last equation for ω and substituting, the maximum rotational speed of the container is determined to be

$$\omega = \sqrt{\frac{4g[z_s(R) - h_0]}{R^2}} = \sqrt{\frac{4(9.81 \text{ m/s}^2)[(0.6 - 0.5) \text{ m}]}{(0.1 \text{ m})^2}} = \mathbf{19.8 \text{ rad/s}}$$

Noting that one complete revolution corresponds to 2π rad, the rotational speed of the container can also be expressed in terms of revolutions per minute (rpm) as

$$\dot{n} = \frac{\omega}{2\pi} = \frac{19.8 \text{ rad/s}}{2\pi \text{ rad/rev}}\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = \mathbf{189 \text{ rpm}}$$

Therefore, the rotational speed of this container should be limited to 189 rpm to avoid any spill of liquid as a result of the centrifugal effect.

Discussion Note that the analysis is valid for any liquid since the result is independent of density or any other fluid property. We should also verify that our assumption of no dry spots is valid. The liquid height at the center is

$$z_s(0) = h_0 - \frac{\omega^2 R^2}{4g} = 0.4 \text{ m}$$

Since $z_s(0)$ is positive, our assumption is validated.

SUMMARY

The normal force exerted by a fluid per unit area is called *pressure,* and its unit is the *pascal,* 1 Pa ≡ 1 N/m^2. The pressure relative to absolute vacuum is called the *absolute pressure,* and the difference between the absolute pressure and the local atmospheric pressure is called the *gage pressure.* Pressures below atmospheric pressure are called *vacuum pressures.* The absolute, gage, and vacuum pressures are related by

$$P_{\text{gage}} = P_{\text{abs}} - P_{\text{atm}}$$

$$P_{\text{vac}} = P_{\text{atm}} - P_{\text{abs}}$$

The pressure at a point in a fluid has the same magnitude in all directions. The variation of pressure with elevation in a fluid at rest is given by

$$\frac{dP}{dz} = -\rho g$$

where the positive z-direction is taken to be upward. When the density of the fluid is constant, the pressure difference across a fluid layer of thickness Δz is

$$\Delta P = P_2 - P_1 = \rho g \, \Delta z$$

The absolute and gage pressures in a static liquid open to the atmosphere at a depth h from the free surface are

$$P = P_{\text{atm}} + \rho g h \qquad \text{and} \qquad P_{\text{gage}} = \rho g h$$

The pressure in a fluid at rest remains constant in the horizontal direction. *Pascal's law* states that the pressure applied to a confined fluid increases the pressure throughout by the same amount. The atmospheric pressure is measured by a *barometer* and is given by

$$P_{\text{atm}} = \rho g h$$

where h is the height of the liquid column.

Fluid statics deals with problems associated with fluids at rest, and it is called *hydrostatics* when the fluid is a liquid. The magnitude of the resultant force acting on a plane surface of a completely submerged plate in a homogeneous fluid is equal to the product of the pressure P_C at the centroid of the surface and the area A of the surface and is expressed as

$$F_R = (P_0 + \rho g h_C)A = P_C A = P_{\text{ave}} A$$

where $h_C = y_C \sin\theta$ is the *vertical distance* of the centroid from the free surface of the liquid. The pressure P_0 is usually the atmospheric pressure, which cancels out in most cases since it acts on both sides of the plate. The point of intersection of the line of action of the resultant force and the surface is the *center of pressure.* The vertical location of the line of action of the resultant force is given by

$$y_P = y_C + \frac{I_{xx,\,C}}{[y_C + P_0/(\rho g \sin\theta)]A}$$

where $I_{xx,\,C}$ is the second moment of area about the x-axis passing through the centroid of the area.

A fluid exerts an upward force on a body immersed in it. This force is called the *buoyant force* and is expressed as

$$F_B = \rho_f g V$$

where V is the volume of the body. This is known as *Archimedes' principle* and is expressed as: the buoyant force acting on a body immersed in a fluid is equal to the weight of the fluid displaced by the body; it acts upward through the centroid of the displaced volume. With constant density, the buoyant force is independent of the distance of the body from the free surface. For *floating* bodies, the submerged volume fraction of the body is equal to the ratio of the average density of the body to the density of the fluid.

The general *equation of motion* for a fluid that acts as a rigid body is

$$\vec{\nabla} P + \rho g \vec{k} = -\rho \vec{a}$$

When gravity is aligned in the $-z$-direction, it is expressed in scalar form as

$$\frac{\partial P}{\partial x} = -\rho a_x, \qquad \frac{\partial P}{\partial y} = -\rho a_y, \qquad \text{and} \qquad \frac{\partial P}{\partial z} = -\rho(g + a_z)$$

where a_x, a_y, and a_z are accelerations in the x-, y-, and z-directions, respectively. During *linearly accelerating motion* in the xz-plane, the pressure distribution is expressed as

$$P = P_0 - \rho a_x x - \rho(g + a_z)z$$

The surfaces of constant pressure (including the free surface) in a liquid with constant acceleration in linear motion are parallel surfaces whose slope in a xz-plane is

$$\text{Slope} = \frac{dz_{\text{isobar}}}{dx} = -\frac{a_x}{g + a_z} = -\tan\theta$$

During rigid-body motion of a liquid in a *rotating cylinder*, the surfaces of constant pressure are *paraboloids of revolution*. The equation for the free surface is

$$z_s = h_0 - \frac{\omega^2}{4g}(R^2 - 2r^2)$$

where z_s is the distance of the free surface from the bottom of the container at radius r and h_0 is the original height of the fluid in the container with no rotation. The variation of pressure in the liquid is expressed as

$$P = P_0 + \frac{\rho\omega^2}{2} r^2 - \rho g z$$

where P_0 is the pressure at the origin ($r = 0$, $z = 0$).

Pressure is a fundamental property, and it is hard to imagine a significant fluid flow problem that does not involve pressure. Therefore, you will see this property in all chapters in the rest of this book. The consideration of hydrostatic forces acting on plane or curved surfaces, however, is mostly limited to this chapter.

REFERENCES AND SUGGESTED READING

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5. M. C. Potter and D. C. Wiggert. *Mechanics of Fluids,* 2nd ed. Upper Saddle River, NJ: Prentice Hall, 1997.
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PROBLEMS*

Pressure, Manometer, and Barometer

3–1C What is the difference between gage pressure and absolute pressure?

3–2C Explain why some people experience nose bleeding and some others experience shortness of breath at high elevations.

3–3C Someone claims that the absolute pressure in a liquid of constant density doubles when the depth is doubled. Do you agree? Explain.

* Problems designated by a "C" are concept questions, and students are encouraged to answer them all. Problems designated by an "E" are in English units, and the SI users can ignore them. Problems with the icon are solved using EES, and complete solutions together with parametric studies are included on the enclosed DVD. Problems with the icon are comprehensive in nature and are intended to be solved with a computer, preferably using the EES software that accompanies this text.