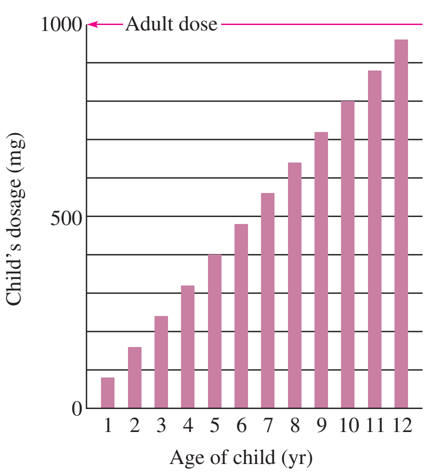
CHAPTER 2

Linear Equations in One Variable



Some ancient peoples chewed on leaves to cure their headaches. Thousands of years ago, the Egyptians used honey, salt, cedar oil, and sycamore bark to cure illnesses. Currently, some of the indigenous people of North America use black birch as a pain reliever.

Today, we are grateful for the miracles of modern medicine and the seemingly simple cures for illnesses. From our own experiences we know that just the right amount of a drug can work wonders but too much of a drug can do great harm. Even though physicians often prescribe the same drug for children and adults, the amount given must be tailored to the individual. The portion of a drug given to children is usually reduced on the basis of factors such as the weight and height of the child. Likewise, older adults frequently need a lower dosage of medication than what would be prescribed for a younger, more active person.

Various algebraic formulas have been developed for determining the proper dosage for a child and an older adult. In Exercises 83 and 84 of Section 2.4 you will see two formulas that are used to determine a child's dosage by using the adult dosage and the child's age.

2.1 THE ADDITION AND MULTIPLICATION PROPERTIES OF EQUALITY

In this section

- The Addition Property of Equality
- The Multiplication Property of Equality
- Variables on Both Sides
- Applications

In Section 1.6, you learned that an equation is a statement that two expressions are equal. You also learned how to determine whether a number is a solution to an equation. In this section you will learn systematic procedures for finding solutions to equations.

The Addition Property of Equality

The equations that we work with in this section and the next two are called linear equations.

Linear Equation

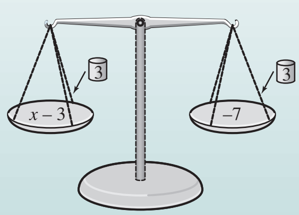
A **linear equation in one variable** x is an equation that can be written in the form

$$ax + b = 0,$$

where a and b are real numbers and $a \neq 0$.

helpful hint

Think of an equation like a balance scale. To keep the scale in balance, what you add to one side you must also add to the other side.



An equation such as $2x + 3 = 0$ is a linear equation. We also refer to equations such as

$$x + 8 = 0, \quad 3x = 7, \quad 2x + 5 = 9 - 5x, \quad \text{and} \quad 3 + 5(x - 1) = -7 + x$$

as linear equations, because these equations could be written in the form $ax + b = 0$ using the properties of equality, which we are about to discuss.

If two workers have equal salaries and each gets a \$1000 raise, then they still have equal salaries after the raise. This example illustrates the addition property of equality.

The Addition Property of Equality

Adding the same number to both sides of an equation does not change the solution to the equation. In symbols, if $a = b$, then

$$a + c = b + c.$$

To **solve** an equation means to find all of the solutions to the equation. The set of all solutions to an equation is the **solution set** to the equation. Equations that have the same solution set are **equivalent equations.** In our first example, we will use the addition property of equality to solve an equation.

EXAMPLE 1 **Adding the same number to both sides**

Solve $x - 3 = -7$.

Solution

We can remove the 3 from the left side of the equation by adding 3 to each side of the equation:

$$x - 3 = -7$$
$$x - 3 + 3 = -7 + 3 \quad \text{Add 3 to each side.}$$
$$x + 0 = -4 \quad \text{Simplify each side.}$$
$$x = -4 \quad \text{Zero is the additive identity.}$$

Since -4 satisfies the last equation, it should also satisfy the original equation because all of the previous equations are equivalent. Check that -4 satisfies the original equation by replacing x by -4:

$$x - 3 = -7 \quad \text{Original equation}$$
$$\mathbf{-4} - 3 = -7 \quad \text{Replace } x \text{ by } -4.$$
$$-7 = -7 \quad \text{Simplify.}$$

Since $-4 - 3 = -7$ is correct, $\{-4\}$ is the solution set to the equation. ■

In Example 1, we used addition to isolate the variable on the left-hand side of the equation. Once the variable is isolated, we can determine the solution to the equation. Because subtraction is defined in terms of addition, we can also use subtraction to isolate the variable.

EXAMPLE 2 **Subtracting the same number from both sides**

Solve $9 + x = -2$.

Solution

We can remove the 9 from the left side by adding -9 to each side or by subtracting 9 from each side of the equation:

$$9 + x = -2$$
$$9 + x \mathbf{- 9} = -2 \mathbf{- 9} \quad \text{Subtract 9 from each side.}$$
$$x = -11 \quad \text{Simplify each side.}$$

Check that -11 satisfies the original equation by replacing x by -11:

$$9 + x = -2 \quad \text{Original equation}$$
$$9 + (\mathbf{-11}) = -2 \quad \text{Replace } x \text{ by } -11.$$

Since $9 + (-11) = -2$ is correct, $\{-11\}$ is the solution set to the equation. ■

study tip

Think! Thinking is the manipulation of facts and principles. Your thinking will be as clear as your understanding of the facts and principles.

Our goal in solving equations is to isolate the variable. In the first two examples, the variable was isolated on the left side of the equation. In the next example, we isolate the variable on the right side of the equation.

EXAMPLE 3 **Isolating the variable on the right side**

Solve $\frac{1}{2} = -\frac{1}{4} + y$.

Solution

We can remove $-\frac{1}{4}$ from the right side by adding $\frac{1}{4}$ to both sides of the equation:

$$\frac{1}{2} = -\frac{1}{4} + y$$
$$\frac{1}{2} + \mathbf{\frac{1}{4}} = -\frac{1}{4} + y + \mathbf{\frac{1}{4}} \quad \text{Add } \tfrac{1}{4} \text{ to each side.}$$
$$\frac{3}{4} = y \quad \text{Simplify each side.}$$

study tip

Don't simply work exercises to get answers. Keep reminding yourself of what it is that you are doing. Keep trying to get the big picture. How does this section relate to what we did in the last section? Where are we going next? When is the picture complete?

Check that $\frac{3}{4}$ satisfies the original equation by replacing y by $\frac{3}{4}$:

$$\frac{1}{2} = -\frac{1}{4} + y \quad \text{Original equation}$$

$$\frac{1}{2} = -\frac{1}{4} + \frac{3}{4} \quad \text{Replace } y \text{ by } \tfrac{3}{4}.$$

$$\frac{1}{2} = \frac{2}{4} \quad \text{Simplify.}$$

Since $\frac{1}{2} = \frac{2}{4}$ is correct, $\left\{\frac{3}{4}\right\}$ is the solution set to the equation.

The Multiplication Property of Equality

To isolate a variable that is involved in a product or a quotient, we need the multiplication property of equality.

The Multiplication Property of Equality

Multiplying both sides of an equation by the same nonzero number does not change the solution to the equation. In symbols, if $a = b$ and $c \neq 0$, then

$$ac = bc.$$

If the variable in an equation is divided by a number, we can isolate the variable by multiplying each side of the equation by the divisor as in the next example.

EXAMPLE 4 **Multiplying both sides by the same number**

Solve $\frac{z}{2} = 6$.

Solution

We isolate the variable z by multiplying each side of the equation by 2.

$$\frac{z}{2} = 6 \quad \text{Original equation}$$

$$2 \cdot \frac{z}{2} = 2 \cdot 6 \quad \text{Multiply each side by 2.}$$

$$1z = 12 \quad \text{Because } 2 \cdot \tfrac{z}{2} = 2 \cdot \tfrac{1}{2}z = 1z$$

$$z = 12 \quad \text{Multiplicative identity}$$

Because $\frac{12}{2} = 6$, {12} is the solution set to the equation.

Because dividing by a number is the same as multiplying by its reciprocal, the multiplication property of equality allows us to divide each side of the equation by any nonzero number.

EXAMPLE 5 **Dividing both sides by the same number**

Solve $-5w = 30$.

Solution

Since w is multiplied by -5, we can isolate w by multiplying by $-\frac{1}{5}$ or by dividing each side by -5:

$$-5w = 30 \quad \text{Original equation}$$

$$\frac{-5w}{-5} = \frac{30}{-5} \quad \text{Divide each side by } -5.$$

$$1 \cdot w = -6 \quad \text{Because } \tfrac{-5}{-5} = 1$$

$$w = -6 \quad \text{Multiplicative identity}$$

Because $-5(-6) = 30$, $\{-6\}$ is the solution set to the equation. ■

In the next example, the coefficient of the variable is a fraction. We could divide each side by the coefficient as we did in Example 5, but it is easier to multiply each side by the reciprocal of the coefficient.

EXAMPLE 6 **Multiplying by the reciprocal**

Solve $\frac{2}{3}p = 40$.

helpful hint

You could solve this equation by multiplying each side by 3 to get $2p = 120$, and then dividing each side by 2 to get $p = 60$.

Solution

Multiply each side by $\frac{3}{2}$, the reciprocal of $\frac{2}{3}$, to isolate p on the left side.

$$\frac{2}{3}p = 40$$

$$\frac{3}{2} \cdot \frac{2}{3}p = \frac{3}{2} \cdot 40 \quad \text{Multiply each side by } \tfrac{3}{2}.$$

$$1 \cdot p = 60 \quad \text{Multiplicative inverses}$$

$$p = 60 \quad \text{Multiplicative identity}$$

Because $\frac{2}{3} \cdot 60 = 40$, we can be sure that the solution set is $\{60\}$. ■

If the coefficient of the variable is an integer, we usually divide each side by that integer, as in Example 5. If the coefficient of the variable is a fraction, we usually multiply each side by the reciprocal of the fraction as in Example 6.

If $-x$ appears in an equation, we can multiply by -1 to get x, because $-1(-x) = -(-x) = x$.

EXAMPLE 7 **Multiplying by −1**

Solve $-h = 12$.

Solution

Multiply each side by -1 to get h on the left side.

$$-h = 12$$

$$-1(-h) = -1 \cdot 12$$

$$h = -12$$

Since $-(-12) = 12$, the solution set is $\{-12\}$. ■

Variables on Both Sides

In the next example, the variable occurs on both sides of the equation. Because the variable represents a real number, we can still isolate the variable by using the addition property of equality.

EXAMPLE 8 **Subtracting an algebraic expression from both sides**

Solve $-9 + 6y = 7y$.

helpful hint

It does not matter whether the variable ends up on the left or right side of the equation. Whether we get $y = -9$ or $-9 = y$ we can still conclude that the solution is -9.

Solution

The expression $6y$ can be removed from the left side of the equation by subtracting $6y$ from both sides.

$$-9 + 6y = 7y$$

$$-9 + 6y - 6y = 7y - 6y \quad \text{Subtract } 6y \text{ from each side.}$$

$$-9 = y \quad \text{Simplify each side.}$$

Check by replacing y by -9 in the original equation:

$$-9 + 6(-9) = 7(-9)$$

$$-63 = -63$$

The solution set to the equation is $\{-9\}$. ■

Applications

In the next example, we use the multiplication property of equality in an applied situation.

EXAMPLE 9 **Population density**

In 1990, San Francisco had $\frac{2}{3}$ as many people per hectare as New York (U.S. Bureau of Census, www.census.gov). The population density of San Francisco was 60 people per hectare. What was the population density of New York?

Solution

If p represents the population density of New York, then $\frac{2}{3}p = 60$. To find p, solve the equation:

$$\frac{2}{3}p = 60$$

$$\frac{3}{2} \cdot \frac{2}{3}p = \frac{3}{2} \cdot 60 \quad \text{Multiply each side by } \tfrac{3}{2}.$$

$$p = 90 \quad \text{Simplify.}$$

So the population density of New York was 90 people per hectare. ■

WARM-UPS

True or false? Explain your answer.

1. The solution to $x - 5 = 5$ is 10.
2. The equation $\frac{x}{2} = 4$ is equivalent to the equation $x = 8$.
3. To solve $\frac{3}{4}y = 12$, we should multiply each side by $\frac{3}{4}$.
4. The equation $\frac{x}{7} = 4$ is equivalent to $\frac{1}{7}x = 4$.
5. Multiplying each side of an equation by any real number will result in an equation that is equivalent to the original equation.
6. To isolate t in $2t = 7 + t$, subtract t from each side.
7. To solve $\frac{2r}{3} = 30$, we should multiply each side by $\frac{3}{2}$.
8. Adding any real number to both sides of an equation will result in an equation that is equivalent to the original equation.
9. The equation $5x = 0$ is equivalent to $x = 0$.
10. The solution to $2x - 3 = x + 1$ is 4.

2.1 EXERCISES

Reading and Writing *After reading this section, write out the answers to these questions. Use complete sentences.*

1. What does the addition property of equality say?
2. What are equivalent equations?
3. What is the multiplication property of equality?
4. What is a linear equation in one variable?
5. How can you tell if your solution to an equation is correct?
6. To obtain an equivalent equation, what are you not allowed to do to both sides of the equation?

Solve each equation. Show your work and check your answer. See Example 1.

7. $x - 6 = -5$ **8.** $x - 7 = -2$

9. $-13 + x = -4$ **10.** $-8 + x = -12$

11. $y - \frac{1}{2} = \frac{1}{2}$ **12.** $y - \frac{1}{4} = \frac{1}{2}$

13. $w - \frac{1}{3} = \frac{1}{3}$ **14.** $w - \frac{1}{3} = \frac{1}{2}$

Solve each equation. Show your work and check your answer. See Example 2.

15. $x + 3 = -6$ **16.** $x + 4 = -3$

17. $12 + x = -7$ **18.** $19 + x = -11$

19. $t + \frac{1}{2} = \frac{3}{4}$ **20.** $t + \frac{1}{3} = 1$

21. $\frac{1}{19} + m = \frac{1}{19}$ **22.** $\frac{1}{3} + n = \frac{1}{2}$

Solve each equation. Show your work and check your answer. See Example 3.

23. $2 = x + 7$ **24.** $3 = x + 5$

25. $-13 = y - 9$ **26.** $-14 = z - 12$

27. $0.5 = -2.5 + x$ **28.** $0.6 = -1.2 + x$

29. $\frac{1}{8} = -\frac{1}{8} + r$ **30.** $\frac{1}{6} = -\frac{1}{6} + h$

Solve each equation. Show your work and check your answer. See Example 4.

31. $\frac{x}{2} = -4$ **32.** $\frac{x}{3} = -6$

33. $0.03 = \frac{y}{60}$ **34.** $0.05 = \frac{y}{80}$

35. $\frac{a}{2} = \frac{1}{3}$

36. $\frac{b}{2} = \frac{1}{5}$

37. $\frac{1}{6} = \frac{c}{3}$

38. $\frac{1}{12} = \frac{d}{3}$

Solve each equation. Show your work and check your answer. See Example 5.

39. $-3x = 15$

40. $-5x = -20$

41. $20 = 4y$

42. $18 = -3a$

43. $2w = 2.5$

44. $-2x = -5.6$

45. $5 = 20x$

46. $-3 = 27d$

Solve each equation. Show your work and check your answer. See Example 6.

47. $\frac{3}{2}x = -3$

48. $\frac{2}{3}x = -8$

49. $90 = \frac{3y}{4}$

50. $14 = \frac{7y}{8}$

51. $-\frac{3}{5}w = -\frac{1}{3}$

52. $-\frac{5}{2}t = -\frac{3}{5}$

53. $\frac{2}{3} = -\frac{4x}{3}$

54. $\frac{1}{14} = -\frac{6p}{7}$

Solve each equation. Show your work and check your answer. See Example 7.

55. $-x = 8$

56. $-x = 4$

57. $-y = -\frac{1}{3}$

58. $-y = -\frac{7}{8}$

59. $3.4 = -z$

60. $4.9 = -t$

61. $-k = -99$

62. $-m = -17$

Solve each equation. Show your work and check your answer. See Example 8.

63. $4x = 3x - 7$

64. $3x = 2x + 9$

65. $9 - 6y = -5y$

66. $12 - 18w = -17w$

67. $-6x = 8 - 7x$

68. $-3x = -6 - 4x$

69. $\frac{1}{2}c = 5 - \frac{1}{2}c$

70. $-\frac{1}{2}h = 13 - \frac{3}{2}h$

Use the appropriate property of equality to solve each equation.

71. $12 = x + 17$

72. $-3 = x + 6$

73. $\frac{3}{4}y = -6$

74. $\frac{5}{9}z = -10$

75. $-3.2 + x = -1.2$

76. $t - 3.8 = -2.9$

77. $2a = \frac{1}{3}$

78. $-3w = \frac{1}{2}$

79. $-9m = 3$

80. $-4h = -2$

81. $-b = -44$

82. $-r = 55$

83. $\frac{2}{3}x = \frac{1}{2}$

84. $\frac{3}{4}x = \frac{1}{3}$

85. $-5x = 7 - 6x$

86. $-\frac{1}{2} + 3y = 4y$

87. $\frac{5a}{7} = -10$

88. $\frac{7r}{12} = -14$

89. $\frac{1}{2}v = -\frac{1}{2}v + \frac{3}{8}$

90. $\frac{1}{3}s + \frac{7}{9} = \frac{4}{3}s$

Solve each problem by writing and solving an equation. See Example 9.

91. ***Cigarette consumption.*** In 1999, cigarette consumption in the U.S. was 125 packs per capita. This rate of consumption was $\frac{5}{8}$ of what it was in 1980. Find the rate of consumption in 1980.

92. ***World grain demand.*** Freeport McMoRan projects that in 2010 world grain supply will be 1.8 trillion metric tons and the supply will be only $\frac{3}{4}$ of world grain demand. What will world grain demand be in 2010?

FIGURE FOR EXERCISE 92

93. ***Advancers and decliners.*** On Thursday, $\frac{13}{25}$ of the stocks traded on the New York Stock Exchange advanced in price. If 1495 stocks advanced, then how many stocks were traded on that day?

94. ***Accidental deaths.*** In 1996, $\frac{23}{50}$ of all accidental deaths in the U.S. were the result of automobile accidents (National Center for Health Statistics, www.nchs.gov). If there were 43,194 deaths due to automobile accidents, then how many accidental deaths were there in 1996?

In this section

- Equations of the Form $ax + b = 0$
- Equations of the Form $ax + b = cx + d$
- Equations with Parentheses
- Applications

2.2 SOLVING GENERAL LINEAR EQUATIONS

All of the equations that we solved in Section 2.1 required only a single application of a property of equality. In this section you will solve equations that require more than one application of a property of equality.

Equations of the Form $ax + b = 0$

To solve an equation of the form $ax + b = 0$ we might need to apply both the addition property of equality and the multiplication property of equality.

EXAMPLE 1

Using the addition and multiplication properties of equality

Solve $3r - 5 = 0$.

Solution

To isolate r, first add 5 to each side, then divide each side by 3.

$$3r - 5 = 0 \quad \text{Original equation}$$

$$3r - 5 + 5 = 0 + 5 \quad \text{Add 5 to each side.}$$

$$3r = 5 \quad \text{Combine like terms.}$$

$$\frac{3r}{3} = \frac{5}{3} \quad \text{Divide each side by 3.}$$

$$r = \frac{5}{3} \quad \text{Simplify.}$$

Checking $\frac{5}{3}$ in the original equation gives

$$3 \cdot \frac{5}{3} - 5 = 5 - 5 = 0.$$

So $\left\{\frac{5}{3}\right\}$ is the solution set to the equation. ■

helpful hint

If we divide by 3 first, we would get $r - \frac{5}{3} = 0$. Then add $\frac{5}{3}$ to each side to get $r = \frac{5}{3}$. Although we get the correct answer, we usually save division to the last step so that fractions do not appear until necessary.

CAUTION It is usually best to use the addition property of equality first and the multiplication property last.

EXAMPLE 2

Using the addition and multiplication properties of equality

Solve $-\frac{2}{3}x + 8 = 0$.

Solution

To isolate x, first subtract 8 from each side, then multiply each side by $-\frac{3}{2}$.

$$-\frac{2}{3}x + 8 = 0 \quad \text{Original equation}$$

$$-\frac{2}{3}x + 8 - 8 = 0 - 8 \quad \text{Subtract 8 from each side.}$$

$$-\frac{2}{3}x = -8 \quad \text{Combine like terms.}$$

$$-\frac{3}{2}\left(-\frac{2}{3}x\right) = -\frac{3}{2}(-8) \quad \text{Multiply each side by } -\frac{3}{2}.$$

$$x = 12 \quad \text{Simplify.}$$

study tip

As you leave class, talk to a classmate about what happened in class. What was the class about? What new terms were mentioned and what do they mean? How does this lesson fit in with the last lesson?

Checking 12 in the original equation gives

$$-\frac{2}{3}(12) + 8 = -8 + 8 = 0.$$

So {12} is the solution set to the equation. ■

Equations of the Form $ax + b = cx + d$

In solving equations our goal is to isolate the variable. We use the addition property of equality to eliminate unwanted terms. Note that it does not matter whether the variable ends up on the right or left side. For some equations we will perform fewer steps if we isolate the variable on the right side.

EXAMPLE 3 **Isolating the variable on the right side**

Solve $3w - 8 = 7w$.

Solution

To eliminate the $3w$ from the left side, we can subtract $3w$ from both sides.

$$3w - 8 = 7w \quad \text{Original equation}$$
$$3w - 8 - 3w = 7w - 3w \quad \text{Subtract } 3w \text{ from each side.}$$
$$-8 = 4w \quad \text{Simplify each side.}$$
$$-\frac{8}{4} = \frac{4w}{4} \quad \text{Divide each side by 4.}$$
$$-2 = w \quad \text{Simplify.}$$

To check, replace w with -2 in the original equation:

$$3w - 8 = 7w \quad \text{Original equation}$$
$$3(-2) - 8 = 7(-2)$$
$$-14 = -14$$

Since -2 satisfies the original equation, the solution set is $\{-2\}$. ■

You should solve the equation in Example 3 by isolating the variable on the left side to see that it takes more steps. In the next example, it is simplest to isolate the variable on the left side.

EXAMPLE 4 **Isolating the variable on the left side**

Solve $\frac{1}{2}b - 8 = 12$.

Solution

To eliminate the 8 from the left side, we add 8 to each side.

$$\frac{1}{2}b - 8 = 12 \quad \text{Original equation}$$
$$\frac{1}{2}b - 8 + 8 = 12 + 8 \quad \text{Add 8 to each side.}$$
$$\frac{1}{2}b = 20 \quad \text{Simplify each side.}$$
$$2 \cdot \frac{1}{2}b = 2 \cdot 20 \quad \text{Multiply each side by 2.}$$
$$b = 40 \quad \text{Simplify.}$$

To check, replace b with 40 in the original equation:

$$\frac{1}{2}b - 8 = 12 \quad \text{Original equation}$$
$$\frac{1}{2}(40) - 8 = 12$$
$$12 = 12$$

Since 40 satisfies the original equation, the solution set is {40}.

It does not matter whether the variable is isolated on the left side or the right side. However, you should decide where you want the variable isolated before you begin to solve the equation.

EXAMPLE 5

Solving $ax + b = cx + d$

Solve $2m - 4 = 4m - 10$.

Solution

First, we decide to isolate the variable on the left side. So we must eliminate the 4 from the left side and eliminate $4m$ from the right side:

$$2m - 4 = 4m - 10$$
$$2m - 4 + 4 = 4m - 10 + 4 \quad \text{Add 4 to each side.}$$
$$2m = 4m - 6 \quad \text{Simplify each side.}$$
$$2m - 4m = 4m - 6 - 4m \quad \text{Subtract } 4m \text{ from each side.}$$
$$-2m = -6 \quad \text{Simplify each side.}$$
$$\frac{-2m}{-2} = \frac{-6}{-2} \quad \text{Divide each side by } -2.$$
$$m = 3 \quad \text{Simplify.}$$

To check, replace m by 3 in the original equation:

$$2m - 4 = 4m - 10 \quad \text{Original equation}$$
$$2 \cdot 3 - 4 = 4 \cdot 3 - 10$$
$$2 = 2$$

Since 3 satisfies the original equation, the solution set is {3}.

study tip

Take good notes. Good note taking is the key to mastering the material. It helps you to concentrate in class and provides a source for review. Learn to listen effectively.

Equations with Parentheses

Equations that contain parentheses or like terms on the same side should be simplified as much as possible before applying any properties of equality.

EXAMPLE 6

Simplifying before using properties of equality

Solve $2(q - 3) + 5q = 8(q - 1)$.

Solution

First remove parentheses and combine like terms on each side of the equation.

$$2(q - 3) + 5q = 8(q - 1) \quad \text{Original equation}$$
$$2q - 6 + 5q = 8q - 8 \quad \text{Distributive property}$$
$$7q - 6 = 8q - 8 \quad \text{Combine like terms.}$$
$$7q - 6 + 6 = 8q - 8 + 6 \quad \text{Add 6 to each side.}$$
$$7q = 8q - 2 \quad \text{Combine like terms.}$$
$$7q - 8q = 8q - 2 - 8q \quad \text{Subtract } 8q \text{ from each side.}$$

$$-q = -2$$
$$-1(-q) = -1(-2) \quad \text{Multiply each side by } -1.$$
$$q = 2 \quad \text{Simplify.}$$

To check, we replace q by 2 in the original equation and simplify:

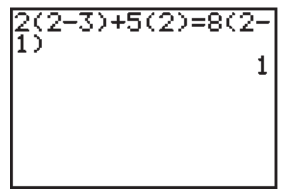
$$2(q - 3) + 5q = 8(q - 1) \quad \text{Original equation}$$
$$2(2 - 3) + 5(2) = 8(2 - 1) \quad \text{Replace } q \text{ by 2.}$$
$$2(-1) + 10 = 8(1)$$
$$8 = 8$$

Because both sides have the same value, the solution set is $\{2\}$.

calculator close-up

You can check an equation by entering the equation on the home screen as shown here. The equal sign is in the TEST menu.

When you press ENTER, the calculator returns the number 1 if the equation is true or 0 if the equation is false. Since the calculator shows a 1, we can be sure that 2 is the solution.



Linear equations can vary greatly in appearance, but there is a strategy that you can use for solving any of them. The following strategy summarizes the techniques that we have been using in the examples. Keep it in mind when you are solving linear equations.

Strategy for Solving Equations

1. Remove parentheses and combine like terms to simplify each side as much as possible.
2. Use the addition property of equality to get like terms from opposite sides onto the same side so that they may be combined.
3. The multiplication property of equality is generally used last.
4. Multiply each side of $-x = a$ by -1 to get $x = -a$.
5. Check that the solution satisfies the original equation.

Applications

Linear equations occur in business situations where there is a fixed cost and a per item cost. A mail order company might charge \$3 plus \$2 per CD for shipping and handling. A lawyer might charge \$300 plus \$65 per hour for handling your lawsuit. AT&T might charge 10 cents per minute plus \$4.95 for long distance calls. The next example illustrates the kind of problem that can be solved in this situation.

EXAMPLE 7 **Long distance charges**

With AT&T's One Rate plan you are charged 10 cents per minute plus \$4.95 for long distance service for one month. If a long distance bill is \$8.65, then what is the number of minutes used?

Solution

Let x represent the number of minutes of calls in the month. At \$0.10 per minute, the cost for x minutes is the product $0.10x$ dollars. Since there is a fixed cost of \$4.95, an expression for the total cost is $0.10x + 4.95$ dollars. Since the total cost is \$8.65, we have $0.10x + 4.95 = 8.65$. Solve this equation to find x.

$$0.10x + 4.95 = 8.65$$

$$0.10x + 4.95 - 4.95 = 8.65 - 4.95 \quad \text{Subtract 4.95 from each side.}$$

$$0.10x = 3.70 \quad \text{Simplify.}$$

$$\frac{0.10x}{0.10} = \frac{3.70}{0.10} \quad \text{Divide each side by 0.10.}$$

$$x = 37 \quad \text{Simplify.}$$

So the bill is for 37 minutes. ■

WARM-UPS

True or false? Explain your answer.

1. The solution to $4x - 3 = 3x$ is 3.
2. The equation $2x + 7 = 8$ is equivalent to $2x = 1$.
3. To solve $3x - 5 = 8x + 7$, you should add 5 to each side and subtract $8x$ from each side.
4. To solve $5 - 4x = 9 + 7x$, you should subtract 9 from each side and then subtract $7x$ from each side.
5. Multiplying each side of an equation by the same nonzero real number will result in an equation that is equivalent to the original equation.
6. To isolate y in $3y - 7 = 6$, divide each side by 3 and then add 7 to each side.
7. To solve $\frac{3w}{4} = 300$, we should multiply each side by $\frac{4}{3}$.
8. The equation $-n = 9$ is equivalent to $n = -9$.
9. The equation $-y = -7$ is equivalent to $y = 7$.
10. The solution to $7x = 5x$ is 0.

2.2 EXERCISES

Reading and Writing *After reading this section, write out the answers to these questions. Use complete sentences.*

1. What properties of equality do you apply to solve $ax + b = 0$?
2. Which property of equality is usually applied last?
3. What property of equality is used to solve $-x = 8$?
4. What is usually the first step in solving an equation involving parentheses?

Solve each equation. Show your work and check your answer. See Examples 1 and 2.

5. $5a - 10 = 0$
6. $8y + 24 = 0$
7. $-3y - 6 = 0$
8. $-9w - 54 = 0$
9. $3x - 2 = 0$
10. $5y + 1 = 0$
11. $2p + 5 = 0$
12. $9z - 8 = 0$
13. $\frac{1}{2}w - 3 = 0$
14. $\frac{3}{8}t + 6 = 0$
15. $-\frac{2}{3}x + 8 = 0$
16. $-\frac{1}{7}z - 5 = 0$
17. $-m + \frac{1}{2} = 0$
18. $-y - \frac{3}{4} = 0$

19. $3p + \frac{1}{2} = 0$

20. $9z - \frac{1}{4} = 0$

Solve each equation. See Examples 3 and 4.

21. $6x - 8 = 4x$

22. $9y + 14 = 2y$

23. $4z = 5 - 2z$

24. $3t = t - 3$

25. $4a - 9 = 7$

26. $7r + 5 = 47$

27. $9 = -6 - 3b$

28. $13 = 3 - 10s$

29. $\frac{1}{2}w - 4 = 13$

30. $\frac{1}{3}q + 13 = -5$

31. $6 - \frac{1}{3}d = \frac{1}{3}d$

32. $9 - \frac{1}{2}a = \frac{1}{4}a$

33. $2w - 0.4 = 2$

34. $10h - 1.3 = 6$

35. $x = 3.3 - 0.1x$

36. $y = 2.4 - 0.2y$

Solve each equation. See Example 5.

37. $3x - 3 = x + 5$

38. $9y - 1 = 6y + 5$

39. $4 - 7d = 13 - 4d$

40. $y - 9 = 12 - 6y$

41. $c + \frac{1}{2} = 3c - \frac{1}{2}$

42. $x - \frac{1}{4} = \frac{1}{2} - x$

43. $\frac{2}{3}a - 5 = \frac{1}{3}a + 5$

44. $\frac{1}{2}t - 3 = \frac{1}{4}t - 9$

Solve each equation. See Example 6.

45. $5(a - 1) + 3 = 28$

46. $2(w + 4) - 1 = 1$

47. $2 - 3(q - 1) = 10 - (q + 1)$

48. $-2(y - 6) = 3(7 - y) - 5$

49. $2(x - 1) + 3x = 6x - 20$

50. $3 - (r - 1) = 2(r + 1) - r$

51. $2\left(y - \frac{1}{2}\right) = 4\left(y - \frac{1}{4}\right) + y$

52. $\frac{1}{2}(4m - 6) = \frac{2}{3}(6m - 9) + 3$

Solve each linear equation. Show your work and check your answer.

53. $5t = -2 + 4t$

54. $8y = 6 + 7y$

55. $3x - 7 = 0$

56. $5x + 4 = 0$

57. $-x + 6 = 5$

58. $-x - 2 = 9$

59. $-9 - a = -3$

60. $4 - r = 6$

61. $2q + 5 = q - 7$

62. $3z - 6 = 2z - 7$

63. $-3x + 1 = 5 - 2x$

64. $5 - 2x = 6 - x$

65. $-12 - 5x = -4x + 1$

66. $-3x - 4 = -2x + 8$

67. $3x + 0.3 = 2 + 2x$

68. $2y - 0.05 = y + 1$

69. $k - 0.6 = 0.2k + 1$

70. $2.3h + 6 = 1.8h - 1$

71. $0.2x - 4 = 0.6 - 0.8x$

72. $0.3x = 1 - 0.7x$

73. $-3(k - 6) = 2 - k$

74. $-2(h - 5) = 3 - h$

75. $2(p + 1) - p = 36$

76. $3(q + 1) - q = 23$

77. $7 - 3(5 - u) = 5(u - 4)$

78. $v - 4(4 - v) = -2(2v - 1)$

79. $4(x + 3) = 12$

80. $5(x - 3) = -15$

81. $\frac{w}{5} - 4 = -6$

82. $\frac{q}{2} + 13 = -22$

83. $\frac{2}{3}y - 5 = 7$

84. $\frac{3}{4}u - 9 = -6$

85. $4 - \frac{2n}{5} = 12$

86. $9 - \frac{2m}{7} = 19$

87. $-\frac{1}{3}p - \frac{1}{2} = \frac{1}{2}$

88. $-\frac{3}{4}z - \frac{2}{3} = \frac{1}{3}$

89. $3.5x - 23.7 = -38.75$

90. $3(x - 0.87) - 2x = 4.98$

Solve each problem. See Example 7.

91. ***The practice.*** A lawyer charges \$300 plus \$65 per hour for a divorce. If the total charge for Bill's divorce was \$1405, then for what number of hours did the lawyer work on the case?

92. ***The plumber.*** A plumber charges \$45 plus \$26 per hour to unclog drains. If the bill for unclogging Tamika's drain was \$123, then for how many hours did the plumber work?

93. ***Celsius temperature.*** If the air temperature in Quebec is 68° Fahrenheit, then the solution to the equation $\frac{9}{5}C + 32 = 68$ gives the Celsius temperature of the air. Find the Celsius temperature.

94. ***Fahrenheit temperature.*** Water boils at 212°F.

a) Use the accompanying graph to determine the Celsius temperature at which water boils.

b) Find the Fahrenheit temperature of hot tap water at 70°C by solving the equation

$$70 = \frac{5}{9}(F - 32).$$

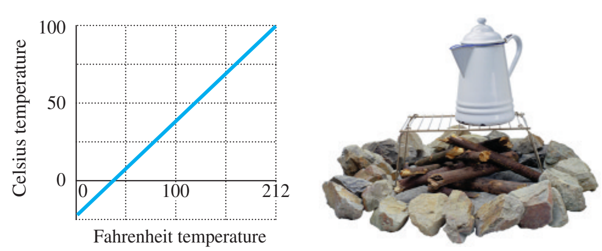


FIGURE FOR EXERCISE 94

95. ***Rectangular patio.*** If a rectangular patio has a length that is 3 feet longer than its width and a perimeter of 42 feet, then the width can be found by solving the equation $2x + 2(x + 3) = 42$. What is the width?

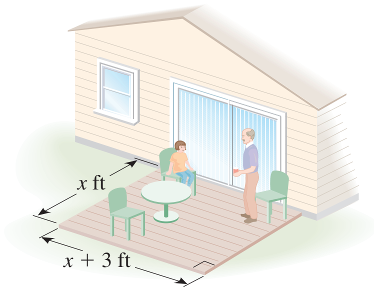


FIGURE FOR EXERCISE 95

96. ***Perimeter of a triangle.*** The perimeter of the triangle shown in the accompanying figure is 12 meters. Determine the values of x, $x + 1$, and $x + 2$ by solving the equation

$$x + (x + 1) + (x + 2) = 12.$$

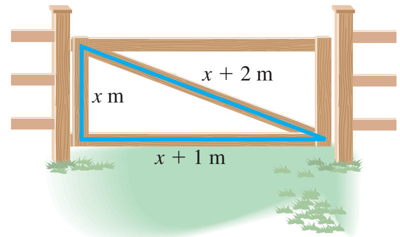


FIGURE FOR EXERCISE 96

97. ***Cost of a car.*** Jane paid 9% sales tax and a \$150 title and license fee when she bought her new Saturn for a total of \$16,009.50. If x represents the price of the car, then x satisfies $x + 0.09x + 150 = 16{,}009.50$. Find the price of the car by solving the equation.

98. ***Cost of labor.*** An electrician charged Eunice \$29.96 for a service call plus \$39.96 per hour for a total of \$169.82 for installing her electric dryer. If n represents the number of hours for labor, then n satisfies

$$39.96n + 29.96 = 169.82.$$

Find n by solving this equation.

2.3 IDENTITIES, CONDITIONAL EQUATIONS, AND INCONSISTENT EQUATIONS

In this section

- Identities
- Conditional Equations
- Inconsistent Equations
- Equations Involving Fractions
- Equations Involving Decimals
- Simplifying the Process

In this section, we will solve more equations of the type that we solved in Sections 2.1 and 2.2. However, some equations in this section have infinitely many solutions, and some have no solution.

Identities

It is easy to find equations that are satisfied by any real number that we choose as a replacement for the variable. For example, the equations

$$x \div 2 = \frac{1}{2}x, \qquad x + x = 2x, \qquad \text{and} \qquad x + 1 = x + 1$$

are satisfied by all real numbers. The equation

$$\frac{5}{x} = \frac{5}{x}$$

is satisfied by any real number except 0 because division by 0 is undefined.

Identity

An equation that is satisfied by every real number for which both sides are defined is called an **identity.**

We cannot recognize that the equation in the next example is an identity until we have simplified each side.

EXAMPLE 1

Solving an identity

Solve $7 - 5(x - 6) + 4 = 3 - 2(x - 5) - 3x + 28$.

Solution

We first use the distributive property to remove the parentheses:

$$7 - 5(x - 6) + 4 = 3 - 2(x - 5) - 3x + 28$$
$$7 - 5x + 30 + 4 = 3 - 2x + 10 - 3x + 28$$
$$41 - 5x = 41 - 5x \quad \text{Combine like terms.}$$

This last equation is true for any value of x because the two sides are identical. So the solution set to the original equation is the set of all real numbers. ■

CAUTION If you get an equation in which both sides are identical, as in Example 1, there is no need to continue to simplify the equation. If you do continue, you will eventually get $0 = 0$, from which you can still conclude that the equation is an identity.

Conditional Equations

The statement $2x + 4 = 10$ is true only on condition that we choose $x = 3$. The equation $x^2 = 4$ is satisfied only if we choose $x = 2$ or $x = -2$. These equations are called conditional equations.

study tip

Life is a game that holds many rewards for those who compete. Winning is never an accident. To win you must know the rules and have a game plan.

Conditional Equation

A **conditional equation** is an equation that is satisfied by at least one real number but is not an identity.

Every equation that we solved in Sections 2.1 and 2.2 is a conditional equation.

Inconsistent Equations

It is easy to find equations that are false no matter what number we use to replace the variable. Consider the equation

$$x = x + 1.$$

If we replace x by 3, we get $3 = 3 + 1$, which is false. If we replace x by 4, we get $4 = 4 + 1$, which is also false. Clearly, there is no number that will satisfy $x = x + 1$. Other examples of equations with no solutions include

$$x = x - 2, \qquad x - x = 5, \qquad \text{and} \qquad 0 \cdot x + 6 = 7.$$

Inconsistent Equation

An equation that has no solution is called an **inconsistent equation.**

The solution set to an inconsistent equation has no members. The set with no members is called the **empty set** and it is denoted by the symbol $\emptyset$.

EXAMPLE 2

Solving an inconsistent equation

Solve $2 - 3(x - 4) = 4(x - 7) - 7x$.

Solution

Use the distributive property to remove the parentheses:

$$2 - 3(x - 4) = 4(x - 7) - 7x \quad \text{The original equation}$$
$$2 - 3x + 12 = 4x - 28 - 7x \quad \text{Distributive property}$$
$$14 - 3x = -28 - 3x \quad \text{Combine like terms on each side.}$$
$$14 - 3x + 3x = -28 - 3x + 3x \quad \text{Add } 3x \text{ to each side.}$$
$$14 = -28 \quad \text{Simplify.}$$

The last equation is not true for any x. So the solution set to the original equation is the empty set, $\emptyset$. The equation is inconsistent. ■

Keep the following points in mind in solving equations.

Summary: Identities and Inconsistent Equations

1. An equation that is equivalent to an equation in which both sides are identical is an identity. The equation is satisfied by all real numbers for which both sides are defined.
2. An equation that is equivalent to an equation that is always false is inconsistent. The equation has no solution. The solution set is the empty set, $\emptyset$.

Equations Involving Fractions

We solved some equations involving fractions in Sections 2.1 and 2.2. Here, we will solve equations with fractions by eliminating all fractions in the first step. All of the fractions will be eliminated if we multiply each side by the least common denominator.

EXAMPLE 3 **Multiplying by the least common denominator**

Solve $\frac{y}{2} - 1 = \frac{y}{3} + 1$

Solution

The least common denominator (LCD) for the denominators 2 and 3 is 6. Since both 2 and 3 divide into 6 evenly, multiplying each side by 6 will eliminate the fractions:

$$6\left(\frac{y}{2} - 1\right) = 6\left(\frac{y}{3} + 1\right) \quad \text{Multiply each side by 6.}$$
$$6 \cdot \frac{y}{2} - 6 \cdot 1 = 6 \cdot \frac{y}{3} + 6 \cdot 1 \quad \text{Distributive property}$$
$$3y - 6 = 2y + 6 \quad \text{Simplify: } 6 \cdot \frac{y}{2} = 3y$$
$$3y = 2y + 12 \quad \text{Add 6 to each side.}$$
$$y = 12 \quad \text{Subtract } 2y \text{ from each side.}$$

Check 12 in the original equation:

$$\frac{12}{2} - 1 = \frac{12}{3} + 1$$
$$5 = 5$$

Since 12 satisfies the original equation, the solution set is $\{12\}$. ■

helpful hint

Note that the fractions in Example 3 will be eliminated if you multiply each side of the equation by any number divisible by both 2 and 3. For example, multiplying by 24 yields

$$12y - 24 = 8y + 24$$
$$4y = 48$$
$$y = 12.$$

Equations involving fractions are usually easier to solve if we first multiply each side by the LCD of the fractions.

Equations Involving Decimals

When an equation involves decimal numbers, we can work with the decimal numbers or we can eliminate all of the decimal numbers by multiplying both sides by 10, or 100, or 1000, and so on. Multiplying a decimal number by 10 moves the decimal point one place to the right. Multiplying by 100 moves the decimal point two places to the right, and so on.

EXAMPLE 4

An equation involving decimals

Solve $0.3p + 8.04 = 12.6$.

Solution

The largest number of decimal places appearing in the decimal numbers of the equation is two (in the number 8.04). Therefore we multiply each side of the equation by 100 because multiplying by 100 moves decimal points two places to the right:

$$0.3p + 8.04 = 12.6 \quad \text{Original equation}$$
$$100(0.3p + 8.04) = 100(12.6) \quad \text{Multiplication property of equality}$$
$$100(0.3p) + 100(8.04) = 100(12.6) \quad \text{Distributive property}$$
$$30p + 804 = 1260$$
$$30p + 804 - 804 = 1260 - 804 \quad \text{Subtract 804 from each side.}$$
$$30p = 456$$
$$\frac{30p}{30} = \frac{456}{30} \quad \text{Divide each side by 30.}$$
$$p = 15.2$$

You can use a calculator to check that

$$0.3(15.2) + 8.04 = 12.6.$$

The solution set is $\{15.2\}$.

helpful hint

After you have used one of the properties of equality on each side of an equation, be sure to simplify all expressions as much as possible before using another property of equality. This step is like making sure that all of the injured football players are removed from the field before proceeding to the next play.

EXAMPLE 5

Another equation with decimals

Solve $0.5x + 0.4(x + 20) = 13.4$

Solution

First use the distributive property to remove the parentheses:

$$0.5x + 0.4(x + 20) = 13.4 \quad \text{Original equation}$$
$$0.5x + 0.4x + 8 = 13.4 \quad \text{Distributive property}$$
$$10(0.5x + 0.4x + 8) = 10(13.4) \quad \text{Multiply each side by 10.}$$
$$5x + 4x + 80 = 134 \quad \text{Simplify.}$$
$$9x + 80 = 134 \quad \text{Combine like terms.}$$
$$9x + 80 - 80 = 134 - 80 \quad \text{Subtract 80 from each side.}$$
$$9x = 54 \quad \text{Simplify.}$$
$$x = 6 \quad \text{Divide each side by 9.}$$

Check 6 in the original equation:

$$0.5(6) + 0.4(6 + 20) = 13.4 \quad \text{Replace } x \text{ by 6.}$$
$$3 + 0.4(26) = 13.4$$
$$3 + 10.4 = 13.4$$

Since both sides of the equation have the same value, the solution set is $\{6\}$. ■

CAUTION If you multiply each side by 10 in Example 5 before using the distributive property, be careful how you handle the terms in parentheses:

$$10 \cdot 0.5x + 10 \cdot 0.4(x + 20) = 10 \cdot 13.4$$
$$5x + 4(x + 20) = 134$$

It is not correct to multiply 0.4 by 10 *and also* to multiply $x + 20$ by 10.

Simplifying the Process

It is very important to develop the skill of solving equations in a systematic way, writing down every step as we have been doing. As you become more skilled at solving equations, you will probably want to simplify the process a bit. One way to simplify the process is by writing only the result of performing an operation on each side. Another way is to isolate the variable on the side where the variable has the larger coefficient, when the variable occurs on both sides. We use these ideas in the next example and in future examples in this text.

EXAMPLE 6 **Simplifying the process**

Solve each equation.

a) $2a - 3 = 0$ **b)** $2k + 5 = 3k + 1$

Solution

a) Add 3 to each side, then divide each side by 2:

$$2a - 3 = 0$$
$$2a = 3 \quad \text{Add 3 to each side.}$$
$$a = \frac{3}{2} \quad \text{Divide each side by 2.}$$

Check that $\frac{3}{2}$ satisfies the original equation. The solution set is $\left\{\frac{3}{2}\right\}$.

b) For this equation we can get a single k on the right by subtracting $2k$ from each side. (If we subtract $3k$ from each side, we get $-k$, and then we need another step.)

$$2k + 5 = 3k + 1$$
$$5 = k + 1 \quad \text{Subtract } 2k \text{ from each side.}$$
$$4 = k \quad \text{Subtract 1 from each side.}$$

Check that 4 satisfies the original equation. The solution set is $\{4\}$. ■

study tip

I hear and I forget; I see and I remember; I do and I understand. There is no substitute for doing exercises, lots of exercises.

WARM-UPS

True or false? Explain your answer.

1. The equation $x - x = 99$ has no solution.
2. The equation $2n + 3n = 5n$ is an identity.
3. The equation $2y + 3y = 4y$ is inconsistent.

WARM-UPS

(*continued*)

4. All real numbers satisfy the equation $1 \div x = \frac{1}{x}$.
5. The equation $5a + 3 = 0$ is an inconsistent equation.
6. The equation $2t = t$ is a conditional equation.
7. The equation $w - 0.1w = 0.9w$ is an identity.
8. The equation $0.2x + 0.03x = 8$ is equivalent to $20x + 3x = 8$.
9. The equation $\frac{x}{x} = 1$ is an identity.
10. The solution to $3h - 8 = 0$ is $\frac{8}{3}$.

2.3 EXERCISES

Reading and Writing *After reading this section, write out the answers to these questions. Use complete sentences.*

1. What is an identity?
2. What is a conditional equation?
3. What is an inconsistent equation?
4. What is the usual first step when solving an equation involving fractions?
5. What is a good first step for solving an equation involving decimals?
6. Where should the variable be when you are finished solving an equation?

Solve each equation. Identify each as a conditional equation, an inconsistent equation, or an identity. See Examples 1 and 2.

7. $x + x = 2x$
8. $2x - x = x$
9. $a - 1 = a + 1$
10. $r + 7 = r$
11. $3y + 4y = 12y$
12. $9t - 8t = 7$
13. $-4 + 3(w - 1) = w + 2(w - 2) - 1$
14. $4 - 5(w + 2) = 2(w - 1) - 7w - 4$
15. $3(m + 1) = 3(m + 3)$
16. $5(m - 1) - 6(m + 3) = 4 - m$
17. $x + x = 2$
18. $3x - 5 = 0$
19. $2 - 3(5 - x) = 3x$
20. $3 - 3(5 - x) = 0$
21. $(3 - 3)(5 - z) = 0$
22. $(2 \cdot 4 - 8)p = 0$
23. $\frac{0}{x} = 0$
24. $\frac{2x}{2} = x$
25. $x \cdot x = x^2$
26. $\frac{2x}{2x} = 1$

Solve each equation by first eliminating the fractions. See Example 3.

27. $\frac{x}{2} + 3 = x - \frac{1}{2}$
28. $13 - \frac{x}{2} = x - \frac{1}{2}$
29. $\frac{x}{2} + \frac{x}{3} = 20$
30. $\frac{x}{2} - \frac{x}{3} = 5$
31. $\frac{w}{2} + \frac{w}{4} = 12$
32. $\frac{a}{4} - \frac{a}{2} = -5$
33. $\frac{3z}{2} - \frac{2z}{3} = -10$
34. $\frac{3m}{4} + \frac{m}{2} = -5$
35. $\frac{1}{3}p - 5 = \frac{1}{4}p$
36. $\frac{1}{2}q - 6 = \frac{1}{5}q$
37. $\frac{1}{6}v + 1 = \frac{1}{4}v - 1$
38. $\frac{1}{15}k + 5 = \frac{1}{6}k - 10$

Solve each equation by first eliminating the decimal numbers. See Examples 4 and 5.

39. $x - 0.2x = 72$
40. $x - 0.1x = 63$
41. $0.3x + 1.2 = 0.5x$

42. $0.4x - 1.6 = 0.6x$

43. $0.02x - 1.56 = 0.8x$

44. $0.6x + 10.4 = 0.08x$

45. $0.1a - 0.3 = 0.2a - 8.3$

46. $0.5b + 3.4 = 0.2b + 12.4$

47. $0.05r + 0.4r = 27$

48. $0.08t + 28.3 = 0.5t - 9.5$

49. $0.05y + 0.03(y + 50) = 17.5$

50. $0.07y + 0.08(y - 100) = 44.5$

51. $0.1x + 0.05(x - 300) = 105$

52. $0.2x - 0.05(x - 100) = 35$

Solve each equation. If you feel proficient enough, try simplifying the process, as described in Example 6.

53. $2x - 9 = 0$

54. $3x + 7 = 0$

55. $-2x + 6 = 0$

56. $-3x - 12 = 0$

57. $\frac{z}{5} + 1 = 6$

58. $\frac{s}{2} + 2 = 5$

59. $\frac{c}{2} - 3 = -4$

60. $\frac{b}{3} - 4 = -7$

61. $3 = t + 6$

62. $-5 = y - 9$

63. $5 + 2q = 3q$

64. $-4 - 5p = -4p$

65. $8x - 1 = 9 + 9x$

66. $4x - 2 = -8 + 5x$

67. $-3x + 1 = -1 - 2x$

68. $-6x + 3 = -7 - 5x$

Solve each equation.

69. $3x - 5 = 2x - 9$

70. $5x - 9 = x - 4$

71. $x + 2(x + 4) = 3(x + 3) - 1$

72. $u + 3(u - 4) = 4(u - 5)$

73. $23 - 5(3 - n) = -4(n - 2) + 9n$

74. $-3 - 4(t - 5) = -2(t + 3) + 11$

75. $0.05x + 30 = 0.4x - 5$

76. $x - 0.08x = 460$

77. $-\frac{2}{3}a + 1 = 2$

78. $-\frac{3}{4}t = \frac{1}{2}$

79. $\frac{y}{2} + \frac{y}{6} = 20$

80. $\frac{3w}{5} - 1 = \frac{w}{2} + 1$

81. $0.09x - 0.2(x + 4) = -1.46$

82. $0.08x + 0.5(x + 100) = 73.2$

83. $436x - 789 = -571$

84. $0.08x + 4533 = 10x + 69$

85. $\frac{x}{344} + 235 = 292$

86. $34(x - 98) = \frac{x}{2} + 475$

Solve each problem.

87. ***Sales commission.*** Danielle sold her house through an agent who charged 8% of the selling price. After the commission was paid, Danielle received \$117,760. If x is the selling price, then x satisfies

$$x - 0.08x = 117{,}760.$$

Solve this equation to find the selling price.

88. ***Raising rabbits.*** Before Roland sold two female rabbits, half of his rabbits were female. After the sale, only one-third of his rabbits were female. If x represents his original number of rabbits, then

$$\frac{1}{2}x - 2 = \frac{1}{3}(x - 2).$$

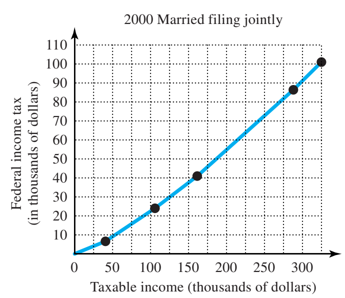
Solve this equation to find the number of rabbits that he had before the sale.

89. ***Eavesdropping.*** Reginald overheard his boss complaining that his federal income tax for 2000 was \$34,276.

a) Use the accompanying graph to estimate his boss's taxable income for 2000.

b) Find his boss's exact taxable income for 2000 by solving the equation

$$23{,}965.5 + 0.31(x - 105{,}950) = 34{,}276.$$



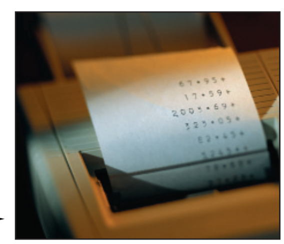


FIGURE FOR EXERCISE 89

90. ***Federal taxes.*** According to Bruce Harrell, CPA, the federal income tax for a class C corporation is found by solving a linear equation. The reason for the equation is that the amount x of federal tax is deducted before the state tax is figured, and the amount of state tax is deducted before the federal tax is figured. To find the amount of federal tax for a corporation with a taxable income of \$200,000, for which the federal tax rate is 25% and the state tax rate is 10%, Bruce must solve

$$x = 0.25[200{,}000 - 0.10(200{,}000 - x)].$$

Solve the equation for Bruce.

2.4 FORMULAS

In this section

- Solving for a Variable
- Finding the Value of a Variable

In this section, you will learn to rewrite formulas using the same properties of equality that we used to solve equations. You will also learn how to find the value of one of the variables in a formula when we know the value of all of the others.

Solving for a Variable

Most drivers know the relationship between distance, rate, and time. For example, if you drive 70 mph for 3 hours, then you will travel 210 miles. At 60 mph a 300-mile trip will take 5 hours. If a 400-mile trip took 8 hours, then you averaged 50 mph. The relationship between distance D, rate R, and time T is expressed by the formula

$$D = R \cdot T.$$

A **formula** or **literal equation** is an equation involving two or more variables.

To find the time for a 300-mile trip at 60 mph, you are using the formula in the form $T = \frac{D}{R}$. The process of rewriting a formula for one variable in terms of the others is called **solving for a certain variable.** To solve for a certain variable, we use the same techniques that we use in solving equations.

EXAMPLE 1 **Solving for a certain variable**

Solve the formula $D = RT$ for T:

Solution

$$D = RT \quad \text{Original formula}$$

$$\frac{D}{R} = \frac{R \cdot T}{R} \quad \text{Divide each side by } R.$$

$$\frac{D}{R} = T \quad \text{Divide out (or cancel) the common factor } R.$$

$$T = \frac{D}{R} \quad \text{It is customary to write the single variable on the left.}$$

The formula $C = \frac{5}{9}(F - 32)$ is used to find the Celsius temperature for a given Fahrenheit temperature. If we solve this formula for F, then we have a formula for finding Fahrenheit temperature for a given Celsius temperature.

EXAMPLE 2 **Solving for a certain variable**

Solve the formula $C = \frac{5}{9}(F - 32)$ for F.

Solution

We could apply the distributive property to the right side of the equation, but it is simpler to proceed as follows:

$$C = \frac{5}{9}(F - 32)$$

$$\frac{9}{5}C = \frac{9}{5} \cdot \frac{5}{9}(F - 32) \quad \text{Multiply each side by } \tfrac{9}{5}, \text{ the reciprocal of } \tfrac{5}{9}.$$

$$\frac{9}{5}C = F - 32 \qquad \text{Simplify.}$$

$$\frac{9}{5}C + 32 = F - 32 + 32 \qquad \text{Add 32 to each side.}$$

$$\frac{9}{5}C + 32 = F \qquad \text{Simplify.}$$

The formula is usually written as $F = \frac{9}{5}C + 32$. ■

When solving for a variable that appears more than once in the equation, we must combine the terms to obtain a single occurrence of the variable. *When a formula has been solved for a certain variable, that variable will not occur on both sides of the equation.*

EXAMPLE 3 **Solving for a variable that appears on both sides**

Solve $5x - b = 3x + d$ for x.

Solution

First get all terms involving x onto one side and all other terms onto the other side:

$$5x - b = 3x + d \qquad \text{Original formula}$$

$$5x - 3x - b = d \qquad \text{Subtract } 3x \text{ from each side.}$$

$$5x - 3x = b + d \qquad \text{Add } b \text{ to each side.}$$

$$2x = b + d \qquad \text{Combine like terms.}$$

$$x = \frac{b + d}{2} \qquad \text{Divide each side by 2.}$$

The formula solved for x is $x = \frac{b + d}{2}$. ■

In Chapter 4, it will be necessary to solve an equation involving x and y for y.

EXAMPLE 4 **Solving for *y***

Solve $x + 2y = 6$ for y. Write the answer in the form $y = mx + b$, where m and b are fixed real numbers.

helpful hint

If we simply wanted to solve $x + 2y = 6$ for y, we could have written

$$y = \frac{6 - x}{2} \text{ or } y = \frac{-x + 6}{2}.$$

However, in Example 4 we requested the form $y = mx + b$. This form is a popular form that we will study in detail in Chapter 4.

Solution

$$x + 2y = 6 \qquad \text{Original equation}$$

$$2y = 6 - x \qquad \text{Subtract } x \text{ from each side.}$$

$$\frac{1}{2} \cdot 2y = \frac{1}{2}(6 - x) \qquad \text{Multiply each side by } \tfrac{1}{2}.$$

$$y = 3 - \frac{1}{2}x \qquad \text{Distributive property}$$

$$y = -\frac{1}{2}x + 3 \qquad \text{Rearrange to get } y = mx + b \text{ form.}$$ ■

Notice that in Example 4 we multiplied each side of the equation by $\frac{1}{2}$, and so we multiplied each term on the right-hand side by $\frac{1}{2}$. Instead of multiplying by $\frac{1}{2}$, we could have divided each side of the equation by 2. We would then divide each term on the right side by 2. This idea is illustrated in the next example.

M A T H A T W O R K

NURSE

Even before the days of Florence Nightingale, nurses around the world were giving comfort and aid to the sick and injured. Continuing in this tradition, Asenet Craffey, staff nurse at the Massachusetts Eye and Ear Infirmary, works in the intensive care unit. During her 12-hour shifts, Ms. Craffey is responsible for the full nursing care of four to eight patients. In the intensive care unit, the nurse-to-patient ratio is usually one to one. When Ms. Craffey is assigned to this unit, she is responsible for over-all care of a patient as well as being prepared for crisis care. Staff scheduling is an additional duty that Ms. Craffey performs, making sure that there is adequate nursing coverage for the day's planned surgeries and quality patient care. Full care means being directly involved in all of the patient's care: monitoring vital signs, changing dressings, helping to feed, following the prescribed orders left by the physicians, and administering drugs.

Many drugs come directly from the pharmacy in the exact dosage for a particular patient. Intravenous (IV) drugs, however, must be monitored so that the correct amount of drops per minute are administered. IV medications can be glucose solutions, antibiotics, or pain killers. Often the prescribed dosage is 1 gram per 100, 200, 500, or 1000 cubic centimeters of liquid. In Exercise 85 of this section you will calculate a drug dosage, just as Ms. Craffey would on the job.

E X A M P L E 5 **Solving for *y***

Solve $2x - 3y = 9$ for y. Write the answer in the form $y = mx + b$, where m and b are real numbers. (When we study lines in Chapter 4 you will see that $y = mx + b$ is the slope-intercept form of the equation of a line.)

Solution

$$2x - 3y = 9 \qquad \text{Original equation}$$

$$-3y = -2x + 9 \qquad \text{Subtract } 2x \text{ from each side.}$$

$$\frac{-3y}{-3} = \frac{-2x + 9}{-3} \qquad \text{Divide each side by } -3.$$

$$y = \frac{-2x}{-3} + \frac{9}{-3} \qquad \text{By the distributive property, each term is divided by } -3.$$

$$y = \frac{2}{3}x - 3 \qquad \text{Simplify.}$$

■

Even though we wrote $y = \frac{2}{3}x - 3$ in Example 5, the equation is still considered to be in the form $y = mx + b$ because we could have written $y = \frac{2}{3}x + (-3)$.

Finding the Value of a Variable

In many situations we know the values of all variables in a formula except one. We use the formula to determine the unknown value.

EXAMPLE 6 **Finding the value of a variable in a formula**

If $2x - 3y = 9$, find y when $x = 6$.

Solution

Method 1: First solve the equation for y. Because we have already solved this equation for y in Example 5 we will not repeat that process in this example. We have

$$y = \frac{2}{3}x - 3.$$

Now replace x by 6 in this equation:

$$y = \frac{2}{3}(6) - 3$$
$$= 4 - 3 = 1$$

So, when $x = 6$, we have $y = 1$.

Method 2: First replace x by 6 in the original equation, then solve for y:

$$2x - 3y = 9 \quad \text{Original equation}$$
$$2 \cdot 6 - 3y = 9 \quad \text{Replace } x \text{ by 6.}$$
$$12 - 3y = 9 \quad \text{Simplify.}$$
$$-3y = -3 \quad \text{Subtract 12 from each side.}$$
$$y = 1 \quad \text{Divide each side by } -3.$$

So when $x = 6$, we have $y = 1$. ■

If we had to find the value of y for many different values of x, it would be best to solve the equation for y, then insert the various values of x. Method 1 of Example 6 would be the better method. If we must find only one value of y, it does not matter which method we use. When doing the exercises corresponding to this example, you should try both methods.

The next example involves the simple interest formula $I = Prt$, where I is the amount of interest, P is the principal or the amount invested, r is the annual interest rate, and t is the time in years. The interest rate is generally expressed as a percent. When using a rate in computations, you must convert it to a decimal.

EXAMPLE 7 **Using the simple interest formula**

If the simple interest is \$120, the principal is \$400, and the time is 2 years, find the rate.

Solution

First, solve the formula $I = Prt$ for r, then insert values of P, I, and t:

$$Prt = I \quad \text{Simple interest formula}$$
$$\frac{Prt}{Pt} = \frac{I}{Pt} \quad \text{Divide each side by } Pt.$$
$$r = \frac{I}{Pt} \quad \text{Simplify.}$$
$$r = \frac{120}{400 \cdot 2} \quad \text{Substitute the values of } I, P, \text{ and } t.$$
$$r = 0.15 \quad \text{Simplify.}$$
$$r = 15\% \quad \text{Move the decimal point two places to the right.}$$
■

helpful hint

All interest computation is based on simple interest. However, depositors do not like to wait two years to get interest as in Example 7. More often the time is $\frac{1}{12}$ year or $\frac{1}{365}$ year. Simple interest computed every month is said to be compounded monthly. Simple interest computed every day is said to be compounded daily.

In solving a geometric problem, it is always helpful to draw a diagram, as we do in the next example.

EXAMPLE 8 **Using a geometric formula**

The perimeter of a rectangle is 36 feet. If the width is 6 feet, then what is the length?

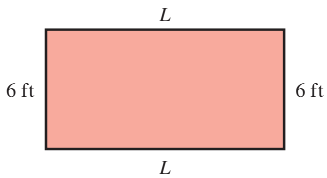


FIGURE 2.1

Solution

First, put the given information on a diagram as shown in Fig. 2.1. Substitute the given values into the formula for the perimeter of a rectangle found at the back of the book, and then solve for L. (We could solve for L first and then insert the given values.)

$$P = 2L + 2W \quad \text{Perimeter of a rectangle}$$
$$36 = 2L + 2 \cdot 6 \quad \text{Substitute 36 for } P \text{ and 6 for } W.$$
$$36 = 2L + 12 \quad \text{Simplify.}$$
$$24 = 2L \quad \text{Subtract 12 from each side.}$$
$$12 = L \quad \text{Divide each side by 2.}$$

Check: If $L = 12$ and $W = 6$, then $P = 2(12) + 2(6) = 36$ feet. So we can be certain that the length is 12 feet.

The next example involves the sale-price formula $S = L - rL$, where S is the selling price, L is the list or original price, and r is the rate of discount. The rate of discount is generally expressed as a percent. In computations, rates must be written as decimals (or fractions).

EXAMPLE 9 **Finding the original price**

What was the original price of a stereo that sold for \$560 after a 20% discount.

Solution

Express 20% as the decimal 0.20 or 0.2 and use the formula $S = L - rL$:

$$\text{Selling price} = \text{list price} - \text{amount of discount}$$
$$560 = L - 0.2L$$
$$10(560) = 10(L - 0.2L) \quad \text{Multiply each side by 10.}$$
$$5600 = 10L - 2L \quad \text{Remove the parentheses.}$$
$$5600 = 8L \quad \text{Combine like terms.}$$
$$\frac{5600}{8} = \frac{8L}{8} \quad \text{Divide each side by 8.}$$
$$700 = L$$

Check: We find that 20% of \$700 is \$140 and \$700 − \$140 = \$560, the selling price. So we are certain that the original price was \$700.

study tip

Don't wait for inspiration to strike, it probably won't. Algebra is learned one tiny step at a time.

WARM-UPS

True or false? Explain your answer.

1. If we solve $D = R \cdot T$ for T, we get $T \cdot R = D$.
2. If we solve $a - b = 3a - m$ for a, we get $a = 3a - m + b$.
3. Solving $A = LW$ for L, we get $L = \frac{W}{A}$.
4. Solving $D = RT$ for R, we get $R = \frac{d}{t}$.
5. The perimeter of a rectangle is the product of its length and width.
6. The volume of a shoe box is the product of its length, width, and height.

WARM-UPS

(continued)

7. The sum of the length and width of a rectangle is one-half of its perimeter.
8. Solving $y - x = 5$ for y gives us $y = x + 5$.
9. If $x = -1$ and $y = -3x + 6$, then $y = 3$.
10. The circumference of a circle is the product of its diameter and the number π.

2.4 EXERCISES

Reading and Writing *After reading this section, write out the answers to these questions. Use complete sentences.*

1. What is a formula?
2. What is a literal equation?
3. What does it mean to solve a formula for a certain variable?
4. How do you solve a formula for a variable that appears on both sides?
5. What are the two methods shown for finding the value of a variable in a formula?
6. What formula expresses the perimeter of a rectangle in terms of its length and width?

Solve each formula for the specified variable. See Examples 1 and 2.

7. $D = RT$ for R
8. $A = LW$ for W
9. $C = \pi D$ for D
10. $F = ma$ for a
11. $I = Prt$ for P
12. $I = Prt$ for t
13. $F = \frac{9}{5}C + 32$ for C
14. $y = \frac{3}{4}x - 7$ for x
15. $A = \frac{1}{2}bh$ for h
16. $A = \frac{1}{2}bh$ for b
17. $P = 2L + 2W$ for L
18. $P = 2L + 2W$ for W
19. $A = \frac{1}{2}(a + b)$ for a
20. $A = \frac{1}{2}(a + b)$ for b
21. $S = P + Prt$ for r
22. $S = P + Prt$ for t
23. $A = \frac{1}{2}h(a + b)$ for a
24. $A = \frac{1}{2}h(a + b)$ for b

Solve each equation for x. See Example 3.

25. $5x + a = 3x + b$
26. $2c - x = 4x + c - 5b$
27. $4(a + x) - 3(x - a) = 0$
28. $-2(x - b) - (5a - x) = a + b$
29. $3x - 2(a - 3) = 4x - 6 - a$
30. $2(x - 3w) = -3(x + w)$
31. $3x + 2ab = 4x - 5ab$
32. $x - a = -x + a + 4b$

Solve each equation for y. See Examples 4 and 5.

33. $x + y = -9$
34. $3x + y = -5$
35. $x + y - 6 = 0$
36. $4x + y - 2 = 0$
37. $2x - y = 2$
38. $x - y = -3$
39. $3x - y + 4 = 0$

40. $-2x - y + 5 = 0$

41. $x + 2y = 4$

42. $3x + 2y = 6$

43. $2x - 2y = 1$

44. $3x - 2y = -6$

45. $y + 2 = 3(x - 4)$

46. $y - 3 = -3(x - 1)$

47. $y - 1 = \frac{1}{2}(x - 2)$

48. $y - 4 = -\frac{2}{3}(x - 9)$

49. $\frac{1}{2}x - \frac{1}{3}y = -2$

50. $\frac{x}{2} + \frac{y}{4} = \frac{1}{2}$

For each equation that follows, find y given that $x = 2$. See Example 6.

51. $y = 3x - 4$

52. $y = -2x + 5$

53. $3x - 2y = -8$

54. $4x + 6y = 8$

55. $\frac{3x}{2} - \frac{5y}{3} = 6$

56. $\frac{2y}{5} - \frac{3x}{4} = \frac{1}{2}$

57. $y - 3 = \frac{1}{2}(x - 6)$

58. $y - 6 = -\frac{3}{4}(x - 2)$

59. $y - 4.3 = 0.45(x - 8.6)$

60. $y + 33.7 = 0.78(x - 45.6)$

Solve each of the following problems. Appendix A contains some geometric formulas that may be helpful. See Examples 7–9.

61. ***Finding the rate.*** If the simple interest on \$5000 for 3 years is \$600, then what is the rate?

62. ***Finding the rate.*** Wayne paid \$420 in simple interest on a loan of \$1000 for 7 years. What was the rate?

63. ***Finding the time.*** Kathy paid \$500 in simple interest on a loan of \$2500. If the annual interest rate was 5%, then what was the time?

64. ***Finding the time.*** Robert paid \$240 in simple interest on a loan of \$1000. If the annual interest rate was 8%, then what was the time?

65. ***Finding the length.*** The area of a rectangle is 28 square yards. The width is 4 yards. Find the length.

66. ***Finding the width.*** The area of a rectangle is 60 square feet. The length is 4 feet. Find the width.

67. ***Finding the length.*** If it takes 600 feet of wire fencing to fence a rectangular feed lot that has a width of 75 feet, then what is the length of the lot?

68. ***Finding the depth.*** If it takes 500 feet of fencing to enclose a rectangular lot that is 104 feet wide, then how deep is the lot?

69. ***Finding the original price.*** Find the original price if there is a 15% discount and the sale price is \$255.

70. ***Finding the list price.*** Find the list price if there is a 12% discount and the sale price is \$4400.

71. ***Rate of discount.*** Find the rate of discount if the discount is \$40 and the original price is \$200.

72. ***Rate of discount.*** Find the rate of discount if the discount is \$20 and the original price is \$250.

73. ***Width of a football field.*** The perimeter of a football field in the NFL, excluding the end zones, is 920 feet. How wide is the field?

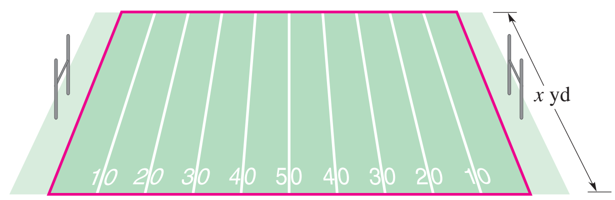


FIGURE FOR EXERCISE 73

74. ***Perimeter of a frame.*** If a picture frame is 16 inches by 20 inches, then what is its perimeter?

75. ***Volume of a box.*** A rectangular box measures 2 feet wide, 3 feet long, and 4 feet deep. What is its volume?

76. ***Volume of a refrigerator.*** The volume of a rectangular refrigerator is 20 cubic feet. If the top measures 2 feet by 2.5 feet, then what is the height?

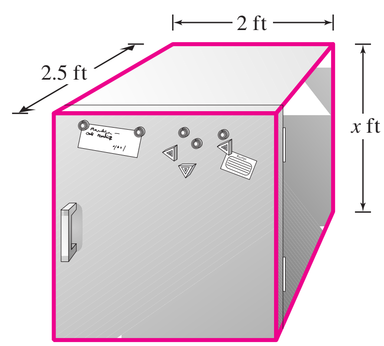


FIGURE FOR EXERCISE 76

77. ***Radius of a pizza.*** If the circumference of a pizza is 8π inches, then what is the radius?

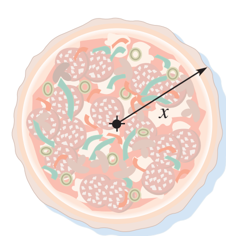


FIGURE FOR EXERCISE 77

78. ***Diameter of a circle.*** If the circumference of a circle is 4π meters, then what is the diameter?

79. ***Height of a banner.*** If a banner in the shape of a triangle has an area of 16 square feet with a base of 4 feet, then what is the height of the banner?

FIGURE FOR EXERCISE 79

80. ***Length of a leg.*** If a right triangle has an area of 14 square meters and one leg is 4 meters in length, then what is the length of the other leg?

81. ***Length of the base.*** A trapezoid with height 20 inches and lower base 8 inches has an area of 200 square inches. What is the length of its upper base?

82. ***Height of a trapezoid.*** The end of a flower box forms the shape of a trapezoid. The area of the trapezoid is 300 square centimeters. The bases are 16 centimeters and 24 centimeters in length. Find the height.

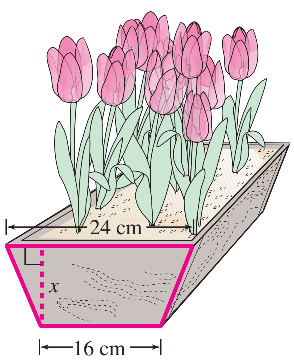


FIGURE FOR EXERCISE 82

83. ***Fried's rule.*** Doctors often prescribe the same drugs for children as they do for adults. The formula $d = 0.08aD$ (Fried's rule) is used to calculate the child's dosage d, where a is the child's age and D is the adult dosage. If a doctor prescribes 1000 milligrams of acetaminophen for an adult, then how many milligrams would the doctor prescribe for an eight-year-old child? Use the bar graph to determine the age at which a child would get the same dosage as an adult.

84. ***Cowling's rule.*** Cowling's rule is another method for determining the dosage of a drug to prescribe to a child.

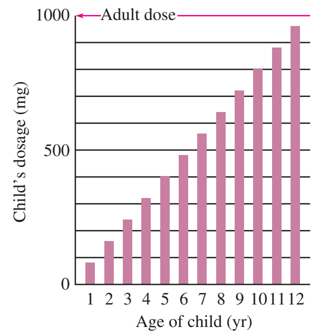


FIGURE FOR EXERCISE 83

For this rule, the formula

$$d = \frac{D(a + 1)}{24}$$

gives the child's dosage d, where D is the adult dosage and a is the age of the child in years. If the adult dosage of a drug is 600 milligrams and a doctor uses this formula to determine that a child's dosage is 200 milligrams, then how old is the child?

85. ***Administering Vancomycin.*** A patient is to receive 750 mg of the antibiotic Vancomycin. However, Vancomycin comes in a solution containing 1 gram (available dose) of Vancomycin per 5 milliliters (quantity) of solution. Use the formula

$$\text{Amount} = \frac{\text{desired dose}}{\text{available dose}} \times \text{quantity}$$

to find the amount of this solution that should be administered to the patient.

86. ***International communications.*** The global investment in telecom infrastructure since 1990 can be modeled by the formula

$$I = 7.5t + 115,$$

where I is in billions of dollars and t is the number of years since 1990 (*Fortune,* September 8, 1997). See the accompanying figure.

a) Use the formula to find the global investment in 1994.

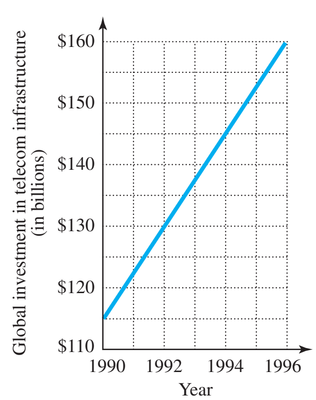


FIGURE FOR EXERCISE 86

b) Use the formula to predict the global investment in 2001.

c) Find the year in which the global investment will reach $250 billion.

87. ***The 2.4-meter rule.*** A 2.4-meter sailboat is a one-person boat that is about 13 feet in length, has a displacement of about 550 pounds, and a sail area of about 81 square feet. To compete in the 2.4-meter class, a boat must satisfy the formula

$$2.4 = \frac{L + 2D - F\sqrt{S}}{2.37},$$

where L = length, F = freeboard, D = girth, and S = sail area. Solve the formula for L.

PHOTO FOR EXERCISE 87

2.5 TRANSLATING VERBAL EXPRESSIONS INTO ALGEBRAIC EXPRESSIONS

In this section

- Writing Algebraic Expressions
- Pairs of Numbers
- Consecutive Integers
- Using Formulas
- Writing Equations

You translated some verbal expressions into algebraic expressions in Section 1.6; in this section you will study translating in more detail.

Writing Algebraic Expressions

The following box contains a list of some frequently occurring verbal expressions and their equivalent algebraic expressions.

Translating Words into Algebra

	Verbal Phrase	**Algebraic Expression**
Addition:	The sum of a number and 8	$x + 8$
	Five is added to a number	$x + 5$
	Two more than a number	$x + 2$
	A number increased by 3	$x + 3$
Subtraction:	Four is subtracted from a number	$x - 4$
	Three less than a number	$x - 3$
	The difference between 7 and a number	$7 - x$
	A number decreased by 2	$x - 2$
Multiplication:	The product of 5 and a number	$5x$
	Twice a number	$2x$
	One-half of a number	$\frac{1}{2}x$
	Five percent of a number	$0.05x$
Division:	The ratio of a number to 6	$\frac{x}{6}$
	The quotient of 5 and a number	$\frac{5}{x}$
	Three divided by some number	$\frac{3}{x}$

EXAMPLE 1 **Writing algebraic expressions**

Translate each verbal expression into an algebraic expression.

a) The sum of a number and 9 **b)** Eighty percent of a number

c) A number divided by 4 **d)** The result of a number subtracted from 5

e) Three less than a number

Solution

a) If x is the number, then the sum of a number and 9 is $x + 9$.

b) If w is the number, then eighty percent of the number is $0.80w$.

c) If y is the number, then the number divided by 4 is $\frac{y}{4}$.

d) If z is the number, then the result of subtracting z from 5 is $5 - z$.

e) If a is the number, then 3 less than a is $a - 3$. ■

helpful hint

We know that x and $10 - x$ have a sum of 10 for any value of x. We can easily check that fact by adding:

$$x + 10 - x = 10$$

In general it is not true that x and $x - 10$ have a sum of 10, because

$$x + x - 10 = 2x - 10.$$

For what value of x is the sum of x and $x - 10$ equal to 10?

Pairs of Numbers

There is often more than one unknown quantity in a problem, but a relationship between the unknown quantities is given. For example, if one unknown number is 5 more than another unknown number, we can use

$$x \quad \text{and} \quad x + 5,$$

to represent them. Note that x and $x + 5$ can also be used to represent two unknown numbers that differ by 5, for if two numbers differ by 5, one of the numbers is 5 more than the other.

How would you represent two numbers that have a sum of 10? If one of the numbers is 2, the other is certainly $10 - 2$, or 8. Thus if x is one of the numbers, then $10 - x$ is the other. The expressions

$$x \quad \text{and} \quad 10 - x$$

have a sum of 10 for any value of x.

EXAMPLE 2 **Algebraic expressions for pairs of numbers**

Write algebraic expressions for each pair of numbers.

a) Two numbers that differ by 12 **b)** Two numbers with a sum of -8

Solution

a) The expressions x and $x - 12$ represent two numbers that differ by 12. We can check by subtracting:

$$x - (x - 12) = x - x + 12 = 12$$

Of course, x and $x + 12$ also differ by 12 because $x + 12 - x = 12$.

b) The expressions x and $-8 - x$ have a sum of -8. We can check by addition:

$$x + (-8 - x) = x - 8 - x = -8$$ ■

Pairs of numbers occur in geometry in discussing measures of angles. You will need the following facts about degree measures of angles.

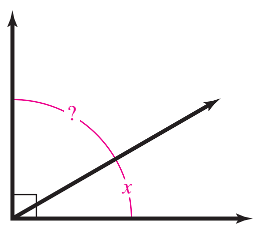
Degree Measures of Angles

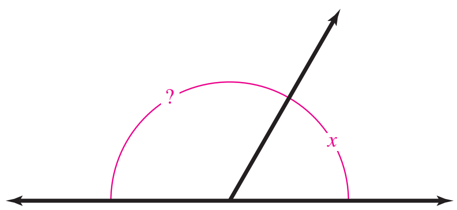
Two angles are called **complementary** if the sum of their degree measures is 90°.
Two angles are called **supplementary** if the sum of their degree measures is 180°.
The sum of the degree measures of the three angles of any triangle is 180°.

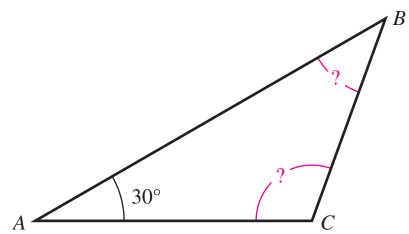
For complementary angles, we use x and $90 - x$ for their degree measures. For supplementary angles, we use x and $180 - x$. Complementary angles that share a common side form a right angle. Supplementary angles that share a common side form a straight angle or straight line.

EXAMPLE 3 **Degree measures**

Write algebraic expressions for each pair of angles shown.

a)


b)


c)


Solution

a) Since the angles shown are complementary, we can use x to represent the degree measure of the smaller angle and $90 - x$ to represent the degree measure of the larger angle.

b) Since the angles shown are supplementary, we can use x to represent the degree measure of the smaller angle and $180 - x$ to represent the degree measure of the larger angle.

c) If we let x represent the degree measure of angle B, then $180 - x - 30$, or $150 - x$, represents the degree measure of angle C. ■

Consecutive Integers

To gain practice in problem solving, we will solve problems about consecutive integers in Section 2.6. Note that each integer is 1 larger than the previous integer, while consecutive even integers as well as consecutive odd integers differ by 2.

EXAMPLE 4 **Expressions for integers**

Write algebraic expressions for the following unknown integers.

a) Three consecutive integers, the smallest of which is w

b) Three consecutive even integers, the smallest of which is z

helpful hint

If x is even, then x, $x + 2$, and $x + 4$ represent three consecutive even integers. If x is odd, then x, $x + 2$, and $x + 4$ represent three consecutive odd integers. Is it possible for x, $x + 1$, and $x + 3$ to represent three consecutive even integers or three consecutive odd integers?

Solution

a) Since each integer is 1 larger than the preceding integer, we can use w, $w + 1$, and $w + 2$ to represent them.

b) Since consecutive even integers differ by 2, these integers can be represented by z, $z + 2$, and $z + 4$. ■

Using Formulas

In writing expressions for unknown quantities, we often use standard formulas such as those given at the back of the book.

EXAMPLE 5 **Writing algebraic expressions using standard formulas**

Find an algebraic expression for

a) the distance if the rate is 30 miles per hour and the time is T hours.

b) the discount if the rate is 40% and the original price is p dollars.

Solution

a) Using the formula $D = RT$, we have $D = 30T$. So $30T$ is an expression that represents the distance in miles.

b) Since the discount is the rate times the original price, an algebraic expression for the discount is $0.40p$ dollars.

Writing Equations

To solve a problem using algebra, we describe or **model** the problem with an equation. In this section we write the equations only, and in Section 2.6 we write and solve them. Sometimes we must write an equation from the information given in the problem and sometimes we use a standard model to get the equation. Some standard models are shown in the following box.

study tip

It is a good idea to work with others, but don't be misled. Working a problem with help is not the same as working a problem on your own. In the end, mathematics is personal. Make sure that you can do it.

Uniform Motion Model

Distance = Rate · Time $\qquad D = R \cdot T$

Selling Price and Discount Model

Discount = Rate of discount · Original price
Selling Price = Original price − Discount

Real Estate Commission Model

Commission = Rate of commission · Selling price
Amount for owner = Selling price − Commission

Percentage Models

What number is 5% of 40? $\qquad x = 0.05 \cdot 40$
Ten is what percent of 80? $\qquad 10 = x \cdot 80$
Twenty is 4% of what number? $\qquad 20 = 0.04 \cdot x$

Geometric Models for Area

Rectangle: $A = LW$ $\qquad$ Square: $A = s^2$
Parallelogram: $A = bh$ $\qquad$ Triangle: $A = \frac{1}{2}bh$

Geometric Models for Perimeter

Perimeter of any figure = the sum of the lengths of the sides
Rectangle: $P = 2L + 2W$ $\qquad$ Square: $P = 4s$

More geometric formulas can be found in Appendix A.

EXAMPLE 6 **Writing equations**

Identify the variable and write an equation that describes each situation.

a) Find two numbers that have a sum of 14 and a product of 45.

b) A coat is on sale for 25% off the list price. If the sale price is \$87, then what is the list price?

c) What percent of 8 is 2?

d) The value of x dimes and $x - 3$ quarters is \$2.05.

Solution

a) Let $x =$ one of the numbers and $14 - x =$ the other number. Since their product is 45, we have

$$x(14 - x) = 45.$$

b) Let $x =$ the list price and $0.25x =$ the amount of discount. We can write an equation expressing the fact that the selling price is the list price minus the discount:

$$\text{List price} - \text{discount} = \text{selling price}$$
$$x - 0.25x = 87$$

c) If we let x represent the percentage, then the equation is $x \cdot 8 = 2$, or $8x = 2$.

d) The value of x dimes at 10 cents each is $10x$ cents. The value of $x - 3$ quarters at 25 cents each is $25(x - 3)$ cents. We can write an equation expressing the fact that the total value of the coins is 205 cents:

$$\text{Value of dimes} + \text{value of quarters} = \text{total value}$$
$$10x + 25(x - 3) = 205$$

helpful hint

At this point we are simply learning to write equations that model certain situations. Don't worry about solving these equations now. In Section 2.6 we will solve problems by writing an equation and solving it.

CAUTION The value of the coins in Example 6(d) is either 205 cents or 2.05 dollars. If the total value is expressed in dollars, then all of the values must be expressed in dollars. So we could also write the equation as

$$0.10x + 0.25(x - 3) = 2.05.$$

WARM-UPS

True or false? Explain your answer.

1. For any value of x, the numbers x and $x + 6$ differ by 6.
2. For any value of a, a and $10 - a$ have a sum of 10.
3. If Jack ran at x miles per hour for 3 hours, he ran $3x$ miles.
4. If Jill ran at x miles per hour for 10 miles, she ran for $10x$ hours.
5. If the realtor gets 6% of the selling price and the house sells for x dollars, the owner gets $x - 0.06x$ dollars.
6. If the owner got \$50,000 and the realtor got 10% of the selling price, the house sold for \$55,000.
7. Three consecutive odd integers can be represented by x, $x + 1$, and $x + 3$.
8. The value in cents of n nickels and d dimes is $0.05n + 0.10d$.
9. If the sales tax rate is 5% and x represents the price of the goods purchased, then the total bill is $1.05x$.
10. If the length of a rectangle is 4 feet more than the width w, then the perimeter is $w + (w + 4)$ feet.

2.5 EXERCISES

Reading and Writing *After reading this section, write out the answers to these questions. Use complete sentences.*

1. What are the different ways of verbally expressing the operation of addition?

2. How can you algebraically express two numbers using only one variable?

3. What are complementary angles?

4. What are supplementary angles?

5. What is the relationship between distance, rate, and time?

6. What is the difference between expressing consecutive even integers and consecutive odd integers algebraically?

Translate each verbal expression into an algebraic expression. See Example 1.

7. The sum of a number and 3

8. Two more than a number

9. Three less than a number

10. Four subtracted from a number

11. The product of a number and 5

12. Five divided by some number

13. Ten percent of a number

14. Eight percent of a number

15. The ratio of a number and 3

16. The quotient of 12 and a number

17. One-third of a number

18. Three-fourths of a number

Write algebraic expressions for each pair of numbers. See Example 2.

19. Two numbers with a difference of 15

20. Two numbers that differ by 9

21. Two numbers with a sum of 6

22. Two numbers with a sum of 5

23. Two numbers with a sum of -4

24. Two numbers with a sum of -8

25. Two numbers such that one is 3 larger than the other

26. Two numbers such that one is 8 smaller than the other

27. Two numbers such that one is 5% of the other

28. Two numbers such that one is 40% of the other

29. Two numbers such that one is 30% more than the other

30. Two numbers such that one is 20% smaller than the other

Each of the following figures shows a pair of angles. Write algebraic expressions for the degree measures of each pair of angles. See Example 3.

31.

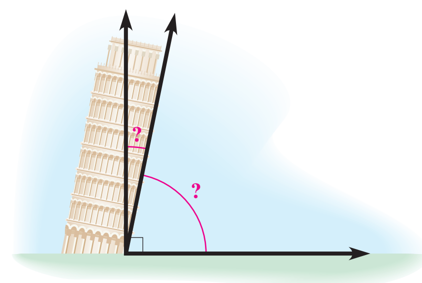


FIGURE FOR EXERCISE 31

32.

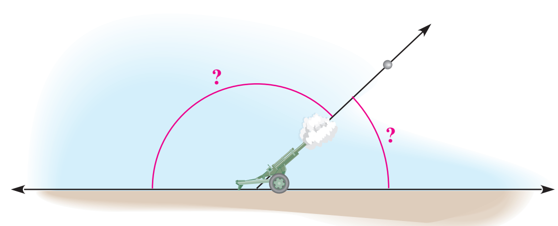


FIGURE FOR EXERCISE 32

33.

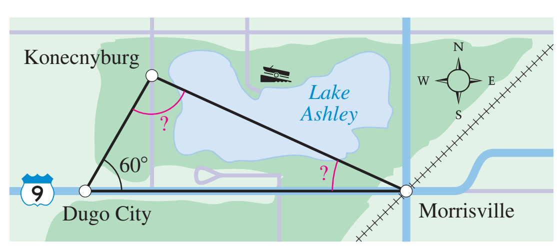


FIGURE FOR EXERCISE 33

34.

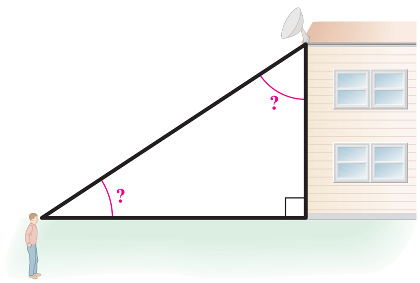


FIGURE FOR EXERCISE 34

Write algebraic expressions for the following unknown integers. See Example 4.

35. Two consecutive even integers, the smallest of which is n

36. Two consecutive odd integers, the smallest of which is x

37. Two consecutive integers

38. Three consecutive even integers

39. Three consecutive odd integers

40. Three consecutive integers

41. Four consecutive even integers

42. Four consecutive odd integers

Find an algebraic expression for the quantity in italics using the given information. See Example 5.

43. The *distance,* given that the rate is x miles per hour and the time is 3 hours

44. The *distance,* given that the rate is $x + 10$ miles per hour and the time is 5 hours

45. The *discount,* given that the rate is 25% and the original price is q dollars

46. The *discount,* given that the rate is 10% and the original price is t yen

47. The *time,* given that the distance is x miles and the rate is 20 miles per hour

48. The *time,* given that the distance is 300 kilometers and the rate is $x + 30$ kilometers per hour

49. The *rate,* given that the distance is $x - 100$ meters and the time is 12 seconds

50. The *rate,* given that the distance is 200 feet and the time is $x + 3$ seconds

51. The *area* of a rectangle with length x meters and width 5 meters

52. The *area* of a rectangle with sides b yards and $b - 6$ yards

53. The *perimeter* of a rectangle with length $w + 3$ inches and width w inches

54. The *perimeter* of a rectangle with length r centimeters and width $r - 1$ centimeters

55. The *width* of a rectangle with perimeter 300 feet and length x feet

56. The *length* of a rectangle with area 200 square feet and width w feet

57. The *length* of a rectangle, given that its width is x feet and its length is 1 foot longer than twice the width

58. The *length* of a rectangle, given that its width is w feet and its length is 3 feet shorter than twice the width

59. The *area* of a rectangle, given that the width is x meters and the length is 5 meters longer than the width

60. The *perimeter* of a rectangle, given that the length is x yards and the width is 10 yards shorter

61. The *simple interest,* given that the principal is $x + 1000$, the rate is 18%, and the time is 1 year

62. The *simple interest,* given that the principal is $3x$, the rate is 6%, and the time is 1 year

63. The *price per pound* of peaches, given that x pounds sold for \$16.50

64. The *rate per hour* of a mechanic who gets \$480 for working x hours

65. The *degree measure* of an angle, given that its complementary angle has measure x degrees

66. The *degree measure* of an angle, given that its supplementary angle has measure x degrees

Identify the variable and write an equation that describes each situation. Do not solve the equation. See Example 6.

67. Two numbers differ by 5 and have a product of 8

68. Two numbers differ by 6 and have a product of -9

69. Herman's house sold for x dollars. The real estate agent received 7% of the selling price, and Herman received \$84,532.

70. Gwen sold her car on consignment for x dollars. The saleswoman's commission was 10% of the selling price, and Gwen received \$6570.

71. What percent of 500 is 100?

72. What percent of 40 is 120?

73. The value of x nickels and $x + 2$ dimes is \$3.80.

74. The value of d dimes and $d - 3$ quarters is \$6.75.

75. The sum of a number and 5 is 13.

76. Twelve subtracted from a number is -6.

77. The sum of three consecutive integers is 42.

78. The sum of three consecutive odd integers is 27.

79. The product of two consecutive integers is 182.

80. The product of two consecutive even integers is 168.

81. Twelve percent of Harriet's income is \$3000.

82. If 9% of the members buy tickets, then we will sell 252 tickets to this group.

83. Thirteen is 5% of what number?

84. Three hundred is 8% of what number?

85. The length of a rectangle is 5 feet longer than the width, and the area is 126 square feet.

86. The length of a rectangle is 1 yard shorter than twice the width, and the perimeter is 298 yards.

87. The value of n nickels and $n - 1$ dimes is 95 cents.

88. The value of q quarters, $q + 1$ dimes, and $2q$ nickels is 90 cents.

89. The measure of an angle is 38° smaller than the measure of its supplementary angle.

90. The measure of an angle is 16° larger than the measure of its complementary angle.

91. ***Target heart rate.*** For a cardiovascular work out, fitness experts recommend that you reach your target heart rate and stay at that rate for at least 20 minutes (*Cycling,* Burkett and Darst). To find your target heart rate, find the sum of your age and your resting heart rate, then subtract that sum from 220. Find 60% of that result and add it to your resting heart rate.

a) Write an equation with variable r expressing the fact that the target heart rate for 30-year-old Bob is 144.

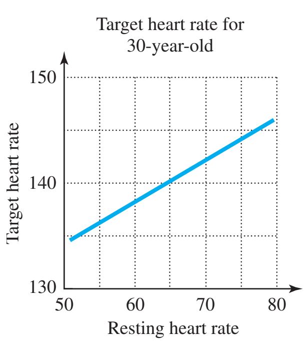


FIGURE FOR EXERCISE 91

b) Judging from the accompanying graph, does the target heart rate for a 30-year-old increase or decrease as the resting heart rate increases?

92. ***Adjusting the saddle.*** The saddle height on a bicycle should be 109% of the rider's inside leg measurement L (*Cycling,* Burkett and Darst). See the figure. Write an equation expressing the fact that the saddle height for Brenda is 36 in.

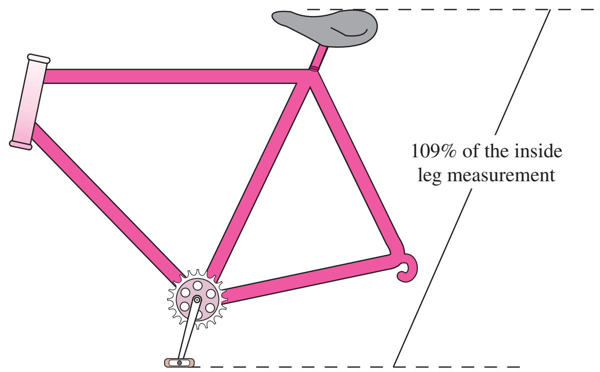
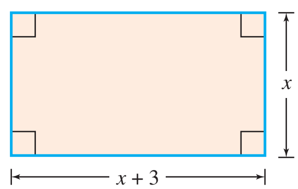


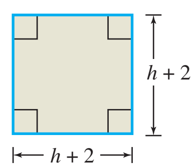
FIGURE FOR EXERCISE 92

Given that the area of each figure is 24 square feet, use the dimensions shown to write an equation expressing this fact. Do not solve the equation.

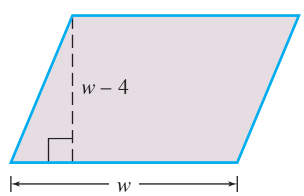
93.



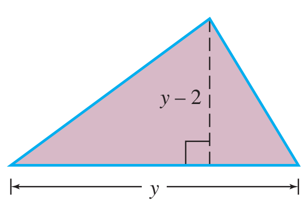
94.



95.



96.



2.6 NUMBER, GEOMETRIC, AND UNIFORM MOTION APPLICATIONS

In this section

- Number Problems
- General Strategy for Solving Verbal Problems
- Geometric Problems
- Uniform Motion Problems

In this section, we apply the ideas of Section 2.5 to solving problems. Many of the problems can be solved by using arithmetic only and not algebra. However, remember that we are not just trying to find the answer, we are trying to learn how to apply algebra. So even if the answer is obvious to you, set the problem up and solve it by using algebra as shown in the examples.

Number Problems

Algebra is often applied to problems involving time, rate, distance, interest, or discount. **Number problems** do not involve any physical situation. In number problems we simply find some numbers that satisfy some given conditions. Number problems can provide good practice for solving more complex problems.

EXAMPLE 1 **A consecutive integer problem**

The sum of three consecutive integers is 48. Find the integers.

Solution

We first represent the unknown quantities with variables. Let x, $x + 1$, and $x + 2$ represent the three consecutive integers. We now write an equation that describes the problem and solve it. The equation expresses the fact that the sum of the integers is 48.

$$x + (x + 1) + (x + 2) = 48$$

$$3x + 3 = 48 \quad \text{Combine like terms.}$$

$$3x = 45 \quad \text{Subtract 3 from each side.}$$

$$x = 15 \quad \text{Divide each side by 3.}$$

$$x + 1 = 16 \quad \text{If } x \text{ is 15, then } x + 1 \text{ is 16 and } x + 2 \text{ is 17.}$$

$$x + 2 = 17$$

Because $15 + 16 + 17 = 48$, the three consecutive integers that have a sum of 48 are 15, 16, and 17. ■

helpful hint

Making a guess can be a good way to get familiar with the problem. For example, let's guess that the answers to Example 1 are 20, 21, and 22. Since $20 + 21 + 22 = 63$, these are not the correct numbers. But now we realize that we should use x, $x + 1$, and $x + 2$ and that the equation should be

$$x + x + 1 + x + 2 = 48.$$

General Strategy for Solving Verbal Problems

You should use the following steps as a guide for solving problems.

Strategy for Solving Problems

1. Read the problem as many times as necessary. Guessing the answer and checking it will help you understand the problem.
2. If possible, draw a diagram to illustrate the problem.
3. Choose a variable and *write* what it represents.
4. Write algebraic expressions for any other unknowns in terms of that variable.
5. Write an equation that describes the situation.
6. Solve the equation.
7. Answer the original question.
8. Check your answer in the original problem (not the equation).

Geometric Problems

Geometric problems involve geometric figures. For these problems you should always draw the figure and label it.

EXAMPLE 2

A perimeter problem

The length of a rectangular piece of property is 1 foot less than twice the width. If the perimeter is 748 feet, find the length and width.

helpful hint

To get familiar with the problem, guess that the width is 50 ft. Then the length is $2 \cdot 50 - 1$ or 99. The perimeter would be

$$2(50) + 2(99) = 298,$$

which is too small. But now we realize that we should let x be the width, $2x - 1$ be the length, and we should solve

$$2x + 2(2x - 1) = 748.$$

Solution

Let $x =$ the width. Since the length is 1 foot less than twice the width, $2x - 1 =$ the length. Draw a diagram as in Fig. 2.2. We know that $2L + 2W = P$ is the formula for perimeter of a rectangle. Substituting $2x - 1$ for L and x for W in this formula yields an equation in x:

$$2L + 2W = P$$
$$2(2x - 1) + 2(x) = 748 \quad \text{Replace } L \text{ by } 2x - 1 \text{ and } W \text{ by } x.$$
$$4x - 2 + 2x = 748 \quad \text{Remove the parentheses.}$$
$$6x - 2 = 748 \quad \text{Combine like terms.}$$
$$6x = 750 \quad \text{Add 2 to each side.}$$
$$x = 125 \quad \text{Divide each side by 6.}$$
$$2x - 1 = 249 \quad \text{If } x = 125\text{, then } 2x - 1 = 2(125) - 1 = 249.$$

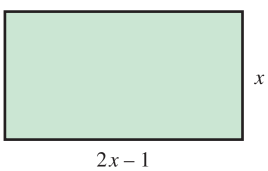


FIGURE 2.2

Check these answers by computing $2L + 2W$:

$$2(249) + 2(125) = 748$$

So the width is 125 feet, and the length is 249 feet. ■

The next geometric example involves the degree measures of angles. For this problem, the figure is given.

EXAMPLE 3

Complementary angles

In Fig. 2.3 the angle formed by the guy wire and the ground is 3.5 times as large as the angle formed by the guy wire and the antenna. Find the degree measure of each of these angles.

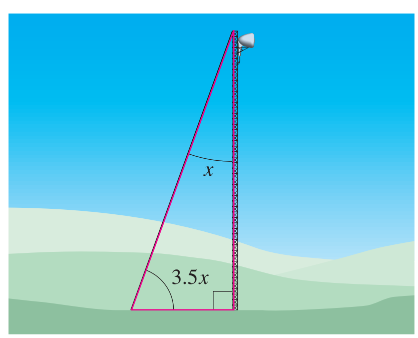


FIGURE 2.3

Solution

Let $x =$ the degree measure of the smaller angle, and let $3.5x =$ the degree measure of the larger angle. Since the antenna meets the ground at a 90° angle, the sum of the degree measures of the other two angles of the right triangle is 90°. (They are complementary angles.) So we have the following equation:

$$x + 3.5x = 90$$
$$4.5x = 90 \quad \text{Combine like terms.}$$
$$x = 20 \quad \text{Divide each side by 4.5.}$$
$$3.5x = 70 \quad \text{Find the other angle.}$$

Check: 70° is 3.5 · 20° and 20° + 70° = 90°. So the smaller angle is 20°, and the larger angle is 70°.

Uniform Motion Problems

Problems involving motion at a constant rate are called **uniform motion problems.** In uniform motion problems we often use an average rate when the actual rate is not constant. For example, you can drive all day and average 50 miles per hour, but you are not driving at a constant 50 miles per hour.

EXAMPLE 4 **Finding the rate**

Bridgette drove her car for 2 hours on an icy road. When the road cleared up, she increased her speed by 35 miles per hour and drove 3 more hours, completing her 255-mile trip. How fast did she travel on the icy road?

Solution

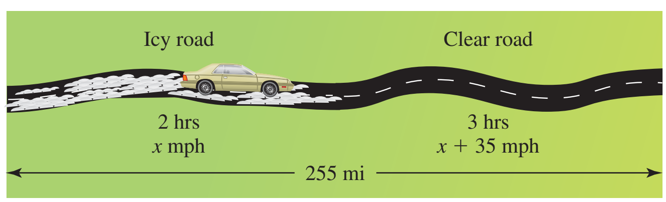
It is helpful to draw a diagram and then make a table to classify the given information. Remember that $D = RT$.

helpful hint

To get familiar with the problem, guess that she traveled 20 mph on the icy road and 55 mph (20 + 35) on the clear road. Her total distance would be

$$20 \cdot 2 + 55 \cdot 3 = 205 \text{ mi.}$$

Of course this is not correct, but now you are familiar with the problem.



	Rate	Time	Distance
Icy road	$x \frac{\text{mi}}{\text{hr}}$	2 hr	$2x$ mi
Clear road	$x + 35 \frac{\text{mi}}{\text{hr}}$	3 hr	$3(x + 35)$ mi

The equation expresses the fact that her total distance traveled was 255 miles:

$$\text{Icy road distance} + \text{clear road distance} = \text{total distance}$$

$$2x + 3(x + 35) = 255$$
$$2x + 3x + 105 = 255$$
$$5x + 105 = 255$$
$$5x = 150$$
$$x = 30$$
$$x + 35 = 65$$

If she drove at 30 miles per hour for 2 hours on the icy road, she went 60 miles. If she drove at 65 miles per hour for 3 hours on the clear road, she went 195 miles. Since $60 + 195 = 255$, we can be sure that her speed on the icy road was 30 mph. ■

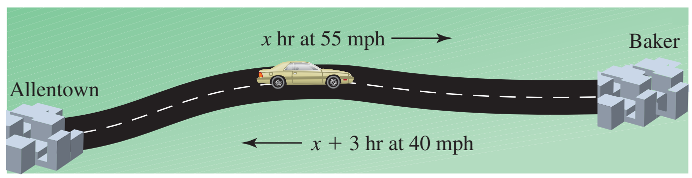
In the next uniform motion problem we find the time.

EXAMPLE 5 **Finding the time**

Pierce drove from Allentown to Baker, averaging 55 miles per hour. His journey back to Allentown using the same route took 3 hours longer because he averaged only 40 miles per hour. How long did it take him to drive from Allentown to Baker? What is the distance between Allentown and Baker?

Solution

Draw a diagram and then make a table to classify the given information. Remember that $D = RT$.



	Rate	Time	Distance
Going	$55 \frac{\text{mi}}{\text{hr}}$	x hr	$55x$ mi
Returning	$40 \frac{\text{mi}}{\text{hr}}$	$x + 3$ hr	$40(x + 3)$ mi

We can write an equation expressing the fact that the distance either way is the same:

$$\text{Distance going} = \text{distance returning}$$
$$55x = 40(x + 3)$$
$$55x = 40x + 120$$
$$15x = 120$$
$$x = 8$$

The trip from Allentown to Baker took 8 hours. The distance between Allentown and Baker is $55 \cdot 8$, or 440 miles. ■

study tip

When taking a test, put a check mark beside every question that you have answered and checked. When you have finished the test, then you can go back and spend the remaining time on the problems that are not yet checked. You won't waste time reworking problems that you know are correct.

WARM-UPS

True or false? Explain your answer.

1. The first step in solving a word problem is to write the equation.
2. You should always write down what the variable represents.
3. Diagrams and tables are used as aids in solving problems.
4. To represent two consecutive odd integers, we use x and $x + 1$.
5. If $5x$ is 2 miles more than $3(x + 20)$, then $5x + 2 = 3(x + 20)$.
6. We can represent two numbers with a sum of 6 by x and $6 - x$.
7. Two numbers that differ by 7 can be represented by x and $x + 7$.

WARM-UPS

(continued)

8. The degree measures of two complementary angles can be represented by x and $90 - x$.

9. The degree measures of two supplementary angles can be represented by x and $x + 180$.

10. If x is half as large as $x + 50$, then $2x = x + 50$.

2.6 EXERCISES

Reading and Writing *After reading this section, write out the answers to these questions. Use complete sentences.*

1. What types of problems are discussed in this section?

2. Why do we solve number problems?

3. What is uniform motion?

4. What are supplementary angles?

5. What are complementary angles?

6. What should you always do when solving a geometric problem?

Show a complete solution to each problem. See Example 1.

7. ***Consecutive integers.*** Find three consecutive integers whose sum is 141.

8. ***Consecutive even integers.*** Find three consecutive even integers whose sum is 114.

9. ***Consecutive odd integers.*** Two consecutive odd integers have a sum of 152. What are the integers?

10. ***Consecutive odd integers.*** Four consecutive odd integers have a sum of 120. What are the integers?

11. ***Consecutive integers.*** Find four consecutive integers whose sum is 194.

12. ***Consecutive even integers.*** Find four consecutive even integers whose sum is 340.

Show a complete solution to each problem. See Examples 2 and 3.

13. ***Olympic swimming.*** If an Olympic swimming pool is twice as long as it is wide and the perimeter is 150 meters, then what are the length and width?

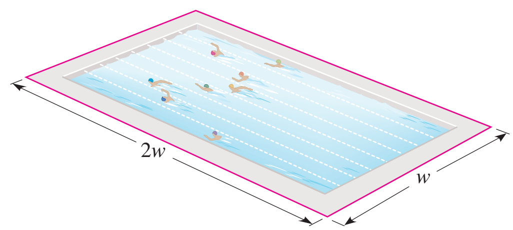


FIGURE FOR EXERCISE 13

study tip

Don't spend too much time on a single problem. If you get stuck on a problem, look at some examples in the text, move on to the next problem, or get help. It is often helpful to work some other problems and then come back to that one pesky problem.

14. ***Wimbledon tennis.*** If the perimeter of a tennis court is 228 feet and the length is 6 feet longer than twice the width, then what are the length and width?

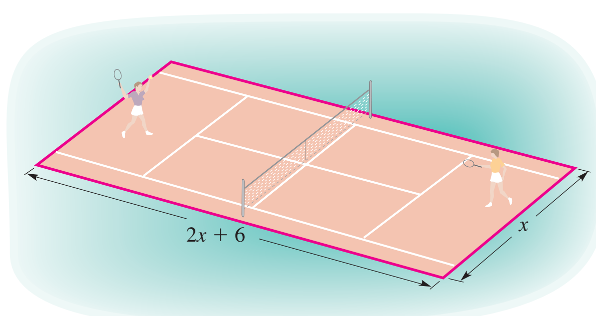


FIGURE FOR EXERCISE 14

15. ***Framed.*** Julia framed an oil painting that her uncle gave her. The painting was 4 inches longer than it was wide, and it took 176 inches of frame molding. What were the dimensions of the picture?

16. ***Industrial triangle.*** Geraldo drove his truck from Indianapolis to Chicago, then to St. Louis, and then back to Indianapolis. He observed that the second side of his triangular route was 81 miles short of being twice as long as the first side and that the third side was 61 miles longer than the first side. If he traveled a total of 720 miles, then how long is each side of this triangular route?

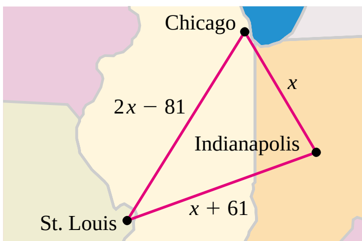


FIGURE FOR EXERCISE 16

17. ***Triangular banner.*** A banner in the shape of an isosceles triangle has a base that is 5 inches shorter than either of the equal sides. If the perimeter of the banner is 34 inches, then what is the length of the equal sides?

18. ***Border paper.*** Dr. Good's waiting room is 8 feet longer than it is wide. When Vincent wallpapered Dr. Good's waiting room, he used 88 feet of border paper. What are the dimensions of Dr. Good's waiting room?

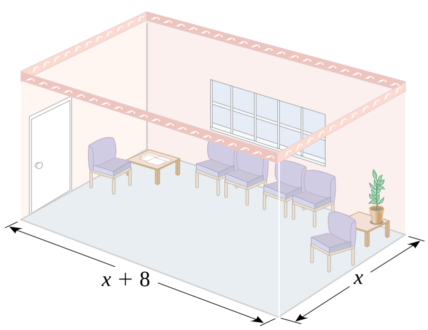


FIGURE FOR EXERCISE 18

19. ***Ramping up.*** A civil engineer is planning a highway overpass as shown in the figure. Find the degree measure of the angle marked *w*.

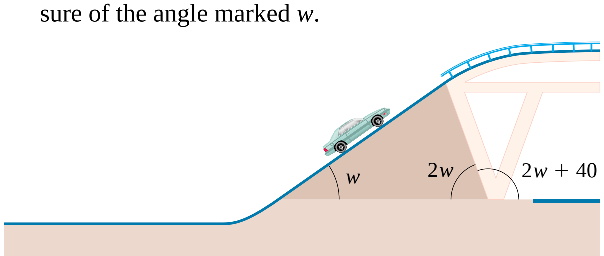


FIGURE FOR EXERCISE 19

20. ***Ramping down.*** For the other side of the overpass, the engineer has drawn the plans shown in the figure. Find the degree measure of the angle marked *z*.

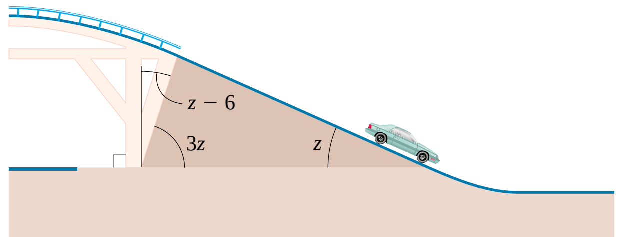


FIGURE FOR EXERCISE 20

Show a complete solution to each problem. See Examples 4 and 5.

21. ***Highway miles.*** Bret drove for 4 hours on the freeway, then decreased his speed by 20 miles per hour and drove for 5 more hours on a country road. If his total trip was 485 miles, then what was his speed on the freeway?

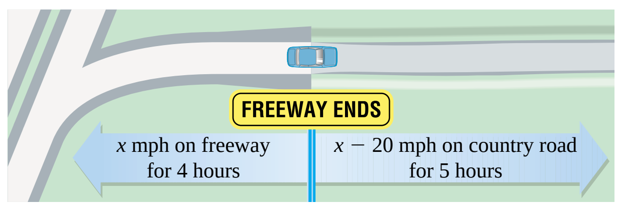


FIGURE FOR EXERCISE 21

22. ***Walking and running.*** On Saturday morning, Lynn walked for 2 hours and then ran for 30 minutes. If she ran twice as fast as she walked and she covered 12 miles altogether, then how fast did she walk?

23. ***Driving all night.*** Kathryn drove her rig 5 hours before dawn and 6 hours after dawn. If her average speed was 5 miles per hour more in the dark and she covered 630 miles altogether, then what was her speed after dawn?

24. ***Commuting to work.*** On Monday, Roger drove to work in 45 minutes. On Tuesday he averaged 12 miles per hour more, and it took him 9 minutes less to get to work. How far does he travel to work?

25. ***Head winds.*** A jet flew at an average speed of 640 mph from Los Angeles to Chicago. Because of head winds the jet averaged only 512 mph on the return trip, and the return trip took 48 minutes longer. How many hours was the flight from Chicago to Los Angeles? How far is it from Chicago to Los Angeles?

26. ***Ride the Peaks.*** Penny's bicycle trip from Colorado Springs to Pikes Peak took 1.5 hours longer than the return trip to Colorado Springs. If she averaged 6 mph on the way to Pikes Peak and 15 mph for the return trip, then how long was the ride from Colorado Springs to Pikes Peak?

Solve each problem.

27. ***Super Bowl score.*** The 1977 Super Bowl was played in the Rose Bowl in Pasadena. In that football game the Oakland Raiders scored 18 more points than the Minnesota Vikings. If the total number of points scored was 46, then what was the final score for the game?

28. ***Top three companies.*** Revenues for the top three companies in 1997, General Motors, Ford, and Exxon, totaled \$453 billion (Fortune 500 List, www.fortune.com). If Ford's revenue was \$31 billion greater that Exxon's, and General Motor's revenue was \$25 billion greater than Ford's, then what was the 1997 revenue for each company?

29. ***Idabel to Lawton.*** Before lunch, Sally drove from Idabel to Ardmore, averaging 50 mph. After lunch she continued on to Lawton, averaging 53 mph. If her driving time after lunch was 1 hour less than her driving time before lunch and the total trip was 256 miles, then how many hours did she drive before lunch? How far is it from Ardmore to Lawton?

30. ***Norfolk to Chadron.*** On Monday, Chuck drove from Norfolk to Valentine, averaging 47 mph. On Tuesday, he continued on to Chadron, averaging 69 mph. His driving time on Monday was 2 hours longer than his driving time on Tuesday. If the total distance from Norfolk to Chadron is 326 miles, then how many hours did he drive on Monday? How far is it from Valentine to Chadron?

31. ***Golden oldies.*** Joan Crawford, John Wayne, and James Stewart were born in consecutive years (*Doubleday Almanac*). Joan Crawford was the oldest of the three, and James Stewart was the youngest. In 1950, after all three had their birthdays, the sum of their ages was 129. In what years were they born?

32. ***Leading men.*** Bob Hope was born 2 years after Clark Gable and 2 years before Henry Fonda (*Doubleday Almanac*). In 1951, after all three of them had their birthdays, the sum of their ages was 144. In what years were they born?

33. ***Trimming a garage door.*** A carpenter used 30 ft of molding in three pieces to trim a garage door. If the long piece was 2 ft longer than twice the length of each shorter piece, then how long was each piece?

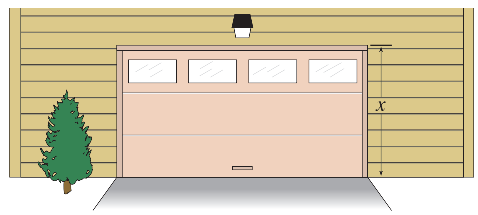


FIGURE FOR EXERCISE 33

34. ***Fencing dog pens.*** Clint is constructing two adjacent rectangular dog pens. Each pen will be three times as long as it is wide, and the pens will share a common long side. If Clint has 65 ft of fencing, what are the dimensions of each pen?

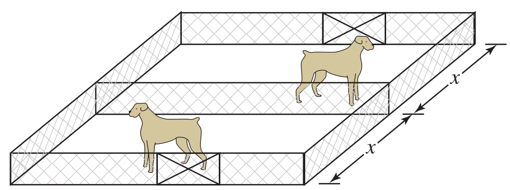


FIGURE FOR EXERCISE 34

2.7 DISCOUNT, INVESTMENT, AND MIXTURE APPLICATIONS

In this section

- Discount Problems
- Commission Problems
- Investment Problems
- Mixture Problems

In this section, we continue our study of applications of algebra. The problems in this section involve percents.

Discount Problems

When an item is sold at a discount, the amount of the discount is usually described as being a percentage of the original price. The percentage is called the **rate of discount.** Multiplying the rate of discount and the original price gives the amount of the discount.

EXAMPLE 1 **Finding the original price**

Ralph got a 12% discount when he bought his new 1999 Corvette Coupe. If the amount of his discount was \$4584, then what was the original price of the Corvette?

Solution

Let x represent the original price. The discount is found by multiplying the 12% rate of discount and the original price:

$$\begin{aligned} \text{rate of discount} \cdot \text{original price} &= \text{amount of discount} \\ 0.12x &= 4584 \\ x &= \frac{4584}{0.12} \quad \text{Divide each side by 0.12.} \\ x &= 38{,}200 \end{aligned}$$

To check, find 12% of \$38,200. Since $0.12 \cdot 38{,}200 = 4584$, the original price of the Corvette was \$38,200.

EXAMPLE 2 **Finding the original price**

When Susan bought her new car, she also got a discount of 12%. She paid \$17,600 for her car. What was the original price of Susan's car?

Solution

helpful hint

To get familiar with the problem, guess that the original price was \$30,000. Then her discount is 0.12(30,000) or \$3600. The price she paid would be 30,000 − 3600 or \$26,400, which is incorrect.

Let x represent the original price for Susan's car. The amount of discount is 12% of x, or $0.12x$. We can write an equation expressing the fact that the original price minus the discount is the price Susan paid.

$$\begin{aligned} \text{Original price} - \text{discount} &= \text{sale price} \\ x - 0.12x &= 17{,}600 \\ 0.88x &= 17{,}600 \quad 1.00x - 0.12x = 0.88x \\ x &= \frac{17{,}600}{0.88} \quad \text{Divide each side by 0.88.} \\ x &= \$20{,}000 \end{aligned}$$

Check: 12% of \$20,000 is \$2400, and \$20,000 − \$2400 = \$17,600. The original price of Susan's car was \$20,000.

Commission Problems

A salesperson's commission for making a sale is often a percentage of the selling price. **Commission problems** are very similar to other problems involving percents. The commission is found by multiplying the rate of commission and the selling price.

EXAMPLE 3 **Real estate commission**

Sarah is selling her house through a real estate agent whose commission rate is 7%. What should the selling price be so that Sarah can get the \$83,700 she needs to pay off the mortgage?

Solution

Let x be the selling price. The commission is 7% of x (not 7% of \$83,700). Sarah receives the selling price less the sales commission:

$$\begin{aligned} \text{Selling price} - \text{commission} &= \text{Sarah's share} \\ x - 0.07x &= 83{,}700 \\ 0.93x &= 83{,}700 \quad 1.00x - 0.07x = 0.93x \\ x &= \frac{83{,}700}{0.93} \\ x &= 90{,}000 \end{aligned}$$

Check: 7% of \$90,000 is \$6300, and \$90,000 − \$6300 = \$83,700. So the house should sell for \$90,000.

Investment Problems

The interest on an investment is a percentage of the investment, just as the sales commission is a percentage of the sale amount. However, in **investment problems** we must often account for more than one investment at different rates. So it is a good idea to make a table, as in the next example.

EXAMPLE 4

Diversified investing

Ruth Ann invested some money in a certificate of deposit with an annual yield of 9%. She invested twice as much in a mutual fund with an annual yield of 10%. Her interest from the two investments at the end of the year was \$232. How much was invested at each rate?

helpful hint

To get familiar with the problem, guess that she invested \$1000 at 9% and \$2000 at 10%. Then her interest in one year would be

$$0.09(1000) + 0.10(2000)$$

or \$290, which is close but incorrect.

Solution

When there are many unknown quantities, it is often helpful to identify them in a table. Since the time is 1 year, the amount of interest is the product of the interest rate and the amount invested.

	Interest rate	Amount invested	Interest for 1 year
CD	9%	x	$0.09x$
Mutual fund	10%	$2x$	$0.10(2x)$

Since the total interest from the investments was \$232, we can write the following equation:

$$\text{CD interest} + \text{mutual fund interest} = \text{total interest}$$
$$0.09x + 0.10(2x) = 232$$
$$0.09x + 0.20x = 232$$
$$0.29x = 232$$
$$x = \frac{232}{0.29}$$
$$x = \$800$$
$$2x = \$1600$$

To check, we find the total interest:

$$0.09(800) + 0.10(1600) = 72 + 160$$
$$= 232$$

So Ruth Ann invested \$800 at 9% and \$1600 at 10%.

study tip

Finding out what happened in class and attending class are not the same. Attend every class and be attentive. Don't just take notes and let your mind wander. Use class time as a learning time.

Mixture Problems

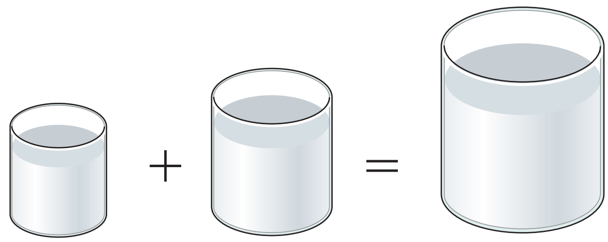
Mixture problems are concerned with the result of mixing two quantities, each of which contains another substance. Notice how similar the following mixture problem is to the last investment problem.

EXAMPLE 5 **Mixing milk**

How many gallons of milk containing 4% butterfat must be mixed with 80 gallons of 1% milk to obtain 2% milk?

Solution

It is helpful to draw a diagram and then make a table to classify the given information.



	Percentage of fat	Amount of milk	Amount of fat
4% milk	4%	x	$0.04x$
1% milk	1%	80	$0.01(80)$
2% milk	2%	$x + 80$	$0.02(x + 80)$

The equation expresses the fact that the total fat from the first two types of milk is the same as the fat in the mixture:

Fat in 4% milk + fat in 1% milk = fat in 2% milk

$$0.04x + 0.01(80) = 0.02(x + 80)$$
$$0.04x + 0.8 = 0.02x + 1.6 \quad \text{Simplify.}$$
$$100(0.04x + 0.8) = 100(0.02x + 1.6) \quad \text{Multiply each side by 100.}$$
$$4x + 80 = 2x + 160 \quad \text{Distributive property.}$$
$$2x + 80 = 160 \quad \text{Subtract } 2x \text{ from each side.}$$
$$2x = 80 \quad \text{Subtract 80 from each side.}$$
$$x = 40 \quad \text{Divide each side by 2.}$$

To check, calculate the total fat:

$$2\% \text{ of 120 gallons} = 0.02(120) = 2.4 \text{ gallons of fat}$$
$$0.04(40) + 0.01(80) = 1.6 + 0.8 = 2.4 \text{ gallons of fat}$$

So we mix 40 gallons of 4% milk with 80 gallons of 1% milk to get 120 gallons of 2% milk. ■

helpful hint

To get familiar with the problem, guess that we need 100 gal of 4% milk. Mixing that with 80 gal of 1% milk would produce 180 gal of 2% milk. Now the two milks separately have

$$0.04(100) + 0.01(80)$$

or 4.8 gal of fat. Together the amount of fat is 0.02(180) or 3.6 gal. Since these amounts are not equal, our guess is incorrect.

In mixture problems, the solutions might contain fat, alcohol, salt, or some other substance. We always assume that the substance neither appears nor disappears in the process. For example, if there are 3 grams of salt in one glass of water and 2 grams in another, then there are exactly 5 grams in a mixture of the two.

study tip

Don't expect to understand a new topic the first time that you see it. Learning mathematics takes time, patience, and repetition. Keep reading the text, asking questions, and working problems. Someone once said, "All mathematics is easy once you understand it."

WARM-UPS

True or false? Explain your answer.

1. If the original price is w and the discount is 8%, then the selling price is $w - 0.08w$.
2. If x is the selling price and the commission is 8% of the selling price, then the commission is $0.08x$.
3. If you need \$40,000 for your house and the agent gets 10% of the selling price, then the agent gets \$4000, and the house sells for \$44,000.
4. If you mix 10 liters of a 20% acid solution with x liters of a 30% acid solution, then the total amount of acid is $2 + 0.3x$ liters.
5. A 10% acid solution mixed with a 14% acid solution results in a 24% acid solution.
6. If a TV costs x dollars and sales tax is 5%, then the total bill is $1.05x$ dollars.

2.7 EXERCISES

Reading and Writing *After reading this section, write out the answers to these questions. Use complete sentences.*

1. What types of problems are discussed in this section?
2. What is the difference between discount and rate of discount?
3. What is the relationship between discount, original price, rate of discount, and sale price?
4. What do mixture problems and investment problems have in common?
5. Why do we make a table when solving certain problems.
6. What is the relationship between amount of interest, amount invested, and interest rate?

Show a complete solution to each problem. See Examples 1 and 2.

7. ***Close-out sale.*** At a 25% off sale, Jose saved \$80 on a 19-inch Panasonic TV. What was the original price of the television.
8. ***Big bike.*** A 12% discount on a Giant Perigee saved Melanie \$46.68. What was the original price of the bike?
9. ***Circuit city.*** After getting a 20% discount, Robert paid \$320 for a Pioneer CD player for his car. What was the original price of the CD player?
10. ***Chrysler Sebring.*** After getting a 15% discount on the price of a new Chrysler Sebring convertible, Helen paid \$27,000. What was the original price of the convertible?

Show a complete solution to each problem. See Example 3.

11. ***Selling price of a home.*** Kirk wants to get \$72,000 for his house. The real estate agent gets a commission equal to 10% of the selling price for selling the house. What should the selling price be?

FIGURE FOR EXERCISE 11

12. ***Horse trading.*** Gene is selling his palomino at an auction. The auctioneer's commission is 10% of the selling price. If Gene still owes \$810 on the horse, then what must the horse sell for so that Gene can pay off his loan?

13. ***Sales tax collection.*** Merilee sells tomatoes at a roadside stand. Her total receipts including the 7% sales tax were \$462.24. What amount of sales tax did she collect?

14. ***Toyota Corolla.*** Gwen bought a new Toyota Corolla. The selling price plus the 8% state sales tax was \$15,714. What was the selling price?

Show a complete solution to each problem. See Example 4.

15. ***Wise investments.*** Wiley invested some money in the Berger 100 Fund and \$3000 more than that amount in the Berger 101 Fund. For the year he was in the fund, the 100 Fund paid 18% simple interest and the 101 Fund paid 15% simple interest. If the income from the two investments totaled \$3750 for one year, then how much did he invest in each fund?

16. ***Loan shark.*** Becky lent her brother some money at 8% simple interest, and she lent her sister twice as much at twice the interest rate. If she received a total of 20 cents interest, then how much did she lend to each of them?

17. ***Investing in bonds.*** David split his \$25,000 inheritance between Fidelity Short-Term Bond Fund with an annual yield of 5% and T. Rowe Price Tax-Free Short-Intermediate Fund with an annual yield of 4%. If his total income for one year on the two investments was \$1140, then how much did he invest in each fund?

18. ***High-risk funds.*** Of the \$50,000 that Natasha pocketed on her last real estate deal, \$20,000 went to charity. She invested part of the remainder in Dreyfus New Leaders Fund with an annual yield of 16% and the rest in Templeton Growth Fund with an annual yield of 25%. If she made \$6060 on these investments in one year, then how much did she invest in each fund?

Show a complete solution to each problem. See Example 5.

19. ***Mixing milk.*** How many gallons of milk containing 1% butterfat must be mixed with 30 gallons of milk containing 3% butterfat to obtain a mixture containing 2% butterfat?

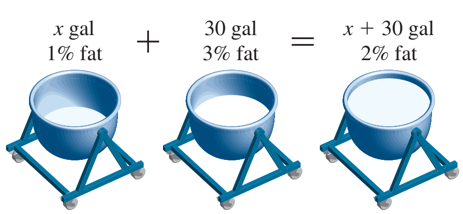


FIGURE FOR EXERCISE 19

20. ***Acid solutions.*** How many gallons of a 5% acid solution should be mixed with 30 gallons of a 10% acid solution to obtain a mixture that is 8% acid?

21. ***Alcohol solutions.*** Gus has on hand a 5% alcohol solution and a 20% alcohol solution. He needs 30 liters of a 10% alcohol solution. How many liters of each solution should he mix together to obtain the 30 liters?

22. ***Adjusting antifreeze.*** Angela needs 20 quarts of 50% antifreeze solution in her radiator. She plans to obtain this by mixing some pure antifreeze with an appropriate amount of a 40% antifreeze solution. How many quarts of each should she use?

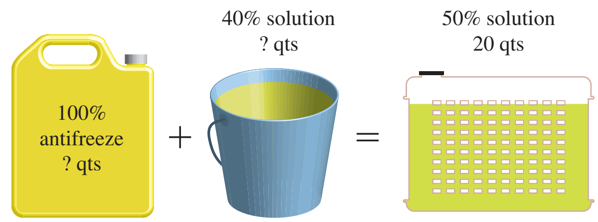


FIGURE FOR EXERCISE 22

Solve each problem.

23. ***Registered voters.*** If 60% of the registered voters of Lancaster County voted in the November election and 33,420 votes were cast, then how many registered voters are there in Lancaster County?

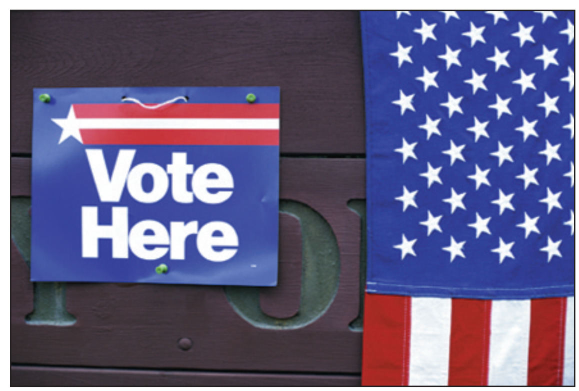


FIGURE FOR EXERCISE 23

24. ***Tough on crime.*** In a random sample of voters, 594 respondents said that they favored passage of a \$33 billion crime bill. If the number in favor of the crime bill was 45% of the number of voters in the sample, then how many voters were in the sample?

25. ***Ford Taurus.*** At an 8% sales tax rate, the sales tax on Peter's new Ford Taurus was \$1200. What was the price of the car?

26. ***Taxpayer blues.*** Last year, Faye paid 24% of her income to taxes. If she paid \$9600 in taxes, then what was her income?

27. ***Making a profit.*** A retail store buys shirts for \$8 and sells them for \$14. What percent increase is this?

28. ***Monitoring AIDS.*** If 28 new AIDS cases were reported in Landon County this year and 35 new cases were reported last year, then what percent decrease in new cases is this?

29. ***High school integration.*** Wilson High School has 400 students, of whom 20% are African American. The school board plans to merge Wilson High with Jefferson High. This one school will then have a student population that is 44% African American. If Jefferson currently has a student population that is 60% African American, then how many students are at Jefferson?

30. ***Junior high integration.*** The school board plans to merge two junior high schools into one school of 800 students in which 40% of the students will be Caucasian. One of the schools currently has 58% Caucasian students; the other has only 10% Caucasian students. How many students are in each of the two schools?

31. ***Hospital capacity.*** When Memorial Hospital is filled to capacity, it has 18 more people in semiprivate rooms (two patients to a room) than in private rooms. The room rates are $200 per day for a private room and $150 per day for a semiprivate room. If the total receipts for rooms is $17,400 per day when all are full, then how many rooms of each type does the hospital have?

32. ***Public relations.*** Memorial Hospital is planning an advertising campaign. It costs the hospital $3000 each time a television ad is aired and $2000 each time a radio ad is aired. The administrator wants to air 60 more television ads than radio ads. If the total cost of airing the ads is $580,000, then how many ads of each type will be aired?

33. ***Mixed nuts.*** Cashews sell for $4.80 per pound, and pistachios sell for $6.40 per pound. How many pounds of pistachios should be mixed with 20 pounds of cashews to get a mixture that sells for $5.40 per pound?

34. ***Premium blend.*** Premium coffee sells for $6.00 per pound, and regular coffee sells for $4.00 per pound. How many pounds of each type of coffee should be blended to obtain 100 pounds of a blend that sells for $4.64 per pound?

35. ***Nickels and dimes.*** Candice paid her library fine with 10 coins consisting of nickels and dimes. If the fine was $0.80, then how many of each type of coin did she use?

36. ***Dimes and quarters.*** Jeremy paid for his breakfast with 36 coins consisting of dimes and quarters. If the bill was $4.50, then how many of each type of coin did he use?

37. ***Cooking oil.*** Crisco Canola Oil is 7% saturated fat. Crisco blends corn oil that is 14% saturated fat with Crisco Canola Oil to get Crisco Canola and Corn Oil, which is 11% saturated fat. How many gallons of corn oil must Crisco mix with 600 gallons of Crisco Canola Oil to get Crisco Canola and Corn Oil?

38. ***Chocolate ripple.*** The Delicious Chocolate Shop makes a dark chocolate that is 35% fat and a white chocolate that is 48% fat. How many kilograms of dark chocolate should be mixed with 50 kilograms of white chocolate to make a ripple blend that is 40% fat?

39. ***Hawaiian Punch.*** Hawaiian Punch is 10% fruit juice. How much water would you have to add to one gallon of Hawaiian Punch to get a drink that is 6% fruit juice?

40. ***VCRs and CDs.*** The manager of a stereo shop placed an order for $10,710 worth of VCRs at $120 each and CD players at $150 each. If the number of VCRs she ordered was three times the number of CD players, then how many of each did she order?

COLLABORATIVE ACTIVITIES

Finding the Better Deal?

For this activity, the students in your group should choose roles. Four standard roles are Moderator (keeps the group on task), Messenger (asks the group's questions to the instructor, tutor, or helper), Quality Manager (checks to see that the work is top quality), and Recorder (records the group's work). See the Instructor's Solution Manual for a description of these roles. After you have chosen roles, read through the activity completely, and answer the questions.

Scenario: You have decided to buy a new car and have asked some friends to help you choose the best deal and the best financing. You have already looked into your finances. You have $1700 from your summer job and $1500 that your parents will give you for a down payment on a car.

Grouping: 2 to 4 students per group

Topic: Percents

You found a car that you really liked that was 10% off the regular $9800 price. Your friends at the student Credit Union tell you it has a 48-month car loan at $7\frac{1}{2}$% annual simple interest.

At a second dealership you find a similar car on sale for $9000 if you finance it through the dealership. The dealer said that after the down payment you could pay it off in 5 years with monthly payments of $140. This second deal sounds good! (You have decided you could afford up to $160 a month in payments.) The idea of having an extra $20 a month is appealing. However, you wonder how much you will actually pay for the second car.

Which car should you buy?

Questions: The following questions will help you to work your way through the problem.

For the first car, if you finance it at the Credit Union:

1. How much will it cost after the discount?
2. How much will you need to borrow?
3. What will the total interest be for the 48 months?
4. How much will the monthly payments be?

For the second car:

5. Find the amount to be financed.
6. Find the interest and the interest rate.

For both cars:

7. Find the total cost for each car.

After reviewing the information, decide which car you would buy.

Extension:

1. What other costs would there be? Find these out for your city.
2. Do a comparison of car loans at your local banks.

WRAP-UP CHAPTER 2

SUMMARY

Equations		**Examples**
Linear equation	An equation of the form $ax + b = 0$ with $a \neq 0$	$3x + 7 = 0$
Identity	An equation that is satisfied by every number for which both sides are defined	$x + x = 2x$
Conditional equation	An equation that has at least one solution but is not an identity	$5x - 10 = 0$
Inconsistent equation	An equation that has no solution	$x = x + 1$
Equivalent equations	Equations that have exactly the same solutions	$2x + 1 = 5$ $2x = 4$
Properties of equality	If the same number is added to or subtracted from each side of an equation, the resulting equation is equivalent to the original equation.	$x - 5 = -9$ $x = -4$
	If each side of an equation is multiplied or divided by the same nonzero number, the resulting equation is equivalent to the original equation.	$9x = 27$ $x = 3$
Solving equations	**1.** Remove parentheses and combine like terms to simplify each side as much as possible. **2.** Get like terms from opposite sides onto the same side so that they may be combined. **3.** Use the multiplication-division property of equality last to isolate the variable. **4.** The equation $-x = a$ is equivalent to $x = -a$. **5.** Check that the solution satisfies the original equation.	$2(x - 3) = -7 + 3(x - 1)$ $2x - 6 = -10 + 3x$ $-x - 6 = -10$ $-x = -4$ $x = 4$ *Check:* $2(4 - 3) = -7 + 3(4 - 1)$ $2 = 2$

Applications

Steps in solving applied problems

1. Read the problem.
2. If possible, draw a diagram to illustrate the problem.
3. Choose a variable and write down what it represents.
4. Represent any other unknowns in terms of that variable.
5. Write an equation that describes the situation.
6. Solve the equation.
7. Answer the original question.
8. Check your answer by using it to solve the original problem (not the equation).

ENRICHING YOUR MATHEMATICAL WORD POWER

For each mathematical term, choose the correct meaning.

1. **linear equation**
   a. an equation in which the terms are in line
   b. an equation of the form $ax + b = 0$ where $a \neq 0$
   c. the equation $a = b$
   d. an equation of the form $a^2 + b^2 = c^2$
2. **identity**
   a. an equation that is satisfied by all real numbers
   b. an equation that is satisfied by every real number
   c. an equation that is identical
   d. an equation that is satisfied by every real number for which both sides are defined
3. **conditional equation**
   a. an equation that has at least one real solution
   b. an equation that is correct
   c. an equation that is satisfied by at least one real number but is not an identity
   d. an equation that we are not sure how to solve
4. **inconsistent equation**
   a. an equation that is wrong
   b. an equation that is only sometimes consistent
   c. an equation that has no solution
   d. an equation with two variables
5. **equivalent equations**
   a. equations that are identical
   b. equations that are correct
   c. equations that are equal
   d. equations that have the same solution
6. **formula**
   a. an equation
   b. a type of race car
   c. a process
   d. an equation involving two or more variables
7. **literal equation**
   a. a formula
   b. an equation with words
   c. a false equation
   d. a fact
8. **complementary angles**
   a. angles that compliment each other
   b. angles whose degree measures total 90°
   c. angles whose degree measures total 180°
   d. angles with the same vertex
9. **supplementary angles**
   a. angles with soft flexible sides
   b. angles whose degree measures total 90°
   c. angles whose degree measures total 180°
   d. angles that form a square
10. **uniform motion**
    a. movement of an army
    b. movement in a straight line
    c. consistent motion
    d. motion at a constant rate

REVIEW EXERCISES

2.1 *Solve each equation and check your answer.*

1. $x - 23 = 12$ **2.** $14 = 18 + y$

3. $\frac{2}{3}u = -4$ **4.** $-\frac{3}{8}r = 15$

5. $-5y = 35$ **6.** $-12 = 6h$

7. $6m = 13 + 5m$ **8.** $19 - 3n = -2n$

study tip

Note how the review exercises are arranged according to the sections in this chapter. If you are having trouble with a certain type of problem, refer back to the appropriate section for examples and explanations.

2.2 *Solve each equation and check your answer.*

9. $2x - 5 = 9$ **10.** $5x - 8 = 38$

11. $3p - 14 = -4p$ **12.** $36 - 9y = 3y$

13. $2z + 12 = 5z - 9$ **14.** $15 - 4w = 7 - 2w$

15. $2(h - 7) = -14$ **16.** $2(t - 7) = 0$

17. $3(w - 5) = 6(w + 2) - 3$

18. $2(a - 4) + 4 = 5(9 - a)$

2.3 *Solve each equation. Identify each equation as a conditional equation, an inconsistent equation, or an identity.*

19. $2(x - 7) - 5 = 5 - (3 - 2x)$

20. $2(x - 7) + 5 = -(9 - 2x)$

21. $2(w - w) = 0$

22. $2y - y = 0$

23. $\frac{3r}{3r} = 1$

24. $\frac{3t}{3} = 1$

25. $\frac{1}{2}a - 5 = \frac{1}{3}a - 1$

26. $\frac{1}{2}b - \frac{1}{2} = \frac{1}{4}b$

27. $0.06q + 14 = 0.3q - 5.2$

28. $0.05(z + 20) = 0.1z - 0.5$

29. $0.05(x + 100) + 0.06x = 115$

30. $0.06x + 0.08(x + 1) = 0.41$

Solve each equation.

31. $2x + \frac{1}{2} = 3x + \frac{1}{4}$ **32.** $5x - \frac{1}{3} = 6x - \frac{1}{2}$

33. $\frac{x}{2} - \frac{3}{4} = \frac{x}{6} + \frac{1}{8}$ **34.** $\frac{1}{3} - \frac{x}{5} = \frac{1}{2} - \frac{x}{10}$

35. $\frac{5}{6}x = -\frac{2}{3}$ **36.** $-\frac{2}{3}x = \frac{3}{4}$

37. $-\frac{1}{2}(x - 10) = \frac{3}{4}x$ **38.** $-\frac{1}{3}(6x - 9) = 23$

39. $3 - 4(x - 1) + 6 = -3(x + 2) - 5$

40. $6 - 5(1 - 2x) + 3 = -3(1 - 2x) - 1$

41. $5 - 0.1(x - 30) = 18 + 0.05(x + 100)$

42. $0.6(x - 50) = 18 - 0.3(40 - 10x)$

2.4 *Solve each equation for x.*

43. $ax + b = 0$ **44.** $mx + e = t$ **45.** $ax - 2 = b$

46. $b = 5 - x$ **47.** $LWx = V$ **48.** $3xy = 6$

49. $2x - b = 5x$ **50.** $t - 5x = 4x$

Solve each equation for y. Write the answer in the form $y = mx + b$, where m and b are real numbers.

51. $5x + 2y = 6$ **52.** $5x - 3y + 9 = 0$

53. $y - 1 = -\frac{1}{2}(x - 6)$ **54.** $y + 6 = \frac{1}{2}(x + 8)$

55. $\frac{1}{2}x + \frac{1}{4}y = 4$ **56.** $-\frac{x}{3} + \frac{y}{2} = 1$

Find the value of y in each formula if $x = -3$.

57. $y = 3x - 4$ **58.** $2x - 3y = -7$

59. $5xy = 6$ **60.** $3xy - 2x = -12$

61. $y - 3 = -2(x - 4)$ **62.** $y + 1 = 2(x - 5)$

2.5 *Translate each verbal expression into an algebraic expression.*

63. The sum of a number and 9

64. The product of a number and 7

65. Two numbers that differ by 8

66. Two numbers with a sum of 12

67. Sixty-five percent of a number

68. One half of a number

Identify the variable, and write an equation that describes each situation. Do not solve the equation.

69. One side of a rectangle is 5 feet longer than the other, and the area is 98 square feet.

70. One side of a rectangle is 1 foot longer than twice the other side, and the perimeter is 56 feet.

71. By driving 10 miles per hour slower than Jim, Barbara travels the same distance in 3 hours as Jim does in 2 hours.

72. Gladys and Ned drove 840 miles altogether, with Gladys averaging 5 miles per hour more in her 6 hours at the wheel than Ned did in his 5 hours at the wheel.

73. The sum of three consecutive even integers is 88.

74. The sum of two consecutive odd integers is 40.

75. The three angles of a triangle have degree measures of t, $2t$, and $t - 10$.

76. Two complementary angles have degree measures p and $3p - 6$.

2.6–7 *Solve each problem.*

77. ***Odd integers.*** If the sum of three consecutive odd integers is 237, then what are the integers?

78. ***Even integers.*** Find two consecutive even integers that have a sum of 450.

79. ***Driving to the shore.*** Lawanda and Betty both drive the same distance to the shore. By driving 15 miles per hour faster than Betty, Lawanda can get there in 3 hours while Betty takes 4 hours. How fast does each of them drive?

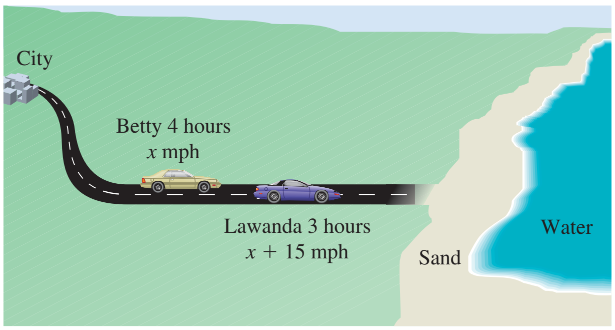


FIGURE FOR EXERCISE 79

80. ***Rectangular lot.*** The length of a rectangular lot is 50 feet more than the width. If the perimeter is 500 feet, then what are the length and width?

81. ***Combined savings.*** Wanda makes \$6000 more per year than her husband does. Wanda saves 10% of her income for retirement, and her husband saves 6%. If together they save \$5400 per year, then how much does each of them make per year?

82. ***Layoffs looming.*** American Products plans to lay off 10% of its employees in its aerospace division and 15% of its employees in its agricultural division. If altogether 12% of the 3000 employees in these two divisions will be laid off, then how many employees are in each division?

MISCELLANEOUS

Use an equation or formula to solve each problem.

83. ***Flat yield curve.*** The accompanying graph shows that the *yield curve* for U.S. Treasury bonds was relatively flat from 2 years out to 30 years on July 13, 1998 (Bloomberg, www.bloomberg.com). In this situation there is not much benefit to be obtained from long-term investing.

a) Use the interest rate in the graph to find the amount of interest earned in the first year on a 2-year bond of \$10,000.

b) How much more interest would you earn in the first year on a 30-year bond of \$10,000?

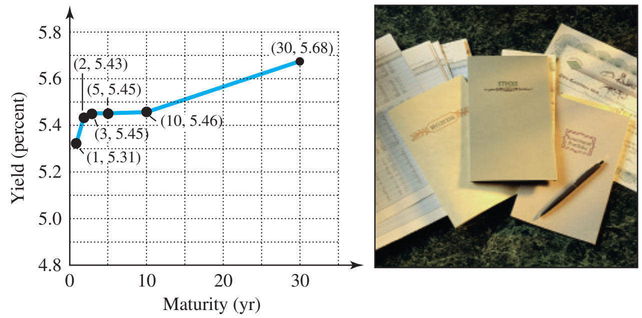


FIGURE FOR EXERCISES 83 AND 84

84. ***Reading the curve.*** Use the accompanying graph to find the maturity of a U.S. Treasury bond that had a yield of 5.46% on July 13, 1998.

85. ***Combined videos.*** The owners of ABC Video discovered that they had no movies in common with XYZ Video and bought XYZ's entire stock. Although XYZ had 200 titles, they had no children's movies, while 60% of ABC's titles were children's movies. If 40% of the movies in the combined stock are children's movies, then how many movies did ABC have before the merger?

86. ***Living comfortably.*** Gary has figured that he needs to take home \$30,400 a year to live comfortably. If the government gets 24% of Gary's income, then what must his income be for him to live comfortably?

87. ***Bracing a gate.*** The diagonal brace on a rectangular gate forms an angle with the horizontal side with degree measure x and an angle with the vertical side with degree measure $2x - 3$. Find x.

88. ***Digging up the street.*** A contractor wants to install a pipeline connecting point A with point C on opposite sides of a road as shown in the figure for this exercise. To save money, the contractor has decided to lay the pipe to point B and then under the road to point C. Find the measure of the angle marked x in the figure.

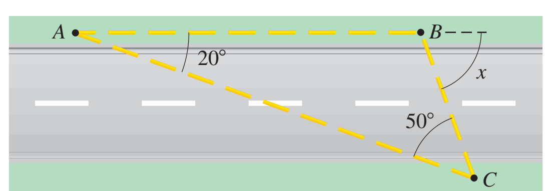


FIGURE FOR EXERCISE 88

CHAPTER 2 TEST

Solve each equation.

1. $6x - 7 = 0$

2. $-10x - 6 + 4x = -4x + 8$

3. $5(2x - 3) = x + 3$

4. $-\frac{2}{3}x + 1 = 7$

5. $2(x + 6) = 2x - 5$

6. $x + 7x = 8x$

7. $x + 0.06x = 742$

8. $\frac{1}{2}x - \frac{1}{3} = \frac{1}{4}x + \frac{1}{6}$

study tip

Before you take an in-class exam on this chapter, work the sample test given here. Set aside 1 hour to work this test and use the answers in the back of this book to grade yourself. Even though your instructor might not ask exactly the same questions, you will get a good idea of your test readiness.

Determine whether each equation is a conditional equation, an inconsistent equation, or an identity.

9. $x + 2 = x$

10. $x + x = 2x$

11. $x + x = 2$

Solve for the indicated variable.

12. $2x - 3y = 9$ for y (in the form $y = mx + b$)

13. $m = aP - w$ for a

14. $2x - 3 = ax$ for x

Write a complete solution to each problem.

15. The perimeter of a rectangle is 72 meters. If the width is 8 meters less than the length, then what is the width of the rectangle?

16. If the area of a triangle is 54 square inches and the base is 12 inches, then what is the height?

17. How many liters of a 20% alcohol solution should Maria mix with 50 liters of a 60% alcohol solution to obtain a 30% solution?

18. If the degree measure of the smallest angle of a triangle is one-half of the degree measure of the second largest angle and one-third of the degree measure of the largest angle, then what is the degree measure of each angle?

Simplify each expression.

1. $3x + 5x$

2. $3x \cdot 5x$

MAKING CONNECTIONS CHAPTERS 1–2

3. $\dfrac{4x + 2}{2}$

4. $5 - 4(3 - x)$

5. $3x + 8 - 5(x - 1)$

6. $(-6)^2 - 4(-3)2$

7. $3^2 \cdot 2^3$

8. $4(-7) - (-6)(3)$

9. $-2x \cdot x \cdot x$

10. $(-1)(-1)(-1)(-1)(-1)$

Solve each equation.

11 $3x + 5x = 8$

12. $3x + 5x = 8x$

13. $3x + 5x = 7x$

14. $3x + 5 = 8$

15. $3x + 1 = 7$

16. $5 - 4(3 - x) = 1$

17. $3x + 8 = 5(x - 1)$

18. $x - 0.05x = 190$

Solve the problem.

19. ***Linear depreciation.*** In computing income taxes, a company is allowed to depreciate a \$20,000 computer system over five years. Using *linear depreciation,* the value V of the computer system at any year t from 0 through 5 is given by

$$V = C - \frac{(C - S)}{5}t,$$

where C is the initial cost of the system and S is the scrap value of the system.

a) What is the value of the computer system after two years if its scrap value is \$4000?

b) If the value of the system after three years is claimed to be \$14,000, then what is the scrap value of the company's system?

c) If the accompanying graph models the depreciation of the system, then what is the scrap value of the system?

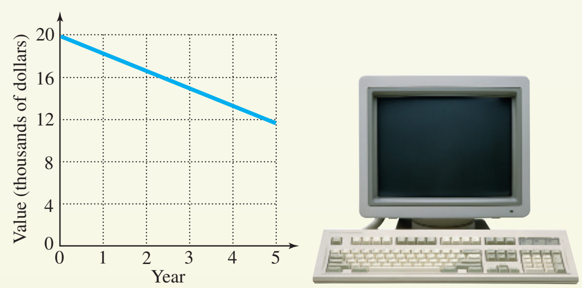


FIGURE FOR EXERCISE 19

study tip

Don't wait until the final exam to review material. Do some review on a regular basis. The Making Connections exercises on this page can be used to review, compare, and contrast different concepts that you have studied. A good time to work these exercises is between a test and the start of new material.