GETTING MORE INVOLVED

52. *Discussion.* When asked to graph the inequality $x + 2y < 12$, a student found that (0, 5) and (8, 0) both satisfied $x + 2y < 12$. The student then drew a dashed line through these two points and shaded the region below the line. What is wrong with this method? Do all of the points graphed by this student satisfy the inequality?

53. *Writing.* Compare and contrast the two methods presented in this section for graphing linear inequalities. What are the advantages and disadvantages of each method? How do you choose which method to use?

8.7 GRAPHING SYSTEMS OF LINEAR INEQUALITIES

In this section

- The Solution to a System of Inequalities
- Graphing a System of Inequalities

In Section 8.6 you learned how to solve a linear inequality. In this section you will solve systems of linear inequalities.

The Solution to a System of Inequalities

A **system of inequalities** consists of two or more inequalities. A point is a solution to a system of inequalities if it satisfies all of the inequalities in the system.

EXAMPLE 1 **Satisfying a system of inequalities**

Determine whether each point is a solution to the system of inequalities:

$$2x + 3y < 6$$
$$y > 2x - 1$$

a) $(-3, 2)$ **b)** $(4, -3)$ **c)** $(5, 1)$

Solution

a) The point $(-3, 2)$ is a solution to the system if it satisfies both inequalities. Let $x = -3$ and $y = 2$ in each inequality:

$$\begin{aligned} 2x + 3y &< 6 & y &> 2x - 1 \\ 2(-3) + 3(2) &< 6 & 2 &> 2(-3) - 1 \\ 0 &< 6 & 2 &> -7 \end{aligned}$$

Because both inequalities are satisfied, the point $(-3, 2)$ is a solution to the system.

b) Let $x = 4$ and $y = -3$ in each inequality:

$$\begin{aligned} 2x + 3y &< 6 & y &> 2x - 1 \\ 2(4) + 3(-3) &< 6 & -3 &> 2(4) - 1 \\ -1 &< 6 & -3 &> 7 \end{aligned}$$

Because only one inequality is satisfied, the point $(4, -3)$ is not a solution to the system.

c) Let $x = 5$ and $y = 1$ in each inequality:

$$\begin{aligned} 2x + 3y &< 6 & y &> 2x - 1 \\ 2(5) + 3(1) &< 6 & 1 &> 2(5) - 1 \\ 13 &< 6 & 1 &> 9 \end{aligned}$$

Because neither inequality is satisfied, the point $(5, 1)$ is not a solution to the system. ■

study tip

Read the text and recite to yourself what you have read. Ask questions and answer them out loud. Listen to your answers to see if they are complete and correct. Would other students understand your answers?

Graphing a System of Inequalities

There are infinitely many points that satisfy a typical system of inequalities. The best way to describe the solution to a system of inequalities is with a graph showing all points that satisfy the system. When we graph the points that satisfy a system, we say that we are graphing the system.

EXAMPLE 2 **Graphing a system of inequalities**

Graph all ordered pairs that satisfy the following system of inequalities:

$$y > x - 2$$
$$y < -2x + 3$$

Solution

We want a graph showing all points that satisfy both inequalities. The lines $y = x - 2$ and $y = -2x + 3$ divide the coordinate plane into four regions as shown in Fig. 8.16. To determine which of the four regions contains points that satisfy the system, we check one point in each region to see whether it satisfies both inequalities. The points are shown in Fig. 8.16.

Check (0, 0):		Check (0, 5):	
$0 > 0 - 2$	Correct	$5 > 0 - 2$	Correct
$0 < -2(0) + 3$	Correct	$5 < -2(0) + 3$	Incorrect
Check (0, −5):		Check (4, 0):	
$-5 > 0 - 2$	Incorrect	$0 > 4 - 2$	Incorrect
$-5 < -2(0) + 3$	Correct	$0 < -2(4) + 3$	Incorrect

The only point that satisfies both inequalities of the system is (0, 0). So every point in the region containing (0, 0) also satisfies both inequalities. The points that satisfy the system are graphed in Fig. 8.17.

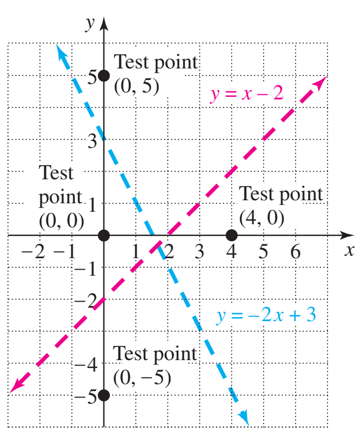

FIGURE 8.16

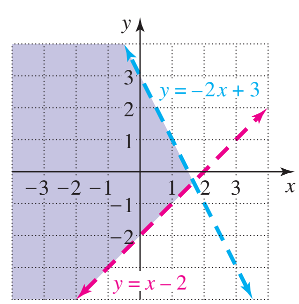

FIGURE 8.17

EXAMPLE 3 **Graphing a system of inequalities**

Graph all ordered pairs that satisfy the following system of inequalities:

$$y > -3x + 4$$
$$2y - x > 2$$

Solution

First graph the equations $y = -3x + 4$ and $2y - x = 2$. Now we select the points (0, 0), (0, 2), (0, 6), and (5, 0). We leave it to you to check each point in the system of inequalities. You will find that only (0, 6) satisfies the system. So only the region containing (0, 6) is shaded in Fig. 8.18.

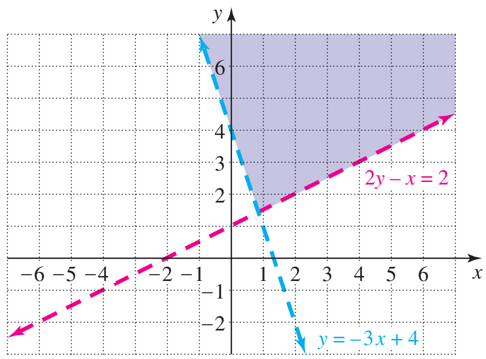

FIGURE 8.18

EXAMPLE 4

Horizontal and vertical boundary lines

Graph the system of inequalities:

$$x > 4$$
$$y < 3$$

Solution

We first graph the vertical line $x = 4$ and the horizontal line $y = 3$. The points that satisfy both inequalities are those points that lie to the right of the vertical line $x = 4$ and below the horizontal line $y = 3$. See Fig. 8.19 for the graph of the system.

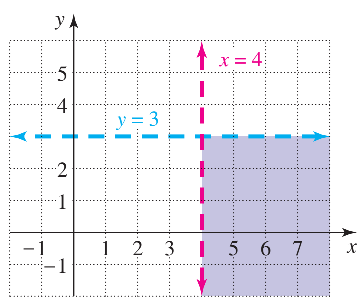

FIGURE 8.19

EXAMPLE 5

Between parallel lines

Graph the system of inequalities:

$$y < x + 4$$
$$y > x - 1$$

helpful hint

We could use the notation from Chapter 3 and write the inequalities in Example 5 as the compound inequality

$$x - 1 < y < x + 4.$$

Solution

First graph the parallel lines $y = x + 4$ and $y = x - 1$. These lines divide the plane into three regions. Check (0, 0), (0, 6), and (0, −4) in the system. Only (0, 0) satisfies the system. So the solution to the system consists of all points in between the parallel lines, as shown in Fig. 8.20.

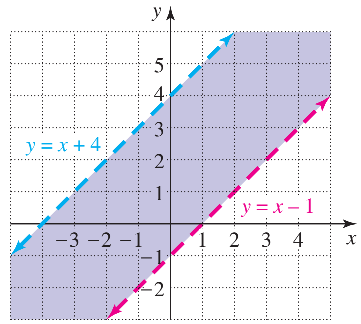

FIGURE 8.20

WARM-UPS

True or false? Explain your answer.

Use the following systems for Exercises 1–7.

a) $y > -3x + 5$
$y < 2x - 3$

b) $y > 2x - 3$
$y < 2x + 3$

c) $x + y > 4$
$x - y < 0$

1. The point $(2, -3)$ is a solution to system (a).

2. The point $(5, 0)$ is a solution to system (a).

3. The point $(0, 0)$ is a solution to system (b).

4. The graph of system (b) is the region between two parallel lines.

5. You can use $(0, 0)$ as a test point for system (c).

6. The point $(2, 2)$ satisfies system (c).

7. The point $(4, 5)$ satisfies system (c).

8. The inequality $x + y > 4$ is equivalent to the inequality $y < -x + 4$.

9. The graph of $y < 2x + 3$ is the region below the line $y = 2x + 3$.

10. There is no ordered pair that satisfies $y < 2x - 3$ and $y > 2x + 3$.

8.7 EXERCISES

Reading and Writing *After reading this section, write out the answers to these questions. Use complete sentences.*

1. What is a system of linear inequalities in two variables?

2. How can you tell if an ordered pair satisfies a system of linear inequalities in two variables?

3. How do we usually describe the solution set to a system of inequalities in two variables?

4. How do you decide whether the boundary lines are solid or dashed?

5. How do you use the test point method for a system of linear inequalities?

6. How do you select test points?

Determine which of the points following each system is a solution to the system. See Example 1.

7. $x - y < 5$ $\quad$ (4, 3), (8, 2), (−3, 0)
$2x + y > 3$

8. $x + y < 4$ $\quad$ (2, −3), (1, 1), (0, −1)
$2x - y < 3$

9. $y > -2x + 1$ $\quad$ (−3, 2), (−1, 5), (3, 6)
$y < 3x + 5$

10. $y < -x + 7$ $\quad$ (−3, 8), (0, 8), (−5, 15)
$y < -x + 9$

11. $x > 3$ $\quad$ (−5, 4), (9, −5), (6, 0)
$y < -2$

12. $y < -5$ $\quad$ (−2, 4), (0, −7), (6, −9)
$x < 1$

Graph each system of inequalities. See Examples 2–5.

13. $y > -x - 1$
$y > x + 1$

14. $y < x + 3$
$y < -2x + 4$

15. $y < 2x - 3$
$y > -x + 2$

16. $y > 2x - 1$
$y < -x - 4$

17. $x + y > 5$
$x - y < 3$

18. $2x + y < 3$
$x - 2y > 2$

19. $2x - 3y < 6$
$x - y > 3$

20. $3x - 2y > 6$
$x + y < 4$

21. $x > 5$
$y > 5$

22. $x < 3$
$y > 2$

23. $y < -1$
$x > -3$

24. $y > -2$
$x < 1$

25. $y > 2x - 4$
$y < 2x + 1$

26. $y < -2x + 3$
$y > -2x$

27. $y > x$
$x > 3$

28. $y < x$
$y < 1$

29. $y > -x$
$x < -1$

30. $y < -x$
$y > -3$

31. $x > 1$
$y - 2x < 3$

32. $y < 2$
$2x + 3y < 6$

33. $2x - 5y < 5$
$x + 2y > 4$

34. $3x + 2y < 2$
$-x - 2y > 4$

35. $x + y > 3$
$x + y > 1$

36. $x - y < 5$
$x - y < 3$

37. $y > 3x + 2$
$y < 3x + 3$

38. $y > x$
$y < -x$

39. $x + y < 5$
$x - y > -1$

40. $2x - y > 4$
$x - 5y < 5$

41. $2x - 3y < 6$
$3x + 4y < 12$

42. $x - 3y > 3$
$x + 2y < 4$

43. $3x - 5y < 15$
$3x + 2y < 12$

44. $x - 4y < 0$
$x + y > 0$

Solve each problem.

45. ***Target heart rate.*** For beneficial exercise, experts recommend that your target heart rate y should be between 65% and 75% of the maximum heart rate for your age x. That is,

$$y > 0.65(220 - x) \quad \text{and} \quad y < 0.75(220 - x).$$

Graph this system of inequalities for $20 < x < 70$.

46. ***Making and storing the tables.*** The Ozark Furniture Company can obtain at most 8000 board feet of oak lumber for making round and rectangular tables. The tables must be stored in a warehouse that has at most 3850 ft^3 of space available for the tables. A round table requires 50 board feet of lumber and 25 ft^3 of warehouse space. A rectangular table requires 80 board feet of lumber and 35 ft^3 of warehouse space. Write a system of inequalities that limits the possible number of tables of each type that can be made and stored. Graph the system.

47. ***Allocating resources.*** Wausaukee Enterprises makes yard barns in two sizes. One small barn requires \$250 in materials and 20 hours of labor, and one large barn requires \$400 in materials and 30 hours of labor. Wausaukee has at most \$4000 to spend on materials and at most 300 hours of labor available. Write a system of inequalities that limits the possible number of barns of each type that can be built. Graph the system.

FIGURE FOR EXERCISE 47

Inequalities can be used to describe limitations on materials used in construction.