2.6 NUMBER, GEOMETRIC, AND UNIFORM MOTION APPLICATIONS

In this section

- Number Problems
- General Strategy for Solving Verbal Problems
- Geometric Problems
- Uniform Motion Problems

In this section, we apply the ideas of Section 2.5 to solving problems. Many of the problems can be solved by using arithmetic only and not algebra. However, remember that we are not just trying to find the answer, we are trying to learn how to apply algebra. So even if the answer is obvious to you, set the problem up and solve it by using algebra as shown in the examples.

Number Problems

Algebra is often applied to problems involving time, rate, distance, interest, or discount. **Number problems** do not involve any physical situation. In number problems we simply find some numbers that satisfy some given conditions. Number problems can provide good practice for solving more complex problems.

EXAMPLE 1

A consecutive integer problem

The sum of three consecutive integers is 48. Find the integers.

Solution

We first represent the unknown quantities with variables. Let x, $x + 1$, and $x + 2$ represent the three consecutive integers. We now write an equation that describes the problem and solve it. The equation expresses the fact that the sum of the integers is 48.

$$x + (x + 1) + (x + 2) = 48$$

$$3x + 3 = 48 \quad \text{Combine like terms.}$$

$$3x = 45 \quad \text{Subtract 3 from each side.}$$

$$x = 15 \quad \text{Divide each side by 3.}$$

$$x + 1 = 16 \quad \text{If } x \text{ is 15, then } x + 1 \text{ is 16 and } x + 2 \text{ is 17.}$$

$$x + 2 = 17$$

Because $15 + 16 + 17 = 48$, the three consecutive integers that have a sum of 48 are 15, 16, and 17.

helpful hint

Making a guess can be a good way to get familiar with the problem. For example, let's guess that the answers to Example 1 are 20, 21, and 22. Since $20 + 21 + 22 = 63$, these are not the correct numbers. But now we realize that we should use x, $x + 1$, and $x + 2$ and that the equation should be

$$x + x + 1 + x + 2 = 48.$$

General Strategy for Solving Verbal Problems

You should use the following steps as a guide for solving problems.

Strategy for Solving Problems

1. Read the problem as many times as necessary. Guessing the answer and checking it will help you understand the problem.
2. If possible, draw a diagram to illustrate the problem.
3. Choose a variable and *write* what it represents.
4. Write algebraic expressions for any other unknowns in terms of that variable.
5. Write an equation that describes the situation.
6. Solve the equation.
7. Answer the original question.
8. Check your answer in the original problem (not the equation).

Geometric Problems

Geometric problems involve geometric figures. For these problems you should always draw the figure and label it.

EXAMPLE 2

A perimeter problem

The length of a rectangular piece of property is 1 foot less than twice the width. If the perimeter is 748 feet, find the length and width.

helpful hint

To get familiar with the problem, guess that the width is 50 ft. Then the length is $2 \cdot 50 - 1$ or 99. The perimeter would be

$$2(50) + 2(99) = 298,$$

which is too small. But now we realize that we should let x be the width, $2x - 1$ be the length, and we should solve

$$2x + 2(2x - 1) = 748.$$

Solution

Let x = the width. Since the length is 1 foot less than twice the width, $2x - 1$ = the length. Draw a diagram as in Fig. 2.2. We know that $2L + 2W = P$ is the formula for perimeter of a rectangle. Substituting $2x - 1$ for L and x for W in this formula yields an equation in x:

$$2L + 2W = P$$

$$2(2x - 1) + 2(x) = 748 \quad \text{Replace } L \text{ by } 2x - 1 \text{ and } W \text{ by } x.$$

$$4x - 2 + 2x = 748 \quad \text{Remove the parentheses.}$$

$$6x - 2 = 748 \quad \text{Combine like terms.}$$

$$6x = 750 \quad \text{Add 2 to each side.}$$

$$x = 125 \quad \text{Divide each side by 6.}$$

$$2x - 1 = 249 \quad \text{If } x = 125, \text{ then } 2x - 1 = 2(125) - 1 = 249.$$

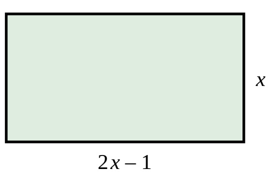


FIGURE 2.2

Check these answers by computing $2L + 2W$:

$$2(249) + 2(125) = 748$$

So the width is 125 feet, and the length is 249 feet. ■

The next geometric example involves the degree measures of angles. For this problem, the figure is given.

EXAMPLE 3

Complementary angles

In Fig. 2.3 the angle formed by the guy wire and the ground is 3.5 times as large as the angle formed by the guy wire and the antenna. Find the degree measure of each of these angles.

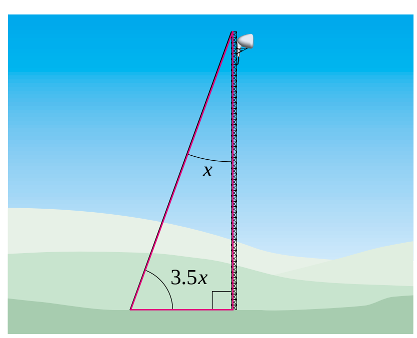


FIGURE 2.3

Solution

Let x = the degree measure of the smaller angle, and let $3.5x$ = the degree measure of the larger angle. Since the antenna meets the ground at a 90° angle, the sum of the degree measures of the other two angles of the right triangle is 90°. (They are complementary angles.) So we have the following equation:

$$x + 3.5x = 90$$
$$4.5x = 90 \quad \text{Combine like terms.}$$
$$x = 20 \quad \text{Divide each side by 4.5.}$$
$$3.5x = 70 \quad \text{Find the other angle.}$$

Check: 70° is 3.5 · 20° and 20° + 70° = 90°. So the smaller angle is 20°, and the larger angle is 70°.

Uniform Motion Problems

Problems involving motion at a constant rate are called **uniform motion problems.** In uniform motion problems we often use an average rate when the actual rate is not constant. For example, you can drive all day and average 50 miles per hour, but you are not driving at a constant 50 miles per hour.

EXAMPLE 4

Finding the rate

Bridgette drove her car for 2 hours on an icy road. When the road cleared up, she increased her speed by 35 miles per hour and drove 3 more hours, completing her 255-mile trip. How fast did she travel on the icy road?

helpful hint

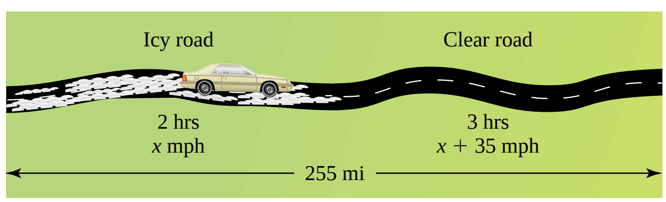
To get familiar with the problem, guess that she traveled 20 mph on the icy road and 55 mph (20 + 35) on the clear road. Her total distance would be

$$20 \cdot 2 + 55 \cdot 3 = 205 \text{ mi.}$$

Of course this is not correct, but now you are familiar with the problem.

Solution

It is helpful to draw a diagram and then make a table to classify the given information. Remember that $D = RT$.



	Rate	Time	Distance
Icy road	$x \frac{\text{mi}}{\text{hr}}$	2 hr	$2x$ mi
Clear road	$x + 35 \frac{\text{mi}}{\text{hr}}$	3 hr	$3(x + 35)$ mi

The equation expresses the fact that her total distance traveled was 255 miles:

$$\text{Icy road distance} + \text{clear road distance} = \text{total distance}$$

$$2x + 3(x + 35) = 255$$
$$2x + 3x + 105 = 255$$
$$5x + 105 = 255$$
$$5x = 150$$
$$x = 30$$
$$x + 35 = 65$$

If she drove at 30 miles per hour for 2 hours on the icy road, she went 60 miles. If she drove at 65 miles per hour for 3 hours on the clear road, she went 195 miles. Since $60 + 195 = 255$, we can be sure that her speed on the icy road was 30 mph.

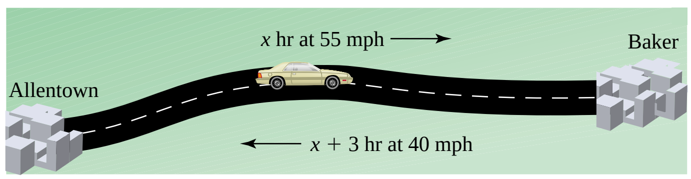
In the next uniform motion problem we find the time.

EXAMPLE 5 **Finding the time**

Pierce drove from Allentown to Baker, averaging 55 miles per hour. His journey back to Allentown using the same route took 3 hours longer because he averaged only 40 miles per hour. How long did it take him to drive from Allentown to Baker? What is the distance between Allentown and Baker?

Solution

Draw a diagram and then make a table to classify the given information. Remember that $D = RT$.



	Rate	Time	Distance
Going	$55 \frac{\text{mi}}{\text{hr}}$	x hr	$55x$ mi
Returning	$40 \frac{\text{mi}}{\text{hr}}$	$x + 3$ hr	$40(x + 3)$ mi

We can write an equation expressing the fact that the distance either way is the same:

$$\text{Distance going} = \text{distance returning}$$
$$55x = 40(x + 3)$$
$$55x = 40x + 120$$
$$15x = 120$$
$$x = 8$$

The trip from Allentown to Baker took 8 hours. The distance between Allentown and Baker is $55 \cdot 8$, or 440 miles.

study tip

When taking a test, put a check mark beside every question that you have answered and checked. When you have finished the test, then you can go back and spend the remaining time on the problems that are not yet checked. You won't waste time reworking problems that you know are correct.

WARM-UPS

True or false? Explain your answer.

1. The first step in solving a word problem is to write the equation.
2. You should always write down what the variable represents.
3. Diagrams and tables are used as aids in solving problems.
4. To represent two consecutive odd integers, we use x and $x + 1$.
5. If $5x$ is 2 miles more than $3(x + 20)$, then $5x + 2 = 3(x + 20)$.
6. We can represent two numbers with a sum of 6 by x and $6 - x$.
7. Two numbers that differ by 7 can be represented by x and $x + 7$.

WARM-UPS

(continued)

8. The degree measures of two complementary angles can be represented by x and $90 - x$.

9. The degree measures of two supplementary angles can be represented by x and $x + 180$.

10. If x is half as large as $x + 50$, then $2x = x + 50$.

2.6 EXERCISES

Reading and Writing *After reading this section, write out the answers to these questions. Use complete sentences.*

1. What types of problems are discussed in this section?

2. Why do we solve number problems?

3. What is uniform motion?

4. What are supplementary angles?

5. What are complementary angles?

6. What should you always do when solving a geometric problem?

Show a complete solution to each problem. See Example 1.

7. ***Consecutive integers.*** Find three consecutive integers whose sum is 141.

8. ***Consecutive even integers.*** Find three consecutive even integers whose sum is 114.

9. ***Consecutive odd integers.*** Two consecutive odd integers have a sum of 152. What are the integers?

10. ***Consecutive odd integers.*** Four consecutive odd integers have a sum of 120. What are the integers?

11. ***Consecutive integers.*** Find four consecutive integers whose sum is 194.

12. ***Consecutive even integers.*** Find four consecutive even integers whose sum is 340.

Show a complete solution to each problem. See Examples 2 and 3.

13. ***Olympic swimming.*** If an Olympic swimming pool is twice as long as it is wide and the perimeter is 150 meters, then what are the length and width?

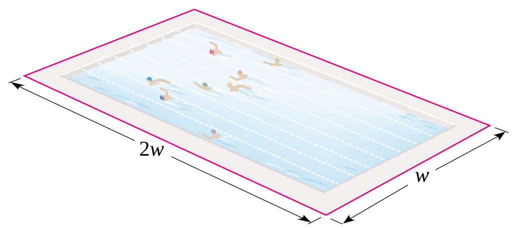


FIGURE FOR EXERCISE 13

study tip

Don't spend too much time on a single problem. If you get stuck on a problem, look at some examples in the text, move on to the next problem, or get help. It is often helpful to work some other problems and then come back to that one pesky problem.

14. ***Wimbledon tennis.*** If the perimeter of a tennis court is 228 feet and the length is 6 feet longer than twice the width, then what are the length and width?

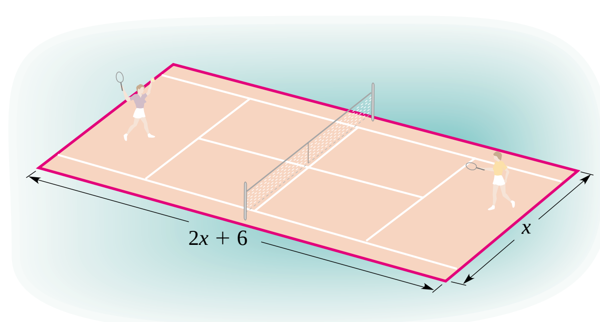


FIGURE FOR EXERCISE 14

15. ***Framed.*** Julia framed an oil painting that her uncle gave her. The painting was 4 inches longer than it was wide, and it took 176 inches of frame molding. What were the dimensions of the picture?

16. ***Industrial triangle.*** Geraldo drove his truck from Indianapolis to Chicago, then to St. Louis, and then back to Indianapolis. He observed that the second side of his triangular route was 81 miles short of being twice as long as the first side and that the third side was 61 miles longer than the first side. If he traveled a total of 720 miles, then how long is each side of this triangular route?

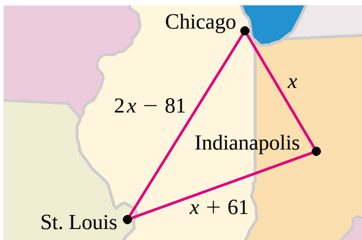


FIGURE FOR EXERCISE 16

17. ***Triangular banner.*** A banner in the shape of an isosceles triangle has a base that is 5 inches shorter than either of the equal sides. If the perimeter of the banner is 34 inches, then what is the length of the equal sides?

18. ***Border paper.*** Dr. Good's waiting room is 8 feet longer than it is wide. When Vincent wallpapered Dr. Good's waiting room, he used 88 feet of border paper. What are the dimensions of Dr. Good's waiting room?

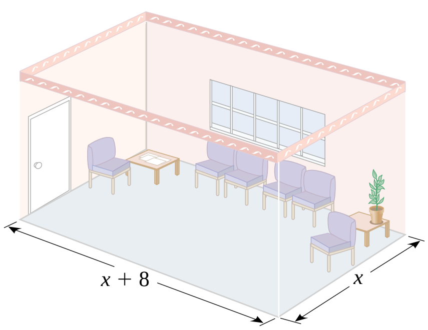


FIGURE FOR EXERCISE 18

19. ***Ramping up.*** A civil engineer is planning a highway overpass as shown in the figure. Find the degree measure of the angle marked w.

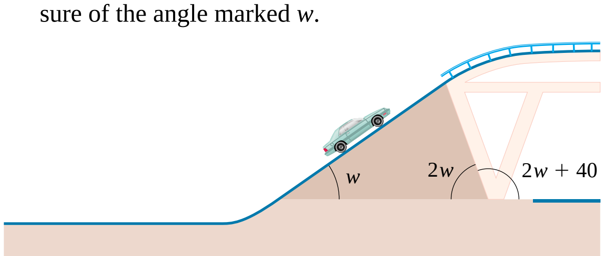


FIGURE FOR EXERCISE 19

20. ***Ramping down.*** For the other side of the overpass, the engineer has drawn the plans shown in the figure. Find the degree measure of the angle marked z.

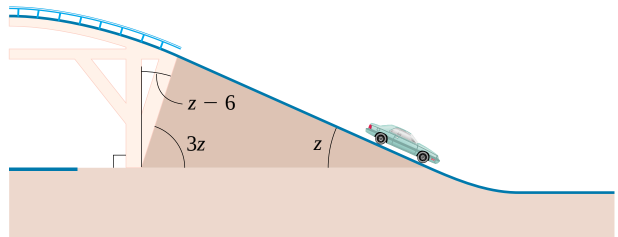


FIGURE FOR EXERCISE 20

Show a complete solution to each problem. See Examples 4 and 5.

21. ***Highway miles.*** Bret drove for 4 hours on the freeway, then decreased his speed by 20 miles per hour and drove for 5 more hours on a country road. If his total trip was 485 miles, then what was his speed on the freeway?

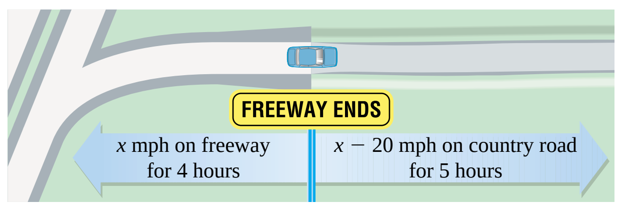


FIGURE FOR EXERCISE 21

22. ***Walking and running.*** On Saturday morning, Lynn walked for 2 hours and then ran for 30 minutes. If she ran twice as fast as she walked and she covered 12 miles altogether, then how fast did she walk?

23. ***Driving all night.*** Kathryn drove her rig 5 hours before dawn and 6 hours after dawn. If her average speed was 5 miles per hour more in the dark and she covered 630 miles altogether, then what was her speed after dawn?

24. ***Commuting to work.*** On Monday, Roger drove to work in 45 minutes. On Tuesday he averaged 12 miles per hour more, and it took him 9 minutes less to get to work. How far does he travel to work?

25. ***Head winds.*** A jet flew at an average speed of 640 mph from Los Angeles to Chicago. Because of head winds the jet averaged only 512 mph on the return trip, and the return trip took 48 minutes longer. How many hours was the flight from Chicago to Los Angeles? How far is it from Chicago to Los Angeles?

26. ***Ride the Peaks.*** Penny's bicycle trip from Colorado Springs to Pikes Peak took 1.5 hours longer than the return trip to Colorado Springs. If she averaged 6 mph on the way to Pikes Peak and 15 mph for the return trip, then how long was the ride from Colorado Springs to Pikes Peak?

Solve each problem.

27. ***Super Bowl score.*** The 1977 Super Bowl was played in the Rose Bowl in Pasadena. In that football game the Oakland Raiders scored 18 more points than the Minnesota Vikings. If the total number of points scored was 46, then what was the final score for the game?

28. ***Top three companies.*** Revenues for the top three companies in 1997, General Motors, Ford, and Exxon, totaled $453 billion (Fortune 500 List, www.fortune.com). If Ford's revenue was $31 billion greater that Exxon's, and General Motor's revenue was $25 billion greater than Ford's, then what was the 1997 revenue for each company?

29. ***Idabel to Lawton.*** Before lunch, Sally drove from Idabel to Ardmore, averaging 50 mph. After lunch she continued on to Lawton, averaging 53 mph. If her driving time after lunch was 1 hour less than her driving time before lunch and the total trip was 256 miles, then how many hours did she drive before lunch? How far is it from Ardmore to Lawton?

30. ***Norfolk to Chadron.*** On Monday, Chuck drove from Norfolk to Valentine, averaging 47 mph. On Tuesday, he continued on to Chadron, averaging 69 mph. His driving time on Monday was 2 hours longer than his driving time on Tuesday. If the total distance from Norfolk to Chadron is 326 miles, then how many hours did he drive on Monday? How far is it from Valentine to Chadron?

31. ***Golden oldies.*** Joan Crawford, John Wayne, and James Stewart were born in consecutive years (*Doubleday Almanac*). Joan Crawford was the oldest of the three, and James Stewart was the youngest. In 1950, after all three had their birthdays, the sum of their ages was 129. In what years were they born?

32. ***Leading men.*** Bob Hope was born 2 years after Clark Gable and 2 years before Henry Fonda (*Doubleday Almanac*). In 1951, after all three of them had their birthdays, the sum of their ages was 144. In what years were they born?

33. ***Trimming a garage door.*** A carpenter used 30 ft of molding in three pieces to trim a garage door. If the long piece was 2 ft longer than twice the length of each shorter piece, then how long was each piece?

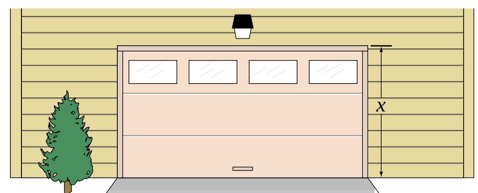


FIGURE FOR EXERCISE 33

34. ***Fencing dog pens.*** Clint is constructing two adjacent rectangular dog pens. Each pen will be three times as long as it is wide, and the pens will share a common long side. If Clint has 65 ft of fencing, what are the dimensions of each pen?

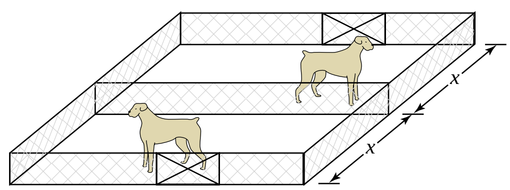


FIGURE FOR EXERCISE 34

2.7 DISCOUNT, INVESTMENT, AND MIXTURE APPLICATIONS

In this section

- Discount Problems
- Commission Problems
- Investment Problems
- Mixture Problems

In this section, we continue our study of applications of algebra. The problems in this section involve percents.

Discount Problems

When an item is sold at a discount, the amount of the discount is usually described as being a percentage of the original price. The percentage is called the **rate of discount.** Multiplying the rate of discount and the original price gives the amount of the discount.

EXAMPLE 1 **Finding the original price**

Ralph got a 12% discount when he bought his new 1999 Corvette Coupe. If the amount of his discount was $4584, then what was the original price of the Corvette?