83. $-\frac{1}{5}+\frac{1}{6}$

84. $-\frac{3}{5}-\frac{1}{4}$

85. $\left(-\frac{3}{4}\right)\left(\frac{2}{15}\right)$

86. $-1 \div \left(-\frac{1}{4}\right)$

Use a calculator to perform the indicated operations. Round answers to three decimal places.

87. $\frac{45.37}{6}$

88. $(-345) \div (28)$

89. $(-4.3)(-4.5)$

90. $\frac{-12.34}{-3}$

91. $\frac{0}{6.345}$

92. $0 \div (34.51)$

93. $199.4 \div 0$

94. $\frac{23.44}{0}$

GETTING MORE INVOLVED

95. ***Discussion.*** If you divide \$0 among five people, how much does each person get? If you divide \$5 among zero people, how much does each person get? What do these questions illustrate?

96. ***Discussion.*** What is the difference between the non-negative numbers and the positive numbers?

97. ***Writing.*** Why do we learn multiplication of signed numbers before division?

98. ***Writing.*** Try to rewrite the rules for multiplying and dividing signed numbers without using the idea of absolute value. Are your rewritten rules clearer than the original rules?

1.5 EXPONENTIAL EXPRESSIONS AND THE ORDER OF OPERATIONS

In this section

- Arithmetic Expressions
- Exponential Expressions
- The Order of Operations

In Sections 1.3 and 1.4 you learned how to perform operations with a pair of real numbers to obtain a third real number. In this section you will learn to evaluate expressions involving several numbers and operations.

Arithmetic Expressions

The result of writing numbers in a meaningful combination with the ordinary operations of arithmetic is called an **arithmetic expression** or simply an **expression.** Consider the expressions

$$(3 + 2) \cdot 5 \qquad \text{and} \qquad 3 + (2 \cdot 5).$$

The parentheses are used as **grouping symbols** and indicate which operation to perform first. Because of the parentheses, these expressions have different values:

$$(3 + 2) \cdot 5 = 5 \cdot 5 = 25$$
$$3 + (2 \cdot 5) = 3 + 10 = 13$$

Absolute value symbols and fraction bars are also used as grouping symbols. The numerator and denominator of a fraction are treated as if each is in parentheses.

EXAMPLE 1 **Using grouping symbols**

Evaluate each expression.

a) $(3 - 6)(3 + 6)$

b) $|3 - 4| - |5 - 9|$

c) $\frac{4 - (-8)}{5 - 9}$

Solution

a) $(3 - 6)(3 + 6) = (-3)(9)$ Evaluate within parentheses first.

$= -27$ Multiply.

b) $|3 - 4| - |5 - 9| = |-1| - |-4|$ Evaluate within absolute value symbols.

$= 1 - 4$ Find the absolute values.

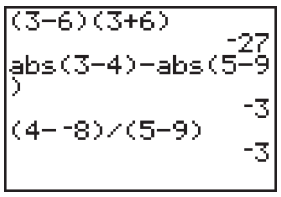
$= -3$ Subtract.

c) $\dfrac{4 - (-8)}{5 - 9} = \dfrac{12}{-4}$ Evaluate the numerator and denominator.

$= -3$ Divide. ■

calculator close-up

One advantage of a graphing calculator is that you can enter an entire expression on its display and then evaluate it. If your calculator does not allow built-up form for fractions, then you must use parentheses around the numerator and denominator as shown here.



Exponential Expressions

study tip

If you need help, do not hesitate to get it. Math has a way of building upon the past. What you learn today will be used tomorrow, and what you learn tomorrow will be used the day after. If you don't straighten out problems immediately, then you can get hopelessly lost. If you are having trouble, see your instructor to find out what help is available.

An arithmetic expression with repeated multiplication can be written by using exponents. For example,

$$2 \cdot 2 \cdot 2 = 2^3 \quad \text{and} \quad 5 \cdot 5 = 5^2.$$

The 3 in 2^3 is the number of times that 2 occurs in the product $2 \cdot 2 \cdot 2$, while the 2 in 5^2 is the number of times that 5 occurs in $5 \cdot 5$. We read 2^3 as "2 cubed" or "2 to the third power." We read 5^2 as "5 squared" or "5 to the second power." In general, an expression of the form a^n is called an **exponential expression** and is defined as follows.

Exponential Expression

For any counting number n,

$$a^n = \underbrace{a \cdot a \cdot a \cdot \ldots \cdot a}_{n \text{ factors}}.$$

We call a the **base** and n the **exponent.**

The expression a^n is read "a to the nth power." If the exponent is 1, it is usually omitted. For example, $9^1 = 9$.

EXAMPLE 2 **Using exponential notation**

Write each product as an exponential expression.

a) $6 \cdot 6 \cdot 6 \cdot 6 \cdot 6$ **b)** $(-3)(-3)(-3)(-3)$ **c)** $\dfrac{3}{2} \cdot \dfrac{3}{2} \cdot \dfrac{3}{2}$

Solution

a) $6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 = 6^5$

b) $(-3)(-3)(-3)(-3) = (-3)^4$

c) $\frac{3}{2} \cdot \frac{3}{2} \cdot \frac{3}{2} = \left(\frac{3}{2}\right)^3$

EXAMPLE 3 **Writing an exponential expression as a product**

Write each exponential expression as a product without exponents.

a) y^6 **b)** $(-2)^4$ **c)** $\left(\frac{5}{4}\right)^3$ **d)** $(-0.1)^2$

Solution

a) $y^6 = y \cdot y \cdot y \cdot y \cdot y \cdot y$

b) $(-2)^4 = (-2)(-2)(-2)(-2)$

c) $\left(\frac{5}{4}\right)^3 = \frac{5}{4} \cdot \frac{5}{4} \cdot \frac{5}{4}$

d) $(-0.1)^2 = (-0.1)(-0.1)$

To evaluate an exponential expression, write the base as many times as indicated by the exponent, then multiply the factors from left to right.

EXAMPLE 4 **Evaluating exponential expressions**

Evaluate.

a) 3^3 **b)** $(-2)^3$ **c)** $\left(\frac{2}{3}\right)^4$ **d)** $(0.4)^2$

Solution

a) $3^3 = 3 \cdot 3 \cdot 3 = 9 \cdot 3 = 27$

b) $(-2)^3 = (-2)(-2)(-2)$
$= 4(-2)$
$= -8$

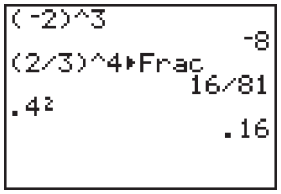
c) $\left(\frac{2}{3}\right)^4 = \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3}$
$= \frac{4}{9} \cdot \frac{2}{3} \cdot \frac{2}{3}$
$= \frac{8}{27} \cdot \frac{2}{3}$
$= \frac{16}{81}$

d) $(0.4)^2 = (0.4)(0.4) = 0.16$

CAUTION Note that $3^3 \neq 9$. We do not multiply the exponent and the base when evaluating an exponential expression.

calculator close-up

You can use the power key for any power. Most calculators also have an x^2 key that gives the second power. Note that parentheses must be used when raising a fraction to a power.



Be especially careful with exponential expressions involving negative numbers. An exponential expression with a negative base is written with parentheses around the base as in $(-2)^4$:

$$(-2)^4 = (-2)(-2)(-2)(-2) = 16$$

To evaluate $-(2^4)$, use the base 2 as a factor four times, then find the opposite:

$$-(2^4) = -(2 \cdot 2 \cdot 2 \cdot 2) = -(16) = -16$$

We often omit the parentheses in $-(2^4)$ and simply write -2^4. So

$$-2^4 = -(2^4) = -16.$$

To evaluate $-(-2)^4$, use the base -2 as a factor four times, then find the opposite:

$$-(-2)^4 = -(16) = -16$$

EXAMPLE 5 **Evaluating exponential expressions involving negative numbers**

Evaluate.

a) $(-10)^4$ **b)** -10^4

c) $-(-0.5)^2$ **d)** $-(5-8)^2$

Solution

a) $(-10)^4 = (-10)(-10)(-10)(-10)$ Use -10 as a factor four times.
$= 10{,}000$

b) $-10^4 = -(10^4)$ Rewrite using parentheses.
$= -(10{,}000)$ Find 10^4.
$= -10{,}000$ Then find the opposite of 10,000.

c) $-(-0.5)^2 = -(-0.5)(-0.5)$ Use -0.5 as a factor two times.
$= -(0.25)$
$= -0.25$

d) $-(5-8)^2 = -(-3)^2$ Evaluate within parentheses first.
$= -(9)$ Square -3 to get 9.
$= -9$ Take the opposite of 9 to get -9.

helpful hint

"Please Excuse My Dear Aunt Sally" is often used as a memory aid for the order of operations. Do Parentheses, Exponents, Multiplication, and Division, then Addition and Subtraction. Multiplication and division have equal priority. The same goes for addition and subtraction.

The Order of Operations

When we evaluate expressions, operations within grouping symbols are always performed first. For example

$$(3+2) \cdot 5 = (5) \cdot 5 = 10 \quad \text{and} \quad (2 \cdot 3)^2 = 6^2 = 36.$$

To make expressions look simpler, we often omit some or all parentheses. In this case we must agree on the order in which to perform the operations. We agree to do multiplication before addition and exponential expressions before multiplication. So

$$3 + 2 \cdot 5 = 3 + 10 = 13 \quad \text{and} \quad 2 \cdot 3^2 = 2 \cdot 9 = 18.$$

We state the complete **order of operations** in the following box.

Order of Operations

If no grouping symbols are present, evaluate expressions in the following order:

1. Evaluate each exponential expression (in order from left to right).
2. Perform multiplication and division (in order from left to right).
3. Perform addition and subtraction (in order from left to right).

For operations within grouping symbols, use the above order within the grouping symbols.

Multiplication and division have equal priority in the order of operations. If both appear in an expression, they are performed in order from left to right. The same holds for addition and subtraction. For example,

$$8 \div 4 \cdot 3 = 2 \cdot 3 = 6 \quad \text{and} \quad 9 - 3 + 5 = 6 + 5 = 11.$$

EXAMPLE 6 **Using the order of operations**

Evaluate each expression.

a) $2^3 \cdot 3^2$ **b)** $2 \cdot 5 - 3 \cdot 4 + 4^2$ **c)** $2 \cdot 3 \cdot 4 - 3^3 + \frac{8}{2}$

Solution

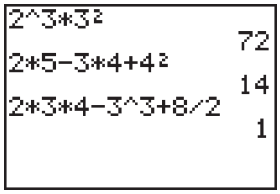
a) $2^3 \cdot 3^2 = 8 \cdot 9$ Evaluate exponential expressions before multiplying.
$= 72$

b) $2 \cdot 5 - 3 \cdot 4 + 4^2 = 2 \cdot 5 - 3 \cdot 4 + 16$ Exponential expressions first
$= 10 - 12 + 16$ Multiplication second
$= 14$ Addition and subtraction from left to right

c) $2 \cdot 3 \cdot 4 - 3^3 + \frac{8}{2} = 2 \cdot 3 \cdot 4 - 27 + \frac{8}{2}$ Exponential expressions first
$= 24 - 27 + 4$ Multiplication and division second
$= 1$ Addition and subtraction from left to right ■

calculator close-up

Most calculators follow the same order of operations shown here. Evaluate these expressions with your calculator.



When grouping symbols are used, we perform operations within grouping symbols first. The order of operations is followed within the grouping symbols.

EXAMPLE 7 **Grouping symbols and the order of operations**

Evaluate.

a) $3 - 2(7 - 2^3)$ **b)** $3 - |7 - 3 \cdot 4|$ **c)** $\frac{9 - 5 + 8}{-5^2 - 3(-7)}$

Solution

a) $3 - 2(7 - 2^3) = 3 - 2(7 - 8)$ Evaluate within parentheses first.
$= 3 - 2(-1)$
$= 3 - (-2)$ Multiply.
$= 5$ Subtract.

b) $3 - |7 - 3 \cdot 4| = 3 - |7 - 12|$ Evaluate within the absolute value symbols first.
$= 3 - |-5|$
$= 3 - 5$ Evaluate the absolute value.
$= -2$ Subtract.

c) $\dfrac{9-5+8}{-5^2-3(-7)} = \dfrac{12}{-25+21} = \dfrac{12}{-4} = -3$ Numerator and denominator are treated as if in parentheses.

When grouping symbols occur within grouping symbols, we evaluate within the innermost grouping symbols first and then work outward. In this case, brackets [ ] can be used as grouping symbols along with parentheses to make the grouping clear.

EXAMPLE 8 Grouping within grouping

Evaluate each expression.

a) $6-4[5-(7-9)]$ **b)** $-2|3-(9-5)|-|-3|$

Solution

a) $6-4[5-(7-9)] = 6-4[5-(-2)]$ Innermost parentheses first

$= 6-4[7]$ Next evaluate within the brackets.

$= 6-28$ Multiply.

$= -22$ Subtract.

b) $-2|3-(9-5)|-|-3| = -2|3-4|-|-3|$ Innermost grouping first

$= -2|-1|-|-3|$ Evaluate within the first absolute value.

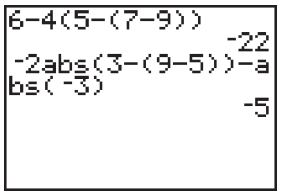
$= -2\cdot 1-3$ Evaluate absolute values.

$= -2-3$ Multiply.

$= -5$ Subtract.

calculator close-up

Graphing calculators can handle grouping symbols within grouping symbols. Since parentheses must occur in pairs, you should have the same number of left parentheses as right parentheses.



WARM-UPS

True or false? Explain your answer.

1. $(-3)^2 = -6$

2. $5-3\cdot 2 = 4$

3. $(5-3)2 = 4$

4. $|5-6| = |5|-|6|$

5. $5+6\cdot 2 = (5+6)\cdot 2$

6. $(2+3)^2 = 2^2+3^2$

7. $5-3^3 = 8$

8. $(5-3)^3 = 8$

9. $6-\dfrac{6}{2} = \dfrac{0}{2}$

10. $\dfrac{6-6}{2} = 0$

1.5 EXERCISES

Reading and Writing *After reading this section write out the answers to these questions. Use complete sentences.*

1. What is an arithmetic expression?

2. What is the purpose of grouping symbols?

3. What is an exponential expression?

4. What is the difference between -3^6 and $(-3)^6$?

5. What is the purpose of the order of operations?

6. What were the different types of grouping symbols used in this section?

Evaluate each expression. See Example 1.

7. $(4 - 3)(5 - 9)$ **8.** $(5 - 7)(-2 - 3)$

9. $|3 + 4| - |-2 - 4|$ **10.** $|-4 + 9| + |-3 - 5|$

11. $\dfrac{7 - (-9)}{3 - 5}$ **12.** $\dfrac{-8 + 2}{-1 - 1}$

13. $(-6 + 5)(7)$ **14.** $-6 + (5 \cdot 7)$

15. $(-3 - 7) - 6$ **16.** $-3 - (7 - 6)$

17. $-16 \div (8 \div 2)$ **18.** $(-16 \div 8) \div 2$

Write each product as an exponential expression. See Example 2.

19. $4 \cdot 4 \cdot 4 \cdot 4$ **20.** $1 \cdot 1 \cdot 1 \cdot 1 \cdot 1$

21. $(-5)(-5)(-5)(-5)$ **22.** $(-7)(-7)(-7)$

23. $(-y)(-y)(-y)$ **24.** $x \cdot x \cdot x \cdot x \cdot x$

25. $\dfrac{3}{7} \cdot \dfrac{3}{7} \cdot \dfrac{3}{7} \cdot \dfrac{3}{7} \cdot \dfrac{3}{7}$ **26.** $\dfrac{y}{2} \cdot \dfrac{y}{2} \cdot \dfrac{y}{2} \cdot \dfrac{y}{2}$

Write each exponential expression as a product without exponents. See Example 3.

27. 5^3 **28.** $(-8)^4$

29. b^2 **30.** $(-a)^5$

31. $\left(-\dfrac{1}{2}\right)^5$

32. $\left(-\dfrac{13}{12}\right)^3$

33. $(0.22)^4$

34. $(1.25)^6$

Evaluate each exponential expression. See Examples 4 and 5.

35. 3^4 **36.** 5^3 **37.** 0^9

38. 0^{12} **39.** $(-5)^4$ **40.** $(-2)^5$

41. $(-6)^3$ **42.** $(-12)^2$ **43.** $(10)^5$

44. $(-10)^6$ **45.** $(-0.1)^3$ **46.** $(-0.2)^2$

47. $\left(\dfrac{1}{2}\right)^3$ **48.** $\left(\dfrac{2}{3}\right)^3$ **49.** $\left(-\dfrac{1}{2}\right)^2$

50. $\left(-\dfrac{2}{3}\right)^2$ **51.** -8^2 **52.** -7^2

53. $-(-8)^4$ **54.** $-(-7)^3$

55. $-(7 - 10)^3$ **56.** $-(6 - 9)^4$

57. $(-2^2) - (3^2)$ **58.** $(-3^4) - (-5^2)$

Evaluate each expression. See Example 6.

59. $3^2 \cdot 2^2$ **60.** $5 \cdot 10^2$

61. $-3 \cdot 2 + 4 \cdot 6$ **62.** $-5 \cdot 4 - 8 \cdot 3$

63. $(-3)^3 + 2^3$ **64.** $3^2 - 5(-1)^3$

65. $-21 + 36 \div 3^2$ **66.** $-18 - 9^2 \div 3^3$

67. $-3 \cdot 2^3 - 5 \cdot 2^2$ **68.** $2 \cdot 5 - 3^2 + 4 \cdot 0$

69. $\dfrac{-8}{2} + 2 \cdot 3 \cdot 5 - 2^3$ **70.** $-4 \cdot 2 \cdot 6 - \dfrac{12}{3} + 3^3$

Evaluate each expression. See Example 7.

71. $(-3 + 4^2)(-6)$ **72.** $-3 \cdot (2^3 + 4) \cdot 5$

73. $(-3 \cdot 2 + 6)^3$ **74.** $5 - 2(-3 + 2)^3$

75. $2 - 5(3 - 4 \cdot 2)$ **76.** $(3 - 7)(4 - 6 \cdot 2)$

77. $3 - 2 \cdot |5 - 6|$ **78.** $3 - |6 - 7 \cdot 3|$

79. $(3^2 - 5) \cdot |3 \cdot 2 - 8|$

80. $|4 - 6 \cdot 3| + |6 - 9|$

81. $\dfrac{3 - 4 \cdot 6}{7 - 10}$ **82.** $\dfrac{6 - (-8)^2}{-3 - (-1)}$

83. $\dfrac{7 - 9 - 3^2}{9 - 7 - 3}$ **84.** $\dfrac{3^2 - 2 \cdot 4}{-30 + 2 \cdot 4^2}$

Evaluate each expression. See Example 8.

85. $3 + 4[9 - 6(2 - 5)]$

86. $9 + 3[5 - (3 - 6)^2]$

87. $6^2 - [(2 + 3)^2 - 10]$

88. $3[(2 - 3)^2 + (6 - 4)^2]$

89. $4 - 5 \cdot |3 - (3^2 - 7)|$

90. $2 + 3 \cdot |4 - (7^2 - 6^2)|$

91. $-2|3 - (7 - 3)| - |-9|$

92. $[3 - (2 - 4)][3 + |2 - 4|]$

Evaluate each expression. Use a calculator to check.

93. $1 + 2^3$ **94.** $(1 + 2)^3$

95. $(-2)^2 - 4(-1)(3)$ **96.** $(-2)^2 - 4(-2)(-3)$

97. $4^2 - 4(1)(-3)$ **98.** $3^2 - 4(-2)(3)$

99. $(-11)^2 - 4(5)(0)$ **100.** $(-12)^2 - 4(3)(0)$

101. $-5^2 - 3 \cdot 4^2$

102. $-6^2 - 5(-3)^2$

103. $[3 + 2(-4)]^2$

104. $[6 - 2(-3)]^2$

105. $|-1| - |-1|$

106. $4 - |1 - 7|$

107. $\dfrac{4 - (-4)}{-2 - 2}$

108. $\dfrac{3 - (-7)}{3 - 5}$

109. $3(-1)^2 - 5(-1) + 4$

110. $-2(1)^2 - 5(1) - 6$

111. $5 - 2^2 + 3^4$

112. $5 + (-2)^2 - 3^2$

113. $-2 \cdot |9 - 6^2|$

114. $8 - 3|5 - 4^2 + 1|$

115. $-3^2 - 5[4 - 2(4 - 9)]$

116. $-2[(3 - 4)^3 - 5] + 7$

117. $1 - 5|5 - (9 + 1)|$

118. $|6 - 3 \cdot 7| + |7 - (5 - 2)|$

Use a calculator to evaluate each expression.

119. $3.2^2 - 4(3.6)(-2.2)$

120. $(-4.5)^2 - 4(-2.8)(-4.6)$

121. $(5.63)^3 - [4.7 - (-3.3)^2]$

122. $9.8^3 - [1.2 - (4.4 - 9.6)^2]$

123. $\dfrac{3.44 - (-8.32)}{6.89 - 5.43}$

124. $\dfrac{-4.56 - 3.22}{3.44 - (-6.26)}$

Solve each problem.

125. ***Population of the United States.*** In 1998 the population of the United States was 270.1 million (U.S. Census Bureau, www.census.gov). If the population continues to grow at an annual rate of 0.86%, then the population in the year 2010 will be $270.1(1.0086)^{12}$ million. Find the predicted population in 2010 to the nearest tenth of a million people.

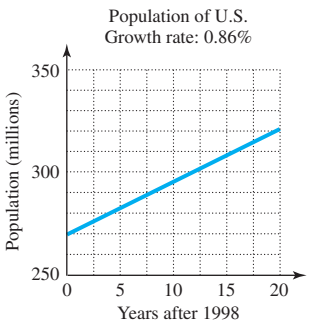


FIGURE FOR EXERCISE 125

126. ***Population of Mexico.*** In 1998 the population of Mexico was 97.2 million (World Resources 1997–1998, www.wri.org). If Mexico's population continues to grow at an annual rate of 2.0%, then the population in the year 2010 will be $97.2(1.02)^{12}$ million.

a) Find the predicted population in the year 2010 to the nearest tenth of a million people.

b) Will the U.S. or Mexico have the greater increase in population between the years 1998 and 2010? (See the previous exercise.)

GETTING MORE INVOLVED

127. ***Discussion.*** How do the expressions $(-5)^3$, $-(5^3)$, -5^3, $-(-5)^3$, and $-1 \cdot 5^3$ differ?

128. ***Discussion.*** How do the expressions $(-4)^4$, $-(4^4)$, -4^4, $-(-4)^4$, and $-1 \cdot 4^4$ differ?

In this section

- Identifying Algebraic Expressions
- Translating Algebraic Expressions
- Evaluating Algebraic Expressions
- Equations
- Applications

1.6 ALGEBRAIC EXPRESSIONS

In Section 1.5 you studied arithmetic expressions. In this section you will study expressions that are more general—expressions that involve variables.

Identifying Algebraic Expressions

Since variables (or letters) are used to represent numbers, we can use variables in arithmetic expressions. The result of combining numbers and variables with the ordinary operations of arithmetic (in some meaningful way) is called an **algebraic**