4.5 APPLICATIONS OF LINEAR EQUATIONS

In this section

- Applied Examples
- Graphing
- Finding a Formula

The linear equation $y = mx + b$ is a formula that determines a value of y for each given value of x. In this section you will study linear equations used as formulas.

Applied Examples

The daily rental charge for renting a 1988 Buick at Wrenta-Wreck is \$30 plus 25 cents per mile. The rental charge depends on the number of miles you drive. If x represents the number of miles driven in one day, then $0.25x + 30$ will give the rental charge in dollars for that day. If we let y represent the rental charge, then we can write the equation

$$y = 0.25x + 30.$$

Because the value of y depends on the value of x, we call x the **independent variable** and y the **dependent variable.** Since $y = 0.25x + 30$ is in slope-intercept form, 0.25 is the slope of the line and (0, 30) is the y-intercept. The rental charge starts at \$30 and is increasing at a rate of 25 cents per mile.

study tip

Relax and don't worry about grades. If you are doing everything you can and should be doing, then there is no reason to worry. If you are neglecting your homework and skipping class, then you should be worried.

For variables in applications, we generally use letters that help us to remember what the variables represent. In the rental example above, we could let m represent the number of miles and R the rental charge. Then R is determined from m by the formula

$$R = 0.25m + 30.$$

There are many examples in which the value of one variable is determined from the value of another variable by means of a linear equation. When this is the case, we say that the dependent variable is a *linear function* of the independent variable. For example, the formula

$$F = \frac{9}{5}C + 32$$

is a linear equation that expresses Fahrenheit temperature in terms of Celsius temperature. In other words, the Fahrenheit temperature F is a linear function of the Celsius temperature C.

EXAMPLE 1 **Distance as a function of time**

A car is averaging 50 miles per hour. Write an equation that expresses the distance it travels as a linear function of the time it travels.

Solution

If the speed is 50 miles per hour, then from the formula $D = RT$, we can write

$$D = 50T.$$

This linear equation expresses D as a linear function of T. ■

Graphing

The graph of a linear equation is a straight line. A linear equation used as a formula is graphed the same way that any linear equation is graphed. If a formula is in slope-intercept form, then we can graph it using the slope and intercept as in

Section 4.3. If we use letters other than x and y for the variables, then we label the axes with these letters. For example, to graph $F = \frac{9}{5}C + 32$ we label the horizontal axis with C and the vertical axis with F. When a formula is solved for a variable, that variable generally goes on the vertical axis of the graph.

EXAMPLE 2 **Graphing a formula**

Graph the linear equation $R = 0.25m + 30$ for $0 \le m \le 500$. R represents the rental charge in dollars, and m represents the number of miles. Find the rental charge for driving 200 miles.

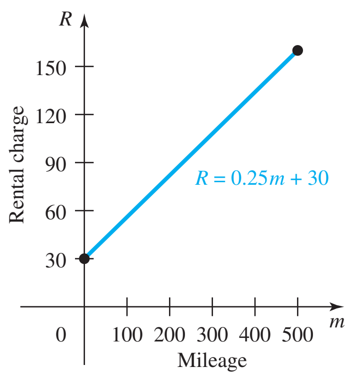

FIGURE 4.28

Solution

We label the x-axis with the letter m and the y-axis with the letter R. See Fig. 4.28. The ordered pairs are of the form (m, R). We adjust the scale on the m-axis to graph the values from 0 to 500. The slope of this line is 0.25, or $\frac{25}{100}$. To sketch the graph, start at the R-intercept (0, 30). Move 100 units to the right and up 25 units to locate a second point on the line. Draw the line as in Fig. 4.28. To find the rental charge for 200 miles, let $m = 200$ in $R = 0.25m + 30$:

$$R = 0.25(200) + 30 = 80$$

So the rental charge for 200 miles is \$80. ■

Finding a Formula

If one variable is a linear function of another, then there is a linear equation expressing one variable in terms of the other. In Section 4.4 we used the point-slope form to find the equation of a line given two points on the line. We can use that same procedure to find a linear function for two variables.

EXAMPLE 3 **Writing a linear function given two points**

A contractor found that his labor cost for installing 100 feet of pipe was \$30. He also found that his labor cost for installing 500 feet of pipe was \$120. If the cost C in dollars is a linear function of the length L in feet, then what is the formula for this function? What would his labor cost be for installing 240 feet of pipe?

Solution

Because C is determined from L, we let C take the place of the dependent variable y and let L take the place of the independent variable x. So the ordered pairs are in the form (L, C). We can use the slope formula to find the slope of the line through the two points (100, 30) and (500, 120) shown in Fig. 4.29.

$$\begin{aligned} m &= \frac{120 - 30}{500 - 100} \\ &= \frac{90}{400} \\ &= \frac{9}{40} \end{aligned}$$

Now we use the point-slope form with the point (100, 30) and slope $\frac{9}{40}$:

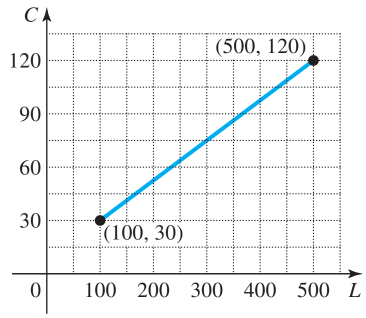

FIGURE 4.29

$$y - y_1 = m(x - x_1)$$
$$C - 30 = \frac{9}{40}(L - 100)$$
$$C - 30 = \frac{9}{40}L - \frac{45}{2}$$
$$C = \frac{9}{40}L - \frac{45}{2} + 30$$
$$C = \frac{9}{40}L + \frac{15}{2} \quad \text{C is a linear function of L.}$$

Now that we have a formula for C in terms of L, we can find C for any value of L. If $L = 240$ feet, then

$$C = \frac{9}{40} \cdot 240 + \frac{15}{2}$$
$$C = 54 + 7.5$$
$$C = 61.5$$

The labor cost to install 240 feet of pipe would be \$61.50.

study tip

When working a test, scan the problems and pick out the ones that are the easiest for you. Do them first. Save the harder problems till last.

WARM-UPS

True or false? Explain your answer.

1. If $z = 3r - 9$, then z is a linear function of r.
2. The circumference of a circle is a linear function of its radius.
3. The area of a circle is a linear function of its radius.
4. The distance driven in 8 hours is a linear function of your average speed.
5. Celsius temperature is a linear function of Fahrenheit temperature.
6. The slope of the line through (1980, 3000) and (1990, 2000) is 100.
7. If your lawyer charges \$90 per hour, then your bill is a linear function of the time spent on the case.
8. The area of a square is a linear function of the length of a side.
9. The perimeter of a square is a linear function of the length of a side.
10. The perimeter of a rectangle with a length of 5 meters is a linear function of its width.

Reading and Writing *After reading this section, write out the answers to these questions. Use complete sentences.*

1. What is the difference between the independent variable and the dependent variable?

4.5 EXERCISES

2. What does it mean to say that one variable is a linear function of another?

3. Why should we use letters other than x and y in applications?

4. Which axis is usually used for the independent variable?

5. What is the procedure for writing a linear function when given two points?

6. Why can we say that Fahrenheit temperature is a linear function of Celsius temperature?

Write an equation that expresses one variable as a linear function of the other. See Example 1.

7. Express length in feet as a linear function of length in yards.

8. Express length in yards as a linear function of length in feet.

9. For a car averaging 65 miles per hour, express the distance it travels as a linear function of the time spent traveling.

10. For a car traveling 6 hours, express the distance it travels as a linear function of its average speed.

11. Express the circumference of a circle as a linear function of its diameter.

12. For a rectangle with a fixed width of 12 feet, express the perimeter as a linear function of its length.

13. Rodney makes \$7.80 per hour. Express his weekly pay as a linear function of the number of hours he works.

14. A triangle has a base of 5 feet. Express its area as a linear function of its height.

Graph each formula for the given values of the independent variable. See Example 2.

15. $P = 40n + 300,\ 0 \le n \le 200$

16. $C = -50r + 500,\ 0 \le r \le 10$

17. $R = 30t + 1000,\ 100 \le t \le 900$

18. $W = 3m - 4000,\ 1000 \le m \le 5000$

19. $C = 2\pi r,\ 1 \le r \le 10$

20. $P = 4s,\ 100 \le s \le 500$

21. $h = -7.5d + 350, 0 \le d \le 40$

22. $a = -50g + 2500, 0 \le g \le 50$

Solve each problem. See Example 2.

23. ***Profit per share.*** In the 1980s, People's Gas had a profit per share, P, that was determined by the equation $P = 0.35x + 4.60$, where x ranges from 0 to 9 corresponding to the years 1980 to 1989. What was the profit per share in 1987? Sketch the graph of this formula for x ranging from 0 to 9.

24. ***Loan value of a car.*** For the first 6 years the loan value of a $30,000 Corvette is determined by the formula $V = -4000a + 30{,}000$, where a is the age in years of the Corvette. What is the loan value of this automobile when it is 5 years old? Sketch the graph of this formula for a between 0 and 6 inclusive.

In Exercises 25–36, solve each problem. See Example 3.

25. ***Plumbing problems.*** When Millie called Pete's Plumbing, Pete worked 2 hours and charged her $70. When her neighbor Rosalee called Pete, he worked 4 hours and charged her $110. Pete's charge is a linear function of the number of hours he works. Find a formula for this function. How much will Pete charge for working 7 hours at Fred's house?

26. ***Interior angles.*** The sum of the measures of the interior angles of a triangle is 180°. The sum of the measures of the interior angles of a square is 360°. The sum S of the measures of the interior angles of any n-sided polygon is a linear function of the number of sides n. Express S as a linear function of n. What is the sum of the measures of the interior angles of an octagon?

FIGURE FOR EXERCISE 26

27. ***If the shoe fits.*** If a child's foot is 7.75 inches long, then the child wears a size 13 shoe. If the child has a foot that is 5.75 inches long, then the child wears a size 7 shoe. The shoe size S is a linear function of the length of the foot L.

a) Write a linear equation expressing S as a function of L.

b) What size shoe fits a child with a 6.25-inch foot?

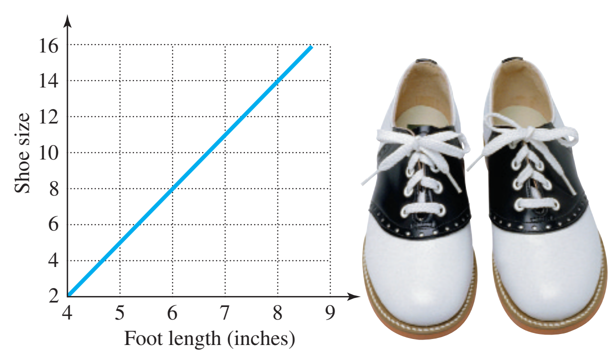

FIGURE FOR EXERCISE 27

28. ***Celsius to Fahrenheit.*** Fahrenheit temperature F is a linear function of Celsius temperature C. When $C = 0$, $F = 32$. When $C = 100$, $F = 212$. Use the point-slope form to write F as a linear function of C. What is the Fahrenheit temperature when $C = 45$?

29. ***Velocity of a projectile.*** The velocity v of a projectile is a linear function of the time t that it is in the air. A ball is thrown downward from the top of a tall building. Its velocity is 42 feet per second after 1 second and 74 feet per second after 2 seconds. Write v as a linear function of t. What is the velocity when $t = 3.5$ seconds?

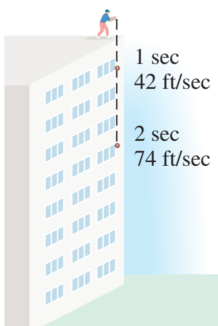

FIGURE FOR EXERCISE 29

30. ***Natural gas.*** The cost C of natural gas is a linear function of the number n of cubic feet of gas used. The cost of 1000 cubic feet of gas is \$39, and the cost of 3000 cubic feet of gas is \$99. Express C as a linear function of n. What is the cost of 2400 cubic feet of gas?

31. ***Expansion joint.*** The width of an expansion joint on the Carl T. Hull bridge is a linear function of the temperature of the roadway. When the temperature is 90°F, the width is 0.75 inch. When the temperature is 30°F, the width is 1.25 inches. Express w as a linear function of t. What is the width of the joint when the temperature is 80°F?

32. ***Perimeter of a rectangle.*** The perimeter P of a rectangle with a fixed width is a linear function of its length. The perimeter is 28 inches when the length is 6.5 inches, and the perimeter is 36 inches when the length is 10.5 inches. Write P as a linear function of L. What is the perimeter when $L = 40$ feet? What is the fixed width of the rectangle?

33. ***Stretching a spring.*** The amount A that a spring stretches beyond its natural length is a linear function of the weight w placed on the spring. A weight of 3 pounds stretches a certain spring 1.8 inches and a weight of 5 pounds stretches the same spring 3 inches. Express A as a linear function of w. How much will the spring stretch with a weight of 6 pounds?

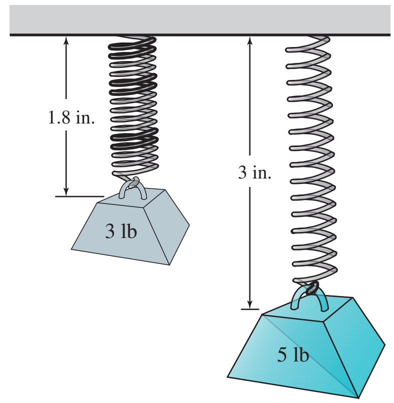

FIGURE FOR EXERCISE 33

34. ***Velocity of a bullet.*** If a gun is fired straight upward, then the velocity v of the bullet is a linear function of the time t that has elapsed since the gun was fired. Suppose that the bullet leaves the gun at 100 feet per second (time $t = 0$) and that after 2 seconds its velocity is 36 feet per second. Express v as a linear function of t. What is the velocity after 3 seconds?

35. ***Enzyme concentration.*** The amount of light absorbed by a certain liquid is a linear function of the concentration of an enzyme in the liquid. A concentration of 2 mg/ml (milligrams per milliliter) produces an absorption of 0.16 and a concentration of 5 mg/ml produces an absorption of 0.40. Express the absorption a as a linear function of the concentration c. What should the absorption be if the concentration is 3 mg/ml? Use the accompanying graph to estimate the concentration when the absorption is 0.50.

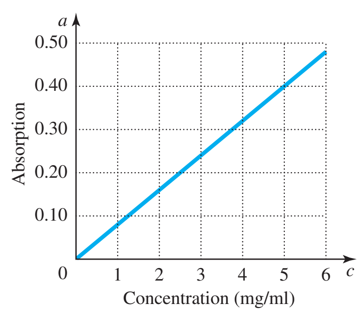

FIGURE FOR EXERCISE 35

36. ***Basal energy requirement.*** The basal energy requirement B is the number of calories that a person needs to maintain the life processes. B depends on the height, weight, and age of the person. For a 28-year-old female with a height of 160 cm, B is a linear function of the person's weight w (in kilograms). For a weight of 45 kg, B is 1300 calories. For a weight of 50 kg, B is 1365 calories. Express B as a linear function of w. What is B for a 28-year-old 160-cm female who weighs 53.2 kg?

GRAPHING CALCULATOR EXERCISES

Most calculators use the variables x and y for graphing. So to graph an equation such as $W = 9R - 21$, *you must graph* $y = 9x - 21$.

37. ***Energy decreasing with age.*** The basal energy requirement (in calories) for a 55-kg 160-cm male at age A is given by $B = 1481 - 4.7A$; the basal energy requirement for a female with the same weight and height is given by $B = 1623 - 6.9A$. Graph these functions on your calculator, and use the graphs to answer the following questions.

a) Which person has a higher basal energy requirement at age 25?

b) Which person has a higher basal energy requirement at age 72?

c) Use the graph to estimate the age at which the basal energy requirements are equal.

d) At what age does the female require no calories?

38. ***Equality of energy.*** The basal energy requirement B for a 70-kg 160-cm male at age A is given by $B = 1620 - 4.7A$. The basal energy requirement B for a 65-kg 165-cm female at age A is given by $B = 1786 - 6.8A$. Graph these linear functions on your calculator, and use the graphs to estimate the age at which these two people have the same basal energy requirement.

4.6 INTRODUCTION TO FUNCTIONS

In this section

- Functions Expressed by Formulas
- Functions Expressed by Tables
- Functions Expressed by Ordered Pairs
- Graphs of Functions
- Domain and Range
- Function Notation

In Section 4.5 you learned that if y is determined from the value of x by an equation of the form $y = mx + b$, then y is a linear function of x. In this section we will discuss other types of functions, but the idea is the same.

Functions Expressed by Formulas

If you get a speeding ticket, then your speed determines the cost of the ticket. You may not know exactly how the judge determines the cost, but the judge is using some rule to determine a cost from knowing your speed. The cost of the ticket is a function of your speed.

Function (as a Rule)

A function is a rule by which any allowable value of one variable (the **independent variable**) determines a *unique* value of a second variable (the **dependent variable**).

helpful hint

According to the dictionary, "determine" means to settle conclusively. If the value of the dependent variable is inconclusive or there is more than one, then the rule is not a function.

One way to express a function is to use a formula. For example, the formula

$$A = \pi r^2$$

gives the area of a circle as a function of its radius. The formula gives us a rule for finding a *unique* area for any given radius. A is the dependent variable, and r is the independent variable. The formula

$$S = -16t^2 + v_0 t + s_0$$

expresses altitude S of a projectile as a function of time t, where v_0 is the initial