Appendix F

MATHEMATICAL NOTATION

If you decided to study Chinese, Russian, Greek, or Arabic, you would have to learn a new alphabet. Because mathematics is a language, you must also learn its "alphabet." Many mathematical symbols are foreign to our everyday lives, so mathematics seems mysterious and foreboding. The purpose of this chapter to familiarize you with many of the mathematical symbols you will encounter in your studies. By becoming comfortable with the symbols, you will be able to focus your attention on the mathematical ideas rather than their notation.

F.1 Arrays

Arrays are ordered lists of numbers. For example, the following sequence of numbers is considered a *one-dimensional array* and has four *elements* within it:

	x_1	x_2	x_3	x_4
$x_i =$	893	730	580	294

The name of this array is x. An index i uniquely identifies each element within the array. In this case, the index has values of 1, 2, 3, and 4.

Arrays can be multidimensional. For example, the following is a *two-dimensional array* with eight elements:

	x_{11}	x_{12}	x_{13}	x_{14}
$x_{ij} =$	893	730	580	294
	649	396	843	693
	x_{21}	x_{22}	x_{23}	x_{24}

In this case, two indices are required. The index i indicates the row and the index j indicates the column.

F.2 Greek Letters

In algebra, letters are used to represent unknown numbers in equations. In engineering and science, the letters in the Latin alphabet (the one we use in English) are insufficient to represent the many quantities encountered. Therefore, the letters we use have been expanded to include the Greek alphabet as well (see Table F.1).

Table F.1 Greek letters

Name	Greek letter	Corresponding Latin letter(s)		Name	Greek letter	Corresponding Latin letter(s)
Alpha	Α α	A a		Nu	Ν ν	N n
Beta	Β β	B b		Xi	Ξ ξ	X x
Gamma	Γ γ	G g		Omicron	Ο ο	O o
Delta	Δ δ	D d		Pi	Π π	P p
Epsilon	Ε ε	E e		Rho	Ρ ρ	R r
Zeta	Ζ ζ	Z z		Sigma	Σ σ	S s
Eta	Η η	E e		Tau	Τ τ	T t
Theta	Θ θ	Th th		Upsilon	Υ υ	U u
Iota	Ι ι	I i		Phi	Φ ϕ	Ph ph
Kappa	Κ κ	K k		Chi	Χ χ	Ch ch
Lambda	Λ λ	L l		Psi	Ψ ψ	Ps ps
Mu	Μ μ	M m		Omega	Ω ω	O o

The capital delta (Δ) is never used in formulas because it is reserved to mean "a change in." For example, Δx means

$$\Delta x = x_{final} - x_{initial} \tag{F-1}$$

The lower case pi (π) is generally reserved to mean the ratio of a circle circumference to the circle diameter.

$$\pi = \frac{\text{circumference}}{\text{diameter}} = 3.14159265358979323846264338327950 2884\cdots \tag{F-2}$$

Pi is an *irrational number* (i.e., it is not the ratio of two integers), so it requires an infinite number of digits to represent it exactly.

The capital sigma (Σ) is generally reserved to mean the summation operator. A properly written summation operator must include an index i, an initial value m (usually 0 or 1), and a final value n.

$$\sum_{i=m}^{n} x_i = x_m + x_{m+1} + \cdots + x_n \tag{F-3}$$

A Brief History of Pi

The ratio of a circle circumference to its diameter is a constant regardless of the size of the circle. In the eighteenth century A.D., this ratio was given the symbol π. Pi has been estimated by a variety of methods described eloquently by the engineer Petr Beckmann in his book *A History of Pi*. A brief summary of π's history is provided below. Much of the history involves increasing the number of known decimal places. Although this may seem to be a fruitless enterprise, new computers are often tested by having them calculate π to many significant figures. Also, the sequence of digits is random and can be used in random number generators.

Date	Source	Value	Correct decimal places
ca. 2000 B.C.	Babylon	$3\frac{1}{8}$	1
ca. 2000 B.C.	Egypt	$4\left(\frac{8}{9}\right)^2$	1
ca. 550 B.C.	Bible (Book of Kings)	3	0
ca. 200 B.C.	Greece's Archimedes	$3\frac{10}{71} < \pi < 3\frac{1}{7}$	2
ca. 15 A.D.	Rome's Vitruvius in *De Architectura*	$3\frac{1}{8}$	1
130 A.D.	Chinese mathematician Hou Han Shu	3.1622	1
ca. 150 A.D.	Greece's Ptolemy	$3\frac{17}{120}$	3
264 A.D.	Chinese mathematician Liu Hui	3.14159	5
380 A.D.	Hindu Siddhantas (system of astronomy)	$3\frac{177}{1250}$	3
ca. 450 A.D.	Chinese mathematicians Tsu Chung-Chih and Tsu Keng-Chih	$3.1415926<\pi<3.1415927$	7
ca. 620 A.D.	Hindu mathematician Brahmagupta	$\sqrt{10}$	1
1220 A.D.	Italy's Fibonacci	$\frac{864}{275}$	3
ca. 1550 A.D.	Holland's Adriaan Anthoniszoon	$\frac{355}{113}$	6
1593 A.D.	France's François Viète	*a*	Arbitrary
1593 A.D.	France's François Viète		9
1593 A.D.	Holland's Adriaen van Rooman		15
1615 A.D.	Holland's Ludolph van Ceulen		35
ca. 1650 A.D.	John Wallis	*b*	Arbitrary
ca. 1650 A.D.	William Brouncker	*c*	Arbitrary
1674 A.D.	James Gregory and Gottfried Wilhelm Leibniz	*d*	Arbitrary
1706 A.D.	England's John Machin	*e*	Arbitrary
1717 A.D.	France's De Lagny		127
ca. 1719 A.D.	Abraham Sharp	*f*	Arbitrary
1722 A.D.	Japan's Takebe		41
ca. 1736 A.D.	Switzerland's Leonhard Euler	*g*, *h*, *i*	Arbitrary
1737 A.D.	Sir Isaac Newton	*j*	Arbitrary

1794 A.D.	Vega		140
1844 A.D.	Mathematician L. K. Schulz von Strassnitzky. He used *idiot savant* Johann Martin Zacharias Dase as a "human computer" to perform the calculations in 2 months.		200
1853 A.D.	Rutherford		440
1853 A.D.	William Shanks (707 decimal places were reported, but 180 were incorrect)		527
1896 A.D.	Störmer	*k*	Arbitrary
1897 A.D.	Indiana General Assembly Bill No. 246	4	0
1947 A.D.	Ferguson		808
1949 A.D.	ENIAC computer (70 hours, Eq. *e*)		2,037
1955 A.D.	Naval Ordnance Research Calculator (13 min)		3,089
1957 A.D.	Pegasus computer (33 hours)		7,480
1958 A.D.	IBM 704 (100 min, Eq. *l*)		10,000
1961 A.D.	IBM 7090 (39 min, Eq. *e*)		20,000
1961 A.D.	IBM 7090 (8 hours 43 min, Eq. *k*)		100,265
1967 A.D.	CDC 6600 (28 hours 10 min, Eq. *k*)		500,000
1989 A.D.	Hitac S-820/80E computer		1,073,740,000
1991 A.D.	Brothers Gregory Volfovich and David Volfovich Chudnovsky using their home-made supercomputer		2,260,321,336

$$a.\ \pi = \frac{2}{\sqrt{\frac{1}{2}}\ \sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1}{2}}}\ \sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1}{2}}}} \times \cdots}$$

$$b.\ \pi = 2\frac{2\cdot 2\cdot 4\cdot 4\cdot 6\cdot 6\ \cdots}{1\cdot 3\cdot 3\cdot 5\cdot 5\cdot 7\cdot 7\ \cdots}$$

$$c.\ \frac{\pi}{4} = 1+\frac{1^2}{2+\frac{3^2}{2+\frac{5^2}{2+\frac{7^2}{2+\cdots}}}}$$

$$d.\ \pi = 4\left(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\ \cdots\right)$$

$$e.\ \pi = 16\arctan\left(\frac{1}{5}\right)-4\arctan\left(\frac{1}{239}\right)$$

$$f.\ \pi = \frac{6}{\sqrt{3}}\left(1-\frac{1}{3\cdot 3}+\frac{1}{3^2\cdot 5}-\frac{1}{3^3\cdot 7}+\ \cdots\right)$$

$$g.\ \pi = \sqrt{6\left(\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots\right)}$$

$$h.\ \pi = \sqrt{8\left(\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots\right)}$$

i. $\pi = \sqrt{6\left(\frac{2^2}{2^2-1} \times \frac{3^2}{3^2-1} \times \frac{5^2}{5^2-1} \times \frac{7^2}{7^2-1} \times \cdots\right)}$	
j. $\pi = 6\left(\frac{1}{2} + \frac{1}{2 \cdot 3 \cdot 2^3} + \frac{1 \cdot 3}{2 \cdot 4 \cdot 5 \cdot 2^5} + \cdots\right)$	
k. $\pi = 24 \arctan\left(\frac{1}{8}\right) + 8 \arctan\left(\frac{1}{57}\right) + 4 \arctan\left(\frac{1}{239}\right)$	
l. $\pi = 16\left(\frac{1}{5} - \frac{1}{3 \cdot 5^3} + \frac{1}{5 \cdot 5^5} - \cdots\right) - 4\left(\frac{1}{239} - \frac{1}{3 \cdot 239^3} + \frac{1}{5 \cdot 239^5} - \cdots\right)$	

(*Note*: Sometimes, the summation operator is written with the short-hand notation $\sum x_i$ in which the index, initial value, and final value are not indicated.) For example, adding the first four values in the previously described array x_i is accomplished as follows:

$$\sum_{i=1}^{4} x_i = x_1 + x_2 + x_3 + x_4 = 893 + 730 + 580 + 294 = 2497 \qquad \text{(F-4)}$$

Sometimes, the summation operator is written so the index i is used as a variable

$$\sum_{i=1}^{4} \frac{1}{i} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = 2.08333 \qquad \text{(F-5)}$$

The capital pi (Π) is generally reserved to mean the multiplication operator. The product of the first four values of the previously described array x_i is accomplished as follows:

$$\prod_{i=1}^{4} x_i = x_1 \cdot x_2 \cdot x_3 \cdot x_4 = 893 \cdot 730 \cdot 580 \cdot 294 = 111,160,282,800 \qquad \text{(F-6)}$$

Sometimes, the multiplication operator is written so the index i is used as a variable

$$\prod_{i=1}^{4} i = 1 \cdot 2 \cdot 3 \cdot 4 = 24 \qquad \text{(F-7)}$$

F.3 Relational Notation

Mathematics is often used to describe how two quantities are related. In English, we might say "A is the same as B," "A is greater than B," "A is not the same as B," etc. In mathematics, very specific symbols are used to convey these ideas as shown in Table F.2.

The *ratio* symbol (:) is an alternative way of saying "divided by." The following two statements are identical:

$$A{:}B \qquad \frac{A}{B} \qquad \text{(F-8)}$$

Imagine we have two rectangles that are *similar* to each other; that is, they have the same shape but differ only in size (see Figure F.1).

Table F.2 Relational notation	
Mathematical notation	Meaning
=	equals
≠	not equal
<	less than
≤	less than or equal to
>	greater than
≥	greater than or equal to
<<	much less than
>>	much greater than
≡	is defined as
~	similar to
≅ or ≈	approximately equals
≍	equivalent to
⊥	perpendicular to
||	parallel to
:	ratio
∝	proportional to

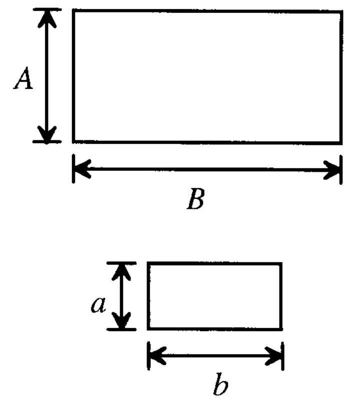


Figure F.1 Two similar rectangles.

We can then say "*A* is to *B* as *a* is to *b*." Mathematically, this can be stated in the following two ways:

$$A : B = a : b \qquad \frac{A}{B} = \frac{a}{b} \tag{F-9}$$

The *proportion* symbol (∝) says that "*y* varies in the same direction as *x*":

$$y \propto x \tag{F-10}$$

This same idea may be stated as

$$y = kx \tag{F-11}$$

where *k* is the proportionality constant. Because *y* increases as *x* increases, *y* is *directly proportional* to *x*. For example, the circumference *C* of a circle is directly proportional to its radius *r*

$$C \propto r \tag{F-12}$$

This can be expressed as

$$C = kr = (2\pi)r \tag{F-13}$$

where the proportionality constant *k* has the value 2π.

Equations F-12 and F-13 show that the circumference is proportional to r raised to the first power (i.e., we would say they are in *direct linear proportion*). Other proportions are possible. For example, the area of a circle is in direct proportion to the square of the radius:

$$A \propto r^2 \tag{F-14}$$

This can be expressed as

$$A = kr^2 = \pi r^2 \tag{F-15}$$

where π is the proportionality constant.

If y increased as x decreased, then we would say that y is *inversely proportional* to x. Mathematically, this may be stated as follows:

$$y \propto \frac{1}{x} \tag{F-16}$$

$$y = \frac{k}{x} \tag{F-17}$$

Other inverse proportions are possible. For example, y could be in inverse proportion to the square of x:

$$y \propto \frac{1}{x^2} \tag{F-18}$$

$$y = \frac{k}{x^2} \tag{F-19}$$

Sometimes we do not know the required mathematical relationships to make the left and right side of an equation equal, but we do know what variables are involved. For example, we know that the area A of a triangle is related to the base length b and height h. However, we may not know the exact relationship without looking up the formula or deriving it. Nonetheless, we can write our understanding of the variables involved as follows:

$$A = f(b, h) \tag{F-20}$$

which says "the area is a function of the base length and height." An alternative notation that is sometimes used is

$$A = A(b, h) \tag{F-21}$$

By drawing a picture (see Figure F.2) and noting that the triangle occupies half the area of a square that circumscribes it, we see that the area is obviously half the base length times the height:

$$A = \tfrac{1}{2}bh \tag{F-22}$$

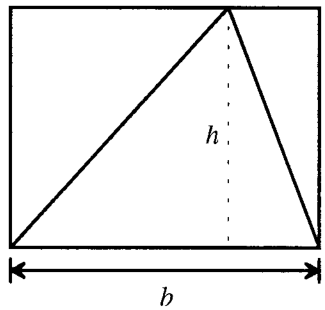
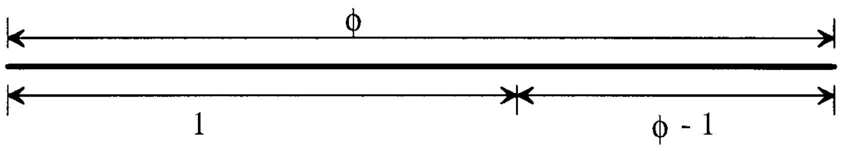


Figure F.2 The area of a triangle is half the area of the square in which it is inscribed.

The Divine Proportion

The *divine proportion* (also known as the *golden section* or *golden ratio*) is an ancient concept that has influenced art, architecture, and music (see H. E. Huntley, *The Divine Proportion*). Many great paintings, the Greek Parthenon, and the violin are all designed according to the divine proportion. Also, the divine proportion is seen in the natural world of crystals, shells, and plants.

The divine proportion is found by dividing a line of total length ϕ into a larger segment of unit length and a smaller segment of length (ϕ - 1). The divine proportion is achieved when the ratio of the larger segment to the total length is the same as the ratio of the smaller segment to the larger segment.



Mathematically, this is stated

$$\frac{1}{\phi} = \frac{\phi - 1}{1}$$

This equation can be manipulated to give

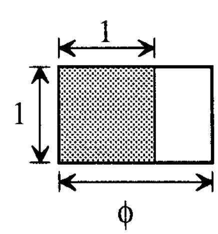
$$\phi^2 - \phi - 1 = 0$$

which is a quadratic equation with the solution

$$\phi = \frac{-(-1) + \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$$

$$\phi = 1.6180339887 \cdots$$

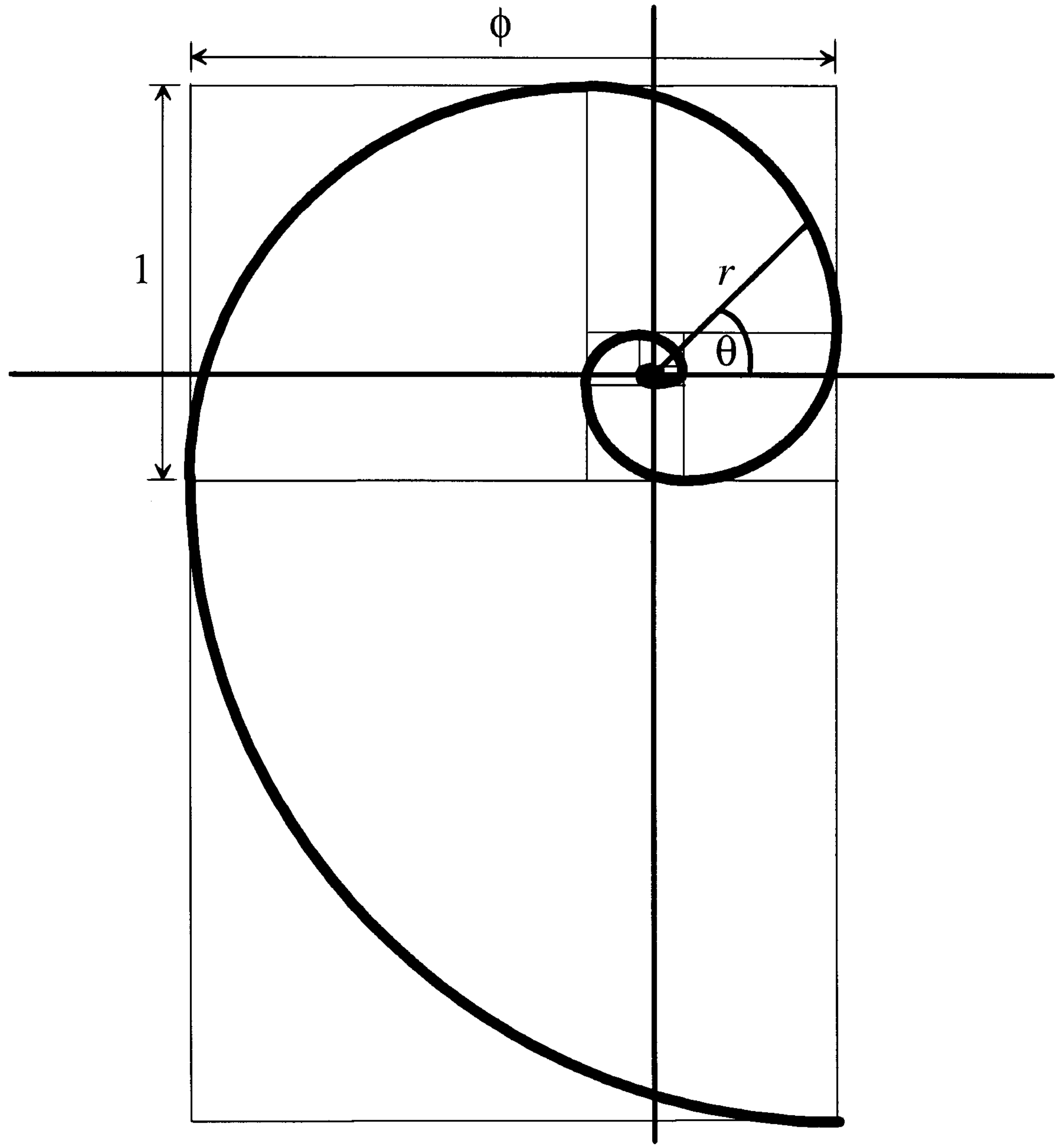
A *golden rectangle* has a length ϕ relative to a unit width. The proportions are such that if a square is removed from it, the remaining rectangle is also a golden rectangle.



From a given smaller golden rectangle of *length* ϕ, it is possible to construct a new larger golden rectangle of *width* ϕ. In the figure below, this process is repeated indefinitely. The multiple golden rectangles circumscribe a *golden spiral* (a.k.a. logarithmic spiral, *spira mirabilis*) defined by the equation

$$r = e^{a\theta}$$

where r is the distance from the origin, θ is the angle, a is a constant, and e is the constant 2.718...



F.4 Miscellaneous Notation

Some common mathematical symbols are shown in Table F.3.

Table F.3 Miscellaneous notation	
Symbol	Meaning
$\pm$	plus or minus
∞	infinity
$\angle$	angle
$\therefore$	therefore
%	percent
$\lvert x \rvert$	absolute value
!	factorial

The percent symbol (%) means "per one hundred." For example, 75% means 75 per 100 or

$$75\% \equiv \frac{75}{100} = 0.75 \qquad \text{(F-23)}$$

There can be no dimensions or units associated with a percentage. For example, if 75 has units of "cm," then 100 has the same units of "cm." Therefore, 75% and 0.75 are both dimensionless numbers.

The absolute value of x, i.e., $|x|$, means "multiply x by negative one if x is negative and leave it alone if it is already positive." More explicitly

$$y = |x| \qquad \text{(F-24)}$$

means

$$y = x \qquad (\text{if } x > 0) \qquad \text{(F-25)}$$

$$y = -x \qquad (\text{if } x < 0) \qquad \text{(F-26)}$$

The factorial sign (!) means

$$n! = \prod_{i=1}^{n} i \qquad \text{(F-27)}$$

where n must be a positive integer. Zero factorial is defined as 1:

$$0! = 1 \qquad \text{(F-28)}$$

The factorial symbol is very appropriate because the numbers get large quickly. For example, 10! is

$$10! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 \cdot 10 = 3,628,800$$

Large factorials are difficult to calculate, so Stirling's approximation is sometimes used:

$$n! \approx \sqrt{2\pi n}\, n^n e^{-n} \tag{F-29}$$

where $e = 2.718 \cdots$.

F.5 Summary

Like many other languages, mathematics has unique notation. In order for you to become facile with mathematics, you must understand its notation.

Arrays are ordered lists of numbers; each element of the list is identified by a subscripted index. Arrays can have one dimension (e.g., x_i), two dimensions (e.g., x_{ij}), or more.

In science, engineering, and mathematics, each quantity in an equation is usually identified by a letter. Unfortunately, the Latin alphabet has only 52 upper- and lowercase letters. (Actually, we effectively have even less. The letter l is avoided because it could be confused with the number 1 and the letter O is avoided because it could be confused with the number 0.) Often, we use Greek letters to increase the number of letters at our disposal. In high school, most students have had little exposure to Greek letters, so you should carefully study the Greek alphabet shown in Table F.1. (This will also help if you decide to pledge a fraternity or sorority.)

The concept of proportion is a recurring theme in science and engineering. Two quantities are directly proportional if one increases as the other increases. Two quantities are inversely proportional if one decreases as the other increases.

Further Readings

Handbook of Mathematical Formulas, Tables, Graphs, Functions, Transforms, Research and Education Association, 1980.

Problems

F.1 Calculate the following:

a. $\sum_{i=0}^{6} 2i - 1$

b. $\sum_{i=1}^{5} (-1)^i$

c. $\sum_{i=1}^{6} (2i)^i$

d. $\prod_{i=0}^{6} (i+1)$

e. $\prod_{i=1}^{3} \frac{1}{i}$

f. $\prod_{i=1}^{4} (-1)^i$

F.2 Calculate 15! according to the definition (Equation F-27) and Stirling's approximation (Equation F-29).

F.3 When a voltage V is applied to an electric resistor with resistance R, the electric current i that flows through the resistor is

$$i = \frac{1}{R} V$$

a. In the above equation, what is the proportionality constant between i and V?

b. At constant R, are V and i directly proportional or inversely proportional?
c. At constant i, are V and R directly proportional or inversely proportional?
d. At constant V, are i and R directly proportional or inversely proportional?
e. At constant resistance, if the voltage is doubled, what happens to the current?
f. At constant current, if the resistance is doubled, what happens to the voltage?
g. At constant voltage, if the resistance is doubled, what happens to the current?
h. The electric power P dissipated by a resistor is $P = i\ V$. Describe the proportionality relationship between power and voltage. What is the proportionality constant between power and voltage?
i. Describe the proportionality relationship between power and current. What is the proportionality constant between power and current?
j. At constant resistance, if the voltage is doubled, what happens to the power dissipation?
k. At constant resistance, if the current is doubled, what happens to the power dissipation?

F.4 When a temperature difference ΔT is applied across a material of thickness x and cross-sectional area A, the heat flow Q can be calculated by the equation

$$Q = kA\frac{\Delta T}{x}$$

where the thermal conductivity k is a property of the material.

a. Is the k for Styrofoam larger than, or smaller than, the k for copper?
b. In the above equation, what is the proportionality constant between Q, A, x, and ΔT?
c. At constant k, A, and x, are Q and ΔT directly or inversely proportional?
d. At constant k, A, and ΔT, are Q and x directly or inversely proportional?
e. At constant k, x, and ΔT, are Q and A directly or inversely proportional?
f. At constant k, A, and x, what happens to Q when ΔT is doubled?
g. At constant k, A, and ΔT, what happens to Q when x is doubled?
h. At constant k, x, and ΔT, what happens to Q when A is doubled?

F.5 In your chemistry class, you have undoubtedly encountered the perfect gas equation (also called the ideal gas equation)

$$PV = nRT$$

For a perfect (ideal) gas, it relates the pressure P, volume V, temperature T, and moles n using the proportionality constant R, which is termed the *universal gas constant.*

a. At constant V and n, are P and T directly proportional or inversely proportional?
b. At constant V and n, are T and P directly proportional or inversely proportional?
c. At constant P and n, are V and T directly proportional or inversely proportional?
d. At constant T and n, are P and V directly proportional or inversely proportional?
e. At constant V and P, are n and T directly proportional or inversely proportional?

F.6 The surface speed s of a rotating drill bit is

$$s = \pi D n$$

where D is the drill bit diameter and n is the revolution rate. If the surface speed exceeds a critical value, the drill bit temperature gets too hot and damages the material being drilled. The maximum allowable revolution rate for a ¼-inch drill bit making a hole in a new polymer is 2000 rpm (revolutions per minute). For a ½-inch drill bit, what is the maximum allowable revolution rate?

F.7 An automobile speedometer does not measure the vehicle speed directly; instead, it measures the rotation rate of the axle. The scale on the speedometer is valid only for a given tire diameter. The speedometer in Fred's new car is calibrated assuming the tire diameter is 24.0 inches. Unfortunately, Fred has not read the owner's manual and he has underfilled his tires, making them all slightly flat. The actual tire diameter is 23.6 inches. Will Fred's speedometer reading be higher than, or lower than, his actual speed? Calculate the fractional error and percentage error between the indicated speed and the actual speed.

F.8 The kinetic energy E of a moving object is

$$E = \frac{1}{2}mv^2$$

where m is the object mass and v is its velocity.

a. At constant m, what is the proportionality constant between E and v?
b. At constant E, what is the proportionality constant between m and v?
c. At constant v, what is the proportionality constant between E and m?
d. At constant m, what happens to E when v is doubled?
e. At constant E, what happens to m when v is doubled?
f. At constant v, what happens to E when m is doubled?
g. The Army is developing a new "kinetic energy" projectile designed to pierce the armor of enemy tanks. What would be more effective, doubling the mass or doubling the velocity?

F.9 The force F required to compress a spring is

$$F = kx$$

where x is the amount of compression and k is the spring constant, a property of the spring. The amount of energy E stored in a compressed spring is

$$E = \frac{1}{2}kx^2$$

a. Which has a greater spring constant, an automobile suspension spring or a ball point pen spring?
b. For a given spring, are the force and compression directly or inversely proportional?
c. For a given spring, are the energy and compression directly or inversely proportional?
d. For a given spring, if the compression is doubled, what happens to the require force?
e. For a given spring, if the compression is doubled, what happens to the stored energy?

F.10 An automobile company wishes to build a new circular test track to test its automobiles. If the length of the track doubles, what happens to the cost of the land on which the track is built?

F.11 Figure F.3 shows a large metal block being dragged across a floor. The force F required to drag the block is $F = \mu mg$ where m is the mass of the metal block, g is the acceleration due to gravity, and μ is the coefficient of friction, a property of both the metal block and floor. The mass of the metal block is $m = \rho V$ where V is the volume of the block and ρ is the density of the metal. The volume of the block is $V = hA$ where A is the surface area of the block contacting the floor and h is the height of the block. In the following questions, assume the choice of metal and floor material are held constant.

a. If the floor and block have a roughened surface, would you expect μ to be larger than, or smaller than, a floor and block with smooth surfaces?
b. The mass of the block is doubled. What happens to the required force?
c. The block and floor are brought to the moon, where the acceleration due to gravity is one-sixth that of earth. What happens to the required force?
d. The volume of the block is doubled. What happens to the required force?
e. While holding the mass of the block constant, the surface area A is doubled. What happens to the required force?
f. While holding the surface area A constant, the height of the block is doubled. What happens to the required force?

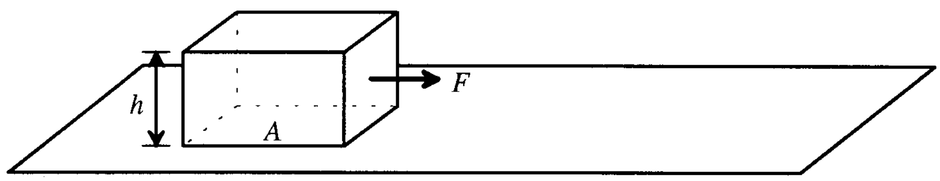


Figure F.3 A large metal block being dragged across the floor.

F.12 Write a computer program that calculates $x!$. It should report values calculated by the definition (Equation F-27) and Stirling's approximation (Equation F-29). The program must ensure that only positive integers are accepted. For example, if the user inputs -3.2 for x, an error message must be generated. Be sure that $0!$ is defined. Once your program is written, determine the largest value of x that can be calculated according to the definition, and the largest value that can be calculated by Stirling's approximation.

F.13 Write a computer program that calculates y, which is defined by

$$y = \sum_{i=1}^{10} (2i + 1)$$

F.14 Write a computer program that calculates y, which is defined by

$$y = \sum_{i=1}^{\infty} \frac{1}{i(i+1)}$$

Notice that the upper limit is infinity, which would take an infinitely long time to calculate. However, as i becomes very large, each successive term becomes much smaller. Therefore, y will reach a limiting value as i approaches infinity. The exact value of y cannot be known (this would take an infinite number of digits), but we can obtain a value that is acceptable for most practical purposes by ending the calculation when the addition of a successive term makes a negligible change in y. Report the approximate value of y

obtained when the addition of a successive term causes less than a 0.0001% (1 part in a million) change in y. Also, report the last value of i obtained when the program stopped.

F.15 Leonardo of Pisa (1180–1250) was better known by his nickname "Fibonacci" for "son of Bonaccio." He developed the sequence of numbers shown below:

$$F_0 = 1,\ F_1 = 1, F_2 = 2, F_3 = 3, F_4 = 5, F_5 = 8, F_6 = 13, F_7 = 21, F_{n-1}, F_n, F_{n+1}$$

The sequence is formed by applying the formula $F_{n+1} = F_n + F_{n-1}$. Write a computer program that calculates the first 50 terms of the Fibonacci sequence. For each term, report the ratio F_n/F_{n-1}. For large n, what does this ratio equal?

F.16 Write a computer program that calculates pi using any two desired series formulas, except those containing an arctan. For each formula, report the value of pi as each successive term is included. Determine which formula converges to the correct value more quickly.