{30}$ or $\frac{2}{3}$ of the job, as shown in Fig. 7.2. The amount of work that each boy does is a fraction of the whole job. So the expressions for work in the last column of the table have a sum of 1:

$$\frac{2}{3} + \frac{20}{x} = 1$$

$$3x \cdot \frac{2}{3} + 3x \cdot \frac{20}{x} = 3x \cdot 1 \quad \text{Multiply each side by } 3x.$$

$$2x + 60 = 3x$$

$$60 = x$$

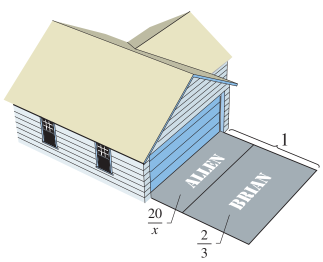


FIGURE 7.2

If it takes Allen 60 minutes to do the job by himself, then he works at the rate of $\frac{1}{60}$ of the job per minute. In 20 minutes he does $\frac{1}{3}$ of the job while Brian does $\frac{2}{3}$. So it would take Allen 60 minutes to shovel the driveway by himself. ■

Notice the similarities between the uniform motion problem in Example 5 and the work problem in Example 6. In both cases it is beneficial to make a table. We use $D = R \cdot T$ in uniform motion problems and $W = R \cdot T$ in work problems. The main points to remember when solving work problems are summarized in the following strategy.

Strategy for Solving Work Problems

1. If a job is completed in x hours, then the rate is $\frac{1}{x}$ job/hr.
2. Make a table showing rate, time, and work completed ($W = R \cdot T$) for each person or machine.
3. The total work completed is the sum of the individual amounts of work completed.
4. If the job is completed, then the total work done is 1 job.

Purchasing Problems

Rates are used in uniform motion and work problems. But rates also occur in purchasing problems. If gasoline is 99.9 cents/gallon, then that is the rate at which your bill is increasing as you pump the gallons into your tank. In purchasing problems the product of the rate and the quantity purchased is the total cost.

EXAMPLE 7

Oranges and grapefruit

Tamara bought 50 pounds of fruit consisting of Florida oranges and Texas grapefruit. She paid twice as much per pound for the grapefruit as she did for the oranges. If Tamara bought $12 worth of oranges and $16 worth of grapefruit, then how many pounds of each did she buy?

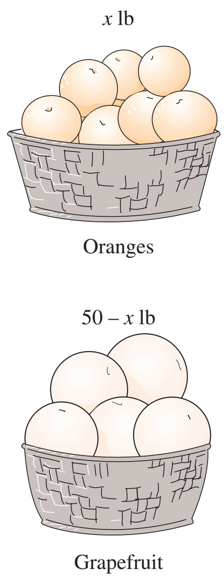


FIGURE 7.3

Solution

Let x represent the number of pounds of oranges and $50 - x$ represent the number of pounds of grapefruit. See Fig. 7.3. Make a table.

	Rate	Quantity	Total cost
Oranges	$\frac{12}{x}$ dollars/pound	x pounds	12 dollars
Grapefruit	$\frac{16}{50 - x}$ dollars/pound	$50 - x$ pounds	16 dollars

Since the price per pound for the grapefruit is twice that for the oranges, we have:

$$2(\text{price per pound for oranges}) = \text{price per pound for grapefruit}$$

$$2\left(\frac{12}{x}\right) = \frac{16}{50 - x}$$

$$\frac{24}{x} = \frac{16}{50 - x}$$

$$16x = 1200 - 24x \quad \text{Extremes-means property}$$

$$40x = 1200$$

$$x = 30$$

$$50 - x = 20$$

If Tamara purchased 20 pounds of grapefruit for $16, then she paid $0.80 per pound. If she purchased 30 pounds of oranges for $12, then she paid $0.40 per pound. Because $0.80 is twice $0.40, we can be sure that she purchased 20 pounds of grapefruit and 30 pounds of oranges. ■

WARM-UPS

True or false? Explain your answer.

1. The formula $t = \frac{1 - t}{m}$, solved for m, is $m = \frac{1 - t}{t}$.
2. To solve $\frac{1}{m} + \frac{1}{n} = \frac{1}{2}$ for m, we multiply each side by $2mn$.
3. If Fiona drives 300 miles in x hours, then her average speed is $\frac{x}{300}$ mph.

WARM-UPS

(continued)

4. If Miguel drives 20 hard bargains in x hours, then he is driving $\frac{20}{x}$ hard bargains per hour.

5. If Fred can paint a house in y days, then he paints $\frac{1}{y}$ of the house per day.

6. If $\frac{1}{x}$ is 1 less than $\frac{2}{x+3}$, then $\frac{1}{x} - 1 = \frac{2}{x+3}$.

7. If a and b are nonzero and $a = \frac{m}{b}$, then $b = am$.

8. If $D = RT$, then $T = \frac{D}{R}$.

9. Solving $P + Prt = I$ for P gives $P = I - Prt$.

10. To solve $3R + yR = m$ for R, we must first factor the left-hand side.

7.8 EXERCISES

Solve each equation for y. See Example 1.

1. $\frac{y-1}{x-3} = 2$

2. $\frac{y-2}{x-4} = -2$

3. $\frac{y-1}{x+6} = -\frac{1}{2}$

4. $\frac{y+5}{x-2} = -\frac{1}{2}$

5. $\frac{y+a}{x-b} = m$

6. $\frac{y-h}{x+k} = a$

7. $\frac{y-1}{x+4} = -\frac{1}{3}$

8. $\frac{y-1}{x+3} = -\frac{3}{4}$

Solve each formula for the indicated variable. See Examples 2 and 3.

9. $A = \frac{B}{C}$ for C

10. $P = \frac{A}{C+D}$ for A

11. $\frac{1}{a} + m = \frac{1}{p}$ for p

12. $\frac{2}{f} + t = \frac{3}{m}$ for m

13. $F = k\frac{m_1m_2}{r^2}$ for m_1

14. $F = \frac{mv^2}{r}$ for r

15. $\frac{1}{a} + \frac{1}{b} = \frac{1}{f}$ for a

16. $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ for R

17. $S = \frac{a}{1-r}$ for r

18. $I = \frac{E}{R+r}$ for R

19. $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$ for P_2

20. $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$ for T_1

21. $V = \frac{4}{3}\pi r^2 h$ for h

22. $h = \frac{S - 2\pi r^2}{2\pi r}$ for S

Find the value of the indicated variable. See Example 4.

23. In the formula of Exercise 9, if $A = 12$ and $B = 5$, find C.

24. In the formula of Exercise 10, if $A = 500$, $P = 100$, and $C = 2$, find D.

25. In the formula of Exercise 11, if $p = 6$ and $m = 4$, find a.

26. In the formula of Exercise 12, if $m = 4$ and $t = 3$, find f.

27. In the formula of Exercise 13, if $F = 32$, $r = 4$, $m_1 = 2$, and $m_2 = 6$, find k.

28. In the formula of Exercise 14, if $F = 10$, $v = 8$, and $r = 6$, find m.

29. In the formula of Exercise 15, if $f = 3$ and $a = 2$, find b.

30. In the formula of Exercise 16, if $R = 3$ and $R_1 = 5$, find R_2.

31. In the formula of Exercise 17, if $S = \frac{3}{2}$ and $r = \frac{1}{5}$, find a.

32. In the formula of Exercise 18, if $I = 15$, $E = 3$, and $R = 2$, find r.

Show a complete solution to each problem. See Example 5.

33. ***Fast walking.*** Marcie can walk 8 miles in the same time as Frank walks 6 miles. If Marcie walks 1 mile per hour faster than Frank, then how fast does each person walk?

34. ***Upstream, downstream.*** Junior's boat will go 15 miles per hour in still water. If he can go 12 miles downstream in the same amount of time as it takes to go 9 miles upstream, then what is the speed of the current?

35. ***Delivery routes.*** Pat travels 70 miles on her milk route, and Bob travels 75 miles on his route. Pat travels 5 miles per hour slower than Bob, and her route takes her one-half hour longer than Bob's. How fast is each one traveling?

36. ***Ride the peaks.*** Smith bicycled 45 miles going east from Durango, and Jones bicycled 70 miles. Jones averaged 5 miles per hour more than Smith, and his trip took one-half hour longer than Smith's. How fast was each one traveling?

37. ***Walking and running.*** Raffaele ran 8 miles and then walked 6 miles. If he ran 5 miles per hour faster than he walked and the total time was 2 hours, then how fast did he walk?

38. ***Triathlon.*** Luisa participated in a triathlon in which she swam 3 miles, ran 5 miles, and then bicycled 10 miles. Luisa ran twice as fast as she swam, and she cycled three times as fast as she swam. If her total time for the triathlon was 1 hour and 46 minutes, then how fast did she swim?

FIGURE FOR EXERCISE 36

Show a complete solution to each problem. See Example 6.

39. ***Fence painting.*** Kiyoshi can paint a certain fence in 3 hours by himself. If Red helps, the job takes only 2 hours. How long would it take Red to paint the fence by himself?

40. ***Envelope stuffing.*** Every week, Linda must stuff 1000 envelopes. She can do the job by herself in 6 hours. If Laura helps, they get the job done in $5\frac{1}{2}$ hours. How long would it take Laura to do the job by herself?

41. ***Garden destroying.*** Mr. McGregor has discovered that a large dog can destroy his entire garden in 2 hours and that a small boy can do the same job in 1 hour. How long would it take the large dog and the small boy working together to destroy Mr. McGregor's garden?

42. ***Draining the vat.*** With only the small valve open, all of the liquid can be drained from a large vat in 4 hours. With only the large valve open, all of the liquid can be drained from the same vat in 2 hours. How long would it take to drain the vat with both valves open?

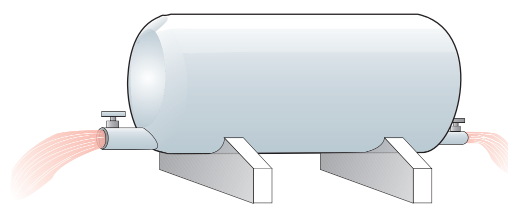


FIGURE FOR EXERCISE 42

43. ***Cleaning sidewalks.*** Edgar can blow the leaves off the sidewalks around the capitol building in 2 hours using a gasoline-powered blower. Ellen can do the same job in 8 hours using a broom. How long would it take them working together?

44. ***Computer time.*** It takes a computer 8 days to print all of the personalized letters for a national sweepstakes. A new computer is purchased that can do the same job in 5 days. How long would it take to do the job with both computers working on it?

Show a complete solution to each problem. See Example 7.

45. ***Apples and bananas.*** Bertha bought 18 pounds of fruit consisting of apples and bananas. She paid \$9 for the apples and \$2.40 for the bananas. If the price per pound of the apples was 3 times that of the bananas, then how many pounds of each type of fruit did she buy?

46. ***Running backs.*** In the playoff game the ball was carried by either Anderson or Brown on 21 plays. Anderson gained 36 yards, and Brown gained 54 yards. If Brown averaged twice as many yards per carry as Anderson, then on how many plays did Anderson carry the ball?

47. ***Fuel efficiency.*** Last week, Joe's Electric Service used 110 gallons of gasoline in its two trucks. The large truck was driven 800 miles, and the small truck was driven 600 miles. If the small truck gets twice as many miles per gallon as the large truck, then how many gallons of gasoline did the large truck use?

48. ***Repair work.*** Sally received a bill for a total of 8 hours labor on the repair of her bulldozer. She paid \$50 to the master mechanic and \$90 to his apprentice. If the master mechanic gets \$10 more per hour than his apprentice, then how many hours did each work on the bulldozer?

FIGURE FOR EXERCISE 46

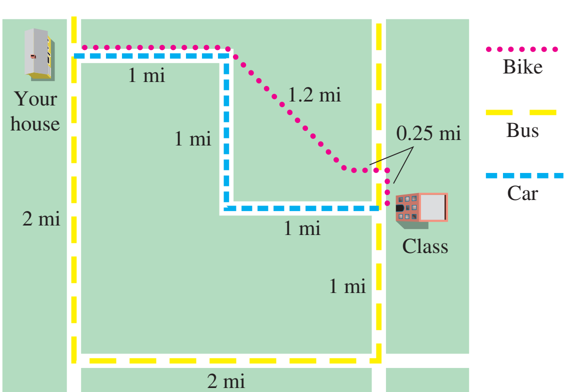
COLLABORATIVE ACTIVITIES

How Do I Get There from Here?

You want to decide whether to ride your bicycle, drive your car, or take the bus to school this year. The best thing to do is to analyze each of your options. Read all the information given below. Then have each person in your group pick one mode of transportation. Working individually, answer the questions using the given information, the map, and the distance formula $d = rt$. Present a case to your group for your type of transportation.

Information available (see the map for distances):

Bicycle: You would need to buy a new bike lock for \$15 and two new tubes at \$2.50 apiece. Determine how fast you would have to bike to beat the car.



Grouping: 3 students
Topic: Distance formula

Car: You would need to pay for a parking permit, which costs \$40. Traffic has increased, so it takes 12 minutes to get to school. What is your average speed?

Bus: The bus stops at the end of your block and has a new student rate of \$1 a week. It leaves at 8:30 A.M. and will get to the college at 8:55 A.M. On Mondays and Wednesdays you have a 9:00 A.M. class, four blocks from the bus stop. Find the average speed of the bus and figure the cost for the 16-week semester.

When presenting your case: Include the time needed to get there, speed, cost, and convenience. Have at least three reasons why this would be the best way to travel. Consider unique features of your area such as traffic, weather, and terrain.

After each of you has presented your case, decide as a group which type of transportation you would choose.