Appendix E

ALGEBRA

Engineering is a quantitative discipline that uses the language of mathematics to make predictions. Imagine the difficulty of designing a technology as complex as the space shuttle without using mathematics to quantify its mass, aerodynamic shape, and trajectory. Qualitative descriptions such as "it's heavy, pointy, and goes in a loop around the earth" are not very useful in the engineering world.

When engineers quantify relationships, they generally use algebra; thus, we begin our study of applied mathematics with a review of high school algebra.

E.1 Operators/Antioperators

An *operator* may be thought of as a mathematical rule that uniquely links numbers to other numbers. The following example uses the addition operator to link any x with a unique y:

$$y = 3 + x$$

x		y
1	$\Rightarrow$	4
2	$\Rightarrow$	5
3	$\Rightarrow$	6
4	$\Rightarrow$	7
etc.		etc.

The *antioperator* is the inverse of the operator. In this example, the antioperator would have to accomplish the following link:

x		y
1	$\Leftarrow$	4
2	$\Leftarrow$	5
3	$\Leftarrow$	6
4	$\Leftarrow$	7
etc.		etc.

The following equation would perform the antioperation:

$$y-3=x$$

Thus, we see that subtraction is the antioperation of addition. (We could also say that addition is the antioperation of subtraction.) By a similar argument, we can conclude that division is the antioperation of multiplication.

Operator	Antioperator
+	-
$\times$	$\div$

The multiplication operator is indicated four different ways; the following four equations are equivalent:

$$y=2\times x$$

$$y=2\cdot x$$

$$y=2x$$

$$y=2(x)$$

The division operator also is indicated three different ways; the following three equations are equivalent:

$$y=3\div x$$

$$y=3/x$$

$$y=\frac{3}{x}$$

E.2 Algebra

Algebra provides methods to solve practical problems by using symbols (usually letters) for unknown quantities. For example, the following problem can be solved by inspection:

$$2x+3=11 \tag{E-1}$$

$$x=4 \text{ (by inspection)}$$

However, many complex problems cannot be solved by inspection, so the rules in Table E.1 are often employed.

Table E.1 Algebra Rules		
Rule	Addition	Multiplication
Commutative	$a+b=b+a$	$ab=ba$
Associative	$(a+b)+c=a+(b+c)=a+b+c$	$(ab)c=a(bc)=abc$
Cancellation	$a+c=b+c \Rightarrow a=b$	$ac=bc$ and $c \neq 0 \Rightarrow a=b$
Distributive	$a(b+c)=ab+ac$	
Identity	$a+0=a$	$a \cdot 1=a$ also $a \cdot (1/a)=1$
Zero	$a+(-a)=a-a=0$	$a \cdot 0=0$

Another important rule is that division by zero is not permitted.

Rather than solve Equation E.1 by inspection, it may be systematically solved by using the rules in Table E.1.

$$2x+3=11$$

$$\Downarrow \text{Think}$$

$$2x+3+(-3)=11+(-3)$$

$$\Downarrow \text{Zero rule}$$

$$2x=8$$

$$\Downarrow \text{Think}$$

$$(\tfrac{1}{2})2x=(\tfrac{1}{2})8$$

$$\Downarrow \text{Identity rule}$$

$$x=4$$

The general procedure can be described as "think of an operation to perform to each side of the equation to simplify it through the various algebraic rules." The important requirement is that whatever is done to one side of the equation *must* be done to the other side. This can be generalized to other operations such as logarithms, exponents, and trigonometric functions that will be discussed later.

E.3 Simultaneous Equations

Frequently in engineering, it is necessary to solve simultaneous algebraic equations. Consider the following two equations:

$$23=4x+5y$$

$$36=6x+8y$$

We are seeking values for x and y that satisfy both equations simultaneously. The first equation may be solved explicitly for x, allowing it to be substituted into the second equation

$$x = \frac{23 - 5\,y}{4}$$

$$36 = 6\left(\frac{23 - 5\,y}{4}\right) + 8\,y \Rightarrow y = 3$$

$$x = \frac{23 - 5(3)}{4} = 2$$

Thus, the solution is $x = 2$ and $y = 3$.

An alternative approach is shown below in which one of the equations is multiplied by a factor that causes one term to be eliminated when the two equations are added.

$$\begin{array}{rclcrcl} -1.5\,[23] & = & -1.5\,[4\,x + 5\,y] & \Rightarrow & -34.5 & = & -6\,x - 7.5\,y \\ 36 & = & 6\,x + 8\,y & \Rightarrow & \underline{36.0} & \underline{=} & \underline{6\,x + 8.0\,y} \\ & & & & 1.5 & = & 0.5\,y \quad \Rightarrow \quad y = 3 \end{array}$$

$$36 = 6\,x + 8\,(3) \quad \Rightarrow \quad x = 2$$

Again, the solution is $x = 2$ and $y = 3$.

In the above example, there were two unknowns which required two equations to determine a solution. If there were three unknowns, then three equations would be required. This can be generalized by saying, "for n unknowns, n equations are required." An important caveat is that the n equations must be *independent*, meaning one equation is not simply a multiple of the other. For example, if there are two unknowns x and y and the following two equations:

$$2 = 4\,x + 5\,y$$
$$4 = 8\,x + 10\,y$$

we cannot solve for x and y because the second equation is simply twice the first equation; no new information is provided.

E.4 Summary

An operator is a rule that links a first number to a second number. An antioperator reverses the operator and links the second number to the first.

Algebraic rules can be used to solve single equations or simultaneous equations. The governing principle is that whatever is done to one side of the equation must also be done to the other side. When solving simultaneous equations, it is necessary for the equations to be independent.

Futher Readings

A. R. Eide, R. D. Jenison, L. H. Mashaw, and L. L. Northup, *Engineering Fundamentals and Problem Solving*, 3rd ed., McGraw-Hill, New York, 1997.

Problems

E.1 Solve for x in the following equations

a. $3.4x + 7.4 = 6.7$
b. $-4 + 5x = 8$
c. $3x + 5 - 2x - 8 = 0$
d. $689(x - 32) = 42$
e. $49x - 78 + x(87 + 43) = 89$
f. $876 = 67x + (98 + 82)x + 17$
g. $56x + 782 = 43x - 17$
h. $94(x + 15) = 67(x - 43)$

E.2 Solve for x and y in the following simultaneous equations

a. $7.8x + 3.7y = 87$
   $9.8x + 5.6y = 13$
b. $5x + 7y = 19$
   $3x + 5y = 10$
c. $8x = 5y - 17$
   $9y = 2x + 5$
d. $22 = 5x + 2y$
   $12 = 2x + 10y$