c) For a fixed sail area, does S increase or decrease as the displacement increases?

GETTING MORE INVOLVED

75. ***Discussion.*** Let $f(x) = \sqrt{x} - 4$ and $g(x) = \sqrt{x}$. Find the domains of f, g, and $g \circ f$.

76. ***Discussion.*** Let $f(x) = \sqrt{x - 4}$ and $g(x) = \sqrt{x - 8}$. Find the domains of f, g, and $f + g$.

GRAPHING CALCULATOR EXERCISES

77. Graph $y_1 = x$, $y_2 = \sqrt{x}$, and $y_3 = x + \sqrt{x}$ in the same screen. Find the domain and range of $y_3 = x + \sqrt{x}$ by examining its graph. (On some graphing calculators you can enter y_3 as $y_3 = y_1 + y_2$.)

78. Graph $y_1 = |x|$, $y_2 = |x - 3|$, and $y_3 = |x| + |x - 3|$. Find the domain and range of $y_3 = |x| + |x - 3|$ by examining its graph.

11.4 INVERSE FUNCTIONS

In this section

- Inverse of a Function
- Identifying Inverse Functions
- Switch-and-Solve Strategy
- Even Roots or Even Powers
- Graphs of f and f^{-1}

In Section 11.3 we introduced the idea of a pair of functions such that $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$. Each function reverses what the other function does. In this section we explore that idea further.

Inverse of a Function

You can buy a 6-, 7-, or 8-foot conference table in the K-LOG Catalog for \$299, \$329, or \$349, respectively. The set

$$f = \{(6, 299), (7, 329), (8, 349)\}$$

gives the price as a function of the length. We use the letter f as a name for this set or function, just as we use the letter f as a name for a function in the function notation. In the function f, lengths in the domain $\{6, 7, 8\}$ are paired with prices in the range $\{299, 329, 349\}$. The **inverse** of the function f, denoted f^{-1}, is a function whose ordered pairs are obtained from f by interchanging the x- and y-coordinates:

$$f^{-1} = \{(299, 6), (329, 7), (349, 8)\}$$

We read f^{-1} as "f inverse." The domain of f^{-1} is $\{299, 329, 349\}$, and the range of f^{-1} is $\{6, 7, 8\}$. The inverse function reverses what the function does: it pairs prices in the range of f with lengths in the domain of f. For example, to find the cost of a 7-foot table, we use the function f to get $f(7) = 329$. To find the length of a table, that costs \$349, we use the function f^{-1} to get $f^{-1}(349) = 8$. Of course, we could find the length of a \$349 table by looking at the function f, but f^{-1} is a function whose input is price and whose output is length. In general, *the domain of f^{-1} is the range of f, and the range of f^{-1} is the domain of f.* See Fig. 11.23.

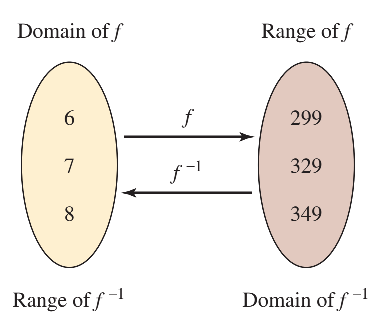


FIGURE 11.23

CAUTION The -1 in f^{-1} is not read as an exponent. It does not mean $\frac{1}{f}$.

The cost per ribbon for Apple Imagewriter ribbons is a function of the number of boxes purchased:

$$g = \{(1, 4.85), (2, 4.60), (3, 4.60), (4, 4.35)\}$$

If we interchange the first and second coordinates in the ordered pairs of this function, we get

$$\{(4.85, 1), (4.60, 2), (4.60, 3), (4.35, 4)\}.$$

This set of ordered pairs is not a function because it contains ordered pairs with the same first coordinates and different second coordinates. So g does not have an inverse function. A function is **invertible** if you obtain a function when the coordinates of all ordered pairs are reversed. So f is invertible and g is not invertible. The function g is not invertible because the definition of function allows more than one number of the domain to be paired with the same number in the range. Of course, when this pairing is reversed, the definition of function is violated.

helpful hint

Consider the universal product codes (UPC) and the prices for all of the items in your favorite grocery store. The price of an item is a function of the UPC because every UPC determines a price. This function is not invertible because you cannot determine the UPC from a given price.

One-to-One Function

If a function is such that no two ordered pairs have different x-coordinates and the same y-coordinate, then the function is called a **one-to-one** function.

In a one-to-one function each member of the domain corresponds to just one member of the range, and each member of the range corresponds to just one member of the domain. *Functions that are one-to-one are invertible functions.*

Inverse Function

The inverse of a one-to-one function f is the function f^{-1}, which is obtained from f by interchanging the coordinates in each ordered pair of f.

EXAMPLE 1 **Identifying invertible functions**

Determine whether each function is invertible. If it is invertible, then find the inverse function.

a) $f = \{(2, 4), (-2, 4), (3, 9)\}$

b) $g = \left\{\left(2, \frac{1}{2}\right), \left(5, \frac{1}{5}\right), \left(7, \frac{1}{7}\right)\right\}$

c) $h = \{(3, 5), (7, 9)\}$

Solution

a) Since (2, 4) and $(-2, 4)$ have the same y-coordinate, this function is not one-to-one, and it is not invertible.

b) This function is one-to-one, and so it is invertible.

$$g^{-1} = \left\{\left(\frac{1}{2}, 2\right), \left(\frac{1}{5}, 5\right), \left(\frac{1}{7}, 7\right)\right\}$$

c) This function is invertible, and $h^{-1} = \{(5, 3), (9, 7)\}$. ■

You learned to use the vertical-line test in Section 4.6 to determine whether a graph is the graph of a function. The **horizontal-line test** is a similar visual test for

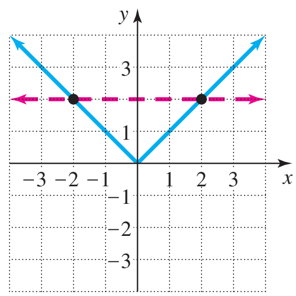


FIGURE 11.24

determining whether a function is invertible. If a horizontal line crosses a graph two (or more) times, as in Fig. 11.24, then there are two points on the graph, say (x_1, y) and (x_2, y), that have different x-coordinates and the same y-coordinate. So the function is not one-to-one, and the function is not invertible.

Horizontal-Line Test

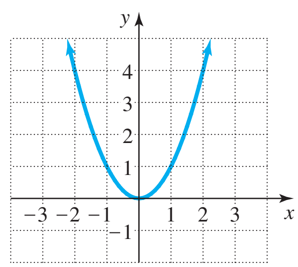
A function is invertible if and only if no horizontal line crosses its graph more than once.

EXAMPLE 2

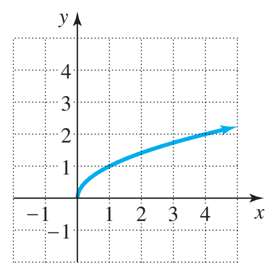
Using the horizontal-line test

Determine whether each function is invertible by examining its graph.

a)



b)



Solution

a) This function is not invertible because a horizontal line can be drawn so that it crosses the graph at (2, 4) and (−2, 4).

b) This function is invertible because every horizontal line that crosses the graph crosses it only once.

helpful hint

Tests such as the vertical-line test and the horizontal-line test are certainly not accurate in all cases. We discuss these tests to get a visual idea of what graphs of functions and invertible functions look like.

Identifying Inverse Functions

Consider the one-to-one function $f(x) = 3x$. The inverse function must reverse the ordered pairs of the function. Because division by 3 undoes multiplication by 3, we could guess that $g(x) = \frac{x}{3}$ is the inverse function. To verify our guess, we can use the following rule for determining whether two given functions are inverses of each other.

Identifying Inverse Functions

Functions f and g are inverses of each other if and only if

$(g \circ f)(x) = x$ for every number x in the domain of f and

$(f \circ g)(x) = x$ for every number x in the domain of g.

In the next example we verify that $f(x) = 3x$ and $g(x) = \frac{x}{3}$ are inverses.

EXAMPLE 3

Identifying inverse functions

Determine whether the functions f and g are inverses of each other.

a) $f(x) = 3x$ and $g(x) = \dfrac{x}{3}$

b) $f(x) = 2x - 1$ and $g(x) = \dfrac{1}{2}x + 1$

c) $f(x) = x^2$ and $g(x) = \sqrt{x}$

Solution

a) Find $g \circ f$ and $f \circ g$:

$$(g \circ f)(x) = g(f(x)) = g(3x) = \frac{3x}{3} = x$$

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{x}{3}\right) = 3 \cdot \frac{x}{3} = x$$

Because each of these equations is true for any real number x, f and g are inverses of each other. We write $g = f^{-1}$ or $f^{-1}(x) = \frac{x}{3}$.

b) Find the composition of g and f:

$$(g \circ f)(x) = g(f(x))$$
$$= g(2x - 1) = \frac{1}{2}(2x - 1) + 1 = x + \frac{1}{2}$$

So f and g are not inverses of each other.

c) If x is any real number, we can write

$$(g \circ f)(x) = g(f(x))$$
$$= g(x^2) = \sqrt{x^2} = |x|.$$

The domain of f is $(-\infty, \infty)$, and $|x| \neq x$ if x is negative. So g and f are not inverses of each other. Note that $f(x) = x^2$ is not a one-to-one function, since both $(3, 9)$ and $(-3, 9)$ are ordered pairs of this function. Thus $f(x) = x^2$ does not have an inverse.

study tip

Personal issues can have a tremendous effect on your progress in any course. So do not hesitate to deal with personal problems. If you need help, get it. Most schools have counseling centers that can help you to overcome personal issues that are affecting your studies.

Switch-and-Solve Strategy

If an invertible function is defined by a list of ordered pairs, as in Example 1, then the inverse function is found by simply interchanging the coordinates in the ordered pairs. If an invertible function is defined by a formula, then the inverse function must reverse or undo what the function does. Because the inverse function interchanges the roles of x and y, we interchange x and y in the formula and then solve the new formula for y to undo what the original function did. This **switch-and-solve** strategy is illustrated in the next two examples.

EXAMPLE 4 **The switch-and-solve strategy**

Find the inverse of $h(x) = 2x + 1$.

Solution

First write the function as $y = 2x + 1$, then interchange x and y:

$$y = 2x + 1$$
$$x = 2y + 1 \quad \text{Interchange } x \text{ and } y.$$
$$x - 1 = 2y \quad \text{Solve for } y.$$
$$\frac{x - 1}{2} = y$$
$$h^{-1}(x) = \frac{x - 1}{2} \quad \text{Replace } y \text{ by } h^{-1}(x).$$

We can verify that h and h^{-1} are inverses by using composition:

$$(h^{-1} \circ h)(x) = h^{-1}(h(x)) = h^{-1}(2x + 1) = \frac{2x + 1 - 1}{2} = \frac{2x}{2} = x$$

$$(h \circ h^{-1})(x) = h(h^{-1}(x)) = h\left(\frac{x - 1}{2}\right) = 2 \cdot \frac{x - 1}{2} + 1 = x - 1 + 1 = x$$

EXAMPLE 5 **The switch-and-solve strategy**

If $f(x) = \dfrac{x + 1}{x - 3}$, find $f^{-1}(x)$.

Solution

Replace $f(x)$ by y, interchange x and y, then solve for y:

$$\begin{aligned}
y &= \frac{x + 1}{x - 3} && \text{Use } y \text{ in place of } f(x). \\
x &= \frac{y + 1}{y - 3} && \text{Switch } x \text{ and } y. \\
x(y - 3) &= y + 1 && \text{Multiply each side by } y - 3. \\
xy - 3x &= y + 1 && \text{Distributive property} \\
xy - y &= 3x + 1 && \\
y(x - 1) &= 3x + 1 && \text{Factor out } y. \\
y &= \frac{3x + 1}{x - 1} && \text{Divide each side by } x - 1. \\
f^{-1}(x) &= \frac{3x + 1}{x - 1} && \text{Replace } y \text{ by } f^{-1}(x).
\end{aligned}$$

You should check that $(f \circ f^{-1})(x) = x$ and $(f^{-1} \circ f)(x) = x$.

The strategy for finding the inverse of a function $f(x)$ is summarized as follows.

Switch-and-Solve Strategy for Finding f^{-1}

1. Replace $f(x)$ by y.
2. Interchange x and y.
3. Solve the equation for y.
4. Replace y by $f^{-1}(x)$.

helpful hint

You should know from memory the inverses of simple functions that involve one or two operations. For example, the inverse of $f(x) = x + 99$ is $f^{-1}(x) = x - 99$. The inverse of $f(x) = x/33 + 22$ is $f^{-1}(x) = 33(x - 22)$.

If we use the switch-and-solve strategy to find the inverse of $f(x) = x^3$, then we get $f^{-1}(x) = x^{1/3}$. For $h(x) = 6x$ we have $h^{-1}(x) = \frac{x}{6}$. The inverse of $k(x) = x - 9$ is $k^{-1}(x) = x + 9$. For each of these functions there is an appropriate operation of arithmetic that undoes what the function does.

If a function involves two operations, the inverse function undoes those operations in the opposite order from which the function does them. For example, the function $g(x) = 3x - 5$ multiplies x by 3 and then subtracts 5 from that result.

To undo these operations, we add 5 and then divide the result by 3. So

$$g^{-1}(x) = \frac{x + 5}{3}.$$

Note that $g^{-1}(x) \neq \frac{x}{3} + 5$.

Even Roots or Even Powers

We need to use special care in finding inverses for functions that involve even roots or even powers. We saw in Example 3(c) that $f(x) = x^2$ is not the inverse of $g(x) = \sqrt{x}$. However, because $g(x) = \sqrt{x}$ is a one-to-one function, it has an inverse. The domain of g is $[0, \infty)$, and the range is $[0, \infty)$. So the inverse of g must have domain $[0, \infty)$ and range $[0, \infty)$. See Fig. 11.25. The only reason that

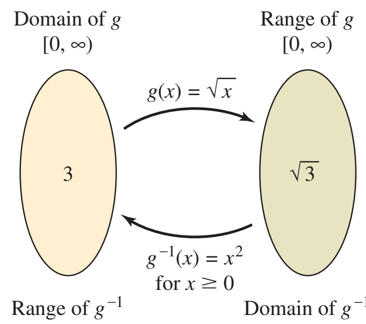
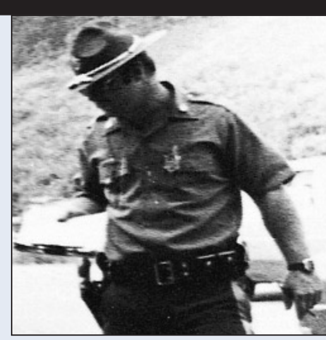


FIGURE 11.25

MATH AT WORK $x^2 + (x + 1)^2 = 5^2$



STATE POLICE OFFICER

When a serious automobile accident occurs in Massachusetts, Stephen Benanti, the Commanding Officer of the Accident Reconstruction Section of the State Police, may be called to examine the physical evidence at the scene. Physical evidence can consist of debris, scrapes, and gouges on the road; damage to fixed objects such as utility poles, trees, or guardrails; and the final resting position of and damage to the vehicles.

One critical type of evidence that is sometimes found are skid marks on the road. Sergeant Benanti can use the lengths of the skid marks to calculate the speeds of accident vehicles. Using a sophisticated laser measuring device, Benanti can store data that later can be downloaded into a computer to reconstruct the accident scene. He must also conduct tests on the road surface to calculate the drag factor, which is the resistance between the tire and the road surface. A smooth, icy surface yields a much lower drag factor than a dry asphalt surface. The minimum speed formula that state troopers use to determine vehicles' speeds is a function of the drag factor and the skid distance: $S = \sqrt{30DF}$, where $S =$ speed, $D =$ distance skidded, and $F =$ drag factor.

In Exercise 73 of this section you will use this minimum speed formula with a given length of skid marks to determine the speed of a vehicle.

$f(x) = x^2$ is not the inverse of g is that it has the wrong domain. So to write the inverse function, we must use the appropriate domain:

$$g^{-1}(x) = x^2 \qquad \text{for} \quad x \geq 0$$

Note that by restricting the domain of g^{-1} to $[0, \infty)$, g^{-1} is one-to-one. With this restriction it is true that $(g \circ g^{-1})(x) = x$ and $(g^{-1} \circ g)(x) = x$ for every nonnegative number x.

EXAMPLE 6

Inverse of a function with an even exponent

Find the inverse of the function $f(x) = (x - 3)^2$ for $x \geq 3$.

Solution

Because of the restriction $x \geq 3$, f is a one-to-one function with domain $[3, \infty)$ and range $[0, \infty)$. The domain of the inverse function is $[0, \infty)$, and its range is $[3, \infty)$. Use the switch-and-solve strategy to find the formula for the inverse:

$$\begin{aligned} y &= (x - 3)^2 \\ x &= (y - 3)^2 \\ y - 3 &= \pm\sqrt{x} \\ y &= 3 \pm \sqrt{x} \end{aligned}$$

Because the inverse function must have range $[3, \infty)$, we use the formula $f^{-1}(x) = 3 + \sqrt{x}$. Because the domain of f^{-1} is assumed to be $[0, \infty)$, no restriction is required on x.

Graphs of f and f^{-1}

Consider $f(x) = x^2$ for $x \geq 0$ and $f^{-1}(x) = \sqrt{x}$. Their graphs are shown in Fig. 11.26. Notice the symmetry. If we folded the paper along the line $y = x$, the two graphs would coincide.

If a point (a, b) is on the graph of the function f, then (b, a) must be on the graph of $f^{-1}(x)$. See Fig. 11.27. The points (a, b) and (b, a) lie on opposite sides of the diagonal line $y = x$ and are the same distance from it. For this reason the graphs of f and f^{-1} are symmetric with respect to the line $y = x$.

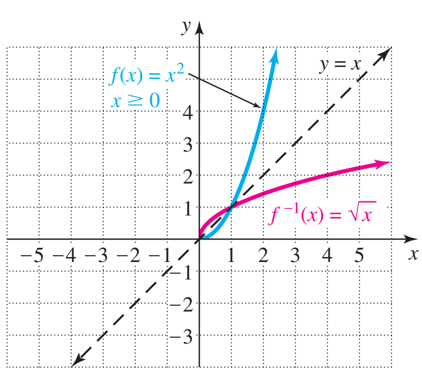


FIGURE 11.26

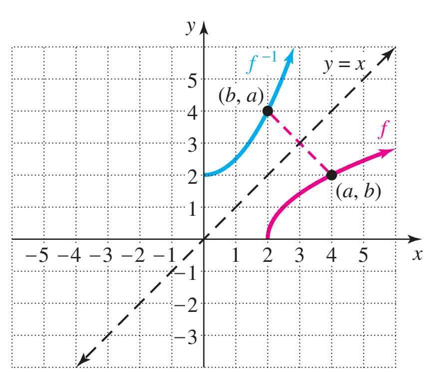


FIGURE 11.27

EXAMPLE 7 **Inverses and their graphs**

Find the inverse of the function $f(x) = \sqrt{x - 1}$ and graph f and f^{-1} on the same pair of axes.

Solution

To find f^{-1}, first switch x and y in the formula $y = \sqrt{x - 1}$:

$$x = \sqrt{y - 1}$$

$$x^2 = y - 1 \quad \text{Square both sides.}$$

$$x^2 + 1 = y$$

Because the range of f is the set of nonnegative real numbers $[0, \infty)$, we must restrict the domain of f^{-1} to be $[0, \infty)$. Thus $f^{-1}(x) = x^2 + 1$ for $x \geq 0$. The two graphs are shown in Fig. 11.28.

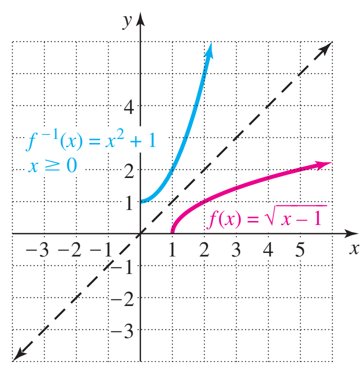


FIGURE 11.28

WARM-UPS

True or false? Explain your answer.

1. The inverse of $\{(1, 3), (2, 5)\}$ is $\{(3, 1), (2, 5)\}$.
2. The function $f(x) = 3$ is a one-to-one function.
3. If $g(x) = 2x$, then $g^{-1}(x) = \frac{1}{2x}$.
4. Only one-to-one functions are invertible.
5. The domain of g is the same as the range of g^{-1}.
6. The function $f(x) = x^4$ is invertible.
7. If $f(x) = -x$, then $f^{-1}(x) = -x$.
8. If h is invertible and $h(7) = -95$, then $h^{-1}(-95) = 7$.
9. If $k(x) = 3x - 6$, then $k^{-1}(x) = \frac{1}{3}x + 2$.
10. If $f(x) = 3x - 4$, then $f^{-1}(x) = x + 4$.

11.4 EXERCISES

Reading and Writing *After reading this section, write out the answers to these questions. Use complete sentences.*

1. What is the inverse of a function?

2. What is the domain of f^{-1}?

3. What is the range of f^{-1}?

4. What does the -1 in f^{-1} mean?

5. What is a one-to-one function?

6. What is the horizontal-line test?

7. What is the switch-and-solve strategy?

8. How are the graphs of f and f^{-1} related?

Determine whether each function is invertible. If it is invertible, then find the inverse. See Example 1.

9. $\{(-3, 3), (-2, 2), (0, 0), (2, 2)\}$

10. $\{(1, 1), (2, 8), (3, 27)\}$

11. $\{(16, 4), (9, 3), (0, 0)\}$

12. $\{(-1, 1), (-3, 81), (3, 81)\}$

13. $\{(0, 5), (5, 0), (6, 0)\}$

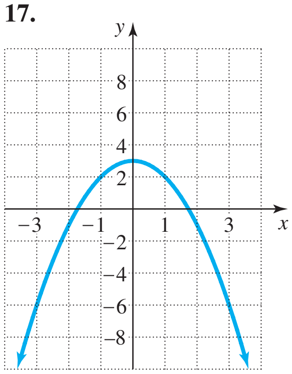
14. $\{(3, -3), (-2, 2), (1, -1)\}$

15. $\{(0, 0), (2, 2), (9, 9)\}$

16. $\{(9, 1), (2, 1), (7, 1), (0, 1)\}$

Determine whether each function is invertible by examining the graph of the function. See Example 2.

17.



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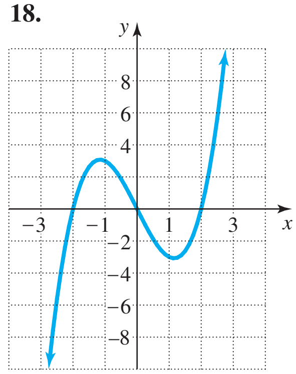

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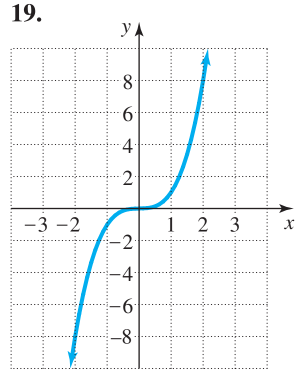

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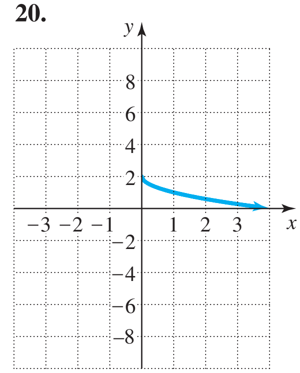

Determine whether each pair of functions f and g are inverses of each other. See Example 3.

21. $f(x) = 2x$ and $g(x) = 0.5x$

22. $f(x) = 3x$ and $g(x) = 0.33x$

23. $f(x) = 2x - 10$ and $g(x) = \frac{1}{2}x + 5$

24. $f(x) = 3x + 7$ and $g(x) = \frac{x - 7}{3}$

25. $f(x) = -x$ and $g(x) = -x$

26. $f(x) = \frac{1}{x}$ and $g(x) = \frac{1}{x}$

27. $f(x) = x^4$ and $g(x) = x^{1/4}$

28. $f(x) = |2x|$ and $g(x) = \left|\frac{x}{2}\right|$

Determine f^{-1} for each function by using the switch-and-solve strategy. Check that $(f \circ f^{-1})(x) = x$ and $(f^{-1} \circ f)(x) = x$. See Examples 4 and 5.

29. $f(x) = 5x$

30. $h(x) = -3x$

31. $g(x) = x - 9$

32. $j(x) = x + 7$

33. $k(x) = 5x - 9$

34. $r(x) = 2x - 8$

35. $m(x) = \frac{2}{x}$

36. $s(x) = \frac{-1}{x}$

37. $f(x) = \sqrt[3]{x - 4}$

38. $f(x) = \sqrt[3]{x + 2}$

39. $f(x) = \frac{3}{x - 4}$

40. $f(x) = \frac{2}{x + 1}$

41. $f(x) = \sqrt[3]{3x + 7}$

42. $f(x) = \sqrt[3]{7 - 5x}$

43. $f(x) = \dfrac{x + 1}{x - 2}$

44. $f(x) = \dfrac{1 - x}{x + 3}$

45. $f(x) = \dfrac{x + 1}{3x - 4}$

46. $g(x) = \dfrac{3x + 5}{2x - 3}$

Find the inverse of each function. See Example 6.

47. $p(x) = \sqrt[6]{x}$

48. $v(x) = \sqrt[6]{x}$

49. $f(x) = (x - 2)^2$ for $x \geq 2$

50. $g(x) = (x + 5)^2$ for $x \geq -5$

51. $f(x) = x^2 + 3$ for $x \geq 0$

52. $f(x) = x^2 - 5$ for $x \geq 0$

53. $f(x) = \sqrt{x + 2}$

54. $f(x) = \sqrt{x - 4}$

In Exercises 55–64, find the inverse of each function and graph f and f^{-1} on the same pair of axes. See Example 7.

55. $f(x) = 2x + 3$

56. $f(x) = -3x + 2$

57. $f(x) = x^2 - 1$ for $x \geq 0$

58. $f(x) = x^2 + 3$ for $x \geq 0$

59. $f(x) = 5x$

60. $f(x) = \dfrac{x}{4}$

61. $f(x) = x^3$

62. $f(x) = 2x^3$

63. $f(x) = \sqrt{x - 2}$

64. $f(x) = \sqrt{x + 3}$

For each pair of functions, find $(f^{-1} \circ f)(x)$

65. $f(x) = x^3 - 1$ and $f^{-1}(x) = \sqrt[3]{x + 1}$

66. $f(x) = 2x^3 + 1$ and $f^{-1}(x) = \sqrt[3]{\dfrac{x - 1}{2}}$

67. $f(x) = \dfrac{1}{2}x - 3$ and $f^{-1}(x) = 2x + 6$

68. $f(x) = 3x - 9$ and $f^{-1}(x) = \dfrac{1}{3}x + 3$

69. $f(x) = \dfrac{1}{x} + 2$ and $f^{-1}(x) = \dfrac{1}{x - 2}$

70. $f(x) = 4 - \dfrac{1}{x}$ and $f^{-1}(x) = \dfrac{1}{4 - x}$

71. $f(x) = \dfrac{x + 1}{x - 2}$ and $f^{-1}(x) = \dfrac{2x + 1}{x - 1}$

72. $f(x) = \dfrac{3x - 2}{x + 2}$ and $f^{-1}(x) = \dfrac{2x + 2}{3 - x}$

Solve each problem.

73. ***Accident reconstruction.*** The distance that it takes a car to stop is a function of the speed and the drag factor. The drag factor is a measure of the resistance between the tire and the road surface. The formula $S = \sqrt{30LD}$ is used to determine the minimum speed S [in miles per hour (mph)] for a car that has left skid marks of length L feet (ft) on a surface with drag factor D.

a) Find the minimum speed for a car that has left skid marks of length 50 ft where the drag factor is 0.75.

b) Does the drag factor increase or decrease for a road surface when it gets wet?

c) Write L as a function S for a road surface with drag factor 1 and graph the function.

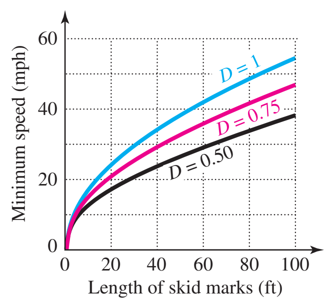

FIGURE FOR EXERCISE 73

74. ***Area of a circle.*** Let x be the radius of a circle and $h(x)$ be the area of the circle. Write a formula for $h(x)$ in terms of x. What does x represent in the notation $h^{-1}(x)$? Write a formula for $h^{-1}(x)$.

75. ***Vehicle cost.*** At Bill Hood Ford in Hammond a sales tax of 9% of the selling price x and a \$125 title and license fee are added to the selling price to get the total cost of a vehicle. Find the function $T(x)$ that the dealer uses to get the total cost as a function of the selling price x. Citizens National Bank will not include sales tax or fees in a loan. Find the function $T^{-1}(x)$ that the bank can use to get the selling price as a function of the total cost x.

76. ***Carpeting cost.*** At the Windrush Trace apartment complex all living rooms are square, but the length of x feet may vary. The cost of carpeting a living room is \$18 per square yard plus a \$50 installation fee. Find the function $C(x)$ that gives the total cost of carpeting a living room of length x. The manager has an invoice for the total cost of a living room carpeting job but does not know in which apartment it was done. Find the function $C^{-1}(x)$ that gives the length of a living room as a function of the total cost of the carpeting job x.

GETTING MORE INVOLVED

77. ***Discussion.*** Let $f(x) = x^n$ for n a positive integer. For which values of n is f an invertible function? Explain.

78. ***Discussion.*** Suppose f is a function with range $(-\infty, \infty)$ and g is a function with domain $(0, \infty)$. Is it possible that g and f are inverse functions? Explain.

GRAPHING CALCULATOR EXERCISES

79. Most graphing calculators can form compositions of functions. Let $f(x) = x^2$ and $g(x) = \sqrt{x}$. To graph the composition $g \circ f$, let $y_1 = x^2$ and $y_2 = \sqrt{y_1}$. The graph of y_2 is the graph of $g \circ f$. Use the graph of y_2 to determine whether f and g are inverse functions.

80. Let $y_1 = x^3 - 4$, $y_2 = \sqrt[3]{x + 4}$, and $y_3 = \sqrt[3]{y_1 + 4}$. The function y_3 is the composition of the first two functions. Graph all three functions on the same screen. What do the graphs indicate about the relationship between y_1 and y_2?

11.5 VARIATION

In this section

- Direct Variation
- Finding the Proportionality Constant
- Inverse Variation
- Joint Variation
- More Variation

If $y = 3x$, then as x varies so does y. Certain functions are customarily expressed in terms of variation. In this section you will learn to write formulas for those functions from verbal descriptions of the functions.

Direct Variation

In a community with an 8% sales tax rate, the amount of tax, t (in dollars), is a function of the amount of the purchase, a (in dollars). This function is expressed by the formula

$$t = 0.08a.$$

If the amount increases, then the tax increases. If a decreases, then t decreases. In this situation we say that *t varies directly with a, or t is directly proportional to a.* The constant tax rate, 0.08, is called the **variation constant** or **proportionality constant.** Notice that t is just a simple linear function of a. We are merely introducing some new terms to express an old idea.

Direct Variation

The statement **y varies directly as x,** or **y is directly proportional to x,** means that

$$y = kx$$

for some constant, k. The constant, k, is a fixed nonzero real number.

Finding the Proportionality Constant

If y varies directly as x and we know corresponding values for x and y, then we can find the proportionality constant.