Solve each problem.

27. Super Bowl score. The 1977 Super Bowl was played in the Rose Bowl in Pasadena. In that football game the Oakland Raiders scored 18 more points than the Minnesota Vikings. If the total number of points scored was 46, then what was the final score for the game?

28. Top three companies. Revenues for the top three companies in 1997, General Motors, Ford, and Exxon, totaled $453 billion (Fortune 500 List, www.fortune.com). If Ford's revenue was $31 billion greater that Exxon's, and General Motor's revenue was $25 billion greater than Ford's, then what was the 1997 revenue for each company?

29. Idabel to Lawton. Before lunch, Sally drove from Idabel to Ardmore, averaging 50 mph. After lunch she continued on to Lawton, averaging 53 mph. If her driving time after lunch was 1 hour less than her driving time before lunch and the total trip was 256 miles, then how many hours did she drive before lunch? How far is it from Ardmore to Lawton?

30. Norfolk to Chadron. On Monday, Chuck drove from Norfolk to Valentine, averaging 47 mph. On Tuesday, he continued on to Chadron, averaging 69 mph. His driving time on Monday was 2 hours longer than his driving time on Tuesday. If the total distance from Norfolk to Chadron is 326 miles, then how many hours did he drive on Monday? How far is it from Valentine to Chadron?

31. Golden oldies. Joan Crawford, John Wayne, and James Stewart were born in consecutive years (*Doubleday Almanac*). Joan Crawford was the oldest of the three, and James Stewart was the youngest. In 1950, after all three had their birthdays, the sum of their ages was 129. In what years were they born?

32. Leading men. Bob Hope was born 2 years after Clark Gable and 2 years before Henry Fonda (*Doubleday Almanac*). In 1951, after all three of them had their birthdays, the sum of their ages was 144. In what years were they born?

33. Trimming a garage door. A carpenter used 30 ft of molding in three pieces to trim a garage door. If the long piece was 2 ft longer than twice the length of each shorter piece, then how long was each piece?
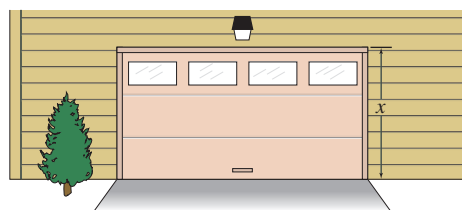

FIGURE FOR EXERCISE 33

34. Fencing dog pens. Clint is constructing two adjacent rectangular dog pens. Each pen will be three times as long as it is wide, and the pens will share a common long side. If Clint has 65 ft of fencing, what are the dimensions of each pen?
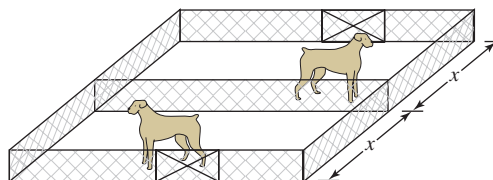

FIGURE FOR EXERCISE 34

2.7 DISCOUNT, INVESTMENT, AND MIXTURE APPLICATIONS

In this section, we continue our study of applications of algebra. The problems in this section involve percents.

Discount Problems

When an item is sold at a discount, the amount of the discount is usually described as being a percentage of the original price. The percentage is called the **rate of discount.** Multiplying the rate of discount and the original price gives the amount of the discount.

EXAMPLE 1

Finding the original price

Ralph got a 12% discount when he bought his new 1999 Corvette Coupe. If the amount of his discount was $4584, then what was the original price of the Corvette?

Solution

Let x represent the original price. The discount is found by multiplying the 12% rate of discount and the original price:

$$\text{rate of discount} \cdot \text{original price} = \text{amount of discount}$$

$$0.12x = 4584$$

$$x = \frac{4584}{0.12} \qquad \text{Divide each side by 0.12.}$$

$$x = 38{,}200$$

To check, find 12% of $38,200. Since $0.12 \cdot 38{,}200 = 4584$, the original price of the Corvette was $38,200. ■

E X A M P L E   2

Finding the original price

When Susan bought her new car, she also got a discount of 12%. She paid $17,600 for her car. What was the original price of Susan's car?

Solution

Let x represent the original price for Susan's car. The amount of discount is 12% of x, or $0.12x$. We can write an equation expressing the fact that the original price minus the discount is the price Susan paid.

$$\text{Original price} - \text{discount} = \text{sale price}$$

$$x - 0.12x = 17{,}600$$

$$0.88x = 17{,}600 \qquad 1.00x - 0.12x = 0.88x$$

$$x = \frac{17{,}600}{0.88} \qquad \text{Divide each side by 0.88.}$$

$$x = \$20{,}000$$

> **helpful hint**
>
> To get familiar with the problem, guess that the original price was $30,000. Then her discount is 0.12(30,000) or $3600. The price she paid would be $30,000 − 3600$ or $26,400, which is incorrect.

Check: 12% of $20,000 is $2400, and $20{,}000 - \$2400 = \$17{,}600$. The original price of Susan's car was $20,000. ■

Commission Problems

A salesperson's commission for making a sale is often a percentage of the selling price. **Commission problems** are very similar to other problems involving percents. The commission is found by multiplying the rate of commission and the selling price.

E X A M P L E   3

Real estate commission

Sarah is selling her house through a real estate agent whose commission rate is 7%. What should the selling price be so that Sarah can get the $83,700 she needs to pay off the mortgage?

Solution

Let x be the selling price. The commission is 7% of x (not 7% of $83,700). Sarah receives the selling price less the sales commission:

$$\text{Selling price} - \text{commission} = \text{Sarah's share}$$

$$x - 0.07x = 83{,}700$$

$$0.93x = 83{,}700 \qquad 1.00x - 0.07x = 0.93x$$

$$x = \frac{83{,}700}{0.93}$$

$$x = 90{,}000$$

Check: 7% of $90,000 is $6300, and $90,000 − $6300 = $83,700. So the house should sell for $90,000. ■

Investment Problems

The interest on an investment is a percentage of the investment, just as the sales commission is a percentage of the sale amount. However, in **investment problems** we must often account for more than one investment at different rates. So it is a good idea to make a table, as in the next example.

E X A M P L E   4

Diversified investing

Ruth Ann invested some money in a certificate of deposit with an annual yield of 9%. She invested twice as much in a mutual fund with an annual yield of 10%. Her interest from the two investments at the end of the year was $232. How much was invested at each rate?

Solution

When there are many unknown quantities, it is often helpful to identify them in a table. Since the time is 1 year, the amount of interest is the product of the interest rate and the amount invested.

	Interest rate	Amount invested	Interest for 1 year
CD	9%	x	$0.09x$
Mutual fund	10%	$2x$	$0.10(2x)$

Since the total interest from the investments was $232, we can write the following equation:

$$\text{CD interest} + \text{mutual fund interest} = \text{total interest}$$
$$0.09x + 0.10(2x) = 232$$
$$0.09x + 0.20x = 232$$
$$0.29x = 232$$
$$x = \frac{232}{0.29}$$
$$x = \$800$$
$$2x = \$1600$$

To check, we find the total interest:

$$0.09(800) + 0.10(1600) = 72 + 160$$
$$= 232$$

So Ruth Ann invested $800 at 9% and $1600 at 10%. ■

Mixture Problems

Mixture problems are concerned with the result of mixing two quantities, each of which contains another substance. Notice how similar the following mixture problem is to the last investment problem.

E X A M P L E   5

Mixing milk

How many gallons of milk containing 4% butterfat must be mixed with 80 gallons of 1% milk to obtain 2% milk?
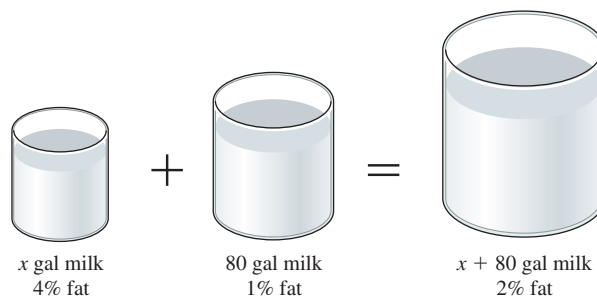
Solution

It is helpful to draw a diagram and then make a table to classify the given information.

x gal milk   80 gal milk   *x* + 80 gal milk
4% fat   1% fat   2% fat

<table>
<thead>
<tr><th></th><th>Percentage of fat</th><th>Amount of milk</th><th>Amount of fat</th></tr>
</thead>
<tbody>
<tr><td>4% milk</td><td>4%</td><td>x</td><td>$0.04x$</td></tr>
<tr><td>1% milk</td><td>1%</td><td>80</td><td>$0.01(80)$</td></tr>
<tr><td>2% milk</td><td>2%</td><td>$x + 80$</td><td>$0.02(x + 80)$</td></tr>
</tbody>
</table>

The equation expresses the fact that the total fat from the first two types of milk is the same as the fat in the mixture:

Fat in 4% milk + fat in 1% milk = fat in 2% milk

$$0.04x + 0.01(80) = 0.02(x + 80)$$
$$0.04x + 0.8 = 0.02x + 1.6 \qquad \text{Simplify.}$$
$$100(0.04x + 0.8) = 100(0.02x + 1.6) \qquad \text{Multiply each side by 100.}$$
$$4x + 80 = 2x + 160 \qquad \text{Distributive property.}$$
$$2x + 80 = 160 \qquad \text{Subtract } 2x \text{ from each side.}$$
$$2x = 80 \qquad \text{Subtract 80 from each side.}$$
$$x = 40 \qquad \text{Divide each side by 2.}$$

To check, calculate the total fat:

$$2\% \text{ of 120 gallons} = 0.02(120) = 2.4 \text{ gallons of fat}$$
$$0.04(40) + 0.01(80) = 1.6 + 0.8 = 2.4 \text{ gallons of fat}$$

So we mix 40 gallons of 4% milk with 80 gallons of 1% milk to get 120 gallons of 2% milk. ■

In mixture problems, the solutions might contain fat, alcohol, salt, or some other substance. We always assume that the substance neither appears nor disappears in the process. For example, if there are 3 grams of salt in one glass of water and 2 grams in another, then there are exactly 5 grams in a mixture of the two.

helpful hint

To get familiar with the problem, guess that we need 100 gal of 4% milk. Mixing that with 80 gal of 1% milk would produce 180 gal of 2% milk. Now the two milks separately have

0.04(100) + 0.01(80)

or 4.8 gal of fat. Together the amount of fat is 0.02(180) or 3.6 gal. Since these amounts are not equal, our guess is incorrect.

study tip

Don't expect to understand a new topic the first time that you see it. Learning mathematics takes time, patience, and repetition. Keep reading the text, asking questions, and working problems. Someone once said, "All mathematics is easy once you understand it."

True or false? Explain your answer.

1. If the original price is w and the discount is 8%, then the selling price is $w - 0.08w$.

2. If x is the selling price and the commission is 8% of the selling price, then the commission is $0.08x$.

3. If you need $40,000 for your house and the agent gets 10% of the selling price, then the agent gets $4000, and the house sells for $44,000.

4. If you mix 10 liters of a 20% acid solution with x liters of a 30% acid solution, then the total amount of acid is $2 + 0.3x$ liters.

5. A 10% acid solution mixed with a 14% acid solution results in a 24% acid solution.

6. If a TV costs x dollars and sales tax is 5%, then the total bill is $1.05x$ dollars.

2.7   EXERCISES

Reading and Writing   *After reading this section, write out the answers to these questions. Use complete sentences.*

1. What types of problems are discussed in this section?

2. What is the difference between discount and rate of discount?

3. What is the relationship between discount, original price, rate of discount, and sale price?

4. What do mixture problems and investment problems have in common?

5. Why do we make a table when solving certain problems.

6. What is the relationship between amount of interest, amount invested, and interest rate?

Show a complete solution to each problem. See Examples 1 and 2.

7. *Close-out sale.* At a 25% off sale, Jose saved $80 on a 19-inch Panasonic TV. What was the original price of the television.

8. *Big bike.* A 12% discount on a Giant Perigee saved Melanie $46.68. What was the original price of the bike?

9. *Circuit city.* After getting a 20% discount, Robert paid $320 for a Pioneer CD player for his car. What was the original price of the CD player?

10. *Chrysler Sebring.* After getting a 15% discount on the price of a new Chrysler Sebring convertible, Helen paid $27,000. What was the original price of the convertible?

Show a complete solution to each problem. See Example 3.

11. *Selling price of a home.* Kirk wants to get $72,000 for his house. The real estate agent gets a commission equal to 10% of the selling price for selling the house. What should the selling price be?

FIGURE FOR EXERCISE 11

12. *Horse trading.* Gene is selling his palomino at an auction. The auctioneer's commission is 10% of the selling price. If Gene still owes $810 on the horse, then what must the horse sell for so that Gene can pay off his loan?

13. *Sales tax collection.* Merilee sells tomatoes at a roadside stand. Her total receipts including the 7% sales tax were $462.24. What amount of sales tax did she collect?

14. *Toyota Corolla.* Gwen bought a new Toyota Corolla. The selling price plus the 8% state sales tax was $15,714. What was the selling price?

Show a complete solution to each problem. See Example 4.

15. *Wise investments.* Wiley invested some money in the Berger 100 Fund and $3000 more than that amount in the Berger 101 Fund. For the year he was in the fund, the 100 Fund paid 18% simple interest and the 101 Fund paid 15% simple interest. If the income from the two investments totaled $3750 for one year, then how much did he invest in each fund?

16. *Loan shark.* Becky lent her brother some money at 8% simple interest, and she lent her sister twice as much at twice the interest rate. If she received a total of 20 cents interest, then how much did she lend to each of them?

17. *Investing in bonds.* David split his $25,000 inheritance between Fidelity Short-Term Bond Fund with an annual yield of 5% and T. Rowe Price Tax-Free Short-Intermediate Fund with an annual yield of 4%. If his total income for one year on the two investments was $1140, then how much did he invest in each fund?

18. *High-risk funds.* Of the $50,000 that Natasha pocketed on her last real estate deal, $20,000 went to charity. She invested part of the remainder in Dreyfus New Leaders Fund with an annual yield of 16% and the rest in Templeton Growth Fund with an annual yield of 25%. If she made $6060 on these investments in one year, then how much did she invest in each fund?

Show a complete solution to each problem. See Example 5.

19. *Mixing milk.* How many gallons of milk containing 1% butterfat must be mixed with 30 gallons of milk containing 3% butterfat to obtain a mixture containing 2% butterfat?
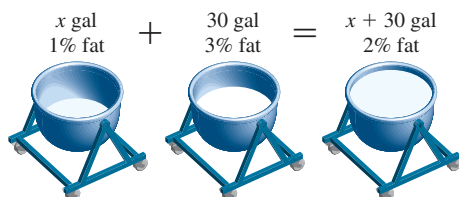

| x gal 1% fat | + | 30 gal 3% fat | = | $x + 30$ gal 2% fat |

FIGURE FOR EXERCISE 19

20. *Acid solutions.* How many gallons of a 5% acid solution should be mixed with 30 gallons of a 10% acid solution to obtain a mixture that is 8% acid?

21. *Alcohol solutions.* Gus has on hand a 5% alcohol solution and a 20% alcohol solution. He needs 30 liters of a 10% alcohol solution. How many liters of each solution should he mix together to obtain the 30 liters?

22. *Adjusting antifreeze.* Angela needs 20 quarts of 50% antifreeze solution in her radiator. She plans to obtain this by mixing some pure antifreeze with an appropriate amount of a 40% antifreeze solution. How many quarts of each should she use?
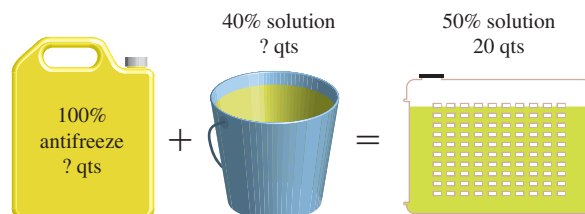

FIGURE FOR EXERCISE 22

Solve each problem.

23. *Registered voters.* If 60% of the registered voters of Lancaster County voted in the November election and 33,420 votes were cast, then how many registered voters are there in Lancaster County?

FIGURE FOR EXERCISE 23

24. *Tough on crime.* In a random sample of voters, 594 respondents said that they favored passage of a $33 billion crime bill. If the number in favor of the crime bill was 45% of the number of voters in the sample, then how many voters were in the sample?

25. *Ford Taurus.* At an 8% sales tax rate, the sales tax on Peter's new Ford Taurus was $1200. What was the price of the car?

26. *Taxpayer blues.* Last year, Faye paid 24% of her income to taxes. If she paid $9600 in taxes, then what was her income?

27. *Making a profit.* A retail store buys shirts for $8 and sells them for $14. What percent increase is this?

28. *Monitoring AIDS.* If 28 new AIDS cases were reported in Landon County this year and 35 new cases were reported last year, then what percent decrease in new cases is this?

29. *High school integration.* Wilson High School has 400 students, of whom 20% are African American. The school board plans to merge Wilson High with Jefferson High. This one school will then have a student population that is 44% African American. If Jefferson currently has a student population that is 60% African American, then how many students are at Jefferson?

30. *Junior high integration.* The school board plans to merge two junior high schools into one school of 800 students in which 40% of the students will be Caucasian. One of the schools currently has 58% Caucasian students; the other has only 10% Caucasian students. How many students are in each of the two schools?

31. *Hospital capacity.* When Memorial Hospital is filled to capacity, it has 18 more people in semiprivate rooms (two patients to a room) than in private rooms. The room rates are $200 per day for a private room and $150 per day for a semiprivate room. If the total receipts for rooms is $17,400 per day when all are full, then how many rooms of each type does the hospital have?

32. *Public relations.* Memorial Hospital is planning an advertising campaign. It costs the hospital $3000 each time a television ad is aired and $2000 each time a radio ad is aired. The administrator wants to air 60 more television ads than radio ads. If the total cost of airing the ads is $580,000, then how many ads of each type will be aired?

33. *Mixed nuts.* Cashews sell for $4.80 per pound, and pistachios sell for $6.40 per pound. How many pounds of pistachios should be mixed with 20 pounds of cashews to get a mixture that sells for $5.40 per pound?

34. *Premium blend.* Premium coffee sells for $6.00 per pound, and regular coffee sells for $4.00 per pound. How many pounds of each type of coffee should be blended to obtain 100 pounds of a blend that sells for $4.64 per pound?

35. *Nickels and dimes.* Candice paid her library fine with 10 coins consisting of nickels and dimes. If the fine was $0.80, then how many of each type of coin did she use?

36. *Dimes and quarters.* Jeremy paid for his breakfast with 36 coins consisting of dimes and quarters. If the bill was $4.50, then how many of each type of coin did he use?

37. *Cooking oil.* Crisco Canola Oil is 7% saturated fat. Crisco blends corn oil that is 14% saturated fat with Crisco Canola Oil to get Crisco Canola and Corn Oil, which is 11% saturated fat. How many gallons of corn oil must Crisco mix with 600 gallons of Crisco Canola Oil to get Crisco Canola and Corn Oil?

38. *Chocolate ripple.* The Delicious Chocolate Shop makes a dark chocolate that is 35% fat and a white chocolate that is 48% fat. How many kilograms of dark chocolate should be mixed with 50 kilograms of white chocolate to make a ripple blend that is 40% fat?

39. *Hawaiian Punch.* Hawaiian Punch is 10% fruit juice. How much water would you have to add to one gallon of Hawaiian Punch to get a drink that is 6% fruit juice?

40. *VCRs and CDs.* The manager of a stereo shop placed an order for $10,710 worth of VCRs at $120 each and CD players at $150 each. If the number of VCRs she ordered was three times the number of CD players, then how many of each did she order?

COLLABORATIVE ACTIVITIES

Finding the Better Deal?

For this activity, the students in your group should choose roles. Four standard roles are Moderator (keeps the group on task), Messenger (asks the group's questions to the instructor, tutor, or helper), Quality Manager (checks to see that the work is top quality), and Recorder (records the group's work). See the Instructor's Solution Manual for a description of these roles. After you have chosen roles, read through the activity completely, and answer the questions.

Scenario: You have decided to buy a new car and have asked some friends to help you choose the best deal and the best financing. You have already looked into your finances. You have $1700 from your summer job and $1500 that your parents will give you for a down payment on a car.

Grouping: 2 to 4 students per group

Topic: Percents

You found a car that you really liked that was 10% off the regular $9800 price. Your friends at the student Credit Union tell you it has a 48-month car loan at $7\frac{1}{2}$% annual simple interest.

At a second dealership you find a similar car on sale for $9000 if you finance it through the dealership. The dealer said that after the down payment you could pay it off in 5 years with monthly payments of $140. This second deal sounds good! (You have decided you could afford up to $160 a month in payments.) The idea of having an extra $20 a month is appealing. However, you wonder how much you will actually pay for the second car.

Which car should you buy?