In this section

- Equations of the Form $ax + b = 0$
- Equations of the Form $ax + b = cx + d$
- Equations with Parentheses
- Applications

2.2 SOLVING GENERAL LINEAR EQUATIONS

All of the equations that we solved in Section 2.1 required only a single application of a property of equality. In this section you will solve equations that require more than one application of a property of equality.

Equations of the Form $ax + b = 0$

To solve an equation of the form $ax + b = 0$ we might need to apply both the addition property of equality and the multiplication property of equality.

EXAMPLE 1

Using the addition and multiplication properties of equality

Solve $3r - 5 = 0$.

Solution

To isolate r, first add 5 to each side, then divide each side by 3.

$$\begin{aligned} 3r - 5 &= 0 && \text{Original equation} \\ 3r - 5 + 5 &= 0 + 5 && \text{Add 5 to each side.} \\ 3r &= 5 && \text{Combine like terms.} \\ \frac{3r}{3} &= \frac{5}{3} && \text{Divide each side by 3.} \\ r &= \frac{5}{3} && \text{Simplify.} \end{aligned}$$

Checking $\frac{5}{3}$ in the original equation gives

$$3 \cdot \frac{5}{3} - 5 = 5 - 5 = 0.$$

So $\left\{\frac{5}{3}\right\}$ is the solution set to the equation. ■

helpful hint

If we divide by 3 first, we would get $r - \frac{5}{3} = 0$. Then add $\frac{5}{3}$ to each side to get $r = \frac{5}{3}$. Although we get the correct answer, we usually save division to the last step so that fractions do not appear until necessary.

CAUTION It is usually best to use the addition property of equality first and the multiplication property last.

EXAMPLE 2

Using the addition and multiplication properties of equality

Solve $-\frac{2}{3}x + 8 = 0$.

Solution

To isolate x, first subtract 8 from each side, then multiply each side by $-\frac{3}{2}$.

$$\begin{aligned} -\frac{2}{3}x + 8 &= 0 && \text{Original equation} \\ -\frac{2}{3}x + 8 - 8 &= 0 - 8 && \text{Subtract 8 from each side.} \\ -\frac{2}{3}x &= -8 && \text{Combine like terms.} \\ -\frac{3}{2}\left(-\frac{2}{3}x\right) &= -\frac{3}{2}(-8) && \text{Multiply each side by } -\tfrac{3}{2}. \\ x &= 12 && \text{Simplify.} \end{aligned}$$

study tip

As you leave class, talk to a classmate about what happened in class. What was the class about? What new terms were mentioned and what do they mean? How does this lesson fit in with the last lesson?

Checking 12 in the original equation gives

$$-\frac{2}{3}(12) + 8 = -8 + 8 = 0.$$

So {12} is the solution set to the equation. ■

Equations of the Form $ax + b = cx + d$

In solving equations our goal is to isolate the variable. We use the addition property of equality to eliminate unwanted terms. Note that it does not matter whether the variable ends up on the right or left side. For some equations we will perform fewer steps if we isolate the variable on the right side.

EXAMPLE 3 **Isolating the variable on the right side**

Solve $3w - 8 = 7w$.

Solution

To eliminate the $3w$ from the left side, we can subtract $3w$ from both sides.

$$3w - 8 = 7w \quad \text{Original equation}$$
$$3w - 8 - 3w = 7w - 3w \quad \text{Subtract } 3w \text{ from each side.}$$
$$-8 = 4w \quad \text{Simplify each side.}$$
$$-\frac{8}{4} = \frac{4w}{4} \quad \text{Divide each side by 4.}$$
$$-2 = w \quad \text{Simplify.}$$

To check, replace w with -2 in the original equation:

$$3w - 8 = 7w \quad \text{Original equation}$$
$$3(-2) - 8 = 7(-2)$$
$$-14 = -14$$

Since -2 satisfies the original equation, the solution set is $\{-2\}$. ■

You should solve the equation in Example 3 by isolating the variable on the left side to see that it takes more steps. In the next example, it is simplest to isolate the variable on the left side.

EXAMPLE 4 **Isolating the variable on the left side**

Solve $\frac{1}{2}b - 8 = 12$.

Solution

To eliminate the 8 from the left side, we add 8 to each side.

$$\frac{1}{2}b - 8 = 12 \quad \text{Original equation}$$
$$\frac{1}{2}b - 8 + 8 = 12 + 8 \quad \text{Add 8 to each side.}$$
$$\frac{1}{2}b = 20 \quad \text{Simplify each side.}$$
$$2 \cdot \frac{1}{2}b = 2 \cdot 20 \quad \text{Multiply each side by 2.}$$
$$b = 40 \quad \text{Simplify.}$$

To check, replace b with 40 in the original equation:

$$\frac{1}{2}b - 8 = 12 \quad \text{Original equation}$$
$$\frac{1}{2}(40) - 8 = 12$$
$$12 = 12$$

Since 40 satisfies the original equation, the solution set is {40}. ■

It does not matter whether the variable is isolated on the left side or the right side. However, you should decide where you want the variable isolated before you begin to solve the equation.

EXAMPLE 5 **Solving $ax + b = cx + d$**

Solve $2m - 4 = 4m - 10$.

Solution

First, we decide to isolate the variable on the left side. So we must eliminate the 4 from the left side and eliminate $4m$ from the right side:

$$2m - 4 = 4m - 10$$
$$2m - 4 + 4 = 4m - 10 + 4 \quad \text{Add 4 to each side.}$$
$$2m = 4m - 6 \quad \text{Simplify each side.}$$
$$2m - 4m = 4m - 6 - 4m \quad \text{Subtract } 4m \text{ from each side.}$$
$$-2m = -6 \quad \text{Simplify each side.}$$
$$\frac{-2m}{-2} = \frac{-6}{-2} \quad \text{Divide each side by } -2.$$
$$m = 3 \quad \text{Simplify.}$$

To check, replace m by 3 in the original equation:

$$2m - 4 = 4m - 10 \quad \text{Original equation}$$
$$2 \cdot 3 - 4 = 4 \cdot 3 - 10$$
$$2 = 2$$

Since 3 satisfies the original equation, the solution set is {3}. ■

study tip

Take good notes. Good note taking is the key to mastering the material. It helps you to concentrate in class and provides a source for review. Learn to listen effectively.

Equations with Parentheses

Equations that contain parentheses or like terms on the same side should be simplified as much as possible before applying any properties of equality.

EXAMPLE 6 **Simplifying before using properties of equality**

Solve $2(q - 3) + 5q = 8(q - 1)$.

Solution

First remove parentheses and combine like terms on each side of the equation.

$$2(q - 3) + 5q = 8(q - 1) \quad \text{Original equation}$$
$$2q - 6 + 5q = 8q - 8 \quad \text{Distributive property}$$
$$7q - 6 = 8q - 8 \quad \text{Combine like terms.}$$
$$7q - 6 + 6 = 8q - 8 + 6 \quad \text{Add 6 to each side.}$$
$$7q = 8q - 2 \quad \text{Combine like terms.}$$
$$7q - 8q = 8q - 2 - 8q \quad \text{Subtract } 8q \text{ from each side.}$$

$$-q = -2$$

$$-1(-q) = -1(-2) \qquad \text{Multiply each side by } -1.$$

$$q = 2 \qquad \text{Simplify.}$$

To check, we replace q by 2 in the original equation and simplify:

$$2(q - 3) + 5q = 8(q - 1) \qquad \text{Original equation}$$

$$2(2 - 3) + 5(2) = 8(2 - 1) \qquad \text{Replace } q \text{ by 2.}$$

$$2(-1) + 10 = 8(1)$$

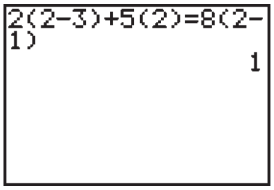
$$8 = 8$$

Because both sides have the same value, the solution set is {2}. ■

calculator close-up

You can check an equation by entering the equation on the home screen as shown here. The equal sign is in the TEST menu.

When you press ENTER, the calculator returns the number 1 if the equation is true or 0 if the equation is false. Since the calculator shows a 1, we can be sure that 2 is the solution.



Linear equations can vary greatly in appearance, but there is a strategy that you can use for solving any of them. The following strategy summarizes the techniques that we have been using in the examples. Keep it in mind when you are solving linear equations.

Strategy for Solving Equations

1. Remove parentheses and combine like terms to simplify each side as much as possible.
2. Use the addition property of equality to get like terms from opposite sides onto the same side so that they may be combined.
3. The multiplication property of equality is generally used last.
4. Multiply each side of $-x = a$ by -1 to get $x = -a$.
5. Check that the solution satisfies the original equation.

Applications

Linear equations occur in business situations where there is a fixed cost and a per item cost. A mail order company might charge \$3 plus \$2 per CD for shipping and handling. A lawyer might charge \$300 plus \$65 per hour for handling your lawsuit. AT&T might charge 10 cents per minute plus \$4.95 for long distance calls. The next example illustrates the kind of problem that can be solved in this situation.

EXAMPLE 7 **Long distance charges**

With AT&T's One Rate plan you are charged 10 cents per minute plus \$4.95 for long distance service for one month. If a long distance bill is \$8.65, then what is the number of minutes used?

Solution

Let x represent the number of minutes of calls in the month. At \$0.10 per minute, the cost for x minutes is the product $0.10x$ dollars. Since there is a fixed cost of \$4.95, an expression for the total cost is $0.10x + 4.95$ dollars. Since the total cost is \$8.65, we have $0.10x + 4.95 = 8.65$. Solve this equation to find x.

$$0.10x + 4.95 = 8.65$$

$$0.10x + 4.95 - 4.95 = 8.65 - 4.95 \quad \text{Subtract 4.95 from each side.}$$

$$0.10x = 3.70 \quad \text{Simplify.}$$

$$\frac{0.10x}{0.10} = \frac{3.70}{0.10} \quad \text{Divide each side by 0.10.}$$

$$x = 37 \quad \text{Simplify.}$$

So the bill is for 37 minutes.

WARM-UPS

True or false? Explain your answer.

1. The solution to $4x - 3 = 3x$ is 3.
2. The equation $2x + 7 = 8$ is equivalent to $2x = 1$.
3. To solve $3x - 5 = 8x + 7$, you should add 5 to each side and subtract $8x$ from each side.
4. To solve $5 - 4x = 9 + 7x$, you should subtract 9 from each side and then subtract $7x$ from each side.
5. Multiplying each side of an equation by the same nonzero real number will result in an equation that is equivalent to the original equation.
6. To isolate y in $3y - 7 = 6$, divide each side by 3 and then add 7 to each side.
7. To solve $\frac{3w}{4} = 300$, we should multiply each side by $\frac{4}{3}$.
8. The equation $-n = 9$ is equivalent to $n = -9$.
9. The equation $-y = -7$ is equivalent to $y = 7$.
10. The solution to $7x = 5x$ is 0.

2.2 EXERCISES

Reading and Writing *After reading this section, write out the answers to these questions. Use complete sentences.*

1. What properties of equality do you apply to solve $ax + b = 0$?
2. Which property of equality is usually applied last?
3. What property of equality is used to solve $-x = 8$?
4. What is usually the first step in solving an equation involving parentheses?

Solve each equation. Show your work and check your answer. See Examples 1 and 2.

5. $5a - 10 = 0$
6. $8y + 24 = 0$
7. $-3y - 6 = 0$
8. $-9w - 54 = 0$
9. $3x - 2 = 0$
10. $5y + 1 = 0$
11. $2p + 5 = 0$
12. $9z - 8 = 0$
13. $\frac{1}{2}w - 3 = 0$
14. $\frac{3}{8}t + 6 = 0$
15. $-\frac{2}{3}x + 8 = 0$
16. $-\frac{1}{7}z - 5 = 0$
17. $-m + \frac{1}{2} = 0$
18. $-y - \frac{3}{4} = 0$

19. $3p + \frac{1}{2} = 0$ **20.** $9z - \frac{1}{4} = 0$

Solve each equation. See Examples 3 and 4.

21. $6x - 8 = 4x$ **22.** $9y + 14 = 2y$

23. $4z = 5 - 2z$ **24.** $3t = t - 3$

25. $4a - 9 = 7$ **26.** $7r + 5 = 47$

27. $9 = -6 - 3b$ **28.** $13 = 3 - 10s$

29. $\frac{1}{2}w - 4 = 13$ **30.** $\frac{1}{3}q + 13 = -5$

31. $6 - \frac{1}{3}d = \frac{1}{3}d$ **32.** $9 - \frac{1}{2}a = \frac{1}{4}a$

33. $2w - 0.4 = 2$ **34.** $10h - 1.3 = 6$

35. $x = 3.3 - 0.1x$ **36.** $y = 2.4 - 0.2y$

Solve each equation. See Example 5.

37. $3x - 3 = x + 5$ **38.** $9y - 1 = 6y + 5$

39. $4 - 7d = 13 - 4d$ **40.** $y - 9 = 12 - 6y$

41. $c + \frac{1}{2} = 3c - \frac{1}{2}$ **42.** $x - \frac{1}{4} = \frac{1}{2} - x$

43. $\frac{2}{3}a - 5 = \frac{1}{3}a + 5$

44. $\frac{1}{2}t - 3 = \frac{1}{4}t - 9$

Solve each equation. See Example 6.

45. $5(a - 1) + 3 = 28$

46. $2(w + 4) - 1 = 1$

47. $2 - 3(q - 1) = 10 - (q + 1)$

48. $-2(y - 6) = 3(7 - y) - 5$

49. $2(x - 1) + 3x = 6x - 20$

50. $3 - (r - 1) = 2(r + 1) - r$

51. $2\left(y - \frac{1}{2}\right) = 4\left(y - \frac{1}{4}\right) + y$

52. $\frac{1}{2}(4m - 6) = \frac{2}{3}(6m - 9) + 3$

Solve each linear equation. Show your work and check your answer.

53. $5t = -2 + 4t$ **54.** $8y = 6 + 7y$

55. $3x - 7 = 0$ **56.** $5x + 4 = 0$

57. $-x + 6 = 5$ **58.** $-x - 2 = 9$

59. $-9 - a = -3$ **60.** $4 - r = 6$

61. $2q + 5 = q - 7$

62. $3z - 6 = 2z - 7$

63. $-3x + 1 = 5 - 2x$

64. $5 - 2x = 6 - x$

65. $-12 - 5x = -4x + 1$ **66.** $-3x - 4 = -2x + 8$

67. $3x + 0.3 = 2 + 2x$ **68.** $2y - 0.05 = y + 1$

69. $k - 0.6 = 0.2k + 1$ **70.** $2.3h + 6 = 1.8h - 1$

71. $0.2x - 4 = 0.6 - 0.8x$ **72.** $0.3x = 1 - 0.7x$

73. $-3(k - 6) = 2 - k$

74. $-2(h - 5) = 3 - h$

75. $2(p + 1) - p = 36$

76. $3(q + 1) - q = 23$

77. $7 - 3(5 - u) = 5(u - 4)$

78. $v - 4(4 - v) = -2(2v - 1)$

79. $4(x + 3) = 12$

80. $5(x - 3) = -15$ **81.** $\frac{w}{5} - 4 = -6$

82. $\frac{q}{2} + 13 = -22$ **83.** $\frac{2}{3}y - 5 = 7$

84. $\frac{3}{4}u - 9 = -6$ **85.** $4 - \frac{2n}{5} = 12$

86. $9 - \frac{2m}{7} = 19$ **87.** $-\frac{1}{3}p - \frac{1}{2} = \frac{1}{2}$

88. $-\frac{3}{4}z - \frac{2}{3} = \frac{1}{3}$

89. $3.5x - 23.7 = -38.75$

90. $3(x - 0.87) - 2x = 4.98$

Solve each problem. See Example 7.

91. ***The practice.*** A lawyer charges \$300 plus \$65 per hour for a divorce. If the total charge for Bill's divorce was \$1405, then for what number of hours did the lawyer work on the case?

92. ***The plumber.*** A plumber charges \$45 plus \$26 per hour to unclog drains. If the bill for unclogging Tamika's drain was \$123, then for how many hours did the plumber work?

93. ***Celsius temperature.*** If the air temperature in Quebec is 68° Fahrenheit, then the solution to the equation $\frac{9}{5}C + 32 = 68$ gives the Celsius temperature of the air. Find the Celsius temperature.

94. ***Fahrenheit temperature.*** Water boils at 212°F.

a) Use the accompanying graph to determine the Celsius temperature at which water boils.

b) Find the Fahrenheit temperature of hot tap water at 70°C by solving the equation

$$70 = \frac{5}{9}(F - 32).$$

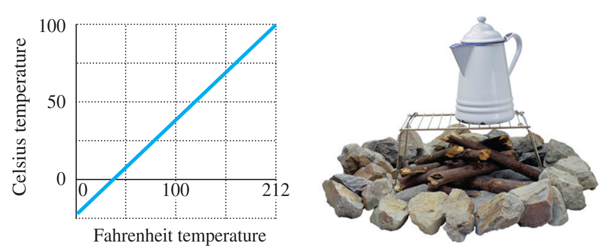


FIGURE FOR EXERCISE 94

95. ***Rectangular patio.*** If a rectangular patio has a length that is 3 feet longer than its width and a perimeter of 42 feet, then the width can be found by solving the equation $2x + 2(x + 3) = 42$. What is the width?

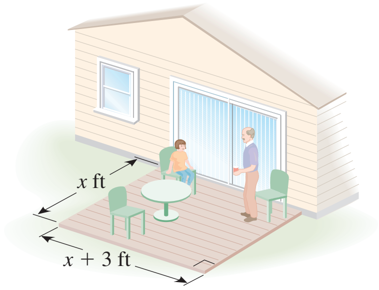


FIGURE FOR EXERCISE 95

96. ***Perimeter of a triangle.*** The perimeter of the triangle shown in the accompanying figure is 12 meters. Determine the values of x, $x + 1$, and $x + 2$ by solving the equation

$$x + (x + 1) + (x + 2) = 12.$$

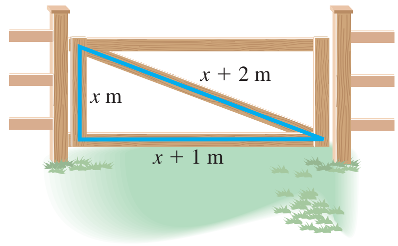


FIGURE FOR EXERCISE 96

97. ***Cost of a car.*** Jane paid 9% sales tax and a \$150 title and license fee when she bought her new Saturn for a total of \$16,009.50. If x represents the price of the car, then x satisfies $x + 0.09x + 150 = 16{,}009.50$. Find the price of the car by solving the equation.

98. ***Cost of labor.*** An electrician charged Eunice \$29.96 for a service call plus \$39.96 per hour for a total of \$169.82 for installing her electric dryer. If n represents the number of hours for labor, then n satisfies

$$39.96n + 29.96 = 169.82.$$

Find n by solving this equation.

2.3 IDENTITIES, CONDITIONAL EQUATIONS, AND INCONSISTENT EQUATIONS

In this section

- Identities
- Conditional Equations
- Inconsistent Equations
- Equations Involving Fractions
- Equations Involving Decimals
- Simplifying the Process

In this section, we will solve more equations of the type that we solved in Sections 2.1 and 2.2. However, some equations in this section have infinitely many solutions, and some have no solution.

Identities

It is easy to find equations that are satisfied by any real number that we choose as a replacement for the variable. For example, the equations

$$x \div 2 = \frac{1}{2}x, \qquad x + x = 2x, \qquad \text{and} \qquad x + 1 = x + 1$$

are satisfied by all real numbers. The equation

$$\frac{5}{x} = \frac{5}{x}$$

is satisfied by any real number except 0 because division by 0 is undefined.

Identity

An equation that is satisfied by every real number for which both sides are defined is called an **identity.**

We cannot recognize that the equation in the next example is an identity until we have simplified each side.