with a height of $160 \mathrm{~cm}, B$ is a linear function of the person's weight $w$ (in kilograms). For a weight of $45 \mathrm{~kg}, B$ is 1300 calories. For a weight of $50 \mathrm{~kg}, B$ is 1365 calories. Express $B$ as a linear function of $w$. What is $B$ for a 28 -year-old $160-\mathrm{cm}$ female who weighs 53.2 kg ?

## GRAPHING CALCULATOR EXERCISES

Most calculators use the variables $x$ and $y$ for graphing. So to graph an equation such as $W=9 R-21$, you must graph $y=9 x-21$.
37. Energy decreasing with age. The basal energy requirement (in calories) for a $55-\mathrm{kg} 160-\mathrm{cm}$ male at age $A$ is given by $B=1481-4.7 A$; the basal energy requirement for a female with the same weight and height is given by $B=1623-6.9 A$. Graph these functions on your calculator, and use the graphs to answer the
following questions.
a) Which person has a higher basal energy requirement at age 25 ?
b) Which person has a higher basal energy requirement at age 72 ?
c) Use the graph to estimate the age at which the basal energy requirements are equal.
d) At what age does the female require no calories?
38. Equality of energy. The basal energy requirement $B$ for a $70-\mathrm{kg} 160-\mathrm{cm}$ male at age $A$ is given by $B=1620-$ 4.7A. The basal energy requirement $B$ for a $65-\mathrm{kg} 165-$ cm female at age $A$ is given by $B=1786-6.8 A$. Graph these linear functions on your calculator, and use the graphs to estimate the age at which these two people have the same basal energy requirement.

## 4.6

## INTRODUCTIONTO F UNCTIONS

## Inthis

section

- Functions Expressed by Formulas
- Functions Expressed by Tables
- Functions Expressed by Ordered Pairs
- Graphs of Functions
- Domain and Range
- Function Notation


## helpful/hint

According to the dictionary, "determine" means to settle conclusively. If the value of the dependent variable is inconclusive or there is more than one, then the rule is not a function.

In Section 4.5 you learned that if $y$ is determined from the value of $x$ by an equation of the form $y=m x+b$, then $y$ is a linear function of $x$. In this section we will discuss other types of functions, but the idea is the same.

## Functions Expressed by Formulas

If you get a speeding ticket, then your speed determines the cost of the ticket. You may not know exactly how the judge determines the cost, but the judge is using some rule to determine a cost from knowing your speed. The cost of the ticket is a function of your speed.

## Function (as a Rule)

A function is a rule by which any allowable value of one variable (the independent variable) determines a unique value of a second variable (the dependent variable).

One way to express a function is to use a formula. For example, the formula

$$
A=\pi r^{2}
$$

gives the area of a circle as a function of its radius. The formula gives us a rule for finding a unique area for any given radius. $A$ is the dependent variable, and $r$ is the independent variable. The formula

$$
S=-16 t^{2}+v_{0} t+s_{0}
$$

expresses altitude $S$ of a projectile as a function of time $t$, where $v_{0}$ is the initial

E X A M P LE 1 ,

A carpet layer charges $\$ 25$ plus $\$ 4$ per square yard for installing carpet. Write the total charge $C$ as a function of the number $n$ of square yards of carpet installed.

## Solution

At $\$ 4$ per square yard, $n$ square yards installed cost $4 n$ dollars. If we include the $\$ 25$ charge, then the total cost is $4 n+25$ dollars. Thus the equation

$$
C=4 n+25
$$

expresses $C$ as a function of $n$.
In the next example, we modify a well-known geometric formula.

## E X A M P L E 2 A function in geometry

Express the area of a circle as a function of its diameter.

## Solution

The area of a circle is given by $A=\pi r^{2}$. Because the radius of a circle is one-half of the diameter, we have $r=\frac{d}{2}$. Now replace $r$ by $\frac{d}{2}$ in the formula $A=\pi r^{2}$ :

$$
\begin{aligned}
A & =\pi\left(\frac{d}{2}\right)^{2} \\
& =\frac{\pi d^{2}}{4}
\end{aligned}
$$

So $A=\frac{\pi d^{2}}{4}$ expresses the area of a circle as a function of its diameter.

## Functions Expressed by Tables

Another way to express a function is with a table. For example, Table 4.1 can be used to determine the cost at United Freight Service for shipping a package that weighs under 100 pounds. For any allowable weight, the table gives us a rule for finding the unique shipping cost. The weight is the independent variable, and the cost is the dependent variable.

| Weight in <br> Pounds | Cost |
| ---: | :---: |
| 0 to 10 | $\$ 4.60$ |
| 11 to 30 | $\$ 12.75$ |
| 31 to 79 | $\$ 32.90$ |
| 80 to 99 | $\$ 55.82$ |

TABLE 4.1

| Weight in <br> Pounds | Cost |
| :---: | :---: |
| 0 to 15 | $\$ 4.60$ |
| 10 to 30 | $\$ 12.75$ |
| 31 to 79 | $\$ 32.90$ |
| 80 to 99 | $\$ 55.82$ |

TABLE 4.2
Now consider Table 4.2. It does not look much different from Table 4.1, but there is an important difference. The cost for shipping a 12-pound package according to Table 4.2 is either $\$ 4.60$ or $\$ 12.75$. Either the table has an error or perhaps $\$ 4.60$ and $\$ 12.75$ are costs for shipping to different destinations. In any case the weight does not determine a unique cost. So Table 4.2 does not express the cost as a function of the weight.

## E X A M P L E 3 Functions defined by tables

Which of the following tables expresses $y$ as a function of $x$ ?

## helpful/hint

In a function, every value for the independent variable determines conclusively a corresponding value for the dependent variable. If there is more than one possible value for the dependent variable, then the set of ordered pairs is not a function.

a) | $\boldsymbol{x}$ | $\boldsymbol{y}$ |
| :---: | ---: |
| 1 | 3 |
| 2 | 6 |
| 3 | 9 |
| 4 | 12 |
| 5 | 15 |

b) | $\boldsymbol{x}$ | $\boldsymbol{y}$ |
| ---: | ---: |
| 1 | 1 |
| -1 | 1 |
| 2 | 2 |
| -2 | 2 |
| 3 | 3 |
| -3 | 3 |

c)

| $\boldsymbol{x}$ | $\boldsymbol{y}$ |
| :---: | :---: |
| 1988 | 27,000 |
| 1989 | 27,000 |
| 1990 | 28,500 |
| 1991 | 29,000 |
| 1992 | 30,000 |
| 1993 | 30,750 |

d)

| $\boldsymbol{x}$ | $\boldsymbol{y}$ |
| :---: | :---: |
| 23 | 48 |
| 35 | 27 |

$19 \quad 28$
$23 \quad 37$
4156
2234

## Solution

In Tables a), b), and c), every value of $x$ corresponds to only one value of $y$. Tables a), b), and c) each express $y$ as a function of $x$. Notice that different values of $x$ may correspond to the same value of $y$. In Table d) we have the value of 23 for $x$ corresponding to two different values of $y, 48$ and 37. So Table d) does not express $y$ as a function of $x$.

## helpful/hint

A computer at your grocery store determines the price of each item from the universal product code. This function consists of a long list of ordered pairs in the computer memory. The function that pairs your total with the amount of tax is also handled by computer, but in this case the computer determines the second coordinate by using a formula.

## Functions Expressed by Ordered Pairs

If the value of the independent variable is written as the first coordinate of an ordered pair and the value of the dependent variable is written as the second coordinate of an ordered pair, then a function can be expressed by a set of ordered pairs. In a function, each value of the independent variable corresponds to a unique value of the dependent variable. So no two ordered pairs can have the same first coordinate and different second coordinates.

## Function (as a Set of Ordered Pairs)

A function is a set of ordered pairs of real numbers such that no two ordered pairs have the same first coordinates and different second coordinates.

## E X A M P L E 4 Functions expressed by a set of ordered pairs

Determine whether each set of ordered pairs is a function.
a) $\{(1,2),(1,5),(-4,6)\}$
b) $\{(-1,3),(0,3),(6,3),(-3,2)\}$

## Solution

a) This set of ordered pairs is not a function because $(1,2)$ and $(1,5)$ have the same first coordinates but different second coordinates.
b) This set of ordered pairs is a function. Note that the same second coordinate with different first coordinates is permitted in a function.

If there are infinitely many ordered pairs in a function, then we can use setbuilder notation from Chapter 1 along with an equation to express the function. For example,

$$
\left\{(x, y) \mid y=x^{2}\right\}
$$

is the set of ordered pairs in which the $y$-coordinate is the square of the $x$-coordinate. Ordered pairs such as $(0,0),(2,4)$, and $(-2,4)$ belong to this set. This set is a function because every value of $x$ determines only one value of $y$.

## E X A M P L E 5 Functions expressed by set-builder notation

Determine whether each set of ordered pairs is a function.
a) $\left\{(x, y) \mid y=3 x^{2}-2 x+1\right\}$
b) $\left\{(x, y) \mid y^{2}=x\right\}$
c) $\{(x, y) \mid x+y=6\}$

## Solution

a) This set is a function because each value we select for $x$ determines only one value for $y$.
b) If $x=9$, then we have $y^{2}=9$. Because both 3 and -3 satisfy $y^{2}=9$, both $(9,3)$ and $(9,-3)$ belong to this set. So the set is not a function.
c) If we solve $x+y=6$ for $y$, we get $y=-x+6$. Because each value of $x$ determines only one value for $y$, this set is a function. In fact, this set is a linear function.

We often omit the set notation when discussing functions. For example, the equation

$$
y=3 x^{2}-2 x+1
$$

expresses $y$ as a function of $x$ because the set of ordered pairs determined by the equation is a function. However, the equation

$$
y^{2}=x
$$

does not express $y$ as a function of $x$ because ordered pairs such as $(9,3)$ and $(9,-3)$ satisfy the equation.

## E X A M P L E 6 Functions expressed by equations

Determine whether each equation expresses $y$ as a function of $x$.
a) $y=|x|$
b) $y=x^{3}$
c) $x=|y|$

## Solution

a) Because every number has a unique absolute value, $y=|x|$ is a function.
b) Because every number has a unique cube, $y=x^{3}$ is a function.

## helpful/hint

To determine whether an equation expresses $y$ as a function of $x$, always select a number for $x$ (the independent variable) and then see if there is more than one corresponding value for $y$ (the dependent variable). If there is more than one corresponding $y$-value, then $y$ is not a function of $x$.


Most calculators graph only functions. If you enter an equation using the $Y=$ key and the calculator accepts it and draws a graph, the equation defines $y$ as a function of $x$.

## E X A M P L E 7

## helpful/hint

Note that the vertical-line test works in theory, but it is certainly limited by the accuracy of the graph.Pictures are often deceptive. However, in spite of its limitations, the vertical-line test gives us a visual idea of what the graph of a function looks like.
c) The equation $x=|y|$ does not express $y$ as a function of $x$ because both $(4,-4)$ and $(4,4)$ satisfy this equation. These ordered pairs have the same first coordinate but different second coordinates.

## Graphs of Functions

Every function determines a set of ordered pairs, and any set of ordered pairs has a graph in the rectangular coordinate system. For example, the set of ordered pairs determined by the linear function $y=2 x-1$ is shown in Fig. 4.30.

Every graph illustrates a set of ordered pairs, but not every graph is a graph of a function. For example, the circle in Fig. 4.31 is not a graph of a function because the ordered pairs $(0,4)$ and $(0,-4)$ are both on the graph, and these two ordered pairs have the same first coordinate and different second coordinates. Whether a graph has such ordered pairs can be determined by a simple visual test called the vertical-line test.


FIGURE 4.30


FIGURE 4.31

## Vertical-Line Test

If it is possible to draw a vertical line that crosses a graph two or more times, then the graph is not the graph of a function.

If there is a vertical line that crosses a graph twice (or more), then we have two points (or more) with the same $x$-coordinate and different $y$-coordinates, and so the graph is not the graph of a function. If you mentally consider every possible vertical line and none of them cross the graph more than once, then you can conclude that the graph is the graph of a function.

## Using the vertical-line test

Which of the following graphs are graphs of functions?
a)

b)



## Solution

Neither a) nor c) is the graph of a function, since we can draw vertical lines that cross these graphs twice. Graph b) is the graph of a function, since no vertical line crosses it twice.

## Domain and Range

The set of all possible numbers that can be used for the independent variable is called the domain of the function. For example, the domain of the function

$$
y=\frac{1}{x}
$$

is the set of all nonzero real numbers because $\frac{1}{x}$ is undefined for $x=0$. For some functions the domain is clearly stated when the function is given. The set of all values of the dependent variable is called the range of the function.

## E X A M P L E 8 Domain and range

State the domain and range of each function.
a) $\{(3,-1),(2,5),(1,5)\}$
b) $y=|x|$
c) $A=\pi r^{2}$ for $r>0$

## Solution

## heIpfulhint

Real-life variables are generally not as simple as the ones we consider. A student's college GPA is not a function of age, because many students with the same age have different GPAs. However, GPA is probably a function of a large number of variables: age, IQ, high school GPA, number of working hours, mother's IQ, and so on.
a) The domain is the set of numbers used as first coordinates, $\{1,2,3\}$. The range is the set of second coordinates, $\{-1,5\}$.
b) Because $|x|$ is a real number for any real number $x$, the domain is the set of all real numbers. The range is the set of numbers that result from taking the absolute value of every real number. Thus the range is the set of nonnegative real numbers, $[0, \infty)$.
c) The condition $r>0$ specifies the domain of the function. The domain is $(0, \infty)$, the positive real numbers. Because $A=\pi r^{2}$, the value of $A$ is also greater than zero. So the range is also the set of positive real numbers.

## Function Notation

When the variable $y$ is a function of $x$, we may use the notation $f(x)$ to represent $y$. The symbol $f(x)$ is read as " $f$ of $x$." So if $x$ is the independent variable, we may use $y$ or $f(x)$ to represent the dependent variable. For example, the function

$$
y=2 x+3
$$

can also be written as

$$
f(x)=2 x+3 .
$$

We use $y$ and $f(x)$ interchangeably. We think of $f$ as the name of the function. We may use letters other than $f$. For example, the function $g(x)=2 x+3$ is the same function as $f(x)=2 x+3$

The expression $f(x)$ represents the second coordinate when the first coordinate is $x$; it does not mean $f$ times $x$. For example, if we replace $x$ by 4 in $f(x)=2 x+3$, we get

$$
f(4)=2 \cdot 4+3=11 .
$$

So if the first coordinate is 4 , then the second coordinate is $f(4)$, or 11 . The ordered pair $(4,11)$ belongs to the function $f$. This statement means that the function $f$ pairs 4 with 11 . We can use the diagram in Fig. 4.32 to picture this situation.


FIGURE 4.32

## E X A M P L E 9 Using function notation

Suppose $f(x)=x^{2}-1$ and $g(x)=-3 x+2$. Find the following:
a) $f(-2)$
b) $f(-1)$
c) $g(0)$
d) $g(6)$

## Solution

a) Replace $x$ by -2 in the formula $f(x)=x^{2}-1$ :

$$
\begin{aligned}
f(-2) & =(-2)^{2}-1 \\
& =4-1 \\
& =3
\end{aligned}
$$

So $f(-2)=3$.
b) Replace $x$ by -1 in the formula $f(x)=x^{2}-1$ :

$$
\begin{aligned}
f(-1) & =(-1)^{2}-1 \\
& =1-1 \\
& =0
\end{aligned}
$$

So $f(-1)=0$.
c) Replace $x$ by 0 in the formula $g(x)=-3 x+2$ :

$$
g(0)=-3 \cdot 0+2=2
$$

So $g(0)=2$.
d) Replace $x$ by 6 in $g(x)=-3 x+2$ to get $g(6)=-16$.

## E X A M P L E 10 Using function notation in an application

The formula $C(n)=0.10 n+4.95$ gives the monthly cost in dollars for $n$ minutes of long-distance calls. Find $C(40)$ and $C(100)$.

## Solution

Replace $n$ with 40 in the formula:

$$
\begin{aligned}
C(n) & =0.10 n+4.95 \\
C(40) & =0.10(40)+4.95 \\
& =8.95
\end{aligned}
$$

So $C(40)=8.95$. The cost for 40 minutes of calls is $\$ 8.95$. Now

$$
C(100)=0.10(100)+4.95=14.95
$$

So $C(100)=14.95$. The cost of 100 minutes of calls is $\$ 14.95$.
C A UTION $C(h)$ is not $C$ times $h$. In the context of formulas, $C(h)$ represents the value of $C$ corresponding to a value of $h$.

## calculator close-up <br> (v) [4) [6] 는

A graphing calculator can be used to evaluate a formula in the same manner as in Example 10. To evaluate
$C=0.10 n+4.95$
enter the formula into your calculator as $y_{1}=$ $0.10 x+4.95$ using the $Y=$ key:


To find the cost of 40 minutes of calls, enter $y_{1}(40)$ on the home screen and press ENTER:


## WARM-UPS

## True or false? Explain your answer.

1. Any set of ordered pairs is a function.
2. The area of a square is a function of the length of a side.
3. The set $\{(-1,3),(-3,1),(-1,-3)\}$ is a function.
4. The set $\{(1,5),(3,5),(7,5)\}$ is a function.
5. The domain of $f(x)=x^{3}$ is the set of all real numbers.
6. The domain of $y=|x|$ is the set of nonnegative real numbers.
7. The range of $y=|x|$ is the set of all real numbers.
8. The set $\{(x, y) \mid x=2 y\}$ is a function.
9. The set $\left\{(x, y) \mid x=y^{2}\right\}$ is a function.
10. If $f(x)=x^{2}-5$, then $f(-2)=-1$.

### 4.6 EXERCISES

Reading and Writing After reading this section, write out the answers to these questions. Use complete sentences.

1. What is a function?
2. What are the different ways to express functions?
3. What do all descriptions of functions have in common?
4. How can you tell at a glance if a graph is a graph of a function?
5. What is the domain of a function?
6. What is function notation?

Write a formula that describes the function for each of the following. See Examples 1 and 2.
7. A small pizza costs $\$ 5.00$ plus 50 cents for each topping. Express the total cost $C$ as a function of the number of toppings $t$.
8. A developer prices condominiums in Florida at $\$ 20,000$ plus $\$ 40$ per square foot of living area. Express the cost $C$ as a function of the number of square feet of living area $s$.
9. The sales tax rate on groceries in Mayberry is $9 \%$. Express the total cost $T$ (including tax) as a function of the total price of the groceries $S$.
10. With a GM MasterCard, $5 \%$ of the amount charged is credited toward a rebate on the purchase of a new car. Express the rebate $R$ as a function of the amount charged $A$.
11. Express the circumference of a circle as a function of its radius.
12. Express the circumference of a circle as a function of its diameter.
13. Express the perimeter $P$ of a square as a function of the length $s$ of a side.
14. Express the perimeter $P$ of a rectangle with width 10 ft as a function of its length $L$.
15. Express the area $A$ of a triangle with a base of 10 m as a function of its height $h$.
16. Express the area $A$ of a trapezoid with bases 12 cm and 10 cm as a function of its height $h$.

Determine whether each table expresses the second variable as a function of the first variable. See Example 3.

17. $x |$| $x$ | $y$ |
| ---: | ---: |
| 1 | 1 |
| 4 | 2 |
| 9 | 3 |
| 16 | 4 |
| 25 | 5 |
| 36 | 6 |
| 49 | 8 |
18. | $x$ | $y$ |
| ---: | ---: |
| 2 | 4 |
| 3 | 9 |
| 4 | 16 |
| 5 | 25 |
| 8 | 36 |
| 9 | 49 |
| 10 | 100 |
19. 

| $t$ | $v$ |
| ---: | ---: |
| 2 | 2 |
| -2 | 2 |
| 3 | 3 |
| -3 | 3 |
| 4 | 4 |
| -4 | 4 |
| 5 | 5 |

20. 

| $s$ | $W$ |
| ---: | ---: |
| 5 | 17 |
| 6 | 17 |
| -1 | 17 |
| -2 | 17 |
| -3 | 17 |
| 7 | 17 |
| 8 | 17 |

21. 

| $a$ | $P$ |
| :--- | ---: |
| 2 | 2 |
| 2 | -2 |
| 3 | 3 |
| 3 | -3 |
| 4 | 4 |
| 4 | -4 |
| 5 | 5 |

22. 

| $n$ | $r$ |
| :--- | ---: |
| 17 | 5 |
| 17 | 6 |
| 17 | -1 |
| 17 | -2 |
| 17 | -3 |
| 17 | -4 |
| 17 | -5 |

23. | $b$ | $q$ |
| :---: | :---: |
| 1970 | 0.14 |
| 1972 | 0.18 |
| 1974 | 0.18 |
| 1976 | 0.22 |
| 1978 | 0.25 |
| 1980 | 0.28 |

Determine whether each set of ordered pairs is a function. See Example 4.
25. $\{(1,2),(2,3),(3,4)\}$
26. $\{(1,-3),(1,3),(2,12)\}$
27. $\{(-1,4),(2,4),(3,4)\}$
28. $\{(1,7),(7,1)\}$
29. $\{(0,-1),(0,1)\}$
30. $\{(1,7),(-2,7),(3,7),(4,7)\}$
31. $\{(50,50)\}$
32. $\{(0,0)\}$

Determine whether each set is a function. See Example 5.
33. $\{(x, y) \mid y=x-3\}$
34. $\left\{(x, y) \mid y=x^{2}-2 x-1\right\}$
35. $\{(x, y)|x=|y|\}$
36. $\left\{(x, y) \mid x=y^{2}+1\right\}$
37. $\{(x, y) \mid x=y+1\}$
38. $\left\{(x, y) \left\lvert\, y=\frac{1}{x}\right.\right\}$
39. $\left\{(x, y) \mid x=y^{2}-1\right\}$
40. $\{(x, y) \mid x=3 y\}$

Determine whether each equation expresses $y$ as a function of $x$. See Example 6.
41. $x=4 y$
42. $x=-3 y$
43. $y=\frac{2}{x}$
44. $y=\frac{x}{2}$
45. $y=x^{3}-1$
46. $y=|x-1|$
47. $x^{2}+y^{2}=25$
48. $x^{2}-y^{2}=9$

Which of the following graphs are graphs of functions? See Example 7.
49.

50.

51.

52.

53.

54.


Determine the domain and range of each function. See Example 8.
55. $\{(3,3),(2,5),(1,7)\}$
56. $\{(0,1),(2,1),(4,1)\}$
57. $y=|x+3|$
58. $y=|x-1|$
59. $y=x$
60. $y=2 x+1$
61. $y=x^{2}$
62. $y=x^{3}$
63. $A=s^{2}$ for $s>0$
64. $S=-16 t^{2}$ for $t \geq 0$

Let $f(x)=2 x-1, g(x)=x^{2}-3$, and $h(x)=|x-1|$.
Find the following. See Example 9.
65. $f(0)$
66. $f(-1)$
67. $f\left(\frac{1}{2}\right)$
68. $f\left(\frac{3}{4}\right)$
69. $g(4)$
70. $g(-4)$
71. $g(0.5)$
72. $g(-1.5)$
73. $h(3)$
74. $h(-1)$
75. $h(0)$
76. $h(1)$

Let $f(x)=x^{3}-x^{2}$ and $g(x)=x^{2}-4.2 x+2.76$.
Find the following. Round each answer to three decimal places.
77. $f(5.68)$
78. $g(-2.7)$
79. $g(3.5)$
80. $f(67.2)$

Solve each problem.
81. Velocity and time. If a ball is thrown straight upward into the air with a velocity of $100 \mathrm{ft} / \mathrm{sec}$, then its velocity $t$ seconds later is given by

$$
v(t)=-32 t+100
$$

a) Find $v(0), v(1)$, and $v(2)$.
b) Is the velocity increasing or decreasing as the time increases?


FIGURE FOR EXERCISE 81
82. Cost and toppings. The cost $c$ in dollars for a pizza with $n$ toppings is given by

$$
c(n)=0.75 n+6.99
$$

a) Find $c(2), c(4)$, and $c(5)$.
b) Is the cost increasing or decreasing as the number of toppings increases?
83. Threshold weight. The threshold weight for an individual is the weight beyond which the risk of death increases significantly. For middle-aged males the
function $W(h)=0.000534 h^{3}$ expresses the threshold weight in pounds as a function of the height $h$ in inches. Find $W(70)$. Find the threshold weight for a $6^{\prime} 2^{\prime \prime}$ middleaged male.
84. Pole vaulting. The height a pole vaulter attains is a function of the vaulter's velocity on the runway. The function

$$
h(v)=\frac{1}{64} v^{2}
$$

gives the height in feet as a function of the velocity $v$ in feet per second.
a) Find $h(35)$ to the nearest tenth of an inch.
b) Who gains more height from an increase of $1 \mathrm{ft} / \mathrm{sec}$ in velocity: a fast runner or a slow runner?


FIGUREFOR EXERCISE84
85. Credit card fees. A certain credit card company gets $4 \%$ of each charge, and the retailer receives the rest. At the end of a billing period the retailer receives a statement showing only the retailer's portion of each transaction. Express the original amount charged $C$ as a function of the retailer's portion $r$.
86. More credit card fees. Suppose that the amount charged on the credit card in the previous exercise includes $8 \%$ sales tax. The credit card company does not get any of the sales tax. In this case the retailer's portion of each transaction includes sales tax on the original cost of the goods. Express the original amount charged $C$ as a function of the retailer's portion.

## GETTING MORE INVOLVED

Discussion In each situation determine whether a is a function of $b, b$ is a function of $a$, or neither. Answers may vary depending on interpretations.
87. $a=$ the price per gallon of regular unleaded.
$b=$ the number of gallons that you get for $\$ 10$.
88. $a=$ the universal product code of an item at Sears.
$b=$ the price of that item.
89. $a=$ a student's score on the last test in this class.
$b=$ the number of hours he/she spent studying.
90. $a=$ a student's score on the last test in this class.
$b=$ the IQ of the student's mother.
91. $a=$ the weight of a package shipped by UPS. $b=$ the cost of shipping that package.
92. $a=$ the Celsius temperature at any time.
$b=$ the Fahrenheit temperature at the same time.
93. $a=$ the weight of a letter.
$b=$ the cost of mailing the letter.
94. $a=$ the cost of a gallon of milk.
$b=$ the amount of sales tax on that gallon.

## COLLABORATIVEACTIVITIES

## Inches or Centimeters?

In this activity you will generate data by measuring in both inches and centimeters the height of each member of your group. Then you will plot the points on a graph and use any two of your points to find the conversion formula for converting inches to centimeters.
Part I: Measure the height of each person in your group and fill out a table like the one shown here:

| Name | Height <br> in inches | Height <br> in centimeters |
| :---: | :---: | :---: |
|  |  |  |
|  |  |  |
|  |  |  |

Grouping: 3 to 4 students
Topic: Plotting points, graphing lines
Part II: The numbers for inches and centimeters from the table will give you three or four ordered pairs to graph. Plot these points on a graph. Let inches be the horizontal $x$-axis and centimeters be the vertical $y$-axis. Let each mark on the axes represent 10 units. When graphing, you will need to estimate the place to plot fractional values.
Part III: Use any two of your points to find an equation of the line you have graphed. What is the slope of your line? Where does it cross the horizontal axis?
Extension: Look up the conversion formula for converting inches to centimeters. Is it the same as the one you found by measuring? If it is different, what could account for the difference?

