Replace the question mark by an expression that makes the equation correct. Equations involving variables are to be identities.

75. $\dfrac{\sqrt{2}}{\sqrt{3}} = \dfrac{\sqrt{6}}{?}$

76. $\dfrac{2}{?} = \sqrt{2}$

77. $\dfrac{1}{\sqrt{2} - 1} = \dfrac{\sqrt{2} + 1}{?}$

78. $\dfrac{\sqrt{6}}{\sqrt{6} + 2} = \dfrac{?}{2}$

79. $\dfrac{1}{\sqrt{x} - 1} = \dfrac{?}{x - 1}$

80. $\dfrac{5}{3 - \sqrt{x}} = \dfrac{?}{9 - x}$

81. $\dfrac{3}{\sqrt{2} + x} = \dfrac{?}{2 - x^2}$

82. $\dfrac{4}{2\sqrt{3} + a} = \dfrac{?}{12 - a^2}$

Use a calculator to find a decimal approximation for each radical expression. Round your answers to three decimal places.

83. $\sqrt{3} + \sqrt{5}$

84. $\sqrt{5} + \sqrt{7}$

85. $2\sqrt{3} + 5\sqrt{3}$

86. $7\sqrt{3}$

87. $(2\sqrt{3})(3\sqrt{2})$

88. $6\sqrt{6}$

89. $\sqrt{5}(\sqrt{5} + \sqrt{3})$

90. $5 + \sqrt{15}$

91. $\dfrac{-1 + \sqrt{6}}{2}$

92. $\dfrac{-1 - \sqrt{6}}{2}$

93. $\dfrac{4 - \sqrt{10}}{-2}$

94. $\dfrac{4 + \sqrt{10}}{-2}$

GETTING MORE INVOLVED

95. ***Exploration.*** A polynomial is prime if it cannot be factored by using integers, but many prime polynomials can be factored if we use radicals.

a) Find the product $(x - \sqrt[3]{2})(x^2 + \sqrt[3]{2}x + \sqrt[3]{4})$.

b) Factor $x^3 + 5$ using radicals.

c) Find the product

$$(\sqrt[3]{5} - \sqrt[3]{2})(\sqrt[3]{25} + \sqrt[3]{10} + \sqrt[3]{4}).$$

d) Use radicals to factor $a + b$ as a sum of two cubes and $a - b$ as a difference of two cubes.

96. ***Discussion.*** Which one of the following expressions is not equivalent to the others?

a) $(\sqrt[3]{x})^4$ **b)** $\sqrt[4]{x^3}$ **c)** $\sqrt[3]{x^4}$

d) $x^{4/3}$ **e)** $(x^{1/3})^4$

9.5 SOLVING EQUATIONS WITH RADICALS AND EXPONENTS

In this section

- The Odd-Root Property
- The Even-Root Property
- Raising Each Side to a Power
- Equations Involving Rational Exponents
- Summary of Methods
- The Distance Formula

One of our goals in algebra is to keep increasing our knowledge of solving equations because the solutions to equations can give us the answers to various applied questions. In this section we will apply our knowledge of radicals and exponents to solving some new types of equations.

The Odd-Root Property

Because $(-2)^3 = -8$ and $2^3 = 8$, the equation $x^3 = 8$ is equivalent to $x = 2$. The equation $x^3 = -8$ is equivalent to $x = -2$. Because there is only one real odd root of each real number, there is a simple rule for writing an equivalent equation in this situation.

Odd-Root Property

If n is an odd positive integer,

$$x^n = k \quad \text{is equivalent to} \quad x = \sqrt[n]{k}$$

for any real number k.

EXAMPLE 1 **Using the odd-root property**

Solve each equation.

a) $x^3 = 27$ **b)** $x^5 + 32 = 0$ **c)** $(x - 2)^3 = 24$

Solution

a) $x^3 = 27$

$x = \sqrt[3]{27}$ Odd-root property

$x = 3$

Check 3 in the original equation. The solution set is $\{3\}$.

b) $x^5 + 32 = 0$

$x^5 = -32$ Isolate the variable.

$x = \sqrt[5]{-32}$ Odd-root property

$x = -2$

Check -2 in the original equation. The solution set is $\{-2\}$.

c) $(x - 2)^3 = 24$

$x - 2 = \sqrt[3]{24}$ Odd-root property

$x = 2 + 2\sqrt[3]{3}$ $\sqrt[3]{24} = \sqrt[3]{8} \cdot \sqrt[3]{3} = 2\sqrt[3]{3}$

Check. The solution set is $\{2 + 2\sqrt[3]{3}\}$.

The Even-Root Property

In solving the equation $x^2 = 4$, you might be tempted to write $x = 2$ as an equivalent equation. But $x = 2$ is not equivalent to $x^2 = 4$ because $2^2 = 4$ and $(-2)^2 = 4$. So the solution set to $x^2 = 4$ is $\{-2, 2\}$. The equation $x^2 = 4$ is equivalent to the compound sentence $x = 2$ or $x = -2$, which we can abbreviate as $x = \pm 2$. The equation $x = \pm 2$ is read "x equals positive or negative 2."

Equations involving other even powers are handled like the squares. Because $2^4 = 16$ and $(-2)^4 = 16$, the equation $x^4 = 16$ is equivalent to $x = \pm 2$. So $x^4 = 16$ has two real solutions. Note that $x^4 = -16$ has no real solutions. The equation $x^6 = 5$ is equivalent to $x = \pm\sqrt[6]{5}$. We can now state a general rule.

helpful hint

We do not say, "take the square root of each side." We are not doing the same thing to each side of $x^2 = 9$ when we write $x = \pm 3$. This is the third time that we have seen a rule for obtaining an equivalent equation without "doing the same thing to each side." (What were the other two?) Because there is only one odd root of every real number, you can actually take an odd root of each side.

Even-Root Property

Suppose n is a positive even integer.

If $k > 0$, then $x^n = k$ is equivalent to $x = \pm\sqrt[n]{k}$.

If $k = 0$, then $x^n = k$ is equivalent to $x = 0$.

If $k < 0$, then $x^n = k$ has no real solution.

EXAMPLE 2 **Using the even-root property**

Solve each equation.

a) $x^2 = 10$ **b)** $w^8 = 0$ **c)** $x^4 = -4$

Solution

a) $x^2 = 10$

$x = \pm\sqrt{10}$ Even-root property

The solution set is $\{-\sqrt{10}, \sqrt{10}\}$, or $\{\pm\sqrt{10}\}$.

b) $w^8 = 0$

$w = 0$ Even-root property

The solution set is $\{0\}$.

c) By the even-root property, $x^4 = -4$ has no real solution. (The fourth power of any real number is nonnegative.) ■

In the next example the even-root property is used to solve some equations that are a bit more complicated than those of Example 2.

EXAMPLE 3 **Using the even-root property**

Solve each equation.

a) $(x - 3)^2 = 4$ **b)** $2(x - 5)^2 - 7 = 0$ **c)** $x^4 - 1 = 80$

Solution

a) $(x - 3)^2 = 4$

$x - 3 = 2$ or $x - 3 = -2$ Even-root property

$x = 5$ or $x = 1$ Add 3 to each side.

The solution set is $\{1, 5\}$.

b) $2(x - 5)^2 - 7 = 0$

$2(x - 5)^2 = 7$ Add 7 to each side.

$(x - 5)^2 = \frac{7}{2}$ Divide each side by 2.

$x - 5 = \sqrt{\frac{7}{2}}$ or $x - 5 = -\sqrt{\frac{7}{2}}$ Even-root property

$x = 5 + \frac{\sqrt{14}}{2}$ or $x = 5 - \frac{\sqrt{14}}{2}$ $\sqrt{\frac{7}{2}} = \frac{\sqrt{7}\cdot\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}} = \frac{\sqrt{14}}{2}$

$x = \frac{10 + \sqrt{14}}{2}$ or $x = \frac{10 - \sqrt{14}}{2}$

The solution set is $\left\{\frac{10 + \sqrt{14}}{2}, \frac{10 - \sqrt{14}}{2}\right\}$.

c) $x^4 - 1 = 80$

$x^4 = 81$

$x = \pm\sqrt[4]{81} = \pm 3$

The solution set is $\{-3, 3\}$. ■

study tip

Review, review, review! Don't wait until the end of a chapter to review. Do a little review every time you study for this course.

In Chapter 6 we solved quadratic equations by factoring. The quadratic equations that we encounter in this chapter can be solved by using the even-root property as in parts (a) and (b) of Example 3. In Chapter 10 you will learn general methods for solving any quadratic equation.

Raising Each Side to a Power

If we start with the equation $x = 3$ and square both sides, we get $x^2 = 9$. The solution set to $x^2 = 9$ is $\{-3, 3\}$; the solution set to the original equation is $\{3\}$. Squaring both sides of an equation might produce a *nonequivalent* equation that has more solutions than the original equation. We call these additional solutions **extraneous solutions.** However, any solution of the original must be among the solutions to the new equation.

CAUTION When you solve an equation by raising each side to a power, you must check your answers. Raising each side to an odd power will always give an equivalent equation; raising each side to an even power might not.

EXAMPLE 4 **Raising each side to a power to eliminate radicals**

Solve each equation.

a) $\sqrt{2x - 3} - 5 = 0$ **b)** $\sqrt[3]{3x + 5} = \sqrt[3]{x - 1}$ **c)** $\sqrt{3x + 18} = x$

Solution

a) Eliminate the square root by raising each side to the power 2:

$$\sqrt{2x-3} - 5 = 0 \quad \text{Original equation}$$
$$\sqrt{2x-3} = 5 \quad \text{Isolate the radical.}$$
$$(\sqrt{2x-3})^2 = 5^2 \quad \text{Square both sides.}$$
$$2x - 3 = 25$$
$$2x = 28$$
$$x = 14$$

Check by evaluating $x = 14$ in the original equation:

$$\sqrt{2(14) - 3} - 5 = 0$$
$$\sqrt{28 - 3} - 5 = 0$$
$$\sqrt{25} - 5 = 0$$
$$0 = 0$$

The solution set is $\{14\}$.

b)
$$\sqrt[3]{3x+5} = \sqrt[3]{x-1} \quad \text{Original equation}$$
$$(\sqrt[3]{3x+5})^3 = (\sqrt[3]{x-1})^3 \quad \text{Cube each side.}$$
$$3x + 5 = x - 1$$
$$2x = -6$$
$$x = -3$$

Check $x = -3$ in the original equation:

$$\sqrt[3]{3(-3) + 5} = \sqrt[3]{-3 - 1}$$
$$\sqrt[3]{-4} = \sqrt[3]{-4}$$

Note that $\sqrt[3]{-4}$ is a real number. The solution set is $\{-3\}$. In this example we checked for arithmetic mistakes. There was no possibility of extraneous solutions here because we raised each side to an odd power.

calculator close-up

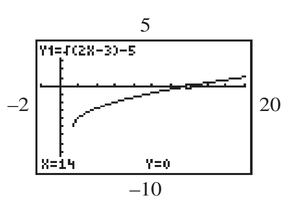
If 14 satisfies the equation

$$\sqrt{2x - 3} - 5 = 0,$$

then (14, 0) is an x-intercept for the graph of

$$y = \sqrt{2x - 3} - 5.$$

So the calculator graph shown here provides visual support for the conclusion that 14 is the only solution to the equation.

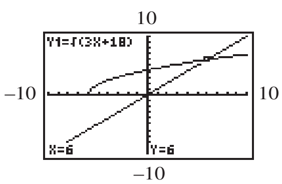


calculator close-up

The graphs of

$y_1 = \sqrt{3x + 18}$

and $y_2 = x$ provide visual support that 6 is the only value of x for which x and $\sqrt{3x + 18}$ are equal.



c)

$$\begin{aligned}
\sqrt{3x + 18} &= x && \text{Original equation}\\
(\sqrt{3x + 18})^2 &= x^2 && \text{Square both sides.}\\
3x + 18 &= x^2 && \text{Simplify.}\\
-x^2 + 3x + 18 &= 0 && \text{Subtract } x^2 \text{ from each side to get zero on one side.}\\
x^2 - 3x - 18 &= 0 && \text{Multiply each side by } -1 \text{ for easier factoring.}\\
(x - 6)(x + 3) &= 0 && \text{Factor.}\\
x - 6 = 0 \quad &\text{or} \quad x + 3 = 0 && \text{Zero factor property}\\
x = 6 \quad &\text{or} \quad x = -3
\end{aligned}$$

Because we squared both sides, we must check for extraneous solutions. If $x = -3$ in the original equation $\sqrt{3x + 18} = x$, we get

$$\begin{aligned}
\sqrt{3(-3) + 18} &= -3\\
\sqrt{9} &= -3\\
3 &= -3,
\end{aligned}$$

which is not correct. If $x = 6$ in the original equation, we get

$$\sqrt{3(6) + 18} = 6,$$

which is correct. The solution set is $\{6\}$. ■

In the next example the radicals are not eliminated after squaring both sides of the equation. In this case we must square both sides a second time. Note that we square the side with two terms the same way we square a binomial.

EXAMPLE 5 **Squaring both sides twice**

Solve $\sqrt{5x - 1} - \sqrt{x + 2} = 1$.

Solution

It is easier to square both sides if the two radicals are not on the same side, so we first rewrite the equation:

$$\begin{aligned}
\sqrt{5x - 1} - \sqrt{x + 2} &= 1 && \text{Original equation}\\
\sqrt{5x - 1} &= 1 + \sqrt{x + 2} && \text{Add } \sqrt{x + 2} \text{ to each side.}\\
(\sqrt{5x - 1})^2 &= (1 + \sqrt{x + 2})^2 && \text{Square both sides.}\\
5x - 1 &= 1 + 2\sqrt{x + 2} + x + 2 && \text{Square the right side like a binomial.}\\
5x - 1 &= 3 + x + 2\sqrt{x + 2} && \text{Combine like terms on the right side.}\\
4x - 4 &= 2\sqrt{x + 2} && \text{Isolate the square root.}\\
2x - 2 &= \sqrt{x + 2} && \text{Divide each side by 2.}\\
(2x - 2)^2 &= (\sqrt{x + 2})^2 && \text{Square both sides.}\\
4x^2 - 8x + 4 &= x + 2 && \text{Square the binomial on the left side.}\\
4x^2 - 9x + 2 &= 0\\
(4x - 1)(x - 2) &= 0\\
4x - 1 = 0 \quad &\text{or} \quad x - 2 = 0\\
x = \frac{1}{4} \quad &\text{or} \quad x = 2
\end{aligned}$$

Check to see whether $\sqrt{5x - 1} - \sqrt{x + 2} = 1$ for $x = \frac{1}{4}$ and for $x = 2$:

$$\sqrt{5 \cdot \frac{1}{4} - 1} - \sqrt{\frac{1}{4} + 2} = \sqrt{\frac{1}{4}} - \sqrt{\frac{9}{4}} = \frac{1}{2} - \frac{3}{2} = -1$$

$$\sqrt{5 \cdot 2 - 1} - \sqrt{2 + 2} = \sqrt{9} - \sqrt{4} = 3 - 2 = 1$$

Because $\frac{1}{4}$ does not satisfy the original equation, the solution set is $\{2\}$. ■

Equations Involving Rational Exponents

Equations involving rational exponents can be solved by combining the methods that you just learned for eliminating radicals and integral exponents. For equations involving rational exponents, always eliminate the root first and the power second.

EXAMPLE 6 **Eliminating the root, then the power**

Solve each equation.

a) $x^{2/3} = 4$

b) $(w - 1)^{-2/5} = 4$

Solution

a) Because the exponent $2/3$ indicates a cube root, raise each side to the power 3:

$x^{2/3} = 4$	Original equation
$(x^{2/3})^3 = 4^3$	Cube each side.
$x^2 = 64$	Multiply the exponents: $\frac{2}{3} \cdot 3 = 2$.
$x = 8$ or $x = -8$	Even-root property

All of the equations are equivalent. Check 8 and -8 in the original equation. The solution set is $\{-8, 8\}$.

b)

$(w - 1)^{-2/5} = 4$	Original equation
$[(w - 1)^{-2/5}]^{-5} = 4^{-5}$	Raise each side to the power -5 to eliminate the negative exponent.
$(w - 1)^2 = \frac{1}{1024}$	Multiply the exponents: $-\frac{2}{5}(-5) = 2$.
$w - 1 = \pm\sqrt{\frac{1}{1024}}$	Even-root property
$w - 1 = \frac{1}{32}$ or $w - 1 = -\frac{1}{32}$	
$w = \frac{33}{32}$ or $w = \frac{31}{32}$	

Check the values in the original equation. The solution set is $\left\{\frac{31}{32}, \frac{33}{32}\right\}$. ■

An equation with a rational exponent might not have a real solution because all even powers of real numbers are nonnegative.

helpful hint

Note how we eliminate the root first by raising each side to an integer power, and then apply the even-root property to get two solutions in Example 6(a). A common mistake is to raise each side to the 3/2 power and get $x = 4^{3/2} = 8$. If you do not use the even-root property you can easily miss the solution -8.

calculator close-up

Check that 31/32 and 33/32 satisfy the original equation.

```
(31/32-1)^(-2/5)
                4
(33/32-1)^(-2/5)
                4
```

EXAMPLE 7

An equation with no solution

Solve $(2t - 3)^{-2/3} = -1$.

Solution

Raise each side to the power -3 to eliminate the root and the negative sign in the exponent:

$$(2t - 3)^{-2/3} = -1 \quad \text{Original equation}$$

$$[(2t - 3)^{-2/3}]^{-3} = (-1)^{-3} \quad \text{Raise each side to the } -3 \text{ power.}$$

$$(2t - 3)^{2} = -1 \quad \text{Multiply the exponents: } -\frac{2}{3}(-3) = 2.$$

By the even-root property this equation has no real solution. The square of every real number is nonnegative.

Summary of Methods

The three most important rules for solving equations with exponents and radicals are restated here.

Strategy for Solving Equations with Exponents and Radicals

1. In raising each side of an equation to an even power, we can create an equation that gives extraneous solutions. We must check all possible solutions in the original equation.
2. When applying the even-root property, remember that there is a positive and a negative even root for any positive real number.
3. For equations with rational exponents, raise each side to a positive or negative integral power first, then apply the even- or odd-root property. (Positive fraction—raise to a positive power; negative fraction—raise to a negative power.)

The Distance Formula

Consider the points (x_1, y_1) and (x_2, y_2) as shown in Fig. 9.1. The distance between these points is the length of the hypotenuse of a right triangle as shown in the figure. The length of side a is $y_2 - y_1$ and the length of side b is $x_2 - x_1$. Using the Pythagorean theorem, we can write

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2.$$

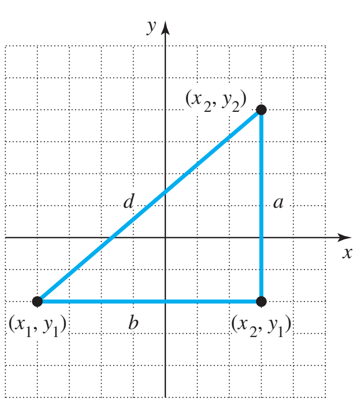


FIGURE 9.1

If we apply the even-root property and omit the negative square root (because the distance is positive), we can express this formula as follows.

Distance Formula

The distance d between (x_1, y_1) and (x_2, y_2) is given by the formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

EXAMPLE 8

Using the distance formula

Find the length of the line segment with endpoints $(-8, -10)$ and $(6, -4)$.

Solution

Let $(x_1, y_1) = (-8, -10)$ and $(x_2, y_2) = (6, -4)$. Now substitute the appropriate values into the distance formula:

$$\begin{aligned} d &= \sqrt{[6 - (-8)]^2 + [-4 - (-10)]^2} \\ &= \sqrt{(14)^2 + (6)^2} \\ &= \sqrt{196 + 36} \\ &= \sqrt{232} \\ &= \sqrt{4 \cdot 58} \\ &= 2\sqrt{58} \quad \text{Simplified form} \end{aligned}$$

The exact length of the segment is $2\sqrt{58}$. ■

In the next example we find the distance between two points without the distance formula. Although we could solve the problem using a coordinate system and the distance formula, that is not necessary.

EXAMPLE 9

Diagonal of a baseball diamond

A baseball diamond is actually a square, 90 feet on each side. What is the distance from third base to first base?

Solution

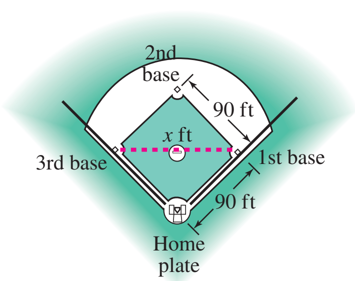


FIGURE 9.2

First make a sketch as in Fig. 9.2. The distance x from third base to first base is the length of the diagonal of the square shown in Fig. 9.2. The Pythagorean theorem can be applied to the right triangle formed from the diagonal and two sides of the square. The sum of the squares of the sides is equal to the diagonal squared:

$$\begin{aligned} x^2 &= 90^2 + 90^2 \\ x^2 &= 8100 + 8100 \\ x^2 &= 16{,}200 \\ x &= \pm\sqrt{16{,}200} = \pm 90\sqrt{2} \end{aligned}$$

The length of the diagonal of a square must be positive, so we disregard the negative solution. Checking the answer in the original equation verifies that the *exact* length of the diagonal is $90\sqrt{2}$ feet. ■

WARM-UPS

True or false? Explain your answer.

1. The equations $x^2 = 4$ and $x = 2$ are equivalent.
2. The equation $x^2 = -25$ has no real solution.
3. There is no solution to the equation $x^2 = 0$.
4. The equation $x^3 = 8$ is equivalent to $x = \pm 2$.
5. The equation $-\sqrt{x} = 16$ has no real solution.
6. To solve $\sqrt{x - 3} = \sqrt{2x + 5}$, first apply the even-root property.

WARM-UPS

(continued)

7. Extraneous solutions are solutions that cannot be found.

8. Squaring both sides of $\sqrt{x} = -7$ yields an equation with an extraneous solution.

9. The equations $x^2 - 6 = 0$ and $x = \pm\sqrt{6}$ are equivalent.

10. Cubing each side of an equation will not produce an extraneous solution.

9.5 EXERCISES

Reading and Writing *After reading this section, write out the answers to these questions. Use complete sentences.*

1. What is the odd-root property?

2. What is the even-root property?

3. What is an extraneous solution?

4. Why can raising each side to a power produce an extraneous solution?

Solve each equation. See Example 1.

5. $x^3 = -1000$

6. $y^3 = 125$

7. $32m^5 - 1 = 0$

8. $243a^5 + 1 = 0$

9. $(y - 3)^3 = -8$

10. $(x - 1)^3 = -1$

11. $\frac{1}{2}x^3 + 4 = 0$

12. $3(x - 9)^7 = 0$

Solve each equation. See Examples 2 and 3.

13. $x^2 = 25$

14. $x^2 = 36$

15. $x^2 - 20 = 0$

16. $a^2 - 40 = 0$

17. $x^2 = -9$

18. $w^2 + 49 = 0$

19. $(x - 3)^2 = 16$

20. $(a - 2)^2 = 25$

21. $(x + 1)^2 - 8 = 0$

22. $(w + 3)^2 - 12 = 0$

23. $\frac{1}{2}x^2 = 5$

24. $\frac{1}{3}x^2 = 6$

25. $(y - 3)^4 = 0$

26. $(2x - 3)^6 = 0$

27. $2x^6 = 128$

28. $3y^4 = 48$

Solve each equation and check for extraneous solutions. See Example 4.

29. $\sqrt{x - 3} - 7 = 0$

30. $\sqrt{a - 1} - 6 = 0$

31. $2\sqrt{w + 4} = 5$

32. $3\sqrt{w + 1} = 6$

33. $\sqrt[3]{2x + 3} = \sqrt[3]{x + 12}$

34. $\sqrt[3]{a + 3} = \sqrt[3]{2a - 7}$

35. $\sqrt{2t + 4} = \sqrt{t - 1}$

36. $\sqrt{w - 3} = \sqrt{4w + 15}$

37. $\sqrt{4x^2 + x - 3} = 2x$

38. $\sqrt{x^2 - 5x + 2} = x$

39. $\sqrt{x^2 + 2x - 6} = 3$

40. $\sqrt{x^2 - x - 4} = 4$

41. $\sqrt{2x^2 - 1} = x$

42. $\sqrt{2x^2 - 3x - 10} = x$

43. $\sqrt{2x^2 + 5x + 6} = x$

44. $\sqrt{5x^2 - 9} = 2x$

Solve each equation and check for extraneous solutions. See Example 5.

45. $\sqrt{x} + \sqrt{x - 3} = 3$

46. $\sqrt{x} + \sqrt{x + 3} = 3$

47. $\sqrt{x + 2} + \sqrt{x - 1} = 3$

48. $\sqrt{x} + \sqrt{x - 5} = 5$

49. $\sqrt{x + 3} - \sqrt{x - 2} = 1$

50. $\sqrt{2x + 1} - \sqrt{x} = 1$

51. $\sqrt{2x + 2} - \sqrt{x - 3} = 2$

52. $\sqrt{3x} - \sqrt{x - 2} = 4$

53. $\sqrt{4 - x} - \sqrt{x + 6} = 2$

54. $\sqrt{6 - x} - \sqrt{x - 2} = 2$

Solve each equation. See Examples 6 and 7.

55. $x^{2/3} = 3$

56. $a^{2/3} = 2$

57. $y^{-2/3} = 9$

58. $w^{-2/3} = 4$

59. $w^{1/3} = 8$

60. $a^{1/3} = 27$

61. $t^{-1/2} = 9$

62. $w^{-1/4} = \frac{1}{2}$

63. $(3a - 1)^{-2/5} = 1$

64. $(r - 1)^{-2/3} = 1$

65. $(t - 1)^{-2/3} = 2$

66. $(w + 3)^{-1/3} = \frac{1}{3}$

67. $(x - 3)^{2/3} = -4$

68. $(x + 2)^{3/2} = -1$

Find the distance between each given pair of points. See Example 8.

69. (6, 5), (4, 2)

70. (7, 3), (5, 1)

71. (3, 5), (1, −3)

72. (6, 2), (3, −5)

73. (4, −2), (−3, −6)

74. (−2, 3), (1, −4)

Solve each equation.

75. $2x^2 + 3 = 7$

76. $3x^2 - 5 = 16$

77. $\sqrt[3]{2w + 3} = \sqrt[3]{w - 2}$

78. $\sqrt[3]{2 - w} = \sqrt[3]{2w - 28}$

79. $(w + 1)^{2/3} = -3$

80. $(x - 2)^{3/4} = 2$

81. $(a + 1)^{1/3} = -2$

82. $(a - 1)^{1/3} = -3$

83. $(4y - 5)^7 = 0$

84. $(5x)^9 = 0$

85. $\sqrt{x^2 + 5x} = 6$

86. $\sqrt{x^2 - 8x} = -3$

87. $\sqrt{4x^2} = x + 2$

88. $\sqrt{9x^2} = x + 6$

89. $(t + 2)^4 = 32$

90. $(w + 1)^4 = 48$

91. $\sqrt{x^2 - 3x} = x$

92. $\sqrt[4]{4x^4 - 48} = -x$

93. $x^{-3} = 8$

94. $x^{-2} = 4$

Solve each problem by writing an equation and solving it. Find the exact answer and simplify it using the rules for radicals. See Example 9.

95. ***Side of a square.*** Find the length of the side of a square whose diagonal is 8 feet.

96. ***Diagonal of a patio.*** Find the length of the diagonal of a square patio with an area of 40 square meters.

97. ***Side of a sign.*** Find the length of the side of a square sign whose area is 50 square feet.

98. ***Side of a cube.*** Find the length of the side of a cubic box whose volume is 80 cubic feet.

99. ***Diagonal of a rectangle.*** If the sides of a rectangle are 30 feet and 40 feet in length, find the length of the diagonal of the rectangle.

100. ***Diagonal of a sign.*** What is the length of the diagonal of a rectangular billboard whose sides are 5 meters and 12 meters?

101. ***Sailboat stability.*** To be considered safe for ocean sailing, the capsize screening value C should be less than 2 (*Sail,* May 1997). For a boat with a beam (or width) b in feet and displacement d in pounds, C is determined by the formula

$$C = 4d^{-1/3}b.$$

a) Find the capsize screening value for the Tartan 4100, which has a displacement of 23,245 pounds and a beam of 13.5 feet.

b) Solve this formula for *d*.

c) The accompanying graph shows *C* as a function of *d* for the Tartan 4100 ($b = 13.5$). For what displacement is the Tartan 4100 safe for ocean sailing?

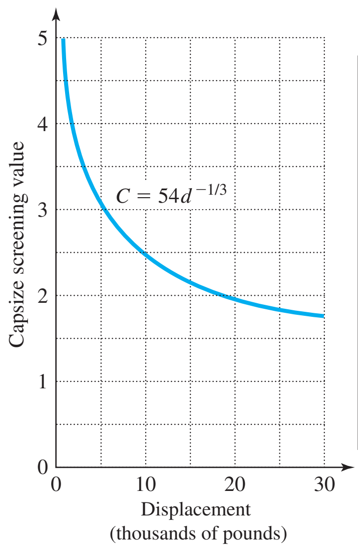


FIGURE FOR EXERCISE 101

102. ***Sailboat speed.*** The sail area-displacement ratio *S* provides a measure of the sail power available to drive a boat. For a boat with a displacement of *d* pounds and a sail area of *A* square feet

$$S = 16Ad^{-2/3}.$$

a) Find *S* for the Tartan 4100, which has a sail area of 810 square feet and a displacement of 23,245 pounds.

b) Solve the formula for *d*.

103. ***Diagonal of a side.*** Find the length of the diagonal of a side of a cubic packing crate whose volume is 2 cubic meters.

104. ***Volume of a cube.*** Find the volume of a cube on which the diagonal of a side measures 2 feet.

105. ***Length of a road.*** An architect designs a public park in the shape of a trapezoid. Find the length of the diagonal road marked *a* in the figure.

106. ***Length of a boundary.*** Find the length of the border of the park marked *b* in the trapezoid shown in the figure.

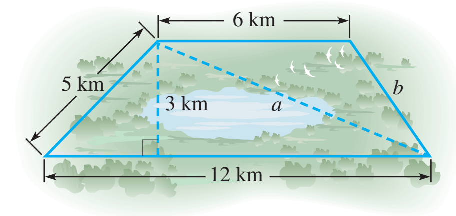


FIGURE FOR EXERCISES 105 AND 106

107. ***Average annual return.*** The formula

$$r = \left(\frac{S}{P}\right)^{1/n} - 1$$

was used to find the average annual return on an investment in Exercise 127 in Section 9.2. Solve the formula for *S* (the amount). Solve it for *P* (the original principal).

108. ***Surface area of a cube.*** The formula $A = 6V^{2/3}$ gives the surface area of a cube in terms of its volume *V*. What is the volume of a cube with surface area 12 square feet?

109. ***Kepler's third law.*** According to Kepler's third law of planetary motion, the ratio $\frac{T^2}{R^3}$ has the same value for every planet in our solar system. *R* is the average radius of the orbit of the planet measured in astronomical units (AU), and *T* is the number of years it takes for one complete orbit of the sun. Jupiter orbits the sun in 11.86 years with an average radius of 5.2 AU, whereas Saturn orbits the sun in 29.46 years. Find the average radius of the orbit of Saturn. (One AU is the distance from the earth to the sun.)

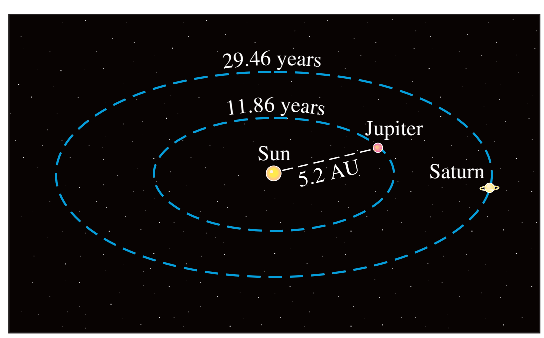


FIGURE FOR EXERCISE 109

110. ***Orbit of Venus.*** If the average radius of the orbit of Venus is 0.723 AU, then how many years does it take for

Venus to complete one orbit of the sun? Use the information in Exercise 109.

Use a calculator to find approximate solutions to the following equations. Round your answers to three decimal places.

111. $x^2 = 3.24$

112. $(x + 4)^3 = 7.51$

113. $\sqrt{x - 2} = 1.73$

114. $\sqrt[3]{x - 5} = 3.7$

115. $x^{2/3} = 8.86$

116. $(x - 1)^{-3/4} = 7.065$

GETTING MORE INVOLVED

117. ***Cooperative learning.*** Work in a small group to write a formula that gives the side of a cube in terms of the volume of the cube and explain the formula to the other groups.

118. ***Cooperative learning.*** Work in a small group to write a formula that gives the side of a square in terms of the diagonal of the square and explain the formula to the other groups.

9.6 COMPLEX NUMBERS

In this section

- Definition
- Addition, Subtraction, and Multiplication
- Division of Complex Numbers
- Square Roots of Negative Numbers
- Imaginary Solutions to Equations

In Chapter 1 we discussed the real numbers and the various subsets of the real numbers. In this section we define a set of numbers that has the real numbers as a subset.

Definition

The equation $2x = 1$ has no solution in the set of integers, but in the set of rational numbers, $2x = 1$ has a solution. The situation is similar for the equation $x^2 = -4$. It has no solution in the set of real numbers because the square of every real number is nonnegative. However, in the set of complex numbers $x^2 = -4$ has two solutions. The complex numbers were developed so that equations such as $x^2 = -4$ would have solutions.

The complex numbers are based on the symbol $\sqrt{-1}$. In the real number system this symbol has no meaning. In the set of complex numbers this symbol is given meaning. We call it i. We make the definition that

$$i = \sqrt{-1} \quad \text{and} \quad i^2 = -1.$$

Complex Numbers

The set of **complex numbers** is the set of all numbers of the form

$$a + bi,$$

where a and b are real numbers, $i = \sqrt{-1}$, and $i^2 = -1$.

In the complex number $a + bi$, a is called the **real part** and b is called the **imaginary part.** If $b \neq 0$, the number $a + bi$ is called an **imaginary number.**

In dealing with complex numbers, we treat $a + bi$ as if it were a binomial, with i being a variable. Thus we would write $2 + (-3)i$ as $2 - 3i$. We agree that $2 + i3$, $3i + 2$, and $i3 + 2$ are just different ways of writing $2 + 3i$ (the standard form). Some examples of complex numbers are

$$-3 - 5i, \quad \frac{2}{3} - \frac{3}{4}i, \quad 1 + i\sqrt{2}, \quad 9 + 0i, \quad \text{and} \quad 0 + 7i.$$