

**EXAMPLE 2-5** The Capillary Rise of Water in a Tube

A 0.6-mm-diameter glass tube is inserted into water at 20°C in a cup. Determine the capillary rise of water in the tube (Fig. 2–27).

**SOLUTION** The rise of water in a slender tube as a result of the capillary effect is to be determined.

**Assumptions** 1 There are no impurities in the water and no contamination on the surfaces of the glass tube. 2 The experiment is conducted in atmospheric air.

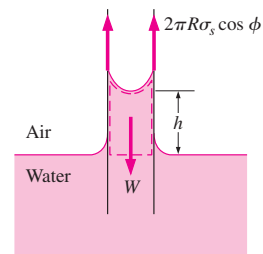
**Properties** The surface tension of water at 20°C is 0.073 N/m (Table 2–3). The contact angle of water with glass is 0° (from preceding text). We take the density of liquid water to be 1000 kg/m<sup>3</sup>.

**Analysis** The capillary rise is determined directly from Eq. 2–15 by substituting the given values, yielding

$$h = \frac{2\sigma_s}{\rho g R} \cos \phi = \frac{2(0.073 \text{ N/m})}{(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.3 \times 10^{-3} \text{ m})} (\cos 0^\circ) \left( \frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right) \\ = 0.050 \text{ m} = \mathbf{5.0 \text{ cm}}$$

Therefore, water rises in the tube 5 cm above the liquid level in the cup.

**Discussion** Note that if the tube diameter were 1 cm, the capillary rise would be 0.3 mm, which is hardly noticeable to the eye. Actually, the capillary rise in a large-diameter tube occurs only at the rim. The center does not rise at all. Therefore, the capillary effect can be ignored for large-diameter tubes.



**FIGURE 2-27**  
Schematic for Example 2–5.

**SUMMARY**

In this chapter various properties commonly used in fluid mechanics are discussed. The mass-dependent properties of a system are called *extensive properties* and the others, *intensive properties*. *Density* is mass per unit volume, and *specific volume* is volume per unit mass. The *specific gravity* is defined as the ratio of the density of a substance to the density of water at 4°C,

$$SG = \frac{\rho}{\rho_{\text{H}_2\text{O}}}$$

The ideal-gas equation of state is expressed as

$$P = \rho RT$$

where  $P$  is the absolute pressure,  $T$  is the thermodynamic temperature,  $\rho$  is the density, and  $R$  is the gas constant.

At a given temperature, the pressure at which a pure substance changes phase is called the *saturation pressure*. For phase-change processes between the liquid and vapor phases of a pure substance, the saturation pressure is commonly called the *vapor pressure*  $P_v$ . Vapor bubbles that form in the

low-pressure regions in a liquid (a phenomenon called *cavitation*) collapse as they are swept away from the low-pressure regions, generating highly destructive, extremely high-pressure waves.

Energy can exist in numerous forms, and their sum constitutes the *total energy*  $E$  (or  $e$  on a unit-mass basis) of a system. The sum of all microscopic forms of energy is called the *internal energy*  $U$  of a system. The energy that a system possesses as a result of its motion relative to some reference frame is called *kinetic energy* expressed per unit mass as  $ke = V^2/2$ , and the energy that a system possesses as a result of its elevation in a gravitational field is called *potential energy* expressed per unit mass as  $pe = gz$ .

The compressibility effects in a fluid are represented by the **coefficient of compressibility**  $\kappa$  (also called **bulk modulus of elasticity**) defined as

$$\kappa = -v \left( \frac{\partial P}{\partial v} \right)_T = \rho \left( \frac{\partial P}{\partial \rho} \right)_T \cong -\frac{\Delta P}{\Delta v/v}$$

The property that represents the variation of the density of a fluid with temperature at constant pressure is the **volume expansion coefficient** (or volume expansivity)  $\beta$ , defined as

$$\beta = \frac{1}{\nu} \left( \frac{\partial \nu}{\partial T} \right)_p = -\frac{1}{\rho} \left( \frac{\partial \rho}{\partial T} \right)_p \cong -\frac{\Delta \rho / \rho}{\Delta T}$$

The *viscosity* of a fluid is a measure of its resistance to deformation. The tangential force per unit area is called *shear stress* and is expressed for simple shear flow between plates (one-dimensional flow) as

$$\tau = \mu \frac{du}{dy}$$

where  $\mu$  is the coefficient of viscosity or the *dynamic* (or *absolute*) *viscosity* of the fluid,  $u$  is the velocity component in the flow direction, and  $y$  is the direction normal to the flow direction. The fluids that obey this linear relationship are called *Newtonian fluids*. The ratio of dynamic viscosity to density is called the *kinematic viscosity*  $\nu$ .

The pulling effect on the liquid molecules at an interface caused by the attractive forces of molecules per unit length is called *surface tension*  $\sigma_s$ . The excess pressure  $\Delta P$  inside a spherical droplet or bubble is given by

$$\Delta P_{\text{droplet}} = P_i - P_o = \frac{2\sigma_s}{R} \quad \text{and} \quad \Delta P_{\text{bubble}} = P_i - P_o = \frac{4\sigma_s}{R}$$

where  $P_i$  and  $P_o$  are the pressures inside and outside the droplet or bubble. The rise or fall of a liquid in a small-diameter tube inserted into the liquid due to surface tension is called the *capillary effect*. The capillary rise or drop is given by

$$h = \frac{2\sigma_s}{\rho g R} \cos \phi$$

where  $\phi$  is the *contact angle*. The capillary rise is inversely proportional to the radius of the tube and is negligible for tubes whose diameter is larger than about 1 cm.

Density and viscosity are two of the most fundamental properties of fluids, and they are used extensively in the chapters that follow. In Chap. 3, the effect of density on the variation of pressure in a fluid is considered, and the hydrostatic forces acting on surfaces are determined. In Chap. 8, the pressure drop caused by viscous effects during flow is calculated and used in the determination of the pumping power requirements. Viscosity is also used as a key property in the formulation and solutions of the equations of fluid motion in Chaps. 9 and 10.

## REFERENCES AND SUGGESTED READING

1. E. C. Bingham. "An Investigation of the Laws of Plastic Flow," *U.S. Bureau of Standards Bulletin*, 13, pp. 309–353, 1916.
2. Y. A. Cengel and M. A. Boles. *Thermodynamics: An Engineering Approach*, 4th ed. New York: McGraw-Hill, 2002.
3. C. T. Crowe, J. A. Roberson, and D. F. Elger. *Engineering Fluid Mechanics*, 7th ed. New York: Wiley, 2001.
4. R. W. Fox and A. T. McDonald. *Introduction to Fluid Mechanics*, 5th ed. New York: Wiley, 1999.
5. D. C. Giancoli. *Physics*, 3rd ed. Upper Saddle River, NJ: Prentice Hall, 1991.
6. M. C. Potter and D. C. Wiggert. *Mechanics of Fluids*, 2nd ed. Upper Saddle River, NJ: Prentice Hall, 1997.
7. Y. S. Touloukian, S. C. Saxena, and P. Hestermans. *Thermophysical Properties of Matter, The TPRC Data Series*, Vol. 11, *Viscosity*. New York: Plenum, 1975.
8. L. Trefethen. "Surface Tension in Fluid Mechanics." In *Illustrated Experiments in Fluid Mechanics*. Cambridge, MA: MIT Press, 1972.
9. *The U.S. Standard Atmosphere*. Washington, DC: U.S. Government Printing Office, 1976.
10. M. Van Dyke. *An Album of Fluid Motion*. Stanford, CA: Parabolic Press, 1982.
11. F. M. White. *Fluid Mechanics*, 5th ed. New York: McGraw-Hill, 2003.
12. C. L. Yaws, X. Lin, and L. Bu. "Calculate Viscosities for 355 Compounds. An Equation Can Be Used to Calculate Liquid Viscosity as a Function of Temperature," *Chemical Engineering*, 101, no. 4, pp. 1110–1128, April 1994.
13. C. L. Yaws. *Handbook of Viscosity*. 3 Vols. Houston, TX: Gulf Publishing, 1994.