1.4 MULTIPLICATION AND DIVISION OF REAL NUMBERS

In this section

- Multiplication of Real Numbers
- Division of Real Numbers
- Division by Zero

In this section we will complete the study of the four basic operations with real numbers.

Multiplication of Real Numbers

The result of multiplying two numbers is referred to as the **product** of the numbers. The numbers multiplied are called **factors.** In algebra we use a raised dot between the factors to indicate multiplication, or we place symbols next to one another to indicate multiplication. Thus $a \cdot b$ or ab are both referred to as the product of a and b. When multiplying numbers, we may enclose them in parentheses to make the meaning clear. To write 5 times 3, we may write it as $5 \cdot 3$, 5(3), (5)3, or (5)(3). In multiplying a number and a variable, no sign is used between them. Thus $5x$ is used to represent the product of 5 and x.

Multiplication is just a short way to do repeated additions. Adding together five 3's gives

$$3 + 3 + 3 + 3 + 3 = 15.$$

So we have the multiplication fact $5 \cdot 3 = 15$. Adding together five -3's gives

$$(-3) + (-3) + (-3) + (-3) + (-3) = -15.$$

So we should have $5(-3) = -15$. We can think of $5(-3) = -15$ as saying that taking on five debts of \$3 each is equivalent to a debt of \$15. Losing five debts of \$3 each is equivalent to gaining \$15, so we should have $(-5)(-3) = 15$.

These examples illustrate the rule for multiplying signed numbers.

helpful hint

The product of two numbers with like signs is positive, but the product of three numbers with like signs could be negative. For example,

$$(-2)(-2)(-2) = 4(-2)$$
$$= -8.$$

Product of Signed Numbers

To find the product of two nonzero real numbers, multiply their absolute values.

- The product is *positive* if the numbers have *like* signs.
- The product is *negative* if the numbers have *unlike* signs.

EXAMPLE 1

Multiplying signed numbers

Evaluate each product.

a) $(-2)(-3)$ **b)** $3(-6)$ **c)** $-5 \cdot 10$

d) $\left(-\frac{1}{3}\right)\left(-\frac{1}{2}\right)$ **e)** $(-0.02)(0.08)$ **f)** $(-300)(-0.06)$

Solution

a) First find the product of the absolute values:

$$|-2| \cdot |-3| = 2 \cdot 3 = 6$$

Because -2 and -3 have the same sign, we get $(-2)(-3) = 6$.

b) First find the product of the absolute values:

$$|3| \cdot |-6| = 3 \cdot 6 = 18$$

Because 3 and -6 have unlike signs, we get $3(-6) = -18$.

calculator close-up

Try finding the products in Example 1 with your calculator.

```
(-2)(-3)
                6
3(-6)
              -18
-5*10
              -50
```

c) $-5 \cdot 10 = -50$

d) $\left(-\frac{1}{3}\right)\left(-\frac{1}{2}\right) = \frac{1}{6}$ Like signs, positive result

e) When multiplying decimals, we total the number of decimal places in the factors to get the number of decimal places in the product. Thus

$$(-0.02)(0.08) = -0.0016.$$

f) $(-300)(-0.06) = 18$

Division of Real Numbers

We say that $10 \div 5 = 2$ because $2 \cdot 5 = 10$. This example illustrates how division is defined in terms of multiplication.

Division of Real Numbers

If a, b, and c are any real numbers with $b \neq 0$, then

$$a \div b = c \qquad \text{provided that} \qquad c \cdot b = a.$$

Using the definition of division, we get

$$10 \div (-2) = -5$$

because $(-5)(-2) = 10$;

$$-10 \div 2 = -5$$

because $(-5)(2) = -10$; and

$$-10 \div (-2) = 5$$

because $(5)(-2) = -10$. From these examples we see that the rule for dividing signed numbers is similar to that for multiplying signed numbers.

Division of Signed Numbers

To find the quotient of nonzero real numbers, divide their absolute values.

- The quotient is *positive* if the numbers have *like* signs.
- The quotient is *negative* if the numbers have *unlike* signs.

Zero divided by any nonzero real number is zero.

EXAMPLE 2 **Dividing signed numbers**

Evaluate.

a) $(-8) \div (-4)$ b) $(-8) \div 8$ c) $8 \div (-4)$

d) $-4 \div \frac{1}{3}$ e) $-2.5 \div 0.05$ f) $0 \div (-6)$

Solution

a) $(-8) \div (-4) = 2$ Same sign, positive result

b) $(-8) \div 8 = -1$ Unlike signs, negative result

c) $8 \div (-4) = -2$

helpful hint

Do not use negative numbers in long division. To find $-378 \div 7$, divide 378 by 7:

$$\begin{array}{r} 54 \\ 7\overline{)378} \\ \underline{35} \\ 28 \\ \underline{28} \\ 0 \end{array}$$

Since a negative divided by a positive is negative

$$-378 \div 7 = -54.$$

d) $-4 \div \frac{1}{3} = -4 \cdot \frac{3}{1}$ Invert and multiply.

$= -4 \cdot 3$

$= -12$

e) $-2.5 \div 0.05 = \frac{-2.5}{0.05}$ Write in fraction form.

$= \frac{-2.5 \cdot 100}{0.05 \cdot 100}$ Multiply by 100 to eliminate the decimals.

$= \frac{-250}{5}$ Simplify.

$= -50$ Divide.

f) $0 \div (-6) = 0$

Division can also be indicated by a fraction bar. For example,

$$24 \div 6 = \frac{24}{6} = 4.$$

If signed numbers occur in a fraction, we use the rules for dividing signed numbers. For example,

$$\frac{-9}{3} = -3, \quad \frac{9}{-3} = -3, \quad \frac{-1}{2} = \frac{1}{-2} = -\frac{1}{2}, \quad \text{and} \quad \frac{-4}{-2} = 2.$$

Note that if one negative sign appears in a fraction, the fraction has the same value whether the negative sign is in the numerator, in the denominator, or in front of the fraction. If the numerator and denominator of a fraction are both negative, then the fraction has a positive value.

study tip

If you don't know how to get started on the exercises, go back to the examples. Cover the solution in the text with a piece of paper and see if you can solve the example. After you have mastered the examples, then try the exercises again.

Division by Zero

Why do we exclude division by zero from the definition of division? If we write $10 \div 0 = c$, we need to find a number c such that $c \cdot 0 = 10$. This is impossible. If we write $0 \div 0 = c$, we need to find a number c such that $c \cdot 0 = 0$. In fact, $c \cdot 0 = 0$ is true for any value of c. Having $0 \div 0$ equal to any number would be confusing in doing computations. Thus $a \div b$ is defined only for $b \neq 0$. Quotients such as

$$8 \div 0, \quad 0 \div 0, \quad \frac{8}{0}, \quad \text{and} \quad \frac{0}{0}$$

are said to be **undefined.**

WARM-UPS

True or false? Explain your answer.

1. The product of 7 and y is written as $7y$.
2. The product of -2 and 5 is 10.
3. The quotient of x and 3 can be written as $x \div 3$ or $\frac{x}{3}$.
4. $0 \div 6$ is undefined.
5. $(-9) \div (-3) = 3$
6. $6 \div (-2) = -3$

WARM-UPS

(continued)

7. $\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right) = \frac{1}{4}$

8. $(-0.2)(0.2) = -0.4$

9. $\left(-\frac{1}{2}\right) \div \left(-\frac{1}{2}\right) = 1$

10. $\frac{0}{0} = 0$

1.4 EXERCISES

Reading and Writing *After reading this section write out the answers to these questions. Use complete sentences.*

1. What operations did we study in this section?
2. What is a product?
3. How do you find the product of two signed numbers?
4. What is the relationship between division and multiplication?
5. How do you find the quotient of nonzero real numbers?
6. Why is division by zero undefined?

Evaluate. See Example 1.

7. $-3 \cdot 9$
8. $6(-4)$
9. $(-12)(-11)$
10. $(-9)(-15)$
11. $-\frac{3}{4} \cdot \frac{4}{9}$
12. $\left(-\frac{2}{3}\right)\left(-\frac{6}{7}\right)$
13. $0.5(-0.6)$
14. $(-0.3)(0.3)$
15. $(-12)(-12)$
16. $(-11)(-11)$
17. $-3 \cdot 0$
18. $0(-7)$

Evaluate. See Example 2.

19. $8 \div (-8)$
20. $-6 \div 2$
21. $(-90) \div (-30)$
22. $(-20) \div (-40)$
23. $\frac{44}{-66}$
24. $\frac{-33}{-36}$
25. $\left(-\frac{2}{3}\right) \div \left(-\frac{4}{5}\right)$
26. $-\frac{1}{3} \div \frac{4}{9}$
27. $\frac{-125}{0}$
28. $-37 \div 0$
29. $0 \div \left(-\frac{1}{3}\right)$
30. $0 \div 43.568$
31. $40 \div (-0.5)$
32. $3 \div (-0.1)$
33. $-0.5 \div (-2)$
34. $-0.75 \div (-0.5)$

Perform the indicated operations.

35. $(25)(-4)$
36. $(5)(-4)$
37. $(-3)(-9)$
38. $(-51) \div (-3)$
39. $-9 \div 3$
40. $86 \div (-2)$
41. $20 \div (-5)$
42. $(-8)(-6)$
43. $(-6)(5)$
44. $(-18) \div 3$
45. $(-57) \div (-3)$
46. $(-30)(4)$
47. $(0.6)(-0.3)$
48. $(-0.2)(-0.5)$
49. $(-0.03)(-10)$
50. $(0.05)(-1.5)$
51. $(-0.6) \div (0.1)$
52. $8 \div (-0.5)$
53. $(-0.6) \div (-0.4)$
54. $(-63) \div (-0.9)$
55. $-\frac{12}{5}\left(-\frac{55}{6}\right)$
56. $-\frac{9}{10} \cdot \frac{4}{3}$
57. $-2\frac{3}{4} \div 8\frac{1}{4}$
58. $-9\frac{1}{2} \div \left(-3\frac{1}{6}\right)$
59. $(0.45)(-365)$
60. $8.5 \div (-0.15)$
61. $(-52) \div (-0.034)$
62. $(-4.8)(5.6)$

Perform the indicated operations. Use a calculator to check.

63. $(-4)(-4)$
64. $-4 - 4$
65. $-4 + (-4)$
66. $-4 \div (-4)$
67. $-4 + 4$
68. $-4 \cdot 4$
69. $-4 - (-4)$
70. $0 \div (-4)$
71. $0.1 - 4$
72. $(0.1)(-4)$
73. $(-4) \div (0.1)$
74. $-0.1 - 4$
75. $(-0.1)(-4)$
76. $-0.1 + 4$
77. $|-0.4|$
78. $|0.4|$
79. $\frac{-0.06}{0.3}$
80. $\frac{2}{-0.04}$
81. $\frac{3}{-0.4}$
82. $\frac{-1.2}{-0.03}$

83. $-\frac{1}{5} + \frac{1}{6}$

84. $-\frac{3}{5} - \frac{1}{4}$

85. $\left(-\frac{3}{4}\right)\left(\frac{2}{15}\right)$

86. $-1 \div \left(-\frac{1}{4}\right)$

Use a calculator to perform the indicated operations. Round answers to three decimal places.

87. $\frac{45.37}{6}$

88. $(-345) \div (28)$

89. $(-4.3)(-4.5)$

90. $\frac{-12.34}{-3}$

91. $\frac{0}{6.345}$

92. $0 \div (34.51)$

93. $199.4 \div 0$

94. $\frac{23.44}{0}$

GETTING MORE INVOLVED

95. ***Discussion.*** If you divide \$0 among five people, how much does each person get? If you divide \$5 among zero people, how much does each person get? What do these questions illustrate?

96. ***Discussion.*** What is the difference between the non-negative numbers and the positive numbers?

97. ***Writing.*** Why do we learn multiplication of signed numbers before division?

98. ***Writing.*** Try to rewrite the rules for multiplying and dividing signed numbers without using the idea of absolute value. Are your rewritten rules clearer than the original rules?

1.5 EXPONENTIAL EXPRESSIONS AND THE ORDER OF OPERATIONS

In this section

- Arithmetic Expressions
- Exponential Expressions
- The Order of Operations

In Sections 1.3 and 1.4 you learned how to perform operations with a pair of real numbers to obtain a third real number. In this section you will learn to evaluate expressions involving several numbers and operations.

Arithmetic Expressions

The result of writing numbers in a meaningful combination with the ordinary operations of arithmetic is called an **arithmetic expression** or simply an **expression.** Consider the expressions

$$(3 + 2) \cdot 5 \qquad \text{and} \qquad 3 + (2 \cdot 5).$$

The parentheses are used as **grouping symbols** and indicate which operation to perform first. Because of the parentheses, these expressions have different values:

$$(3 + 2) \cdot 5 = 5 \cdot 5 = 25$$
$$3 + (2 \cdot 5) = 3 + 10 = 13$$

Absolute value symbols and fraction bars are also used as grouping symbols. The numerator and denominator of a fraction are treated as if each is in parentheses.

EXAMPLE 1 **Using grouping symbols**

Evaluate each expression.

a) $(3 - 6)(3 + 6)$

b) $|3 - 4| - |5 - 9|$

c) $\frac{4 - (-8)}{5 - 9}$