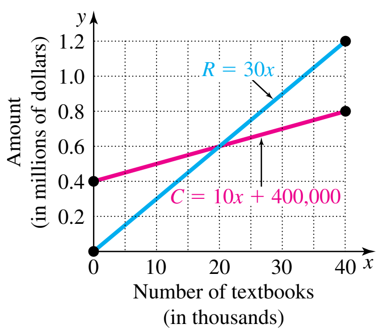
a) Use the accompanying graph to estimate the values of T and B that satisfy both equations.
b) Solve the system algebraically to find the bonus and the amount of tax.

57. ***Textbook case.*** The accompanying graph shows the cost of producing textbooks and the revenue from the sale of those textbooks.
a) What is the cost of producing 10,000 textbooks?
b) What is the revenue when 10,000 textbooks are sold?
c) For what number of textbooks is the cost equal to the revenue?
d) The cost of producing zero textbooks is called the *fixed cost*. Find the fixed cost.



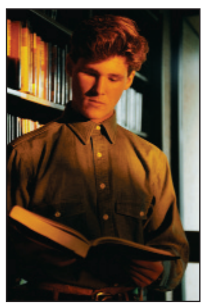


FIGURE FOR EXERCISE 57

58. ***Free market.*** The function $S = 5000 + 200x$ and $D = 9500 - 100x$ express the supply S and the demand D, respectively, for a popular compact disk brand as a function of its price x (in dollars).
a) Graph the functions on the same coordinate system.
b) What happens to the supply as the price increases?
c) What happens to the demand as the price increases?
d) The price at which supply and demand are equal is called the *equilibrium price*. What is the equilibrium price?

GETTING MORE INVOLVED

59. ***Discussion.*** Which of the following equations is not equivalent to $2x - 3y = 6$?

a) $3y - 2x = 6$ b) $y = \frac{2}{3}x - 2$

c) $x = \frac{3}{2}y + 3$ d) $2(x - 5) = 3y - 4$

60. ***Discussion.*** Which of the following equations is inconsistent with the equation $3x + 4y = 8$?

a) $y = \frac{3}{4}x + 2$ b) $6x + 8y = 16$

c) $y = -\frac{3}{4}x + 8$ d) $3x - 4y = 8$

GRAPHING CALCULATOR EXERCISES

61. Solve each system by graphing each pair of equations on a graphing calculator and using the trace feature or intersect feature to estimate the point of intersection. Find the coordinates of the intersection to the nearest tenth.

a) $y = 3.5x - 7.2$
$y = -2.3x + 9.1$

b) $2.3x - 4.1y = 3.3$
$3.4x + 9.2y = 1.3$

In this section

- The Addition Method
- Equations Involving Fractions or Decimals
- Applications

8.2 THE ADDITION METHOD

In Section 8.1 you used substitution to eliminate a variable in a system of equations. In this section we see another method for eliminating a variable in a system of equations.

The Addition Method

In the **addition method** we eliminate a variable by adding the equations.

EXAMPLE 1

An independent system solved by addition

Solve the system by the addition method:

$$3x - 5y = -9$$
$$4x + 5y = 23$$

calculator close-up

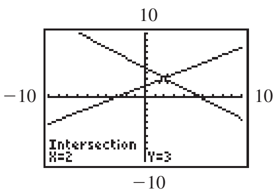
To check Example 1, graph

$y_1 = (-9 - 3x)/-5$

and

$y_2 = (23 - 4x)/5.$

Use the intersect feature to find the point of intersection of the two lines.



Solution

The addition property of equality allows us to add the same number to each side of an equation. We can also use the addition property of equality to add the two left sides and add the two right sides:

$$\begin{aligned} 3x - 5y &= -9 \\ 4x + 5y &= 23 \\ \hline 7x \qquad &= 14 \quad \text{Add.} \\ x &= 2 \end{aligned}$$

The y-term was eliminated when we added the equations because the coefficients of the y-terms were opposites. Now use $x = 2$ in one of the original equations to find y. It does not matter which original equation we use. In this example we will use both equations to see that we get the same y in either case.

$$\begin{aligned} 3x - 5y &= -9 & & 4x + 5y = 23 \\ 3(2) - 5y &= -9 & \text{Replace } x \text{ by 2.} \quad & 4(2) + 5y = 23 \\ 6 - 5y &= -9 & \text{Solve for } y. \quad & 8 + 5y = 23 \\ -5y &= -15 & & 5y = 15 \\ y &= 3 & & y = 3 \end{aligned}$$

Because $3(2) - 5(3) = -9$ and $4(2) + 5(3) = 23$ are both true, $(2, 3)$ satisfies both equations. The solution set is $\{(2, 3)\}$. ■

Actually the addition method can be used to eliminate any variable whose coefficients are opposites. If neither variable has coefficients that are opposites, then we use the multiplication property of equality to change the coefficients of the variables, as shown in Examples 2 and 3.

EXAMPLE 2

Using multiplication and addition

Solve the system by the addition method:

$$\begin{aligned} 2x - 3y &= -13 \\ 5x - 12y &= -46 \end{aligned}$$

Solution

If we multiply both sides of the first equation by -4, the coefficients of y will be 12 and -12, and y will be eliminated by addition.

$$\begin{aligned} (-4)(2x - 3y) &= (-4)(-13) \quad \text{Multiply each side by } -4. \\ 5x - 12y &= -46 \end{aligned}$$

$$\begin{aligned} -8x + 12y &= 52 \\ 5x - 12y &= -46 \quad \text{Add.} \\ \hline -3x \qquad &= 6 \\ x &= -2 \end{aligned}$$

Replace x by -2 in one of the original equations to find y:

$$\begin{aligned} 2x - 3y &= -13 \\ 2(-2) - 3y &= -13 \\ -4 - 3y &= -13 \\ -3y &= -9 \\ y &= 3 \end{aligned}$$

study tip

Keep on reviewing. After you have done your current assignment, go back a section or two and try a few problems. You will be amazed at how much your knowledge will improve with a regular review.

Because $2(-2) - 3(3) = -13$ and $5(-2) - 12(3) = -46$ are both true, the solution set is $\{(-2, 3)\}$. ■

EXAMPLE 3

Multiplying both equations before adding

Solve the system by the addition method:

$$-2x + 3y = 6$$
$$3x - 5y = -11$$

Solution

To eliminate x, we multiply the first equation by 3 and the second by 2:

$$3(-2x + 3y) = 3(6) \quad \text{Multiply each side by 3.}$$
$$2(3x - 5y) = 2(-11) \quad \text{Multiply each side by 2.}$$

$$-6x + 9y = 18$$
$$6x - 10y = -22 \quad \text{Add.}$$
$$-y = -4$$
$$y = 4$$

Note that we could have eliminated y by multiplying by 5 and 3. Now insert $y = 4$ into one of the original equations to find x:

$$-2x + 3(4) = 6 \quad \text{Let } y = 4 \text{ in } -2x + 3y = 6.$$
$$-2x + 12 = 6$$
$$-2x = -6$$
$$x = 3$$

Check that (3, 4) satisfies both equations. The solution set is $\{(3, 4)\}$. ■

We can always use the addition method as long as the equations in a system are in the same form.

EXAMPLE 4

calculator close-up

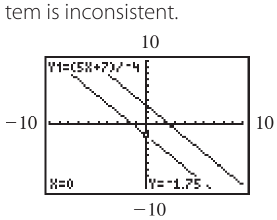
To check Example 4, graph

$y_1 = (5x + 7)/-4$

and

$y_2 = (-5x + 12)/4.$

Since the lines appear to be parallel, the graph supports the conclusion that the system is inconsistent.



Using the addition method for an inconsistent system

Solve the system:

$$-4y = 5x + 7$$
$$4y = -5x + 12$$

Solution

If these equations are added, both variables are eliminated:

$$-4y = 5x + 7$$
$$4y = -5x + 12$$
$$0 = 19$$

Because this equation is inconsistent, the original equations are inconsistent. The solution set to the system is the empty set, $\emptyset$. ■

Equations Involving Fractions or Decimals

When a system of equations involves fractions or decimals, we can use the multiplication property of equality to eliminate the fractions or decimals.

EXAMPLE 5 **A system with fractions**

Solve the system:

$$\frac{1}{2}x - \frac{2}{3}y = 7$$
$$\frac{2}{3}x - \frac{3}{4}y = 11$$

Solution

Multiply the first equation by 6 and the second equation by 12:

$$6\left(\frac{1}{2}x - \frac{2}{3}y\right) = 6(7) \quad \rightarrow \quad 3x - 4y = 42$$
$$12\left(\frac{2}{3}x - \frac{3}{4}y\right) = 12(11) \quad \rightarrow \quad 8x - 9y = 132$$

To eliminate x, multiply the first equation by -8 and the second by 3:

$$\begin{aligned} -8(3x - 4y) = -8(42) \quad &\rightarrow \quad -24x + 32y = -336 \\ 3(8x - 9y) = 3(132) \quad &\rightarrow \quad \underline{24x - 27y = 396} \\ &\qquad\qquad\qquad 5y = 60 \\ &\qquad\qquad\qquad y = 12 \end{aligned}$$

Substitute $y = 12$ into the first of the original equations:

$$\frac{1}{2}x - \frac{2}{3}(12) = 7$$
$$\frac{1}{2}x - 8 = 7$$
$$\frac{1}{2}x = 15$$
$$x = 30$$

Check (30, 12) in the original system. The solution set is $\{(30, 12)\}$. ■

calculator close-up

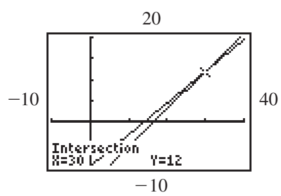
To check Example 5, graph

$y_1 = (7 - (1/2)x)/(-2/3)$

and

$y_2 = (11 - (2/3)x)/(-3/4)$.

The lines appear to intersect at (30, 12).



The strategy for solving a system by addition is summarized as follows.

The Addition Method

1. Write both equations in the same form (usually $Ax + By = C$).
2. Multiply one or both of the equations by appropriate numbers (if necessary) so that one of the variables will be eliminated by addition.
3. Add the equations to get an equation in one variable.
4. Solve the equation in one variable.
5. Substitute the value obtained for one variable into one of the original equations to obtain the value of the other variable.
6. Check the two values in both of the original equations.

Applications

Any system of two linear equations in two variables can be solved by either the addition method or substitution. In applications we use whichever method appears to be the simpler for the problem at hand.

EXAMPLE 6

Fajitas and burritos

At the Cactus Cafe the total price for four fajita dinners and three burrito dinners is \$48, and the total price for three fajita dinners and two burrito dinners is \$34. What is the price of each type of dinner?

helpful hint

You can see from Example 6 that the standard form $Ax + By = C$ occurs naturally in accounting. This form will occur whenever we have the price of each item and a quantity of two items and want to express the total cost.

Solution

Let x represent the price (in dollars) of a fajita dinner, and let y represent the price (in dollars) of a burrito dinner. We can write two equations to describe the given information:

$$4x + 3y = 48$$
$$3x + 2y = 34$$

Because 12 is the least common multiple of 4 and 3 (the coefficients of x), we multiply the first equation by -3 and the second by 4:

$$-3(4x + 3y) = -3(48) \quad \text{Multiply each side by } -3.$$
$$4(3x + 2y) = 4(34) \quad \text{Multiply each side by } 4.$$

$$-12x - 9y = -144$$
$$12x + 8y = 136 \quad \text{Add.}$$
$$-y = -8$$
$$y = 8$$

To find x, use $y = 8$ in the first equation $4x + 3y = 48$:

$$4x + 3(8) = 48$$
$$4x + 24 = 48$$
$$4x = 24$$
$$x = 6$$

So the fajita dinners are \$6 each, and the burrito dinners are \$8 each. Check this solution in the original problem. ■

EXAMPLE 7

Mixing cooking oil

Canola oil is 7% saturated fat, and corn oil is 14% saturated fat. Crisco sells a blend, Crisco Canola and Corn Oil, which is 11% saturated fat. How many gallons of each type of oil must be mixed to get 280 gallons of this blend?

study tip

Play offensive math, not defensive math. A student who says, "Give me a question and I'll see if I can answer it," is playing defensive math. The student is taking a passive approach to learning. A student who takes an active approach and knows the usual questions and answers for each topic is playing offensive math.

Solution

Let x represent the number of gallons of canola oil, and let y represent the number of gallons of corn oil. Make a table to summarize all facts:

	Amount (gallons)	% fat	Amount of fat (gallons)
Canola oil	x	7	$0.07x$
Corn oil	y	14	$0.14y$
Canola and Corn Oil	280	11	0.11(280) or 30.8

We can write two equations to express the following facts: (1) the total amount of oil is 280 gallons and (2) the total amount of fat is 30.8 gallons. Then we can use multiplication and addition to solve the system.

$$\begin{aligned} &(1) & x + y &= 280 & &\text{Multiply by } -0.07. & -0.07x - 0.07y &= -19.6 \\ &(2) & 0.07x + 0.14y &= 30.80 & & & 0.07x + 0.14y &= 30.8 \\ & & & & & & 0.07y &= 11.2 \\ & & & & & & y &= \frac{11.2}{0.07} = 160 \end{aligned}$$

If $y = 160$ and $x + y = 280$, then $x = 120$. Check that $0.07(120) + 0.14(160) = 30.8$. So it takes 120 gallons of canola oil and 160 gallons of corn oil to make 280 gallons of Crisco Canola and Corn Oil.

WARM-UPS

True or false? Explain your answer.

Exercises 1–6 refer to the following systems.

a) $3x - y = 9$, $2x + y = 6$ b) $4x - 2y = 20$, $-2x + y = -10$ c) $x - y = 6$, $x - y = 7$

1. To solve system (a) by addition, we simply add the equations.
2. To solve system (a) by addition, we can multiply the first equation by 2 and the second by 3 and then add.
3. To solve system (b) by addition, we can multiply the second equation by 2 and then add.
4. Both $(0, -10)$ and $(5, 0)$ are in the solution set to system (b).
5. The solution set to system (b) is the set of all real numbers.
6. System (c) has no solution.
7. Both the addition method and substitution method are used to eliminate a variable from a system of two linear equations in two variables.
8. For the addition method, both equations must be in standard form.
9. To eliminate fractions in an equation, we multiply each side by the least common denominator of all fractions involved.
10. We can eliminate either variable by using the addition method.

8.2 EXERCISES

Reading and Writing *After reading this section, write out the answers to these questions. Use complete sentences.*

1. What method is presented in this section for solving a system of linear equations?
2. What are we trying to accomplish by adding the equations?
3. What must we sometimes do before we add the equations?
4. How can you recognize an inconsistent system when solving by addition?
5. How can you recognize a dependent system when solving by addition?
6. For which systems is the addition method easier to use than substitution?

Solve each system by the addition method. See Examples 1–3.

7. $x + y = 7$
$x - y = 9$

8. $3x - 4y = 11$
$-3x + 2y = -7$

9. $x - y = 12$
$2x + y = 3$

10. $x - 2y = -1$
$-x + 5y = 4$

11. $2x - y = -5$
$3x + 2y = 3$

12. $3x + 5y = -11$
$x - 2y = 11$

13. $2x - 5y = 13$
$3x + 4y = -15$

14. $3x + 4y = -5$
$5x + 6y = -7$

15. $2x = 3y + 11$
$7x - 4y = 6$

16. $2x = 2 - y$
$3x + y = -1$

17. $x + y = 48$
$12x + 14y = 628$

18. $x + y = 13$
$22x + 36y = 356$

Solve each system by the addition method. Determine whether the equations are independent, dependent, or inconsistent. See Example 4.

19. $3x - 4y = 9$
$-3x + 4y = 12$

20. $x - y = 3$
$-6x + 6y = 17$

21. $5x - y = 1$
$10x - 2y = 2$

22. $4x + 3y = 2$
$-12x - 9y = -6$

23. $2x - y = 5$
$2x + y = 5$

24. $-3x + 2y = 8$
$3x + 2y = 8$

Solve each system by the addition method. See Example 5.

25. $\frac{1}{4}x + \frac{1}{3}y = 5$
$x - y = 6$

26. $\frac{3x}{2} - \frac{2y}{3} = 10$
$\frac{1}{2}x + \frac{1}{2}y = -1$

27. $\frac{x}{4} - \frac{y}{3} = -4$
$\frac{x}{8} + \frac{y}{6} = 0$

28. $\frac{x}{3} - \frac{y}{2} = -\frac{5}{6}$
$\frac{x}{5} - \frac{y}{3} = -\frac{3}{5}$

29. $\frac{1}{8}x + \frac{1}{4}y = 5$
$\frac{1}{16}x + \frac{1}{2}y = 7$

30. $\frac{3}{7}x + \frac{5}{9}y = 27$
$\frac{1}{9}x + \frac{2}{7}y = 7$

31. $0.05x + 0.10y = 1.30$
$x + y = 19$

32. $0.1x + 0.06y = 9$
$0.09x + 0.5y = 52.7$

33. $x + y = 1200$
$0.12x + 0.09y = 120$

34. $x - y = 100$
$0.20x + 0.06y = 150$

35. $1.5x - 2y = -0.25$
$3x + 1.5y = 6.375$

36. $3x - 2.5y = 7.125$
$2.5x - 3y = 7.3125$

Write a system of two equations in two unknowns for each problem. Solve each system by the method of your choice. See Examples 6 and 7.

37. ***Coffee and doughnuts.*** On Monday, Archie paid \$2.54 for three doughnuts and two coffees. On Tuesday he paid \$2.46 for two doughnuts and three coffees. On Wednesday he was tired of paying the tab and went out for coffee by himself. What was his bill for one doughnut and one coffee?

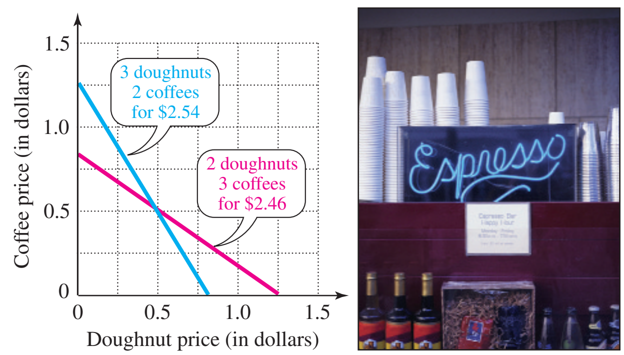


FIGURE FOR EXERCISE 37

38. ***Books and magazines.*** At Gwen's garage sale, all books were one price, and all magazines were another price. Harriet bought four books and three magazines for \$1.45, and June bought two books and five magazines for \$1.25. What was the price of a book and what was the price of a magazine?

39. ***Boys and girls.*** One-half of the boys and one-third of the girls of Freemont High attended the homecoming game, whereas one-third of the boys and one-half of the girls attended the homecoming dance. If there were 570 students at the game and 580 at the dance, then how many students are there at Freemont High?

40. ***Girls and boys.*** There are 385 surfers in Surf City. Two-thirds of the boys are surfers and one-twelfth of the girls are surfers. If there are two girls for every boy, then how many boys and how many girls are there in Surf City?

41. ***Nickels and dimes.*** Winborne has 35 coins consisting of dimes and nickels. If the value of his coins is \$3.30, then how many of each type does he have?

42. ***Pennies and nickels.*** Wendy has 52 coins consisting of nickels and pennies. If the value of the coins is \$1.20, then how many of each type does she have?

43. ***Blending fudge.*** The Chocolate Factory in Vancouver blends its double-dark-chocolate fudge, which is 35% fat, with its peanut butter fudge, which is 25% fat, to obtain double-dark-peanut fudge, which is 29% fat.

a) Use the accompanying graph to estimate the number of pounds of each type that must be mixed to obtain 50 pounds of double-dark-peanut fudge.

b) Write a system of equations and solve it algebraically to find the exact amount of each type that should be used to obtain 50 pounds of double-dark-peanut fudge.

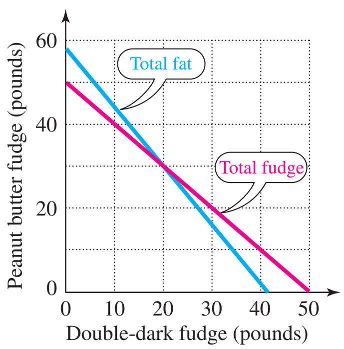


FIGURE FOR EXERCISE 43

44. ***Low-fat yogurt.*** Ziggy's Famous Yogurt blends regular yogurt that is 3% fat with its no-fat yogurt to obtain low-fat yogurt that is 1% fat. How many pounds of regular yogurt and how many pounds of no-fat yogurt should be mixed to obtain 60 pounds of low-fat yogurt?

45. ***Keystone state.*** Judy averaged 42 miles per hour (mph) driving from Allentown to Harrisburg and 51 mph driving from Harrisburg to Pittsburgh. If she drove a total of 288 miles in 6 hours, then how long did it take her to drive from Harrisburg to Pittsburgh?

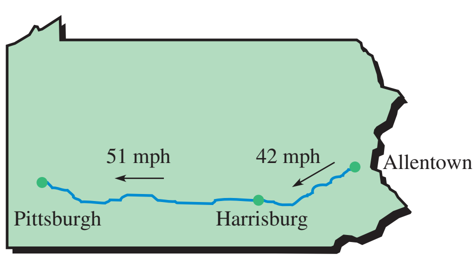


FIGURE FOR EXERCISE 45

46. ***Empire state.*** Spike averaged 45 mph driving from Rochester to Syracuse and 49 mph driving from Syracuse to Albany. If he drove a total of 237 miles in 5 hours, then how far is it from Syracuse to Albany?

47. ***Probability of rain.*** If Valerie Voss states that the probability of rain tomorrow is four times the probability that it doesn't rain, then what is the probability of rain tomorrow? (*Hint:* The probability that it rains plus the probability that it doesn't rain is 1.)

48. ***Super Bowl contender.*** A Las Vegas odds-maker believes that the probability that San Francisco plays in the next Super Bowl is nine times the probability that they do not play in the next Super Bowl. What is the odds-maker's probability that San Francisco plays in the next Super Bowl?

49. ***Rectangular lot.*** The width of a rectangular lot is 75% of its length. If the perimeter is 700 meters, then what are the length and width?

50. ***Fence painting.*** Darren and Douglas must paint the 792-foot fence that encircles their family home. Because Darren is older, he has agreed to paint 20% more than Douglas. How much of the fence will each boy paint?

GETTING MORE INVOLVED

51. ***Discussion.*** Explain how you decide whether it is easier to solve a system by substitution or addition.

52. ***Exploration.*** **a)** Write a linear equation in two variables that is satisfied by $(-3, 5)$.

b) Write another linear equation in two variables that is satisfied by $(-3, 5)$.

c) Are your equations independent or dependent?

d) Explain how to select the second equation so that it will be independent of the first.

53. ***Exploration.*** **a)** Make up a system of two linear equations in two variables such that both $(-1, 2)$ and $(4, 5)$ are in the solution set.

b) Are your equations independent or dependent?

c) Is it possible to find an independent system that is satisfied by both ordered pairs? Explain.