95. ***Rectangular patio.*** If a rectangular patio has a length that is 3 feet longer than its width and a perimeter of 42 feet, then the width can be found by solving the equation $2x + 2(x + 3) = 42$. What is the width?

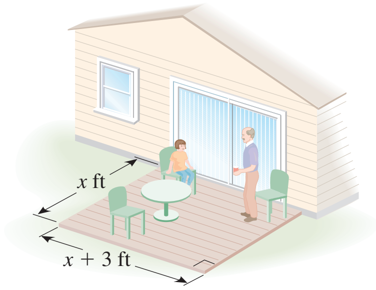

FIGURE FOR EXERCISE 95

96. ***Perimeter of a triangle.*** The perimeter of the triangle shown in the accompanying figure is 12 meters. Determine the values of x, $x + 1$, and $x + 2$ by solving the equation

$$x + (x + 1) + (x + 2) = 12.$$

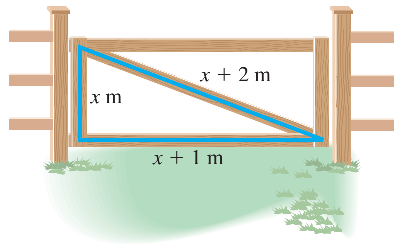

FIGURE FOR EXERCISE 96

97. ***Cost of a car.*** Jane paid 9% sales tax and a \$150 title and license fee when she bought her new Saturn for a total of \$16,009.50. If x represents the price of the car, then x satisfies $x + 0.09x + 150 = 16{,}009.50$. Find the price of the car by solving the equation.

98. ***Cost of labor.*** An electrician charged Eunice \$29.96 for a service call plus \$39.96 per hour for a total of \$169.82 for installing her electric dryer. If n represents the number of hours for labor, then n satisfies

$$39.96n + 29.96 = 169.82.$$

Find n by solving this equation.

2.3 IDENTITIES, CONDITIONAL EQUATIONS, AND INCONSISTENT EQUATIONS

In this section

- Identities
- Conditional Equations
- Inconsistent Equations
- Equations Involving Fractions
- Equations Involving Decimals
- Simplifying the Process

In this section, we will solve more equations of the type that we solved in Sections 2.1 and 2.2. However, some equations in this section have infinitely many solutions, and some have no solution.

Identities

It is easy to find equations that are satisfied by any real number that we choose as a replacement for the variable. For example, the equations

$$x \div 2 = \frac{1}{2}x, \qquad x + x = 2x, \qquad \text{and} \qquad x + 1 = x + 1$$

are satisfied by all real numbers. The equation

$$\frac{5}{x} = \frac{5}{x}$$

is satisfied by any real number except 0 because division by 0 is undefined.

Identity

An equation that is satisfied by every real number for which both sides are defined is called an **identity.**

We cannot recognize that the equation in the next example is an identity until we have simplified each side.

EXAMPLE 1 **Solving an identity**

Solve $7 - 5(x - 6) + 4 = 3 - 2(x - 5) - 3x + 28$.

Solution

We first use the distributive property to remove the parentheses:

$$7 - 5(x - 6) + 4 = 3 - 2(x - 5) - 3x + 28$$
$$7 - 5x + 30 + 4 = 3 - 2x + 10 - 3x + 28$$
$$41 - 5x = 41 - 5x \quad \text{Combine like terms.}$$

This last equation is true for any value of x because the two sides are identical. So the solution set to the original equation is the set of all real numbers.

CAUTION If you get an equation in which both sides are identical, as in Example 1, there is no need to continue to simplify the equation. If you do continue, you will eventually get $0 = 0$, from which you can still conclude that the equation is an identity.

Conditional Equations

The statement $2x + 4 = 10$ is true only on condition that we choose $x = 3$. The equation $x^2 = 4$ is satisfied only if we choose $x = 2$ or $x = -2$. These equations are called conditional equations.

study tip

Life is a game that holds many rewards for those who compete. Winning is never an accident. To win you must know the rules and have a game plan.

Conditional Equation

A **conditional equation** is an equation that is satisfied by at least one real number but is not an identity.

Every equation that we solved in Sections 2.1 and 2.2 is a conditional equation.

Inconsistent Equations

It is easy to find equations that are false no matter what number we use to replace the variable. Consider the equation

$$x = x + 1.$$

If we replace x by 3, we get $3 = 3 + 1$, which is false. If we replace x by 4, we get $4 = 4 + 1$, which is also false. Clearly, there is no number that will satisfy $x = x + 1$. Other examples of equations with no solutions include

$$x = x - 2, \qquad x - x = 5, \qquad \text{and} \qquad 0 \cdot x + 6 = 7.$$

Inconsistent Equation

An equation that has no solution is called an **inconsistent equation.**

The solution set to an inconsistent equation has no members. The set with no members is called the **empty set** and it is denoted by the symbol $\emptyset$.

EXAMPLE 2 **Solving an inconsistent equation**

Solve $2 - 3(x - 4) = 4(x - 7) - 7x$.

Solution

Use the distributive property to remove the parentheses:

$$
\begin{aligned}
2 - 3(x - 4) &= 4(x - 7) - 7x && \text{The original equation} \\
2 - 3x + 12 &= 4x - 28 - 7x && \text{Distributive property} \\
14 - 3x &= -28 - 3x && \text{Combine like terms on each side.} \\
14 - 3x + 3x &= -28 - 3x + 3x && \text{Add } 3x \text{ to each side.} \\
14 &= -28 && \text{Simplify.}
\end{aligned}
$$

The last equation is not true for any x. So the solution set to the original equation is the empty set, $\varnothing$. The equation is inconsistent. ■

Keep the following points in mind in solving equations.

Summary: Identities and Inconsistent Equations

1. An equation that is equivalent to an equation in which both sides are identical is an identity. The equation is satisfied by all real numbers for which both sides are defined.
2. An equation that is equivalent to an equation that is always false is inconsistent. The equation has no solution. The solution set is the empty set, $\varnothing$.

Equations Involving Fractions

We solved some equations involving fractions in Sections 2.1 and 2.2. Here, we will solve equations with fractions by eliminating all fractions in the first step. All of the fractions will be eliminated if we multiply each side by the least common denominator.

EXAMPLE 3 **Multiplying by the least common denominator**

Solve $\frac{y}{2} - 1 = \frac{y}{3} + 1$

Solution

The least common denominator (LCD) for the denominators 2 and 3 is 6. Since both 2 and 3 divide into 6 evenly, multiplying each side by 6 will eliminate the fractions:

$$
\begin{aligned}
6\left(\frac{y}{2} - 1\right) &= 6\left(\frac{y}{3} + 1\right) && \text{Multiply each side by 6.} \\
6 \cdot \frac{y}{2} - 6 \cdot 1 &= 6 \cdot \frac{y}{3} + 6 \cdot 1 && \text{Distributive property} \\
3y - 6 &= 2y + 6 && \text{Simplify: } 6 \cdot \frac{y}{2} = 3y \\
3y &= 2y + 12 && \text{Add 6 to each side.} \\
y &= 12 && \text{Subtract } 2y \text{ from each side.}
\end{aligned}
$$

Check 12 in the original equation:

$$\frac{12}{2} - 1 = \frac{12}{3} + 1$$

$$5 = 5$$

Since 12 satisfies the original equation, the solution set is $\{12\}$. ■

helpful hint

Note that the fractions in Example 3 will be eliminated if you multiply each side of the equation by any number divisible by both 2 and 3. For example, multiplying by 24 yields

$$
\begin{aligned}
12y - 24 &= 8y + 24 \\
4y &= 48 \\
y &= 12.
\end{aligned}
$$

Equations involving fractions are usually easier to solve if we first multiply each side by the LCD of the fractions.

Equations Involving Decimals

When an equation involves decimal numbers, we can work with the decimal numbers or we can eliminate all of the decimal numbers by multiplying both sides by 10, or 100, or 1000, and so on. Multiplying a decimal number by 10 moves the decimal point one place to the right. Multiplying by 100 moves the decimal point two places to the right, and so on.

EXAMPLE 4

An equation involving decimals

Solve $0.3p + 8.04 = 12.6$.

Solution

The largest number of decimal places appearing in the decimal numbers of the equation is two (in the number 8.04). Therefore we multiply each side of the equation by 100 because multiplying by 100 moves decimal points two places to the right:

$$\begin{aligned} 0.3p + 8.04 &= 12.6 && \text{Original equation} \\ 100(0.3p + 8.04) &= 100(12.6) && \text{Multiplication property of equality} \\ 100(0.3p) + 100(8.04) &= 100(12.6) && \text{Distributive property} \\ 30p + 804 &= 1260 \\ 30p + 804 - 804 &= 1260 - 804 && \text{Subtract 804 from each side.} \\ 30p &= 456 \\ \frac{30p}{30} &= \frac{456}{30} && \text{Divide each side by 30.} \\ p &= 15.2 \end{aligned}$$

You can use a calculator to check that

$$0.3(15.2) + 8.04 = 12.6.$$

The solution set is $\{15.2\}$.

helpful hint

After you have used one of the properties of equality on each side of an equation, be sure to simplify all expressions as much as possible before using another property of equality. This step is like making sure that all of the injured football players are removed from the field before proceeding to the next play.

EXAMPLE 5

Another equation with decimals

Solve $0.5x + 0.4(x + 20) = 13.4$

Solution

First use the distributive property to remove the parentheses:

$$\begin{aligned} 0.5x + 0.4(x + 20) &= 13.4 && \text{Original equation} \\ 0.5x + 0.4x + 8 &= 13.4 && \text{Distributive property} \\ 10(0.5x + 0.4x + 8) &= 10(13.4) && \text{Multiply each side by 10.} \\ 5x + 4x + 80 &= 134 && \text{Simplify.} \\ 9x + 80 &= 134 && \text{Combine like terms.} \\ 9x + 80 - 80 &= 134 - 80 && \text{Subtract 80 from each side.} \\ 9x &= 54 && \text{Simplify.} \\ x &= 6 && \text{Divide each side by 9.} \end{aligned}$$

Check 6 in the original equation:

$$0.5(6) + 0.4(6 + 20) = 13.4 \quad \text{Replace } x \text{ by 6.}$$
$$3 + 0.4(26) = 13.4$$
$$3 + 10.4 = 13.4$$

Since both sides of the equation have the same value, the solution set is $\{6\}$. ■

CAUTION If you multiply each side by 10 in Example 5 before using the distributive property, be careful how you handle the terms in parentheses:

$$10 \cdot 0.5x + 10 \cdot 0.4(x + 20) = 10 \cdot 13.4$$
$$5x + 4(x + 20) = 134$$

It is not correct to multiply 0.4 by 10 *and also* to multiply $x + 20$ by 10.

Simplifying the Process

It is very important to develop the skill of solving equations in a systematic way, writing down every step as we have been doing. As you become more skilled at solving equations, you will probably want to simplify the process a bit. One way to simplify the process is by writing only the result of performing an operation on each side. Another way is to isolate the variable on the side where the variable has the larger coefficient, when the variable occurs on both sides. We use these ideas in the next example and in future examples in this text.

EXAMPLE 6 **Simplifying the process**

Solve each equation.

a) $2a - 3 = 0$ **b)** $2k + 5 = 3k + 1$

Solution

a) Add 3 to each side, then divide each side by 2:

$$2a - 3 = 0$$
$$2a = 3 \quad \text{Add 3 to each side.}$$
$$a = \frac{3}{2} \quad \text{Divide each side by 2.}$$

Check that $\frac{3}{2}$ satisfies the original equation. The solution set is $\left\{\frac{3}{2}\right\}$.

b) For this equation we can get a single k on the right by subtracting $2k$ from each side. (If we subtract $3k$ from each side, we get $-k$, and then we need another step.)

$$2k + 5 = 3k + 1$$
$$5 = k + 1 \quad \text{Subtract } 2k \text{ from each side.}$$
$$4 = k \quad \text{Subtract 1 from each side.}$$

Check that 4 satisfies the original equation. The solution set is $\{4\}$. ■

study tip

I hear and I forget; I see and I remember; I do and I understand. There is no substitute for doing exercises, lots of exercises.

WARM-UPS

True or false? Explain your answer.

1. The equation $x - x = 99$ has no solution.
2. The equation $2n + 3n = 5n$ is an identity.
3. The equation $2y + 3y = 4y$ is inconsistent.

WARM-UPS

(continued)

4. All real numbers satisfy the equation $1 \div x = \frac{1}{x}$.
5. The equation $5a + 3 = 0$ is an inconsistent equation.
6. The equation $2t = t$ is a conditional equation.
7. The equation $w - 0.1w = 0.9w$ is an identity.
8. The equation $0.2x + 0.03x = 8$ is equivalent to $20x + 3x = 8$.
9. The equation $\frac{x}{x} = 1$ is an identity.
10. The solution to $3h - 8 = 0$ is $\frac{8}{3}$.

2.3 EXERCISES

Reading and Writing *After reading this section, write out the answers to these questions. Use complete sentences.*

1. What is an identity?
2. What is a conditional equation?
3. What is an inconsistent equation?
4. What is the usual first step when solving an equation involving fractions?
5. What is a good first step for solving an equation involving decimals?
6. Where should the variable be when you are finished solving an equation?

Solve each equation. Identify each as a conditional equation, an inconsistent equation, or an identity. See Examples 1 and 2.

7. $x + x = 2x$
8. $2x - x = x$
9. $a - 1 = a + 1$
10. $r + 7 = r$
11. $3y + 4y = 12y$
12. $9t - 8t = 7$
13. $-4 + 3(w - 1) = w + 2(w - 2) - 1$
14. $4 - 5(w + 2) = 2(w - 1) - 7w - 4$
15. $3(m + 1) = 3(m + 3)$
16. $5(m - 1) - 6(m + 3) = 4 - m$
17. $x + x = 2$
18. $3x - 5 = 0$
19. $2 - 3(5 - x) = 3x$
20. $3 - 3(5 - x) = 0$
21. $(3 - 3)(5 - z) = 0$
22. $(2 \cdot 4 - 8)p = 0$
23. $\frac{0}{x} = 0$
24. $\frac{2x}{2} = x$
25. $x \cdot x = x^2$
26. $\frac{2x}{2x} = 1$

Solve each equation by first eliminating the fractions. See Example 3.

27. $\frac{x}{2} + 3 = x - \frac{1}{2}$
28. $13 - \frac{x}{2} = x - \frac{1}{2}$
29. $\frac{x}{2} + \frac{x}{3} = 20$
30. $\frac{x}{2} - \frac{x}{3} = 5$
31. $\frac{w}{2} + \frac{w}{4} = 12$
32. $\frac{a}{4} - \frac{a}{2} = -5$
33. $\frac{3z}{2} - \frac{2z}{3} = -10$
34. $\frac{3m}{4} + \frac{m}{2} = -5$
35. $\frac{1}{3}p - 5 = \frac{1}{4}p$
36. $\frac{1}{2}q - 6 = \frac{1}{5}q$
37. $\frac{1}{6}v + 1 = \frac{1}{4}v - 1$
38. $\frac{1}{15}k + 5 = \frac{1}{6}k - 10$

Solve each equation by first eliminating the decimal numbers. See Examples 4 and 5.

39. $x - 0.2x = 72$
40. $x - 0.1x = 63$
41. $0.3x + 1.2 = 0.5x$

42. $0.4x - 1.6 = 0.6x$

43. $0.02x - 1.56 = 0.8x$

44. $0.6x + 10.4 = 0.08x$

45. $0.1a - 0.3 = 0.2a - 8.3$

46. $0.5b + 3.4 = 0.2b + 12.4$

47. $0.05r + 0.4r = 27$

48. $0.08t + 28.3 = 0.5t - 9.5$

49. $0.05y + 0.03(y + 50) = 17.5$

50. $0.07y + 0.08(y - 100) = 44.5$

51. $0.1x + 0.05(x - 300) = 105$

52. $0.2x - 0.05(x - 100) = 35$

Solve each equation. If you feel proficient enough, try simplifying the process, as described in Example 6.

53. $2x - 9 = 0$

54. $3x + 7 = 0$

55. $-2x + 6 = 0$

56. $-3x - 12 = 0$

57. $\frac{z}{5} + 1 = 6$

58. $\frac{s}{2} + 2 = 5$

59. $\frac{c}{2} - 3 = -4$

60. $\frac{b}{3} - 4 = -7$

61. $3 = t + 6$

62. $-5 = y - 9$

63. $5 + 2q = 3q$

64. $-4 - 5p = -4p$

65. $8x - 1 = 9 + 9x$

66. $4x - 2 = -8 + 5x$

67. $-3x + 1 = -1 - 2x$

68. $-6x + 3 = -7 - 5x$

Solve each equation.

69. $3x - 5 = 2x - 9$

70. $5x - 9 = x - 4$

71. $x + 2(x + 4) = 3(x + 3) - 1$

72. $u + 3(u - 4) = 4(u - 5)$

73. $23 - 5(3 - n) = -4(n - 2) + 9n$

74. $-3 - 4(t - 5) = -2(t + 3) + 11$

75. $0.05x + 30 = 0.4x - 5$

76. $x - 0.08x = 460$

77. $-\frac{2}{3}a + 1 = 2$

78. $-\frac{3}{4}t = \frac{1}{2}$

79. $\frac{y}{2} + \frac{y}{6} = 20$

80. $\frac{3w}{5} - 1 = \frac{w}{2} + 1$

81. $0.09x - 0.2(x + 4) = -1.46$

82. $0.08x + 0.5(x + 100) = 73.2$

83. $436x - 789 = -571$

84. $0.08x + 4533 = 10x + 69$

85. $\frac{x}{344} + 235 = 292$

86. $34(x - 98) = \frac{x}{2} + 475$

Solve each problem.

87. ***Sales commission.*** Danielle sold her house through an agent who charged 8% of the selling price. After the commission was paid, Danielle received \$117,760. If x is the selling price, then x satisfies

$$x - 0.08x = 117{,}760.$$

Solve this equation to find the selling price.

88. ***Raising rabbits.*** Before Roland sold two female rabbits, half of his rabbits were female. After the sale, only one-third of his rabbits were female. If x represents his original number of rabbits, then

$$\frac{1}{2}x - 2 = \frac{1}{3}(x - 2).$$

Solve this equation to find the number of rabbits that he had before the sale.

89. ***Eavesdropping.*** Reginald overheard his boss complaining that his federal income tax for 2000 was \$34,276.

a) Use the accompanying graph to estimate his boss's taxable income for 2000.

b) Find his boss's exact taxable income for 2000 by solving the equation

$$23{,}965.5 + 0.31(x - 105{,}950) = 34{,}276.$$

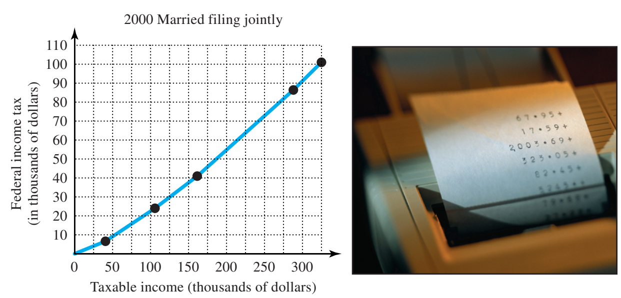

FIGURE FOR EXERCISE 89

90. ***Federal taxes.*** According to Bruce Harrell, CPA, the federal income tax for a class C corporation is found by solving a linear equation. The reason for the equation is that the amount x of federal tax is deducted before the state tax is figured, and the amount of state tax is deducted before the federal tax is figured. To find the amount of federal tax for a corporation with a taxable income of \$200,000, for which the federal tax rate is 25% and the state tax rate is 10%, Bruce must solve

$$x = 0.25[200{,}000 - 0.10(200{,}000 - x)].$$

Solve the equation for Bruce.