1.8 USING THE PROPERTIES TO SIMPLIFY EXPRESSIONS

The properties of the real numbers can be helpful when we are doing computations. In this section we will see how the properties can be applied in arithmetic and algebra.

Using the Properties in Computation

The properties of the real numbers can often be used to simplify computations. For example, to find the product of 26 and 200, we can write

$$(26)(200) = (26)(2 \cdot 100)$$
$$= (26 \cdot 2)(100)$$
$$= 52 \cdot 100$$
$$= 5200$$

It is the associative property that allows us to multiply 26 by 2 to get 52, then multiply 52 by 100 to get 5200.

E X A M P L E   1

Using the properties

Use the appropriate property to aid you in evaluating each expression.

a) $347 + 35 + 65$ **b)** $3 \cdot 435 \cdot \dfrac{1}{3}$ **c)** $6 \cdot 28 + 4 \cdot 28$

Solution

a) Notice that the sum of 35 and 65 is 100. So apply the associative property as follows:

$$347 + (35 + 65) = 347 + 100$$
$$= 447$$

b) Use the commutative and associative properties to rearrange this product. We can then do the multiplication quickly:

$$3 \cdot 435 \cdot \frac{1}{3} = 435\left(3 \cdot \frac{1}{3}\right)$$
$$= 435 \cdot 1$$
$$= 435$$

c) Use the distributive property to rewrite this expression.

$$6 \cdot 28 + 4 \cdot 28 = (6 + 4)28$$
$$= 10 \cdot 28$$
$$= 280$$ ■

Like Terms

An expression containing a number or the product of a number and one or more variables raised to powers is called a **term.** For example,

$$-3, \qquad 5x, \qquad -3x^2y, \qquad a, \qquad \text{and} \qquad -abc$$

are terms. The number preceding the variables in a term is called the **coefficient.** In the term $5x$, the coefficient of x is 5. In the term $-3x^2y$ the coefficient of x^2y is -3. In the term a, the coefficient of a is 1 because $a = 1 \cdot a$. In the term $-abc$ the coefficient of abc is -1 because $-abc = -1 \cdot abc$. If two terms contain the same variables with the same exponents, they are called **like terms.** For example, $3x^2$ and $-5x^2$ are like terms, but $3x^2$ and $-5x^3$ are not like terms.

Combining Like Terms

Using the distributive property on an expression involving the sum of like terms allows us to combine the like terms as shown in the next example.

E X A M P L E   2

Combining like terms

Use the distributive property to perform the indicated operations.

a) $3x + 5x$ **b)** $-5xy - (-4xy)$

Solution

a) $3x + 5x = (3 + 5)x$
$$= 8x$$

Because the distributive property is valid for any real numbers, we have $3x + 5x = 8x$ no matter what number is used for x.

b) $-5xy - (-4xy) = [-5 - (-4)]xy$
$$= -1xy$$
$$= -xy$$

■

Of course, we do not want to write out all of the steps shown in Example 2 every time we combine like terms. We can combine like terms as easily as we can add or subtract their coefficients.

E X A M P L E   3

Combining like terms

Perform the indicated operations.

a) $w + 2w$ **b)** $-3a + (-7a)$ **c)** $-9x + 5x$
d) $7xy - (-12xy)$ **e)** $2x^2 + 4x^2$

Solution

a) $w + 2w = 1w + 2w = 3w$ **b)** $-3a + (-7a) = -10a$
c) $-9x + 5x = -4x$ **d)** $7xy - (-12xy) = 19xy$
e) $2x^2 + 4x^2 = 6x^2$

■

C A U T I O N   There are no like terms in expressions such as
$$2 + 5x, \quad\quad 3xy + 5y, \quad\quad 3w + 5a, \quad\quad \text{and} \quad\quad 3z^2 + 5z$$
The terms in these expressions cannot be combined.

Products and Quotients

In the next example we use the associative property of multiplication to simplify the product of two expressions.

E X A M P L E   4

Finding products

Simplify.

a) $3(5x)$ b) $2\left(\dfrac{x}{2}\right)$ c) $(4x)(6x)$ d) $(-2a)(4b)$

Solution

a) $3(5x) = (3 \cdot 5)x$

$\qquad\quad = (15)x$

$\qquad\quad = 15x$

b) $2\left(\dfrac{x}{2}\right) = 2\left(\dfrac{1}{2} \cdot x\right)$

$\qquad\quad = \left(2 \cdot \dfrac{1}{2}\right)x$

$\qquad\quad = 1 \cdot x$

$\qquad\quad = x$

c) $(4x)(6x) = 4 \cdot 6 \cdot x \cdot x$

$\qquad\qquad = 24x^2$

d) $(-2a)(4b) = -2 \cdot 4 \cdot a \cdot b = -8ab$ ∎

> **study tip**
>
> Note how the exercises are keyed to the examples. This serves two purposes. If you have missed class and are studying on your own, you should study an example and then immediately try to work the corresponding exercises. If you have seen an explanation in class, then you can start the exercises and refer back to the examples as necessary.

C A U T I O N   Be careful with expressions such as $3(5x)$ and $3(5 + x)$. In $3(5x)$ we multiply 5 by 3 to get $3(5x) = 15x$. In $3(5 + x)$, both 5 and x are multiplied by the 3 to get $3(5 + x) = 15 + 3x$.

In Example 4 we showed how the properties are used to simplify products. However, in practice we usually do not write out any steps for these problems—we can write just the answer.

E X A M P L E   5

Finding products quickly

Find each product.

a) $(-3)(4x)$ b) $(-4a)(-7a)$ c) $(-3a)\left(\dfrac{b}{3}\right)$ d) $6 \cdot \dfrac{x}{2}$

Solution

a) $-12x$ b) $28a^2$ c) $-ab$ d) $3x$ ∎

In Section 1.1 we found the quotient of two numbers by inverting the divisor and then multiplying. Since $a \div b = a \cdot \dfrac{1}{b}$, any quotient can be written as a product.

E X A M P L E   6

Simplifying quotients

Simplify.

a) $\dfrac{10x}{5}$ b) $\dfrac{4x + 8}{2}$

Solution

a) Since dividing by 5 is equivalent to multiplying by $\dfrac{1}{5}$, we have

$$\frac{10x}{5} = \frac{1}{5}(10x) = \left(\frac{1}{5} \cdot 10\right)x = (2)x = 2x.$$

Note that you can simply divide 10 by 5 to get 2.

b) Since dividing by 2 is equivalent to multiplying by $\frac{1}{2}$, we have

$$\frac{4x + 8}{2} = \frac{1}{2}(4x + 8) = 2x + 4.$$

Note that both 4 and 8 are divided by 2.   ■

CAUTION   It is not correct to divide only one term in the numerator by the denominator. For example,

$$\frac{4 + 7}{2} \neq 2 + 7$$

because $\frac{4 + 7}{2} = \frac{11}{2}$ and $2 + 7 = 9$.

Removing Parentheses

Multiplying a number by -1 merely changes the sign of the number. For example,

$$(-1)(7) = -7 \qquad \text{and} \qquad (-1)(-8) = 8.$$

So -1 times a number is the *opposite* of the number. Using variables, we write

$$(-1)x = -x \qquad \text{or} \qquad -1(y + 5) = -(y + 5).$$

When a minus sign appears in front of a sum, we can change the minus sign to -1 and use the distributive property. For example,

$$
\begin{aligned}
-(w + 4) &= -1(w + 4) \\
&= (-1)w + (-1)4 \\
&= -w + (-4) \\
&= -w - 4
\end{aligned}
$$

Note how the minus sign in front of the parentheses caused all of the signs to change: $-(w + 4) = -w - 4$. As another example, consider the following:

$$
\begin{aligned}
-(x - 3) &= -1(x - 3) \\
&= (-1)x - (-1)3 \\
&= -x + 3
\end{aligned}
$$

CAUTION   When removing parentheses preceded by a minus sign, you must change the sign of *every* term within the parentheses.

E X A M P L E   7

Removing parentheses

Simplify each expression.

a) $5 - (x + 3)$ **b)** $3x - 6 - (2x - 4)$ **c)** $-6x - (-x + 2)$

Solution

a) $\begin{aligned}5 - (x + 3) &= 5 - x - 3 \\ &= 5 - 3 - x \\ &= 2 - x\end{aligned}$

---

calculator

4   5   6   ✕

close-up

A negative sign in front of parentheses changes the sign of every term inside the parentheses.

```
-(5-3)
                -2
-1(5-3)
                -2
-5+3
                -2
```

b) $3x - 6 - (2x - 4) = 3x - 6 - 2x + 4$
$$= 3x - 2x - 6 + 4$$
$$= x - 2$$

c) $-6x - (-x + 2) = -6x + x - 2$
$$= -5x - 2 \qquad \blacksquare$$

The commutative and associative properties of addition allow us to rearrange the terms so that we may combine the like terms. However, it is not necessary to actually write down the rearrangement. We can identify the like terms and combine them without rearranging.

E X A M P L E   8   **Simplifying algebraic expressions**
Simplify.

a) $(-2x + 3) + (5x - 7)$ **b)** $-3x + 6x + 5(4 - 2x)$

c) $-2x(3x - 7) - (x - 6)$ **d)** $x - 0.02(x + 500)$

Solution

a) $(-2x + 3) + (5x - 7) = 3x - 4$

b) $-3x + 6x + 5(4 - 2x) = -3x + 6x + 20 - 10x$
$$= -7x + 20$$

c) $-2x(3x - 7) - (x - 6) = -6x^2 + 14x - x + 6$
$$= -6x^2 + 13x + 6$$

d) $x - 0.02(x + 500) = 1x - 0.02x - 10$
$$= 0.98x - 10 \qquad \blacksquare$$

W A R M - U P S

True or false? Explain your answer.

A statement involving variables should be marked true only if it is true for all values of the variable.

1. $3(x + 6) = 3x + 18$ **2.** $-3x + 9 = -3(x + 9)$

3. $-1(x - 4) = -x + 4$ **4.** $3a + 4a = 7a$

5. $(3a)(4a) = 12a$ **6.** $3(5 \cdot 2) = 15 \cdot 6$

7. $x + x = x^2$ **8.** $x \cdot x = 2x$

9. $3 + 2x = 5x$ **10.** $-(5x - 2) = -5x + 2$

1.8   E X E R C I S E S

Reading and Writing   *After reading this section write out the answers to these questions. Use complete sentences.*

1. What are like terms?

2. What is the coefficient of a term?

3. What can you do to like terms that you cannot do to unlike terms?

4. What operations can you perform with unlike terms?

5. What is the difference between a positive sign preceding a set of parentheses and a negative sign preceding a set of parentheses?

63. $8y\left(\dfrac{y}{4}\right)$ **64.** $10\left(\dfrac{2a}{5}\right)$ **65.** $\dfrac{6a-3}{3}$

66. $\dfrac{-8x+6}{2}$ **67.** $\dfrac{-9x+6}{-3}$ **68.** $\dfrac{10-5x}{-5}$

6. What happens when a number is multiplied by -1?

Simplify each expression. See Example 7.

69. $x-(3x-1)$ **70.** $4x-(2x-5)$

71. $5-(y-3)$ **72.** $8-(m-6)$

Use the appropriate properties to evaluate the expressions. See Example 1.

73. $2m+3-(m+9)$
74. $7-8t-(2t+6)$
75. $-3-(-w+2)$
76. $-5x-(-2x+9)$

7. $35(200)$ **8.** $15(300)$

9. $\dfrac{4}{3}(0.75)$ **10.** $5(0.2)$

11. $256+78+22$ **12.** $12+88+376$

13. $35\cdot3+35\cdot7$ **14.** $98\cdot478+2\cdot478$

Simplify the following expressions by combining like terms. See Example 8.

77. $3x+5x+6+9$
78. $2x+6x+7+15$

15. $18\cdot4\cdot2\cdot\dfrac{1}{4}$ **16.** $19\cdot3\cdot2\cdot\dfrac{1}{3}$

79. $-2x+3+7x-4$
80. $-3x+12+5x-9$

17. $(120)(300)$ **18.** $150\cdot200$

81. $3a-7-(5a-6)$

19. $12\cdot375(-6+6)$ **20.** $354^2(-2\cdot4+8)$

82. $4m-5-(m-2)$
83. $2(a-4)-3(-2-a)$

21. $78+6+8+4+2$

84. $2(w+6)-3(-w-5)$

22. $-47+12-6-12+6$

85. $-5m+6(m-3)+2m$
86. $-3a+2(a-5)+7a$

Combine like terms where possible. See Examples 2 and 3.

87. $5-3(x+2)-6$

23. $5w+6w$ **24.** $4a+10a$

88. $7+2(k-3)-k+6$
89. $x-0.05(x+10)$

25. $4x-x$ **26.** $a-6a$

90. $x-0.02(x+300)$

27. $2x-(-3x)$ **28.** $2b-(-5b)$

91. $4.5-3.2(x-5.3)-8.75$

29. $-3a-(-2a)$ **30.** $-10m-(-6m)$

92. $0.03(4.5x-3.9)+0.06(9.8x-45)$

31. $-a-a$ **32.** $a-a$

Simplify each expression.

33. $10-6t$ **34.** $9-4w$

93. $3x-(4-x)$ **94.** $2+8x-11x$

35. $3x^2+5x^2$ **36.** $3r^2+4r^2$

37. $-4x+2x^2$ **38.** $6w^2-w$

95. $y-5-(-y-9)$ **96.** $a-(b-c-a)$

39. $5mw^2-12mw^2$ **40.** $4ab^2-19ab^2$

97. $7-(8-2y-m)$ **98.** $x-8-(-3-x)$

Simplify the following products or quotients. See Examples 4–6.

99. $\dfrac{1}{2}(10-2x)+\dfrac{1}{3}(3x-6)$

41. $3(4h)$ **42.** $2(5h)$

43. $6b(-3)$ **44.** $-3m(-1)$

100. $\dfrac{1}{2}(x-20)-\dfrac{1}{5}(x+15)$

45. $(-3m)(3m)$ **46.** $(2x)(-2x)$ **47.** $(-3d)(-4d)$

101. $0.2(x+3)-0.05(x+20)$
102. $0.08x+0.12(x+100)$

48. $(-5t)(-2t)$ **49.** $(-y)(-y)$ **50.** $y(-y)$

103. $2k+1-3(5k-6)-k+4$
104. $2w-3+3(w-4)-5(w-6)$

51. $-3a(5b)$ **52.** $-7w(3r)$ **53.** $-3a(2+b)$

105. $-3m-3[2m-3(m+5)]$

54. $-2x(3+y)$ **55.** $-k(1-k)$ **56.** $-t(t-1)$

106. $6h+4[2h-3(h-9)-(h-1)]$

57. $\dfrac{3y}{3}$ **58.** $\dfrac{-9t}{9}$ **59.** $\dfrac{-15y}{5}$

60. $\dfrac{-12b}{2}$ **61.** $2\left(\dfrac{y}{2}\right)$ **62.** $6\left(\dfrac{m}{3}\right)$

Solve each problem.

107. *Married filing jointly.* The expression

$$0.15(43{,}850) + 0.28(x - 43{,}850)$$

gives the 2000 federal income tax for a married couple filing jointly with a taxable income of x dollars, where x is over \$43,850 but not over \$105,950 (Internal Revenue Service, www.irs.gov).
a) Simplify the expression.
b) Use the expression to find the amount of tax for a couple with a taxable income of \$80,000.
c) Use the graph shown here to estimate the 2000 federal income tax for a couple with a taxable income of \$150,000.
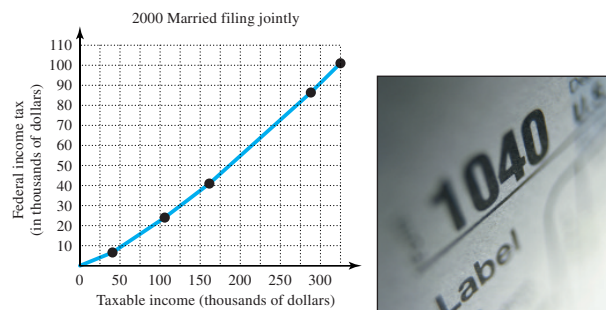**d)** Use the graph to find the approximate taxable income for a couple who paid \$70,000 in federal income tax.

FIGURE FOR EXERCISE 107

108. *Marriage penalty.* The expression

$$0.15(26{,}250) + 0.28(x - 26{,}250)$$

gives the 2000 federal income tax for a single taxpayer with taxable income of x dollars, where x is over \$26,250 but not over \$63,550.
a) Simplify the expression.
b) Find the amount of tax for a single taxpayer with taxable income of \$40,000.

c) Who pays more, two single taxpayers with taxable incomes of \$40,000 each or one married couple with taxable income of \$80,000 together? See Exercise 107.

109. *Perimeter of a corral.* The perimeter of a rectangular corral that has width x feet and length $x + 40$ feet is $2(x) + 2(x + 40)$. Simplify the expression for the perimeter. Find the perimeter if $x = 30$ feet.
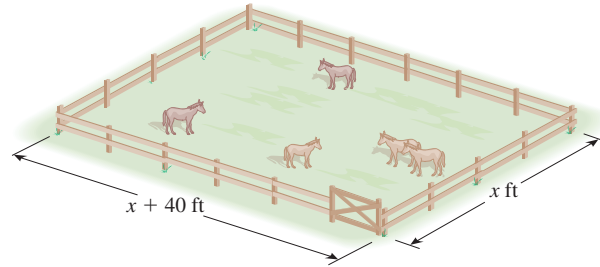

FIGURE FOR EXERCISE 109

GETTING MORE INVOLVED

110. *Discussion.* What is wrong with the way in which each of the following expressions is simplified?
a) $4(2 + x) = 8 + x$
b) $4(2x) = 8 \cdot 4x = 32x$
c) $\dfrac{4 + x}{2} = 2 + x$
d) $5 - (x - 3) = 5 - x - 3 = 2 - x$

111. *Discussion.* An instructor asked his class to evaluate the expression $1/2x$ for $x = 5$. Some students got 0.1; others got 2.5. Which answer is correct and why?

COLLABORATIVE ACTIVITIES

Remembering the Rules

This chapter reviews different types of numbers used in algebra. This activity will review the rules for the basic operations: addition, subtraction, multiplication, and division for fractions, decimals, and real numbers.
Part I: Remembering the rules. Have each member of your group choose an operation: addition, subtraction, multiplication, or division.

1. Fractions:
   a. Write the rules for working a fraction problem using the operation you have chosen. Use your book as a reference

Grouping: 4 students
Topic: Fractions, decimals, and signed numbers

and consider the following sample problems:

$$\frac{1}{2} + \frac{2}{5}, \quad \frac{1}{3} \cdot \frac{6}{7}, \quad \frac{3}{5} - \frac{1}{3}, \quad \frac{1}{3} \div \frac{2}{3}$$

   b. Starting with addition, each of you will share what he or she has written with the other members of the group. Make additions or corrections if needed.
Switch operations. Each member of the group now takes the operation of the person to his or her right.

2. Decimals: Repeat parts 1(a) and 1(b) for the following sample problems:

$$0.012 + 3, \; 2.1 - 0.25, \; 3.2 \cdot 0.23, \; 5.4 \div 1.2$$

Switch operations.   Each member of the group now takes the operation of the person to his or her right.

3. Signed numbers: Repeat parts 1(a) and 1(b) for the following sample problems:

$$-3 + 5, \; 3 - (-2), \; -2 \cdot 3, \; -6 \div -2$$

Part II: Testing your rules.   As a group, work through the following problems together, using the rules you have written from Part I. Add any new rules that come up while you work.

1. $\dfrac{1}{3} + \dfrac{2}{5}$ **2.** $0.076 + 7 + 2.005$

3. $-8 + 17$

4. $2\dfrac{9}{14} - 1\dfrac{5}{7}$

5. $8 - 3.024$

6. $-5 - (-19)$

7. $3\dfrac{1}{3} \cdot 2\dfrac{2}{5}$

8. $0.0723 \cdot 100$

9. $12(-3)(-2)$

10. $1\dfrac{2}{3} \div 5\dfrac{1}{2}$

11. $1.024 \div 3.2$

12. $-405 \div 15$

13. $-2\left(\dfrac{2}{3} - \dfrac{1}{4}\right) \div 2\dfrac{1}{2} + (3.052 - (-0.948))$

Extension: Before each exam, form a study group to review material. Write any new rules or definitions used in each chapter. Save these to study for your final exam.

WRAP-UP

CHAPTER 1

SUMMARY

The Real Numbers		Examples
Counting or natural numbers	$\{1, 2, 3, \ldots\}$	
Whole numbers	$\{0, 1, 2, 3, \ldots\}$	
Integers	$\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$	
Rational numbers	$\left\{\dfrac{a}{b}\,\middle|\, a \text{ and } b \text{ are integers with } b \neq 0\right\}$	$\dfrac{3}{2}, 5, -6, 0$
Irrational numbers	$\{x \mid x \text{ is a real number that is not rational}\}$	$\sqrt{2}, \sqrt{3}, \pi$
Real numbers	The set of real numbers consists of all rational numbers together with all irrational numbers.	

Fractions		Examples
Reducing fractions	$\dfrac{a \cdot c}{b \cdot c} = \dfrac{a}{b}$	$\dfrac{4}{6} = \dfrac{2 \cdot 2}{2 \cdot 3} = \dfrac{2}{3}$
Building up fractions	$\dfrac{a}{b} = \dfrac{a \cdot c}{b \cdot c}$	$\dfrac{3}{8} = \dfrac{3 \cdot 5}{8 \cdot 5} = \dfrac{15}{40}$
Multiplying fractions	$\dfrac{a}{b} \cdot \dfrac{c}{d} = \dfrac{ac}{bd}$	$\dfrac{2}{3} \cdot \dfrac{4}{5} = \dfrac{8}{15}$
Dividing fractions	$\dfrac{a}{b} \div \dfrac{c}{d} = \dfrac{a}{b} \cdot \dfrac{d}{c}$	$\dfrac{2}{3} \div \dfrac{4}{5} = \dfrac{2}{3} \cdot \dfrac{5}{4} = \dfrac{10}{12} = \dfrac{5}{6}$