1.3 # ADDITION AND SUBTRACTION OF REAL NUMBERS

In arithmetic we add and subtract only positive numbers and zero. In Section 1.1 we introduced the concept of absolute value of a number. Now we will use absolute value to extend the operations of addition and subtraction to the real numbers. We will work only with rational numbers in this chapter. You will learn to perform operations with irrational numbers in Chapter 9.

Addition of Two Negative Numbers

A good way to understand positive and negative numbers is to *think of the positive numbers as assets and the negative numbers as debts.* For this illustration we can think of assets simply as cash. For example, if you have $3 and $5 in cash, then your total cash is $8. You get the total by adding two positive numbers.

Think of debts as unpaid bills such as the electric bill or the phone bill. If you have debts of $7 and $8, then your total debt is $15. You can get the total debt by adding negative numbers:

$$(-7) \quad + \quad (-8) \quad = \quad -15$$
$$\uparrow \qquad \uparrow \qquad \uparrow \qquad\qquad \uparrow$$
$$\text{\$7 debt} \quad \text{plus} \quad \text{\$8 debt} \qquad \text{\$15 debt}$$

We think of this addition as adding the absolute values of -7 and -8 ($7 + 8 = 15$), and then putting a negative sign on that result to get -15. These examples illustrate the following rule.

> ### Sum of Two Numbers with Like Signs
>
> To find the sum of two numbers with the same sign, add their absolute values. The sum has the same sign as the given numbers.

EXAMPLE 1

Adding numbers with like signs
Perform the indicated operations.

a) $23 + 56$ **b)** $(-12) + (-9)$ **c)** $(-3.5) + (-6.28)$ **d)** $\left(-\dfrac{1}{2}\right) + \left(-\dfrac{1}{4}\right)$

Solution

a) The sum of two positive numbers is a positive number: $23 + 56 = 79$.

b) The absolute values of -12 and -9 are 12 and 9, and $12 + 9 = 21$. So

$$(-12) + (-9) = -21.$$

c) Add the absolute values of -3.5 and -6.28, and put a negative sign on the sum. Remember to line up the decimal points when adding decimal numbers:

$$
\begin{array}{r}
3.50 \\
6.28 \\
\hline
9.78
\end{array}
$$

So $(-3.5) + (-6.28) = -9.78$.

d) $\left(-\dfrac{1}{2}\right) + \left(-\dfrac{1}{4}\right) = \left(-\dfrac{2}{4}\right) + \left(-\dfrac{1}{4}\right) = -\dfrac{3}{4}$ ■

Addition of Numbers with Unlike Signs

If you have a debt of \$5 and have only \$5 in cash, then your debts equal your assets (in absolute value), and your net worth is \$0. **Net worth** is the total of debts and assets. Symbolically,

$$-5 \quad + \quad 5 \quad = \quad 0.$$

$\uparrow$ \$5 debt $\uparrow$ \$5 cash $\uparrow$ Net worth

For any number a, a and its opposite, $-a$, have a sum of zero. For this reason, a and $-a$ are called **additive inverses** of each other. Note that the words "negative," "opposite," and "additive inverse" are often used interchangeably.

Additive Inverse Property

For any number a,

$$a + (-a) = 0 \quad \text{and} \quad (-a) + a = 0.$$

E X A M P L E   2

Finding the sum of additive inverses

Evaluate.

a) $34 + (-34)$ **b)** $-\dfrac{1}{4} + \dfrac{1}{4}$ **c)** $2.97 + (-2.97)$

Solution

a) $34 + (-34) = 0$

b) $-\dfrac{1}{4} + \dfrac{1}{4} = 0$

c) $2.97 + (-2.97) = 0$ ■

To understand the sum of a positive and a negative number that are not additive inverses of each other, consider the following situation. If you have a debt of \$6 and \$10 in cash, you may have \$10 in hand, but your net worth is only \$4. Your assets exceed your debts (in absolute value), and you have a positive net worth. In symbols,

$$-6 + 10 = 4.$$

Note that to get 4, we actually subtract 6 from 10.

If you have a debt of \$7 but have only \$5 in cash, then your debts exceed your assets (in absolute value). You have a negative net worth of $-\$2$. In symbols,

$$-7 + 5 = -2.$$

Note that to get the 2 in the answer, we subtract 5 from 7.

As you can see from these examples, the sum of a positive number and a negative number (with different absolute values) may be either positive or negative. These examples help us to understand the rule for adding numbers with unlike signs and different absolute values.

helpful hint

We use the illustrations with debts and assets to make the rules for adding signed numbers understandable. However, in the end the carefully written rules tell us exactly how to perform operations with signed numbers, and we must obey the rules.

> ### Sum of Two Numbers with Unlike Signs (and Different Absolute Values)
>
> To find the sum of two numbers with unlike signs (and different absolute values), subtract their absolute values.
>
> - The answer is positive if the number with the larger absolute value is positive.
> - The answer is negative if the number with the larger absolute value is negative.

E X A M P L E   3

Adding numbers with unlike signs

Evaluate.

a) $-5 + 13$ **b)** $6 + (-7)$ **c)** $-6.4 + 2.1$

d) $-5 + 0.09$ **e)** $\left(-\dfrac{1}{3}\right) + \left(\dfrac{1}{2}\right)$ **f)** $\dfrac{3}{8} + \left(-\dfrac{5}{6}\right)$

Solution

a) The absolute values of -5 and 13 are 5 and 13. Subtract them to get 8. Since the number with the larger absolute value is 13 and it is positive, the result is positive:

$$-5 + 13 = 8$$

b) The absolute values of 6 and -7 are 6 and 7. Subtract them to get 1. Since -7 has the larger absolute value, the result is negative:

$$6 + (-7) = -1$$

c) Line up the decimal points and subtract 2.1 from 6.4.

$$\begin{array}{r} 6.4 \\ -2.1 \\ \hline 4.3 \end{array}$$

Since 6.4 is larger than 2.1, and 6.4 has a negative sign, the sign of the answer is negative. So $-6.4 + 2.1 = -4.3$.

d) Line up the decimal points and subtract 0.09 from 5.00.

$$\begin{array}{r} 5.00 \\ -0.09 \\ \hline 4.91 \end{array}$$

Since 5.00 is larger than 0.09, and 5.00 has the negative sign, the sign of the answer is negative. So $-5 + 0.09 = -4.91$.

e) $\left(-\dfrac{1}{3}\right) + \left(\dfrac{1}{2}\right) = \left(-\dfrac{2}{6}\right) + \left(\dfrac{3}{6}\right)$

$= \dfrac{1}{6}$

f) $\dfrac{3}{8} + \left(-\dfrac{5}{6}\right) = \dfrac{9}{24} + \left(-\dfrac{20}{24}\right)$

$= -\dfrac{11}{24}$

■

calculator

close-up

Your calculator can add signed numbers. Most calculators have a key for subtraction and a different key for the negative sign.

```
-5+13
               8
-5+.09
           -4.91
3/8+-5/6▶Frac
           -11/24
```

You should do the exercises in this section by hand and then check with a calculator.

Subtraction of Signed Numbers

Each subtraction problem with signed numbers is solved by doing an equivalent addition problem. So before attempting subtraction of signed numbers be sure that you understand addition of signed numbers.

Now think of subtraction as removing debts or assets, and think of addition as receiving debts or assets. If you have $10 in cash and $3 is taken from you, your resulting net worth is the same as if you have $10 cash and a phone bill for $3 arrives in the mail. In symbols,

$$10 \quad - \quad 3 \quad = \quad 10 \quad + \quad (-3).$$

Remove   Cash   Receive   Debt

Removing cash is equivalent to receiving a debt.

Suppose you have $15 but owe a friend $5. Your net worth is only $10. If the debt of $5 is canceled or forgiven, your net worth will go up to $15, the same as if you received $5 in cash. In symbols,

$$10 \quad - \quad (-5) \quad = \quad 10 \quad + \quad 5.$$

Remove   Debt   Receive   Cash

Removing a debt is equivalent to receiving cash.

Notice that each subtraction problem is equivalent to an addition problem in which we add the opposite of what we want to subtract. In other words, subtracting a number is the same as adding its opposite.

Subtraction of Real Numbers

For any real numbers a and b,

$$a - b = a + (-b).$$

E X A M P L E   4

Subtracting signed numbers

Perform each subtraction.

a) $-5 - 3$ **b)** $5 - (-3)$

c) $-5 - (-3)$ **d)** $\dfrac{1}{2} - \left(-\dfrac{1}{4}\right)$

e) $-3.6 - (-5)$ **f)** $0.02 - 8$

Solution

To do *any* subtraction, we can change it to addition of the opposite.

a) $-5 - 3 = -5 + (-3) = -8$

b) $5 - (-3) = 5 + (3) = 8$

c) $-5 - (-3) = -5 + 3 = -2$

d) $\dfrac{1}{2} - \left(-\dfrac{1}{4}\right) = \dfrac{2}{4} + \dfrac{1}{4} = \dfrac{3}{4}$

e) $-3.6 - (-5) = -3.6 + 5 = 1.4$

f) $0.02 - 8 = 0.02 + (-8) = -7.98$ ■

WARM-UPS

True or false? Explain your answer.

1. $-9 + 8 = -1$
2. $(-2) + (-4) = -6$
3. $0 - 7 = -7$
4. $5 - (-2) = 3$
5. $-5 - (-2) = -7$
6. The additive inverse of -3 is 0.
7. If b is a negative number, then $-b$ is a positive number.
8. The sum of a positive number and a negative number is a negative number.
9. The result of a subtracted from b is the same as b plus the opposite of a.
10. If a and b are negative numbers, then $a - b$ is a negative number.

1.3   EXERCISES

Reading and Writing   *After reading this section write out the answers to these questions. Use complete sentences.*

1. What operations did we study in this section?

2. How do you find the sum of two numbers with the same sign?

3. When can we say that two numbers are additive inverses of each other?

4. What is the sum of two numbers with opposite signs and the same absolute value?

5. How do we find the sum of two numbers with unlike signs?

6. What is the relationship between subtraction and addition?

Perform the indicated operation. See Example 1.

7. $3 + 10$
8. $81 + 19$
9. $(-3) + (-10)$
10. $(-81) + (-19)$
11. $-0.25 + (-0.9)$
12. $-0.8 + (-2.35)$

13. $\left(-\dfrac{1}{3}\right) + \left(-\dfrac{1}{6}\right)$
14. $\dfrac{2}{3} + \dfrac{1}{12}$

Evaluate. See Examples 2 and 3.

15. $-8 + 8$
16. $20 + (-20)$
17. $-\dfrac{17}{50} + \dfrac{17}{50}$
18. $\dfrac{12}{13} + \left(-\dfrac{12}{13}\right)$
19. $-7 + 9$
20. $10 + (-30)$
21. $7 + (-13)$
22. $-8 + 20$
23. $8.6 + (-3)$
24. $-9.5 + 12$
25. $3.9 + (-6.8)$
26. $-5.24 + 8.19$
27. $\dfrac{1}{4} + \left(-\dfrac{1}{2}\right)$
28. $-\dfrac{2}{3} + 2$

Fill in the parentheses to make each statement correct. See Example 4.

29. $8 - 2 = 8 + (?)$
30. $3.5 - 1.2 = 3.5 + (?)$
31. $4 - 12 = 4 + (?)$
32. $\dfrac{1}{2} - \dfrac{5}{6} = \dfrac{1}{2} + (?)$
33. $-3 - (-8) = -3 + (?)$
34. $-9 - (-2.3) = -9 + (?)$
35. $8.3 - (-1.5) = 8.3 + (?)$
36. $10 - (-6) = 10 + (?)$

Perform the indicated operation. See Example 4.

37. $6 - 10$
38. $3 - 19$
39. $-3 - 7$
40. $-3 - 12$
41. $5 - (-6)$
42. $5 - (-9)$

43. $-6 - 5$

44. $-3 - 6$

45. $\dfrac{1}{4} - \dfrac{1}{2}$

46. $\dfrac{2}{5} - \dfrac{2}{3}$

47. $\dfrac{1}{2} - \left(-\dfrac{1}{4}\right)$

48. $\dfrac{2}{3} - \left(-\dfrac{1}{6}\right)$

49. $10 - 3$

50. $13 - 3$

51. $1 - 0.07$

52. $0.03 - 1$

53. $7.3 - (-2)$

54. $-5.1 - 0.15$

55. $-0.03 - 5$

56. $0.7 - (-0.3)$

Perform the indicated operations. Do not use a calculator.

57. $-5 + 8$

58. $-6 + 10$

59. $-6 + (-3)$

60. $(-13) + (-12)$

61. $-80 - 40$

62. $44 - (-15)$

63. $61 - (-17)$

64. $-19 - 13$

65. $(-12) + (-15)$

66. $-12 + 12$

67. $13 + (-20)$

68. $15 + (-39)$

69. $-102 - 99$

70. $-94 - (-77)$

71. $-161 - 161$

72. $-19 - 88$

73. $-16 + 0.03$

74. $0.59 + (-3.4)$

75. $0.08 - 3$

76. $1.8 - 9$

77. $-3.7 + (-0.03)$

78. $0.9 + (-1)$

79. $-2.3 - (-6)$

80. $-7.08 - (-9)$

81. $\dfrac{3}{4} + \left(-\dfrac{3}{5}\right)$

82. $-\dfrac{1}{3} + \dfrac{3}{5}$

83. $-\dfrac{1}{12} - \left(-\dfrac{3}{8}\right)$

84. $-\dfrac{1}{17} - \left(-\dfrac{1}{17}\right)$

Use a calculator to perform the indicated operations.

85. $45.87 + (-49.36)$

86. $-0.357 + (-3.465)$

87. $0.6578 + (-1)$

88. $-2.347 + (-3.5)$

89. $-3.45 - 45.39$

90. $9.8 - 9.974$

91. $-5.79 - 3.06$

92. $0 - (-4.537)$

Solve each problem.

93. *Overdrawn.* Willard opened his checking account with a deposit of $97.86. He then wrote checks and had other charges as shown in his account register. Find his current balance.

Deposit		97.86
Wal-Mart	27.89	
Kmart	42.32	
ATM cash	25.00	
Service charge	3.50	
Check printing	8.00	

FIGURE FOR EXERCISE 93

94. *Net worth.* Melanie has a $125,000 house with a $78,422 mortgage. She has $21,236 in a savings account and has $9,477 in credit card debt. She owes $6,131 to the credit union and figures that her cars and other household items are worth a total of $15,000. What is Melanie's net worth?

95. *Falling temperatures.* At noon the temperature in Montreal was 5°C. By midnight the mercury had fallen 12°. What was the temperature at midnight?

96. *Bitter cold.* The overnight low temperature in Milwaukee was -13°F for Monday night. The temperature went up 20° during the day on Tuesday and then fell 15° to reach Tuesday night's overnight low temperature.
a) What was the overnight low Tuesday night?
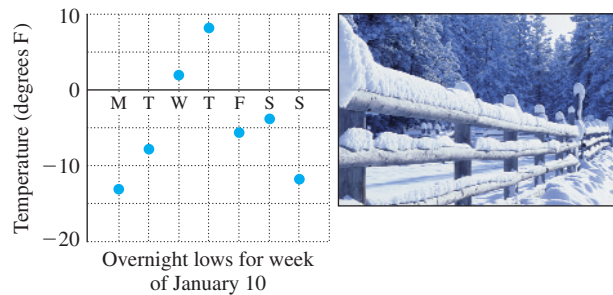b) Judging from the accompanying graph, was the average low for the week above or below 0°F?

Overnight lows for week
of January 10

FIGURE FOR EXERCISE 96

GETTING MORE INVOLVED

97. *Writing.* What does absolute value have to do with adding signed numbers? Can you add signed numbers without using absolute value?

98. *Discussion.* Why do we learn addition of signed numbers before subtraction?

99. *Discussion.* Aimee and Joni are traveling south in separate cars on Interstate 5 near Stockton. While they are speaking to each other on cellular telephones, Aimee gives her location as mile marker x and Joni gives her location as mile marker y. Which of the following expressions gives the distance between them? Explain your answer.
a) $y - x$ b) $x - y$ c) $|x - y|$
d) $|y - x|$ e) $|x| + |y|$