Problems

L.1 The following relationships are given:

$$dx = 0.00000000001 \text{ or } dx = 0.0000000000000001$$

$$dy = 3\,dx$$

$$dz = 7\,dx$$

Determine if the following equations are "valid," that is, do we obtain identical results regardless of whether *dx* is at the 10-zero scale or 15-zero scale?

a. $c = \frac{dy}{dx} + \frac{dz}{dx}$

b. $c = \frac{dy}{dz}$

c. $c = \frac{dy}{dx} + \frac{dx}{dz}$

d. $c = \frac{dy}{dx} - \left(\frac{dz}{dx}\right)^2$

e. $c\,dx = \frac{dy}{dz} + \frac{dz}{dy}$

f. $c = dx + dz$

g. $c\,dz = dy + dx$

h. $c = \sqrt{\frac{dz}{dx} + \sin\frac{dy}{dx}}$

i. $\frac{c}{dx} = dy + dz$

j. $c = \frac{(dy)^2}{dx} + \frac{(dz)^2}{dx}$

L.2 Take the derivative of the following functions:

a. $y = 3x^2$

b. $y = 4x^2 + 2x + 1$

c. $y = 2e^x$

d. $y = 2 \times 10^x$

e. $y = \sin x + \cos x$

f. $y = 10^x \times e^x$

L.3 Write a computer program that numerically determines the derivative of a function. This is done by using the definition of the derivative

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{y_2 - y_1}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x_2) - f(x_1)}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x_1 + \Delta x) - f(x_1)}{\Delta x}$$

For example, in Fortran, the derivative of the function

$$y = 7x^3$$

would be written as

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deriv = ((7.0*(x+delta_x)**3)-(7.0*x**3))/delta_x
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As **`delta_x`** becomes smaller, this equation better approximates the derivative.

Choose one of the functions from Problem L.2. Plot the function from $x = 1$ to $x = 10$ by hand or using a spreadsheet. Find the derivative at **`x`** = 1, 2, 3, · · · , 9, and 10. Evaluate the derivative using **`delta_x`** = 1, 0.01, 0.0001, 0.000001, and 10^{-20}. Compare this numerical derivative to the analytical derivative you determined in Problem L.2. Present your results in a table (see Table L.5) prepared by the computer program.

Table L.5 Output for Problem L.3						
x	delta_x					Analytical
	1	0.01	0.0001	0.000001	0.1E-19	
1						
2						
⇓						
9						
10						

What do you conclude about the effect of decreasing Δx?

L.4 Integrate the following functions from $x = 1$ to 2:

a. $y = \int_1^2 dx$

b. $y = \int_1^2 5x^3 dx$

c. $y = \int_1^2 (5x^3 + 2x^2)\, dx$

d. $y = \int_1^2 e^x\, dx$

e. $y = \int_1^2 \sin x\, dx$ (x is in degrees)

f. $y = \int_1^2 \cos x\, dx$ (x is in radians)

L.5 For each of the functions listed in Problem L.4, determine the average value of y in the interval from $x = 1$ to 2.

L.6 Write a computer program that integrates one of the functions listed in Problem L.4 using the trapezoidal rule. Plot the function by hand, or use a spreadsheet. Use the following values for Δx: 0.1, 0.01, 0.0001, 0.000001, and 10^{-8}. Have the computer prepare a table that looks like the following:

delta_x =	0.1	0.01	0.0001	0.000001	0.1E-07	Analytical
y =						

What do you conclude about the effect of decreasing Δx?

L.7 Repeat Problem L.6, but use Simpson's rule instead of the trapezoidal rule.

L.8 Write a computer program that calculates the arithmetic, geometric, and harmonic mean of the data in the following table. Read the data from a file.

8.5	9.2	8.1	5.8	3.5	8.2	9.5	4.6
2.5	7.4	8.4	6.5	4.2	4.8	9.1	7.3
4.4	5.4	5.5	8.2	2.8	6.4	6.2	9.1
6.4	5.5	3.8	9.3	7.2	7.9	9.6	7.3

L.9 Write a computer function subprogram that calculates one of the following functions using a power series:

a. e^x
b. a^x
c. $\ln x$
d. $\sin x$
e. $\cos x$
f. $\arcsin x$
g. $\arccos x$
h. $\arctan x$
i. $\sinh x$
j. $\cosh x$

Stop the calculation when an additional term from the power series changes the value of the function by less than 0.0001% (1 part in a million). Call your function subprogram from a main program. Compare the value returned by your function subprogram to the one returned by the corresponding intrinsic function.

L.10 Write the following computer function subprograms: factorials, Bernoulli numbers (see Table L.3), Euler numbers (see Table L.3). Use these function subprograms when you write function subprograms for the following:

a. $\tan x$
b. $\cot x$
c. $\sec x$
d. $\csc x$
e. $\tanh x$

Stop the calculation when an additional term from the power series changes the function by less than 0.0001% (1 part in a million). Call your function subprogram from a main program. Compare the value returned by your function subprogram to the one returned by the corresponding intrinsic function. [*Note*: If you use Fortran, it has no intrinsic functions for $\cot x$, $\sec x$, and $\csc x$. These may be calculated as $(\tan x)^{-1}$, $(\cos x)^{-1}$, and $(\sin x)^{-1}$, respectively.]

L.11 Find the volume of a geometric shape using the formula $V = \int A(y)\, dy$ in which $A(y) = 3(5-y)^2$. Evaluate the integral from $y = 0$ to $y = 5$.

L.12 By hand, or using a spreadsheet, plot the following functions in the square brackets. On the same figure, show the indicated limit as a horizontal line. Show that the function approaches the indicated limit.

a. $\lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)^n\right] = e$
b. $\lim_{n\to\infty}\left[\left(1+\frac{z}{n}\right)^n\right] = e^z$ (Evaluate with $z = 2$)
c. $\lim_{x\to 0}\,[x^x] = 1$
d. $\lim_{x\to 0}\left[(1+x)^{1/x}\right] = e$
e. $\lim_{x\to 0}\left[\frac{a^x-1}{x}\right] = \ln a$ (Evaluate with $a = 2$)
f. $\lim_{x\to 0}\left[\frac{\sin x}{x}\right] = 1$
g. $\lim_{n\to\infty} n[x^{1/n}-1] = \ln x$ (Evaluate with $x = 2$)

L.13 On the same figure, plot the following functions:

a. $y = 2^x$ and $\frac{dy}{dx}$
b. $y = 2.718281^x$ and $\frac{dy}{dx}$
c. $y = 3^x$ and $\frac{dy}{dx}$

L.14 Suppose you wish to average 40 mph on a trip and find that when you are half the distance to your destination, you have averaged 30 mph. How fast should you travel in the remaining half of the trip to attain an overall average of 40 mph?