5.3 MULTIPLICATION OF BINOMIALS

In this section

- The FOIL Method
- Multiplying Binomials Quickly

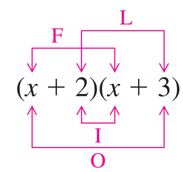
In Section 5.2 you learned to multiply polynomials. In this section you will learn a rule that makes multiplication of binomials simpler.

The FOIL Method

We can use the distributive property to find the product of two binomials. For example,

$$(x + 2)(x + 3) = (x + 2)x + (x + 2)3 \quad \text{Distributive property}$$
$$= x^2 + 2x + 3x + 6 \quad \text{Distributive property}$$
$$= x^2 + 5x + 6 \quad \text{Combine like terms.}$$

There are four terms in $x^2 + 2x + 3x + 6$. The term x^2 is the product of the *first* term of each binomial, x and x. The term $3x$ is the product of the two *outer* terms, 3 and x. The term $2x$ is the product of the two *inner* terms, 2 and x. The term 6 is the product of the last term of each binomial, 2 and 3. We can connect the terms multiplied by lines as follows:



F = First terms
O = Outer terms
I = Inner terms
L = Last terms

If you remember the word FOIL, you can get the product of the two binomials much faster than writing out all of the steps above. This method is called the **FOIL method.** The name should make it easier to remember.

EXAMPLE 1

Using the FOIL method

Find each product.

a) $(x + 2)(x - 4)$ **b)** $(2x + 5)(3x - 4)$

c) $(a - b)(2a - b)$ **d)** $(x + 3)(y + 5)$

Solution

a) $(x + 2)(x - 4) = \overset{F}{x^2} \overset{O}{- 4x} \overset{I}{+ 2x} \overset{L}{- 8}$

$= x^2 - 2x - 8$ Combine the like terms.

b) $(2x + 5)(3x - 4) = 6x^2 - 8x + 15x - 20$

$= 6x^2 + 7x - 20$ Combine the like terms.

c) $(a - b)(2a - b) = 2a^2 - ab - 2ab + b^2$

$= 2a^2 - 3ab + b^2$

d) $(x + 3)(y + 5) = xy + 5x + 3y + 15$ There are no like terms to combine.

helpful hint

You may have to practice FOIL a while to get good at it. However, the better you are at FOIL, the easier you will find factoring in Chapter 6.

FOIL can be used to multiply any two binomials. The binomials in the next example have higher powers than those of Example 1.

EXAMPLE 2

Using the FOIL method

Find each product.

a) $(x^3 - 3)(x^3 + 6)$ **b)** $(2a^2 + 1)(a^2 + 5)$

Solution

a) $(x^3 - 3)(x^3 + 6) = x^6 + 6x^3 - 3x^3 - 18$
$= x^6 + 3x^3 - 18$

b) $(2a^2 + 1)(a^2 + 5) = 2a^4 + 10a^2 + a^2 + 5$
$= 2a^4 + 11a^2 + 5$

study tip

Remember that everything we do in solving problems is based on principles (which are also called rules, theorems, and definitions). These principles justify the steps we take. Be sure that you understand the reasons. If you just memorize procedures without understanding, you will soon forget the procedures.

Multiplying Binomials Quickly

The outer and inner products in the FOIL method are often like terms, and we can combine them without writing them down. Once you become proficient at using FOIL, you can find the product of two binomials without writing anything except the answer.

EXAMPLE 3

Using FOIL to find a product quickly

Find each product. Write down only the answer.

a) $(x + 3)(x + 4)$ **b)** $(2x - 1)(x + 5)$ **c)** $(a - 6)(a + 6)$

Solution

a) $(x + 3)(x + 4) = x^2 + 7x + 12$ Combine like terms: $3x + 4x = 7x$.

b) $(2x - 1)(x + 5) = 2x^2 + 9x - 5$ Combine like terms: $10x - x = 9x$.

c) $(a - 6)(a + 6) = a^2 - 36$ Combine like terms: $6a - 6a = 0$.

EXAMPLE 4

More products

Find each product.

a) $\left(\frac{1}{2}x - 2\right)\left(\frac{1}{3}x + 1\right)$ **b)** $(x - 1)(x + 3)(x - 4)$

Solution

a) $\left(\frac{1}{2}x - 2\right)\left(\frac{1}{3}x + 1\right) = \frac{1}{6}x^2 - \frac{2}{3}x + \frac{1}{2}x - 2$ $\frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$

$= \frac{1}{6}x^2 - \frac{1}{6}x - 2$ $-\frac{2}{3} + \frac{1}{2} = -\frac{1}{6}$

b) Multiply the first two binomials and then multiply that result by $x - 4$:

$$\begin{aligned}(x - 1)(x + 3)(x - 4) &= (x^2 + 2x - 3)(x - 4)\\ &= x(x^2 + 2x - 3) - 4(x^2 + 2x - 3)\\ &= x^3 + 2x^2 - 3x - 4x^2 - 8x + 12\\ &= x^3 - 2x^2 - 11x + 12\end{aligned}$$

EXAMPLE 5 **Area of a garden**

Sheila has a square garden with sides of length x feet. If she increases the length by 7 feet and decreases the width by 2 feet, then what trinomial represents the area of the new rectangular garden?

Solution

The length of the new garden is $x + 7$ and the width is $x - 2$ as shown in Fig. 5.2. The area is $(x + 7)(x - 2)$ or $x^2 + 5x - 14$ square feet.

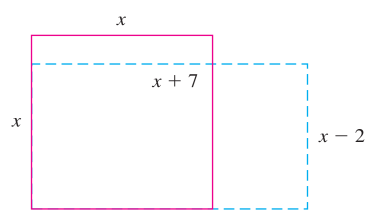


FIGURE 5.2

WARM-UPS

True or false? Answer true only if the equation is true for all values of the variable or variables. Explain your answer.

1. $(x + 3)(x + 2) = x^2 + 6$
2. $(x + 2)(y + 1) = xy + x + 2y + 2$
3. $(3a - 5)(2a + 1) = 6a^2 + 3a - 10a - 5$
4. $(y + 3)(y - 2) = y^2 + y - 6$
5. $(x^2 + 2)(x^2 + 3) = x^4 + 5x^2 + 6$
6. $(3a^2 - 2)(3a^2 + 2) = 9a^2 - 4$
7. $(t + 3)(t + 5) = t^2 + 8t + 15$
8. $(y - 9)(y - 2) = y^2 - 11y - 18$
9. $(x + 4)(x - 7) = x^2 + 4x - 28$
10. It is not necessary to learn FOIL as long as you can get the answer.

5.3 EXERCISES

Reading and Writing *After reading this section, write out the answers to these questions. Use complete sentences.*

1. What property of the real numbers do we usually use to find the product of two binomials?
2. What does FOIL stand for?
3. What is the purpose of FOIL?
4. What is the maximum number of terms that can be obtained when two binomials are multiplied?

Use FOIL to find each product. See Example 1.

5. $(x + 2)(x + 4)$
6. $(x + 3)(x + 5)$
7. $(a - 3)(a + 2)$
8. $(b - 1)(b + 2)$
9. $(2x - 1)(x - 2)$

10. $(2y - 5)(y - 2)$
11. $(2a - 3)(a + 1)$
12. $(3x - 5)(x + 4)$
13. $(w - 50)(w - 10)$
14. $(w - 30)(w - 20)$
15. $(y - a)(y + 5)$
16. $(a + t)(3 - y)$
17. $(5 - w)(w + m)$
18. $(a - h)(b + t)$
19. $(2m - 3t)(5m + 3t)$
20. $(2x - 5y)(x + y)$
21. $(5a + 2b)(9a + 7b)$
22. $(11x + 3y)(x + 4y)$

Use FOIL to find each product. See Example 2.

23. $(x^2 - 5)(x^2 + 2)$
24. $(y^2 + 1)(y^2 - 2)$
25. $(h^3 + 5)(h^3 + 5)$
26. $(y^6 + 1)(y^6 - 4)$
27. $(3b^3 + 2)(b^3 + 4)$
28. $(5n^4 - 1)(n^4 + 3)$
29. $(y^2 - 3)(y - 2)$
30. $(x - 1)(x^2 - 1)$
31. $(3m^3 - n^2)(2m^3 + 3n^2)$
32. $(6y^4 - 2z^2)(6y^4 - 3z^2)$
33. $(3u^2v - 2)(4u^2v + 6)$
34. $(5y^3w^2 + z)(2y^3w^2 + 3z)$

Find each product. Try to write only the answer. See Example 3.

35. $(b + 4)(b + 5)$
36. $(y + 8)(y + 4)$
37. $(x - 3)(x + 9)$
38. $(m + 7)(m - 8)$
39. $(a + 5)(a + 5)$
40. $(t - 4)(t - 4)$
41. $(2x - 1)(2x - 1)$
42. $(3y + 4)(3y + 4)$
43. $(z - 10)(z + 10)$
44. $(3h - 5)(3h + 5)$
45. $(a + b)(a + b)$
46. $(x - y)(x - y)$
47. $(a - b)(a - 2b)$
48. $(b - 8c)(b - c)$
49. $(2x - y)(x + 3y)$
50. $(3y + 5z)(y - 3z)$
51. $(5t - 2)(t - 1)$
52. $(2t - 3)(2t - 1)$
53. $(h - 7)(h - 9)$
54. $(h - 7w)(h - 7w)$
55. $(h + 7w)(h + 7w)$
56. $(h - 7q)(h + 7q)$
57. $(2h^2 - 1)(2h^2 - 1)$
58. $(3h^2 + 1)(3h^2 + 1)$

Perform the indicated operations. See Example 4.

59. $\left(2a + \frac{1}{2}\right)\left(4a - \frac{1}{2}\right)$
60. $\left(3b + \frac{2}{3}\right)\left(6b - \frac{1}{3}\right)$
61. $\left(\frac{1}{2}x - \frac{1}{3}\right)\left(\frac{1}{4}x + \frac{1}{2}\right)$
62. $\left(\frac{2}{3}t - \frac{1}{4}\right)\left(\frac{1}{2}t - \frac{1}{2}\right)$
63. $-2x^4(3x - 1)(2x + 5)$
64. $4xy^3(2x - y)(3x + y)$
65. $(x - 1)(x + 1)(x + 3)$
66. $(a - 3)(a + 4)(a - 5)$
67. $(3x - 2)(3x + 2)(x + 5)$
68. $(x - 6)(9x + 4)(9x - 4)$
69. $(x - 1)(x + 2) - (x + 3)(x - 4)$
70. $(k - 4)(k + 9) - (k - 3)(k + 7)$

Solve each problem. See Example 5.

71. ***Area of a rug.*** Find a trinomial that represents the area of a rectangular rug whose sides are $x + 3$ feet and $2x - 1$ feet.

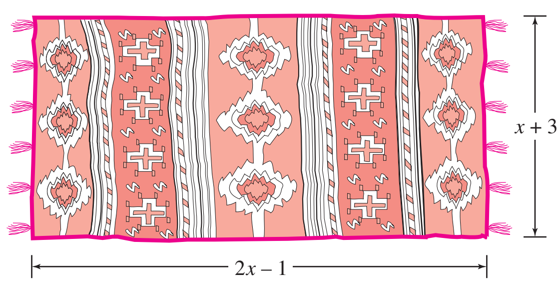


FIGURE FOR EXERCISE 71

72. ***Area of a parallelogram.*** Find a trinomial that represents the area of a parallelogram whose base is $3x + 2$ meters and whose height is $2x + 3$ meters.

73. ***Area of a sail.*** The sail of a tall ship is triangular in shape with a base of $4.57x + 3$ meters and a height of $2.3x - 1.33$ meters. Find a polynomial that represents the area of the triangle.

74. ***Area of a square.*** A square has a side of length $1.732x + 1.414$ meters. Find a polynomial that represents its area.

GETTING MORE INVOLVED

75. ***Exploration.*** Find the area of each of the four regions shown in the figure. What is the total area of the four regions? What does this exercise illustrate?

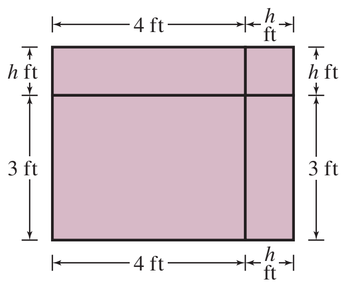


FIGURE FOR EXERCISE 75

76. ***Exploration.*** Find the area of each of the four regions shown in the figure. What is the total area of the four regions? What does this exercise illustrate?

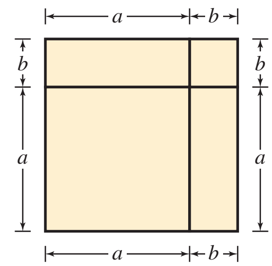


FIGURE FOR EXERCISE 76

In this section

- The Square of a Binomial
- Product of a Sum and a Difference
- Higher Powers of Binomials
- Applications to Area

5.4 SPECIAL PRODUCTS

In Section 5.3 you learned the FOIL method to make multiplying binomials simpler. In this section you will learn rules for squaring binomials and for finding the product of a sum and a difference. These products are called **special products.**

The Square of a Binomial

To compute $(a + b)^2$, the square of a binomial, we can write it as $(a + b)(a + b)$ and use FOIL:

$$\begin{aligned}(a + b)^2 &= (a + b)(a + b)\\ &= a^2 + ab + ab + b^2\\ &= a^2 + 2ab + b^2\end{aligned}$$

So to square $a + b$, *we square the first term* (a^2), *add twice the product of the two terms* $(2ab)$, *then add the square of the last term* (b^2). The square of a binomial occurs so frequently that it is helpful to learn this new rule to find it. The rule for squaring a sum is given symbolically as follows.

The Square of a Sum

$$(a + b)^2 = a^2 + 2ab + b^2$$

EXAMPLE 1 **Using the rule for squaring a sum**

Find the square of each sum.

a) $(x + 3)^2$ **b)** $(2a + 5)^2$