

was worth $P(1+r)^{10}(1+r)^5$ dollars. Which law of exponents can be used to simplify the last expression? Simplify it.

76. **CD rollover.** Ronnie invested P dollars in a 2-year CD with an annual rate of return of r . After the CD rolled over two times, its value was $P(1+r)^2$. Which law of exponents can be used to simplify the expression? Simplify it.

GETTING MORE INVOLVED

77. **Writing.** When we square a product, we square each factor in the product. For example, $(3b)^2 = 9b^2$. Explain why we cannot square a sum by simply squaring each term of the sum.
78. **Writing.** Explain why we define 2^0 to be 1. Explain why $-2^0 \neq 1$.

5.7

NEGATIVE EXPONENTS AND SCIENTIFIC NOTATION

In this section

- Negative Integral Exponents
- Rules for Integral Exponents
- Converting from Scientific Notation
- Converting to Scientific Notation
- Computations with Scientific Notation

We defined exponential expressions with positive integral exponents in Chapter 1 and learned the rules for positive integral exponents in Section 5.6. In this section you will first study negative exponents and then see how positive and negative integral exponents are used in scientific notation.

Negative Integral Exponents

If x is nonzero, the reciprocal of x is written as $\frac{1}{x}$. For example, the reciprocal of 2^3 is written as $\frac{1}{2^3}$. To write the reciprocal of an exponential expression in a simpler way, we use a negative exponent. So $2^{-3} = \frac{1}{2^3}$. In general we have the following definition.

Negative Integral Exponents

If a is a nonzero real number and n is a positive integer, then

$$a^{-n} = \frac{1}{a^n} \quad (\text{If } n \text{ is positive, } -n \text{ is negative.})$$

Since a^{-n} and a^n are reciprocals, their product is 1. Using a negative exponent for the reciprocal allows us to get this result with the product rule for exponents:

$$a^{-n} \cdot a^n = a^{-n+n} = a^0 = 1$$

EXAMPLE 1

Simplifying expressions with negative exponents

Simplify.

a) 2^{-5}

b) $(-2)^{-5}$

c) $\frac{2^{-3}}{3^{-2}}$

Solution

a) $2^{-5} = \frac{1}{2^5} = \frac{1}{32}$

b) $(-2)^{-5} = \frac{1}{(-2)^5}$ Definition of negative exponent
 $= \frac{1}{-32} = -\frac{1}{32}$

c) $\frac{2^{-3}}{3^{-2}} = 2^{-3} \div 3^{-2}$
 $= \frac{1}{2^3} \div \frac{1}{3^2}$
 $= \frac{1}{8} \div \frac{1}{9} = \frac{1}{8} \cdot \frac{9}{1} = \frac{9}{8}$

calculator



close-up

You can evaluate expressions with negative exponents on a calculator as shown here.

```
2^-5*Frac 1/32
(-2)^-5*Frac -1/32
2^-3/3^-2*Frac 9/8
```

CAUTION In simplifying -5^{-2} , the negative sign preceding the 5 is used after 5 is squared and the reciprocal is found. So $-5^{-2} = -(5^{-2}) = -\frac{1}{25}$.

To evaluate a^{-n} , you can first find the n th power of a and then find the reciprocal. However, the result is the same if you first find the reciprocal of a and then find the n th power of the reciprocal. For example,

$$3^{-2} = \frac{1}{3^2} = \frac{1}{9} \quad \text{or} \quad 3^{-2} = \left(\frac{1}{3}\right)^2 = \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}.$$

So the power and the reciprocal can be found in either order. If the exponent is -1 , we simply find the reciprocal. For example,

$$5^{-1} = \frac{1}{5}, \quad \left(\frac{1}{4}\right)^{-1} = 4, \quad \text{and} \quad \left(-\frac{3}{5}\right)^{-1} = -\frac{5}{3}.$$

Because $3^{-2} \cdot 3^2 = 1$, the reciprocal of 3^{-2} is 3^2 , and we have

$$\frac{1}{3^{-2}} = 3^2.$$

These examples illustrate the following rules.

Rules for Negative Exponents

If a is a nonzero real number and n is a positive integer, then

$$a^{-n} = \left(\frac{1}{a}\right)^n, \quad a^{-1} = \frac{1}{a}, \quad \frac{1}{a^{-n}} = a^n, \quad \text{and} \quad \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n.$$

helpful hint

Just because the exponent is negative, it doesn't mean the expression is negative. Note that $(-2)^{-3} = -\frac{1}{8}$ while $(-2)^{-4} = \frac{1}{16}$.

EXAMPLE 2

Using the rules for negative exponents

Simplify.

a) $\left(\frac{3}{4}\right)^{-3}$ b) $10^{-1} + 10^{-1}$ c) $\frac{2}{10^{-3}}$

Solution

a) $\left(\frac{3}{4}\right)^{-3} = \left(\frac{4}{3}\right)^3 = \frac{64}{27}$

b) $10^{-1} + 10^{-1} = \frac{1}{10} + \frac{1}{10} = \frac{2}{10} = \frac{1}{5}$

c) $\frac{2}{10^{-3}} = 2 \cdot \frac{1}{10^{-3}} = 2 \cdot 10^3 = 2 \cdot 1000 = 2000$ ■

Rules for Integral Exponents

Negative exponents are used to make expressions involving reciprocals simpler looking and easier to write. Negative exponents have the added benefit of working in conjunction with all of the rules of exponents that you learned in Section 5.6. For example, we can use the product rule to get

$$x^{-2} \cdot x^{-3} = x^{-2+(-3)} = x^{-5}$$

and the quotient rule to get

$$\frac{y^3}{y^5} = y^{3-5} = y^{-2}.$$

calculator



close-up

You can use a calculator to demonstrate that the product rule for exponents holds when the exponents are negative numbers.

2^-3*2^-5
0.00390625
2^(-3+ -5)
0.00390625

With negative exponents there is no need to state the quotient rule in two parts as we did in Section 5.6. It can be stated simply as

$$\frac{a^m}{a^n} = a^{m-n}$$

for any integers m and n . We list the rules of exponents here for easy reference.

Rules for Integral Exponents

The following rules hold for nonzero real numbers a and b and any integers m and n .

- | | |
|---|-----------------------------|
| 1. $a^0 = 1$ | Definition of zero exponent |
| 2. $a^m \cdot a^n = a^{m+n}$ | Product rule |
| 3. $\frac{a^m}{a^n} = a^{m-n}$ | Quotient rule |
| 4. $(a^m)^n = a^{mn}$ | Power rule |
| 5. $(ab)^n = a^n \cdot b^n$ | Power of a product rule |
| 6. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ | Power of a quotient rule |

EXAMPLE 3

The product and quotient rules for integral exponents

Simplify. Write your answers without negative exponents. Assume that the variables represent nonzero real numbers.

- | | |
|----------------------------|-----------------------------|
| a) $b^{-3}b^5$ | b) $-3x^{-3} \cdot 5x^2$ |
| c) $\frac{m^{-6}}{m^{-2}}$ | d) $\frac{4y^5}{-12y^{-3}}$ |

Solution

$$\begin{aligned} \text{a) } b^{-3}b^5 &= b^{-3+5} && \text{Product rule} \\ &= b^2 && \text{Simplify.} \end{aligned}$$

$$\begin{aligned} \text{b) } -3x^{-3} \cdot 5x^2 &= -15x^{-1} && \text{Product rule} \\ &= -\frac{15}{x} && \text{Definition of negative exponent} \end{aligned}$$

$$\begin{aligned} \text{c) } \frac{m^{-6}}{m^{-2}} &= m^{-6-(-2)} && \text{Quotient rule} \\ &= m^{-4} && \text{Simplify.} \\ &= \frac{1}{m^4} && \text{Definition of negative exponent} \end{aligned}$$

$$\text{d) } \frac{4y^5}{-12y^{-3}} = \frac{y^{5-(-3)}}{-3} = \frac{-y^8}{3}$$

helpful hint

Example 3(c) could be done using the rules for negative exponents and the old quotient rule:

$$\frac{m^{-6}}{m^{-2}} = \frac{m^2}{m^6} = \frac{1}{m^4}$$

It is always good to look at alternative methods. The more tools in your toolbox the better.

In the next example we use the power rules with negative exponents.

EXAMPLE 4 The power rules for integral exponents

Simplify each expression. Write your answers with positive exponents only. Assume that all variables represent nonzero real numbers.

a) $(a^{-3})^2$ b) $(10x^{-3})^{-2}$ c) $\left(\frac{4x^{-5}}{y^2}\right)^{-2}$

Solution

a) $(a^{-3})^2 = a^{-3 \cdot 2}$ Power rule
 $= a^{-6}$
 $= \frac{1}{a^6}$ Definition of negative exponent

b) $(10x^{-3})^{-2} = 10^{-2}(x^{-3})^{-2}$ Power of a product rule
 $= 10^{-2}x^{(-3)(-2)}$ Power rule
 $= \frac{x^6}{10^2}$ Definition of negative exponent
 $= \frac{x^6}{100}$

c) $\left(\frac{4x^{-5}}{y^2}\right)^{-2} = \frac{(4x^{-5})^{-2}}{(y^2)^{-2}}$ Power of a quotient rule
 $= \frac{4^{-2}x^{10}}{y^{-4}}$ Power of a product rule and power rule
 $= 4^{-2} \cdot x^{10} \cdot \frac{1}{y^{-4}}$ Because $\frac{a}{b} = a \cdot \frac{1}{b}$.
 $= \frac{1}{4^2} \cdot x^{10} \cdot y^4$ Definition of negative exponent
 $= \frac{x^{10}y^4}{16}$ Simplify.

helpful hint

The exponent rules in this section apply to expressions that involve only multiplication and division. This is not too surprising since exponents, multiplication, and division are closely related. Recall that $a^3 = a \cdot a \cdot a$ and $a \div b = a \cdot b^{-1}$.

Converting from Scientific Notation

Many of the numbers occurring in science are either very large or very small. The speed of light is 983,569,000 feet per second. One millimeter is equal to 0.000001 kilometer. In scientific notation, numbers larger than 10 or smaller than 1 are written by using positive or negative exponents.

Scientific notation is based on multiplication by integral powers of 10. Multiplying a number by a positive power of 10 moves the decimal point to the right:

$$10(5.32) = 53.2$$

$$10^2(5.32) = 100(5.32) = 532$$

$$10^3(5.32) = 1000(5.32) = 5320$$

Multiplying by a negative power of 10 moves the decimal point to the left:

$$10^{-1}(5.32) = \frac{1}{10}(5.32) = 0.532$$

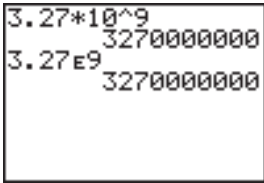
$$10^{-2}(5.32) = \frac{1}{100}(5.32) = 0.0532$$

$$10^{-3}(5.32) = \frac{1}{1000}(5.32) = 0.00532$$

calculator

close-up

On a graphing calculator you can write scientific notation by actually using the power of 10 or press EE to get the letter E, which indicates that the following number is the power of 10.



Note that if the exponent is not too large, scientific notation is converted to standard notation when you press ENTER.

So if n is a positive integer, multiplying by 10^n moves the decimal point n places to the right and multiplying by 10^{-n} moves it n places to the left.

A number in scientific notation is written as a product of a number between 1 and 10 and a power of 10. The times symbol \times indicates multiplication. For example, 3.27×10^9 and 2.5×10^{-4} are numbers in scientific notation. In scientific notation there is one digit to the left of the decimal point.

To convert 3.27×10^9 to standard notation, move the decimal point nine places to the right:

$$3.27 \times 10^9 = 3,270,000,000$$

9 places to the right

Of course, it is not necessary to put the decimal point in when writing a whole number.

To convert 2.5×10^{-4} to standard notation, the decimal point is moved four places to the left:

$$2.5 \times 10^{-4} = 0.00025$$

4 places to the left

In general, we use the following strategy to convert from scientific notation to standard notation.

Strategy for Converting from Scientific Notation to Standard Notation

1. Determine the number of places to move the decimal point by examining the exponent on the 10.
2. Move to the right for a positive exponent and to the left for a negative exponent.

EXAMPLE 5

Converting scientific notation to standard notation

Write in standard notation.

a) 7.02×10^6

b) 8.13×10^{-5}

Solution

a) Because the exponent is positive, move the decimal point six places to the right:

$$7.02 \times 10^6 = 7020000. = 7,020,000$$

b) Because the exponent is negative, move the decimal point five places to the left.

$$8.13 \times 10^{-5} = 0.0000813 \quad \blacksquare$$

Converting to Scientific Notation

To convert a positive number to scientific notation, we just reverse the strategy for converting from scientific notation.

Strategy for Converting to Scientific Notation

1. Count the number of places (n) that the decimal must be moved so that it will follow the first nonzero digit of the number.
2. If the original number was larger than 10, multiply by 10^n .
3. If the original number was smaller than 1, multiply by 10^{-n} .

Remember that the scientific notation for a number larger than 10 will have a positive power of 10 and the scientific notation for a number between 0 and 1 will have a negative power of 10.

EXAMPLE 6**Converting numbers to scientific notation**

Write in scientific notation.

- a) 7,346,200 b) 0.0000348 c) 135×10^{-12}

Solution

- a) Because 7,346,200 is larger than 10, the exponent on the 10 will be positive:

$$7,346,200 = 7.3462 \times 10^6$$

- b) Because 0.0000348 is smaller than 1, the exponent on the 10 will be negative:

$$0.0000348 = 3.48 \times 10^{-5}$$

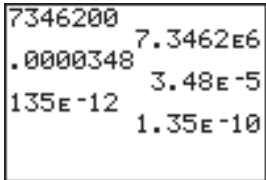
- c) There should be only one nonzero digit to the left of the decimal point:

$$\begin{aligned} 135 \times 10^{-12} &= 1.35 \times 10^2 \times 10^{-12} && \text{Convert 135 to scientific notation.} \\ &= 1.35 \times 10^{-10} && \text{Product rule} \end{aligned}$$

calculator

close-up

To convert to scientific notation, set the mode to scientific. In scientific mode all results are given in scientific notation.


Computations with Scientific Notation

An important feature of scientific notation is its use in computations. Numbers in scientific notation are nothing more than exponential expressions, and you have already studied operations with exponential expressions in this section. We use the same rules of exponents on numbers in scientific notation that we use on any other exponential expressions.

EXAMPLE 7**Using the rules of exponents with scientific notation**

Perform the indicated computations. Write the answers in scientific notation.

- a) $(3 \times 10^6)(2 \times 10^8)$ b) $\frac{4 \times 10^5}{8 \times 10^{-2}}$ c) $(5 \times 10^{-7})^3$

Solution

a) $(3 \times 10^6)(2 \times 10^8) = 3 \cdot 2 \cdot 10^6 \cdot 10^8 = 6 \times 10^{14}$

b) $\frac{4 \times 10^5}{8 \times 10^{-2}} = \frac{4}{8} \cdot \frac{10^5}{10^{-2}} = \frac{1}{2} \cdot 10^{5-(-2)}$ Quotient rule

$$= (0.5)10^7$$

$\frac{1}{2} = 0.5$

$$= 5 \times 10^{-1} \cdot 10^7$$
 Write 0.5 in scientific notation.
$$= 5 \times 10^6$$
 Product rule

c) $(5 \times 10^{-7})^3 = 5^3(10^{-7})^3$ Power of a product rule

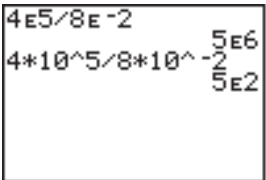
$$= 125 \cdot 10^{-21}$$
 Power rule
$$= 1.25 \times 10^2 \times 10^{-21}$$
 $125 = 1.25 \times 10^2$

$$= 1.25 \times 10^{-19}$$
 Product rule

calculator

close-up

With a calculator's built-in scientific notation, some parentheses can be omitted as shown below. Writing out the powers of 10 can lead to errors.



Try these computations with your calculator.

EXAMPLE 8 Converting to scientific notation for computations

Perform these computations by first converting each number into scientific notation. Give your answer in scientific notation.

a) $(3,000,000)(0.0002)$ b) $(20,000,000)^3(0.0000003)$

Solution

$$\begin{aligned} \text{a) } (3,000,000)(0.0002) &= 3 \times 10^6 \cdot 2 \times 10^{-4} && \text{Scientific notation} \\ &= 6 \times 10^2 && \text{Product rule} \end{aligned}$$

$$\begin{aligned} \text{b) } (20,000,000)^3(0.0000003) &= (2 \times 10^7)^3(3 \times 10^{-7}) && \text{Scientific notation} \\ &= 8 \times 10^{21} \cdot 3 \times 10^{-7} && \text{Power of a product rule} \\ &= 24 \times 10^{14} \\ &= 2.4 \times 10^1 \times 10^{14} && 24 = 2.4 \times 10^1 \\ &= 2.4 \times 10^{15} && \text{Product rule} \end{aligned}$$

WARM-UPS

True or false? Explain your answer.

- $10^{-2} = \frac{1}{100}$
- $\left(-\frac{1}{5}\right)^{-1} = 5$
- $3^{-2} \cdot 2^{-1} = 6^{-3}$
- $\frac{3^{-2}}{3^{-1}} = \frac{1}{3}$
- $23.7 = 2.37 \times 10^{-1}$
- $0.000036 = 3.6 \times 10^{-5}$
- $25 \cdot 10^7 = 2.5 \times 10^8$
- $0.442 \times 10^{-3} = 4.42 \times 10^{-4}$
- $(3 \times 10^{-9})^2 = 9 \times 10^{-18}$
- $(2 \times 10^{-5})(4 \times 10^4) = 8 \times 10^{-20}$

5.7 EXERCISES

Reading and Writing After reading this section, write out the answers to these questions. Use complete sentences.

- What does a negative exponent mean?
- What is the correct order for evaluating the operations indicated by a negative exponent?
- What is the new quotient rule for exponents?
- How do you convert a number from scientific notation to standard notation?
- How do you convert a number from standard notation to scientific notation?

6. Which numbers are not usually written in scientific notation?

Variables in all exercises represent positive real numbers. Evaluate each expression. See Example 1.

7. 3^{-1} 8. 3^{-3} 9. $(-2)^{-4}$

10. $(-3)^{-4}$ 11. -4^{-2} 12. -2^{-4}

13. $\frac{5^{-2}}{10^{-2}}$ 14. $\frac{3^{-4}}{6^{-2}}$

Simplify. See Example 2.

15. $\left(\frac{5}{2}\right)^{-3}$ 16. $\left(\frac{4}{3}\right)^{-2}$ 17. $6^{-1} + 6^{-1}$

18. $2^{-1} + 4^{-1}$ 19. $\frac{10}{5^{-3}}$ 20. $\frac{1}{25 \cdot 10^{-4}}$

21. $\frac{1}{4^{-3}} + \frac{3^2}{2^{-1}}$ 22. $\frac{2^3}{10^{-2}} - \frac{2}{7^{-2}}$

Simplify. Write answers without negative exponents. See Example 3.

23. $x^{-1}x^2$ 24. $y^{-3}y^5$

25. $-2x^2 \cdot 8x^{-6}$ 26. $5y^5(-6y^{-7})$

27. $-3a^{-2}(-2a^{-3})$ 28. $(-b^{-3})(-b^{-5})$

29. $\frac{u^{-5}}{u^3}$ 30. $\frac{w^{-4}}{w^6}$

31. $\frac{8t^{-3}}{-2t^{-5}}$ 32. $\frac{-22w^{-4}}{-11w^{-3}}$

33. $\frac{-6x^5}{-3x^{-6}}$ 34. $\frac{-51y^6}{17y^{-9}}$

Simplify each expression. Write answers without negative exponents. See Example 4.

35. $(x^2)^{-5}$ 36. $(y^{-2})^4$ 37. $(a^{-3})^{-3}$

38. $(b^{-5})^{-2}$ 39. $(2x^{-3})^{-4}$ 40. $(3y^{-1})^{-2}$

41. $(4x^2y^{-3})^{-2}$ 42. $(6s^{-2}t^4)^{-1}$

43. $\left(\frac{2x^{-1}}{y^{-3}}\right)^{-2}$ 44. $\left(\frac{a^{-2}}{3b^3}\right)^{-3}$

45. $\left(\frac{2a^{-3}}{ac^{-2}}\right)^{-4}$ 46. $\left(\frac{3w^2}{w^4x^3}\right)^{-2}$

Simplify. Write answers without negative exponents.

47. $2^{-1} \cdot 3^{-1}$ 48. $2^{-1} + 3^{-1}$

49. $(2 \cdot 3^{-1})^{-1}$ 50. $(2^{-1} + 3)^{-1}$

51. $(x^{-2})^{-3} + 3x^7(-5x^{-1})$

52. $(ab^{-1})^2 - ab(-ab^{-3})$

53. $\frac{a^3b^{-2}}{a^{-1}} + \left(\frac{b^6a^{-2}}{b^5}\right)^{-2}$

54. $\left(\frac{x^{-3}y^{-1}}{2x}\right)^{-3} + \frac{6x^9y^3}{-3x^{-3}}$

Write each number in standard notation. See Example 5.

55. 9.86×10^9 56. 4.007×10^4

57. 1.37×10^{-3} 58. 9.3×10^{-5}

59. 1×10^{-6} 60. 3×10^{-1}

61. 6×10^5 62. 8×10^6

Write each number in scientific notation. See Example 6.

63. 9000

64. 5,298,000

65. 0.00078

66. 0.000214

67. 0.0000085

68. 5,670,000,000

69. 525×10^9

70. 0.0034×10^{-8}

Perform the computations. Write answers in scientific notation. See Example 7.

71. $(3 \times 10^5)(2 \times 10^{-15})$ 72. $(2 \times 10^{-9})(4 \times 10^{23})$

73. $\frac{4 \times 10^{-8}}{2 \times 10^{30}}$ 74. $\frac{9 \times 10^{-4}}{3 \times 10^{-6}}$

75. $\frac{3 \times 10^{20}}{6 \times 10^{-8}}$ 76. $\frac{1 \times 10^{-8}}{4 \times 10^7}$

77. $(3 \times 10^{12})^2$ 78. $(2 \times 10^{-5})^3$

79. $(5 \times 10^4)^3$ 80. $(5 \times 10^{14})^{-1}$

81. $(4 \times 10^{32})^{-1}$ 82. $(6 \times 10^{11})^2$

Perform the following computations by first converting each number into scientific notation. Write answers in scientific notation. See Example 8.

83. $(4300)(2,000,000)$

84. $(40,000)(4,000,000,000)$

85. $(4,200,000)(0.00005)$

86. $(0.00075)(4,000,000)$

87. $(300)^3(0.000001)^5$

88. $(200)^4(0.0005)^3$

89. $\frac{(4000)(90,000)}{0.00000012}$

90. $\frac{(30,000)(80,000)}{(0.000006)(0.002)}$



Perform the following computations with the aid of a calculator. Write answers in scientific notation. Round to three decimal places.

91. $(6.3 \times 10^6)(1.45 \times 10^{-4})$
92. $(8.35 \times 10^9)(4.5 \times 10^3)$
93. $(5.36 \times 10^{-4}) + (3.55 \times 10^{-5})$
94. $(8.79 \times 10^8) + (6.48 \times 10^9)$
95. $\frac{(3.5 \times 10^5)(4.3 \times 10^{-6})}{3.4 \times 10^{-8}}$
96. $\frac{(3.5 \times 10^{-8})(4.4 \times 10^{-4})}{2.43 \times 10^{45}}$
97. $(3.56 \times 10^{85})(4.43 \times 10^{96})$
98. $(8 \times 10^{99}) + (3 \times 10^{99})$



Solve each problem.

99. **Distance to the sun.** The distance from the earth to the sun is 93 million miles. Express this distance in feet. (1 mile = 5280 feet.)

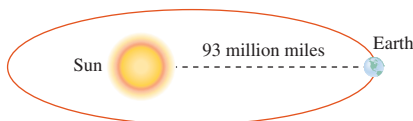


FIGURE FOR EXERCISE 99

100. **Speed of light.** The speed of light is 9.83569×10^8 feet per second. How long does it take light to travel from the sun to the earth? See Exercise 99.
101. **Warp drive, Scotty.** How long does it take a spacecraft traveling at 2×10^{35} miles per hour (warp factor 4) to travel 93 million miles.
102. **Area of a dot.** If the radius of a very small circle is 2.35×10^{-8} centimeters, then what is the circle's area?
103. **Circumference of a circle.** If the circumference of a circle is 5.68×10^9 feet, then what is its radius?
104. **Diameter of a circle.** If the diameter of a circle is 1.3×10^{-12} meters, then what is its radius?
105. **Extracting metals from ore.** Thomas Sherwood studied the relationship between the concentration of a metal in commercial ore and the price of the metal. The accompanying graph shows the Sherwood plot with the locations of several metals marked. Even though the scales on this graph are not typical, the graph can be read in the same manner as other graphs. Note also that a concentration of 100 is 100%.

- a) Use the figure to estimate the price of copper (Cu) and its concentration in commercial ore.
- b) Use the figure to estimate the price of a metal that has a concentration of 10^{-6} percent in commercial ore.
- c) Would the four points shown in the graph lie along a straight line if they were plotted in our usual coordinate system?

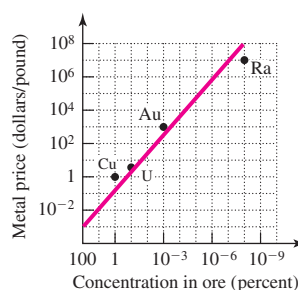


FIGURE FOR EXERCISE 105

106. **Recycling metals.** The accompanying graph shows the prices of various metals that are being recycled and the minimum concentration in waste required for recycling. The straight line is the line from the figure for Exercise 105. Points above the line correspond to metals for which it is economically feasible to increase recycling efforts.
 - a) Use the figure to estimate the price of mercury (Hg) and the minimum concentration in waste required for recycling mercury.
 - b) Use the figure to estimate the price of silver (Ag) and the minimum concentration in waste required for recycling silver.

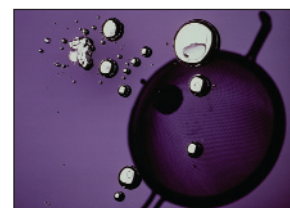
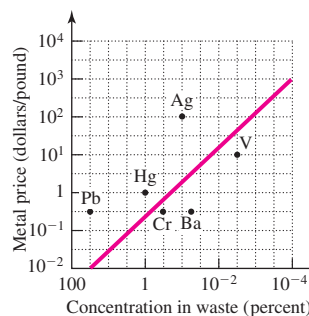


FIGURE FOR EXERCISE 106



107. **Present value.** The present value P that will amount to A dollars in n years with interest compounded annually at annual interest rate r , is given by

$$P = A(1 + r)^{-n}.$$

Find the present value that will amount to \$50,000 in 20 years at 8% compounded annually.

- 108. Investing in stocks.** U.S. small company stocks have returned an average of 14.9% annually for the last 50 years (T. Rowe Price, www.troweprice.com). Use the present value formula from the previous exercise to find the amount invested today in small company stocks that would be worth \$1 million in 50 years,

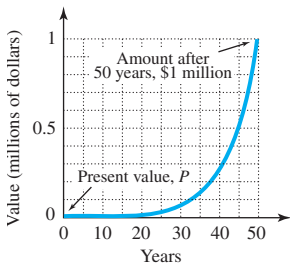


FIGURE FOR EXERCISE 108

assuming that small company stocks continue to return 14.9% annually for the next 50 years.

GETTING MORE INVOLVED

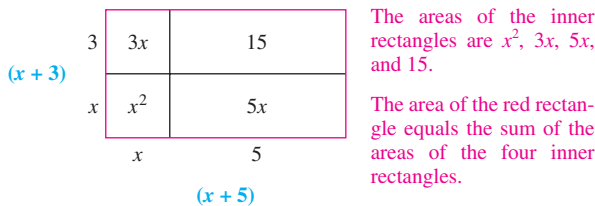
- 109. Exploration.** a) If $w^{-3} < 0$, then what can you say about w ? b) If $(-5)^m < 0$, then what can you say about m ? c) What restriction must be placed on w and m so that $w^m < 0$?

- 110. Discussion.** Which of the following expressions is not equal to -1 ? Explain your answer.
 a) -1^{-1} b) -1^{-2}
 c) $(-1^{-1})^{-1}$ d) $(-1)^{-1}$
 e) $(-1)^{-2}$

COLLABORATIVE ACTIVITIES

Area as a Model of FOIL

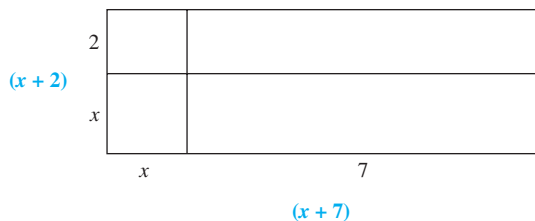
Sometimes we can use drawings to represent mathematical operations. The area of a rectangle can represent the process we use when multiplying binomials. The rectangle below represents the multiplication of the binomials $(x + 3)$ and $(x + 5)$:



Area of red rectangle:

$$(x + 3)(x + 5) = x^2 + 3x + 5x + 15 = x^2 + 8x + 15$$

1. a. With your partner, find the areas of the inner rectangles to find the product $(x + 2)(x + 7)$ below:

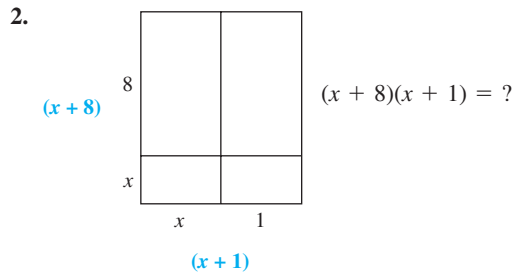


$$(x + 2)(x + 7) = ?$$

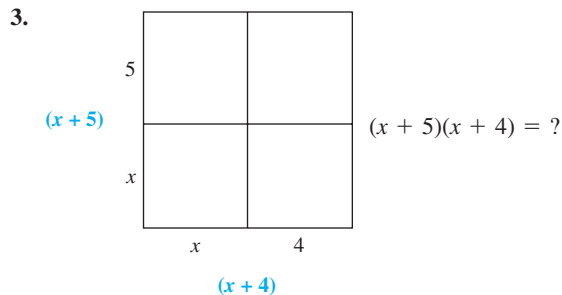
- b. Find the same product $(x + 2)(x + 7)$ using FOIL.

Grouping: Pairs
 Topic: Multiplying polynomials

For problem 2, student A uses FOIL to find the given product while student B finds the area with the diagram.



For problem 3, student B uses FOIL and A uses the diagram.



4. Student A draws a diagram to find the product $(x + 3)(x + 7)$. Student B finds $(x + 3)(x + 7)$ using FOIL.