Falling Objects and Projectile Motion

chapter 3

unit one

chapter overview

Our main purpose in this chapter is to explore how objects move under the influence of the gravitational acceleration near the surface of the earth. Uniform acceleration, introduced in the previous chapter, plays a prominent role. We begin by considering carefully the acceleration of a dropped object, and then we will extend these ideas to thrown objects or objects projected at an angle to the ground.

chapter outline

1 **Acceleration due to gravity.** How does a dropped object move under the influence of the earth's gravitational pull? How is its acceleration measured, and in what sense is it constant?

2 **Tracking a falling object.** How do velocity and distance traveled vary with time for a falling object? How can we quickly estimate these values knowing the gravitational acceleration?

3 **Throwing a ball upward.** What changes when a ball is thrown upward rather than being dropped? Why does the ball appear to hover near the top of its flight?

4 **Projectile motion.** What determines the motion of an object that is fired horizontally? How do the velocity and position of the object change with time in this case?

5 **Hitting a target.** What factors determine the trajectory of a rifle bullet or football that has been launched at some angle to the horizontal to hit a target?

Have you ever watched a leaf or a ball fall to the ground? At times during your first few years of life, you probably amused yourself by dropping an object repeatedly and watching it fall. As we grow older, that experience becomes so common that we usually do not stop to think about it or to ask why objects fall as they do. Yet this question has intrigued scientists and philosophers for centuries.

To understand nature, we must first carefully observe it. If we control the conditions under which we make our observations, we are doing an experiment. The observations of falling objects that you performed as a young child were a simple form of experiment, and we would like to rekindle that interest in experimentation here. Progress in science has depended on carefully controlled experiments, and your own progress in understanding nature will depend on your active testing of ideas through experiments. You may be amazed at what you discover.

Look around for some small, compact objects. A short pencil, a rubber eraser, a paper clip, or a small ball will all do nicely. Holding two objects at arm's length, release them simultaneously and watch them fall to the floor (fig. 3.1). Be careful to release them from the same height above the floor without giving either one an upward or downward push.

How would you describe the motion of these falling objects? Is their motion accelerated? Do they reach the floor at the same time? Does the motion depend on the shape and composition of the object? To explore this last question, you might take a small piece of paper and drop it at the same time as an eraser or a ball. First, drop the paper unfolded. Then, try folding it or crumpling it into a ball. What difference does this make?

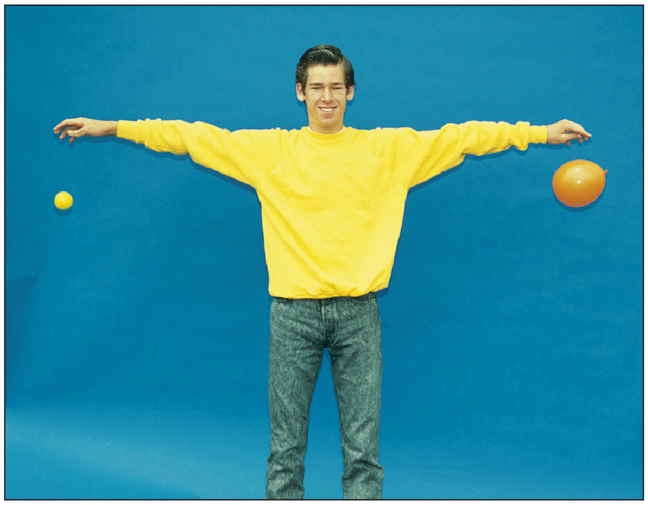


figure **3.1** An experimenter dropping objects of different mass. Do they reach the ground at the same time?

From these simple experiments, we can draw some general conclusions about the motion of falling objects. We can also try throwing or projecting objects at different angles to study the motion of a projectile. We will find that a constant downward gravitational acceleration is involved in all of these cases. This acceleration affects virtually everything that we do when we move or play on the surface of this earth.

3.1 Acceleration Due to Gravity

If you dropped a few objects as suggested in the introduction, you already know the answer to one of the questions posed there. Are the falling objects accelerated? Think for a moment about whether the velocity is changing. Before you release an object, its velocity is zero, but an instant after the object is released, the velocity has some value different from zero. There has been a change in velocity. If the velocity is changing, there is an acceleration.

Things happen so rapidly that it is difficult, just from watching the fall, to say much about the acceleration. It does appear to be large, because the velocity increases rapidly. Does the object reach a large velocity instantly, or does the acceleration occur more uniformly? To answer this question, we must slow the motion down somehow so that our eyes and brains can keep up with what is happening.

How can we measure the gravitational acceleration?

There are several ways to slow down the action. One was pioneered by the Italian scientist, Galileo Galilei (1564–1642), who was the first to accurately describe the *acceleration due to gravity.* Galileo's method was to roll or slide objects down a slightly inclined plane. This allows only a small portion of the gravitational acceleration to come into play, just that part in the direction of motion along the plane. Thus a smaller acceleration results. Other methods (not available to Galileo) use time-lapse photography, spark timers, or video recording to locate the position of the falling object at different times.

If you happen to have a grooved ruler and a small ball or marble handy, you can make an inclined plane yourself. Lift one end of the ruler slightly by placing a pencil under one end, and let the ball or marble roll down the ruler under the influence of gravity (fig. 3.2). Can you see it gradually pick up speed as it rolls? Is it clearly moving faster at the bottom of the incline than it was halfway down?

Galileo was handicapped by a lack of accurate timing devices. He often had to use his own pulse as a timer. Despite this limitation, he was able to establish that the acceleration was uniform, or constant, with time and to estimate its value using inclined planes. We are more fortunate. We have devices that allow us to study the motion

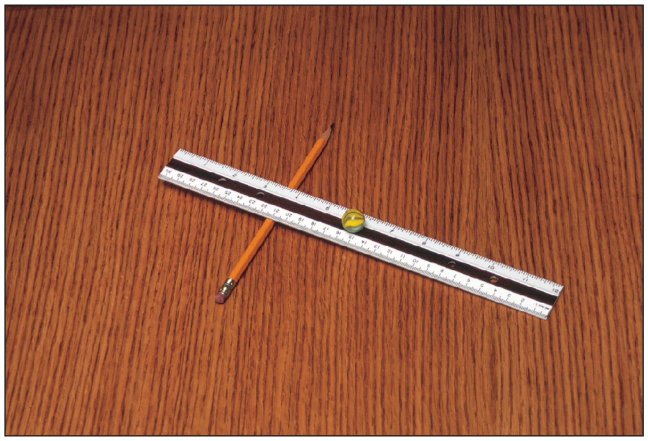


figure **3.2** A marble rolling down a ruler serving as an inclined plane. Does the velocity of the marble increase as it rolls down the incline?

of a falling object more directly. One such device is a stroboscope, a rapidly blinking light whose flashes occur at regular intervals in time. Figure 3.3 is a photograph taken using a stroboscope to illuminate an object as it falls. The position of the object is pinpointed every time the light flashes.

If you look closely at figure 3.3, you will notice that the distance covered in successive time intervals increases regularly. The time intervals between successive positions of the ball are all equal. (If the stroboscope light flashes every 1⁄20 of a second, you are seeing the position of the ball every 1⁄20 of a second.) Since the distance covered by the ball in equal time intervals is increasing, the velocity must be increasing. Figure 3.3 shows a ball whose velocity is steadily increasing in the downward direction.

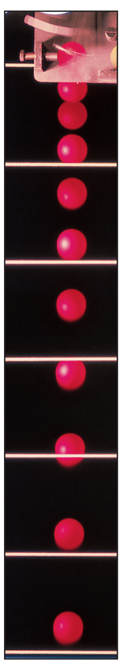


figure **3.3** A falling ball is illuminated by a rapidly blinking stroboscope. The stroboscope blinks at regular time intervals.

Computing values of the average velocity for each time interval will make this even clearer. The computation can be done if we know the time interval between flashes and can measure the position of the ball from the photograph, knowing the distance between the grid marks. Table 3.1 displays data obtained in this manner. It shows the position of a ball at intervals of 1⁄20 of a second (0.05 second).

To see that the velocity is indeed increasing, we compute the average velocity for each successive time interval. For example, between the second and third flashes, the ball traveled a distance of 3.6 centimeters, which is found by subtracting 1.2 centimeters from 4.8 centimeters. Dividing this distance by the time interval of 0.05 second yields the average size of the velocity:

$$v = \frac{3.6 \text{ cm}}{0.05 \text{ s}} = 72 \text{ cm/s}.$$

You could verify the other values shown in the third column of table 3.1 by doing similar computations.

It is clear in table 3.1 that the velocity values steadily increase. To see that velocity is increasing at a constant rate, we can plot velocity against time (fig. 3.4). Notice that each velocity data point is plotted at the midpoint between the two times (or flashes) from which it was computed. This is because these values represent the average velocity for the short time intervals between flashes. For constant acceleration, the average velocity for any time interval is equal to the instantaneous velocity at the midpoint of that interval.

Did you notice that the slope of the line is constant in figure 3.4? The velocity values all fall approximately on a constant-slope straight line. Since acceleration is the slope of the velocity-versus-time graph, the acceleration must also be constant. The velocity increases uniformly with time.

table **3.1**

Distance and Velocity Values for a Falling Ball

Time	Distance	Velocity
0	0	
		24 cm/s
0.05 s	1.2 cm	
		72 cm/s
0.10 s	4.8 cm	
		124 cm/s
0.15 s	11.0 cm	
		174 cm/s
0.20 s	19.7 cm	
		218 cm/s
0.25 s	30.6 cm	
		268 cm/s
0.30 s	44.0 cm	
		320 cm/s
0.35 s	60.0 cm	
		368 cm/s
0.40 s	78.4 cm	
		416 cm/s
0.45 s	99.2 cm	
		464 cm/s
0.50 s	122.4 cm	

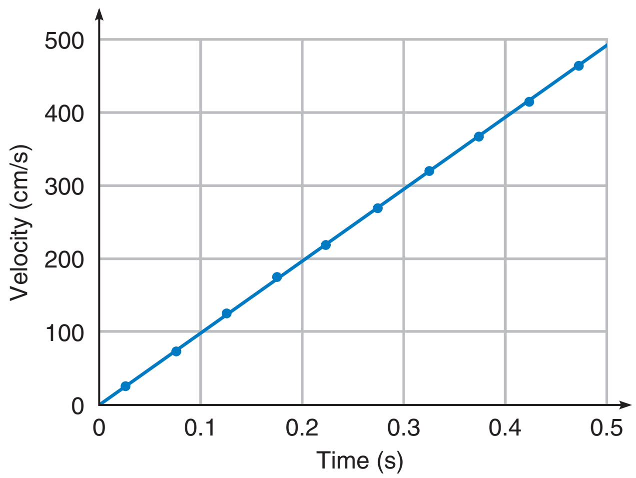


figure **3.4** Velocity plotted against time for the falling ball. The velocity values are those shown in table 3.1.

To find the value of the acceleration, we choose two velocity values that lie on the straight line and calculate how rapidly the velocity is changing. For example, the last velocity value, 464 cm/s, and the second value, 72 cm/s, are separated by a time interval corresponding to 8 flashes or 0.40 second. The increase in velocity Δv is found by subtracting 72 cm/s from 464 cm/s, obtaining 392 cm/s. To find the acceleration, we divide this change in velocity by the time interval ($a = \Delta v/t$),

$$a = \frac{392 \text{ cm/s}}{0.4 \text{ s}} = 980 \text{ cm/s}^2 = 9.8 \text{ m/s}^2.$$

This result gives us the **acceleration due to gravity** for objects falling near the earth's surface. Its value actually varies slightly from point to point on the earth's surface because of differences in altitude and other effects. This acceleration is used so often that it is given its own symbol g where

$$g = 9.8 \text{ m/s}^2.$$

Called the *gravitational acceleration* or *acceleration due to gravity,* it is valid only near the surface of the earth and thus is not a fundamental constant.

How did Galileo's ideas on falling objects differ from Aristotle's?

There is another sense in which the gravitational acceleration is constant, which takes us back to the experiments suggested in the chapter opener, p. 39. When you drop objects of different sizes and weights, do they reach the floor at the same time? Except for an unfolded piece of paper, it is likely that all of the objects that you test, regardless of their weight, reach the floor at the same time when released simultaneously. This finding suggests that the gravitational acceleration does not depend on the weight of the object.

Galileo used similar experiments to prove this point. His experiments contradicted Aristotle's view that heavier objects fall more rapidly. How could Aristotle's idea have been accepted for so long when simple experiments can disprove it? Experimentation was not part of the intellectual outlook of Aristotle and his followers; they valued pure thought and logic more highly. Galileo and other scientists of his time broke new ground by using experiments as an aid to thinking. A new tradition was emerging.

On the other hand, Aristotle's view agrees with our intuition that heavy objects do fall more rapidly than some lighter objects. If, for example, we drop a brick together with a feather or unfolded piece of paper (fig. 3.5), the brick will reach the floor first. The paper or feather will not fall in a straight line but instead will flutter to the floor much as a leaf falls from a tree. What is happening here?

You will probably recognize that the effects of **air resistance** impede the fall of the feather or paper much more than the fall of the brick, a steel ball, or a paper clip. When we crumple the piece of paper into a ball and drop it simultaneously with a brick or other heavy object, the two objects reach the floor at approximately the same time. We live at the bottom of a sea of air, and the effects of air resistance can be substantial for objects like leaves, feathers, or pieces of paper. These effects produce a slower and less regular flight for light objects that have a large surface area.

If we drop a feather and a brick simultaneously in a vacuum or in the very thin atmosphere of the moon, they do reach the ground at the same time. Moonlike conditions are not part of our everyday experience, however, so we are used to seeing feathers fall more slowly than rocks or bricks. Galileo's insight was that the gravitational acceleration is the same for all objects, regardless of their weight, provided that the effects of air resistance are not significant. Aristotle did not separate the effect of air resistance from that of gravity in his observations.

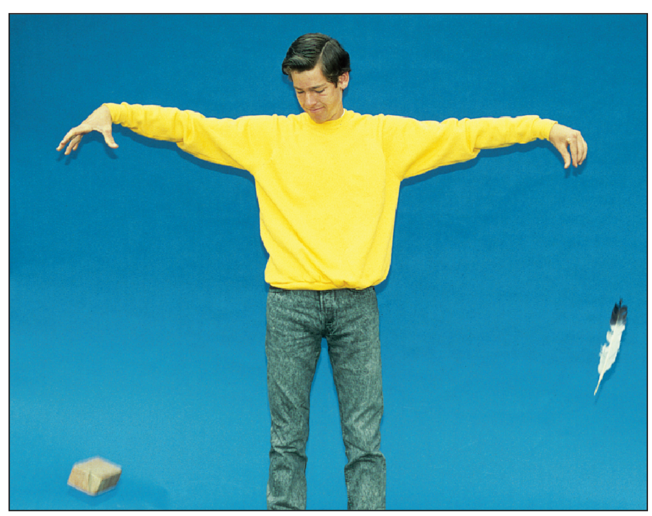


figure **3.5** The brick reaches the floor first when a brick and a feather are dropped at the same time.

The gravitational acceleration for objects near the surface of the earth is uniform and has the value of 9.8 m/s^2. It can be measured by using stroboscopes or similar techniques to record the position of a falling object at regular, very small time intervals. This acceleration is constant in time. Contrary to Aristotle's belief, it also has the same value for objects of different weight.

3.2 Tracking a Falling Object

Imagine yourself dropping a ball from a sixth-story window, as in figure 3.6. How long does it take for the ball to reach the ground below? How fast is it traveling when it gets there? Things happen quickly, so the answers to these questions are not obvious.

If we assume that air-resistance effects are small for the object we are tracking, we know that it accelerates toward the ground at the constant rate of 9.8 m/s^2. Let's make some quick estimates of how these values change with time without doing detailed computations.

How does the velocity vary with time?

In making estimates of velocity and distance for a falling object, we often take advantage of the fact that the gravitational-acceleration value of 9.8 m/s^2 is almost 10 m/s^2 and round it up. This makes the numerical values easier to calculate without sacrificing much in accuracy. Multiplying by 10 is quicker than multiplying by 9.8.

How fast is our dropped ball moving after 1 second? An acceleration of 10 m/s^2 means that the velocity is increasing by 10 m/s each second. If its original velocity is zero, then after 1 second its velocity has increased to 10 m/s, in 2 seconds to 20 m/s, and in 3 seconds to 30 m/s. For each additional second, the ball gains 10 m/s in velocity.*

To help you appreciate these values, look back at table 2.1, which shows unit comparisons for familiar speeds. A velocity of 30 m/s is roughly 70 MPH, so after 3 seconds the ball is moving quickly. After just 1 second, it is moving with a downward velocity of 10 m/s, which is over 20 MPH. The ball gains velocity at a faster rate than is possible for a high-powered automobile on a level surface.

How far does the ball fall in different times?

The high velocities are more meaningful if we examine how far the ball falls during these times. As the ball falls, it gains speed, so it travels farther in each successive time interval, as in the photograph in figure 3.3. Because of uniform

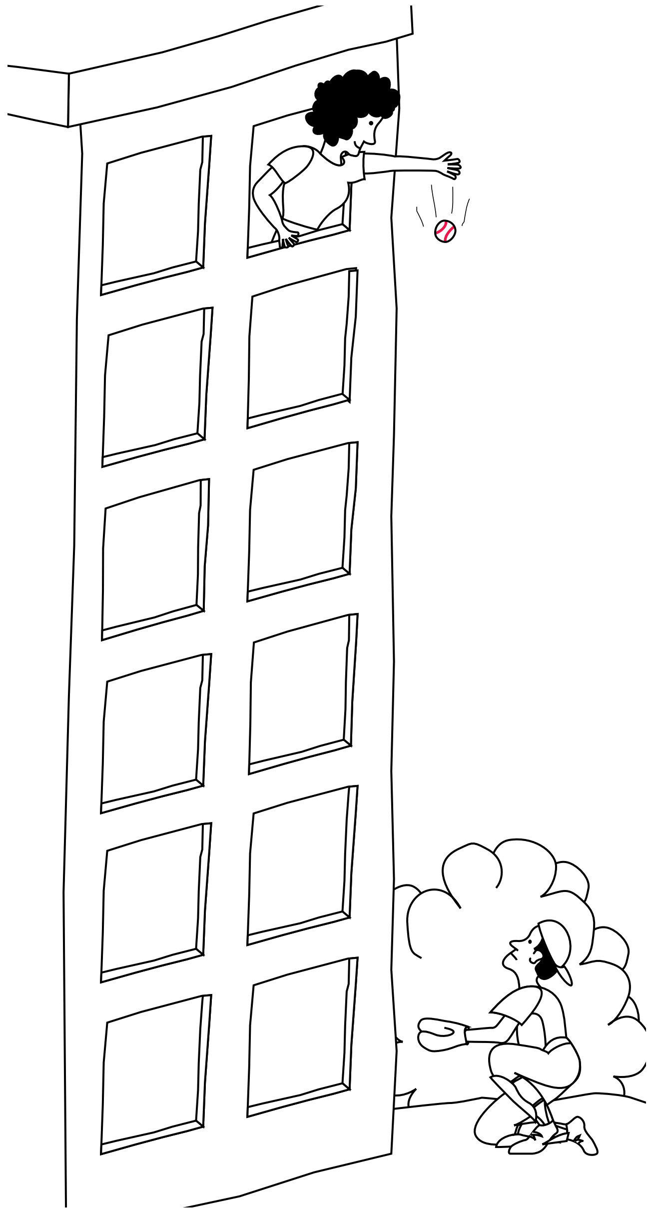


figure **3.6** A ball is dropped from a sixth-story window. How long does it take to reach the ground?

acceleration, the distance increases at an ever-increasing rate.

During the first second of motion, the velocity of the ball increases from zero to 10 m/s. Its average velocity during that first second is 5 m/s, and it travels a distance of 5 meters in that second. This can also be found by using the relationship between distance, acceleration, and time in section 2.5. If the starting velocity is zero, we found that $d = \frac{1}{2}at^2$. After 1 second, the ball has fallen a distance

$$d = \frac{1}{2}(10 \text{ m/s}^2)(1 \text{ s})^2 = 5 \text{ m}.$$

Since the height of a typical story of a multistory building is less than 4 meters, the ball falls more than one story in just a second.

*In section 2.5, we noted that the velocity of an object moving with uniform acceleration is $v = v_0 + at$, where v_0 is the original velocity and the second term is the change in velocity, $\Delta v = at$. When a ball is dropped, $v_0 = 0$, so v is just at, the change in velocity.

During the next second of motion, the velocity increases from 10 m/s to 20 m/s, yielding an average velocity of 15 m/s for that interval. The ball travels 15 meters in that second, which, when added to the 5 meters covered in the first second, yields a total of 20 meters. After 2 seconds, the distance fallen is four times as large as the 5 meters traveled after 1 second.* Since 20 meters is roughly five stories in height, the ball dropped from the sixth story will be near the ground after 2 seconds.

Figure 3.7 gives the velocity and distance fallen at half-second time intervals for a ball dropped from a six-story building. Notice that in just half a second, the ball falls 1.25 meters. An object dropped to the floor from an outstretched arm therefore hits the floor in roughly half a second. This makes it difficult to time with a stopwatch. (See Try This Box 3.1.)

The change in velocity is proportional to the size of the time interval selected. In 1 second the change in velocity is 10 m/s, so in half a second the change in velocity is 5 m/s. In each half-second the ball gains approximately 5 m/s in velocity, illustrated in figure 3.7. As the velocity gets larger, the arrows representing the velocity vectors grow. If we plotted these velocity values against time, we would get a simple upward-sloping straight-line graph as in figure 3.4.

What does the graph of the distance values look like? The distance values increase in proportion to the square of the time, which means that they increase more and more rapidly as time elapses. Instead of being a straight-line graph, the graph of the distance values curves upward as in figure 3.8. The rate of change of distance with time is itself increasing with time.

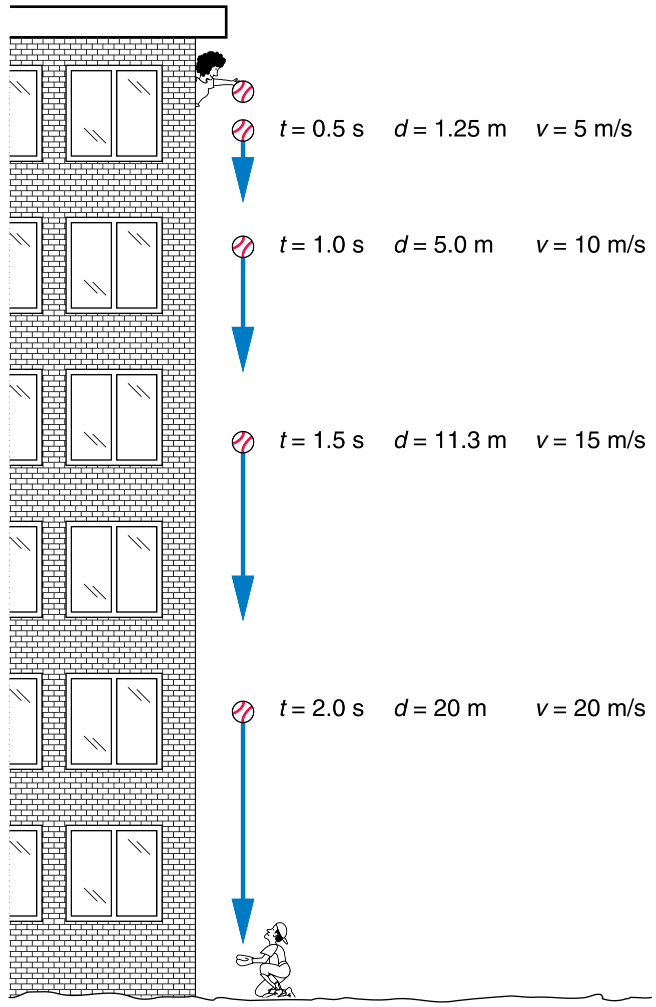


figure 3.7 Velocity and distance values for the dropped ball shown at half-second time intervals.

Throwing a ball downward

Suppose that instead of just dropping the ball, we throw it straight down, giving it a starting velocity v_0 different from zero. How does this affect the results? Will the ball reach the ground more rapidly and with a larger velocity? You would probably guess correctly that the answer is yes.

In the case of the velocity values, the effect of the starting velocity is not difficult to see. The ball is still being accelerated by gravity so that the change in velocity for each second of motion is still $\Delta v = 10$ m/s, or for a half-second, 5 m/s. If the initial downward velocity is 20 m/s, after half a second, the velocity is 25 m/s, and after 1 second, it is 30 m/s. We simply add the change in velocity to the initial velocity as indicated by the formula $v = v_0 + at$.

In the case of distance, however, the values increase more rapidly. The full expression for distance traveled by a uniformly accelerated object (introduced in section 2.5) is

$$d = v_0 t + \frac{1}{2}at^2.$$

*This is a result of the time being squared in the formula for distance. Putting 2 s in place of 1 s in the formula $d = \frac{1}{2}at^2$ multiplies the result by a factor of 4 ($2^2 = 4$), yielding a distance of 20 m.

try this box 3.1

Sample Question: Using a Pulse Rate to Time a Falling Object

Question: Suppose that Galileo's resting pulse rate was 60 beats per minute. Would his pulse be a useful timer for getting position-versus-time data for an object dropped from the height of 2 to 3 meters?

Answer: A pulse rate of 60 beats per minute corresponds to 1 beat per second. In the time of 1 second, a dropped object falls a distance of approximately 5 m. (It falls 1.22 m in just half a second as seen in table 3.1.) Thus this pulse rate (or most pulse rates) would not be an adequate timer for an object dropped from a height of a few meters. It could be slightly more effective for an object dropped from a tower several stories in height.

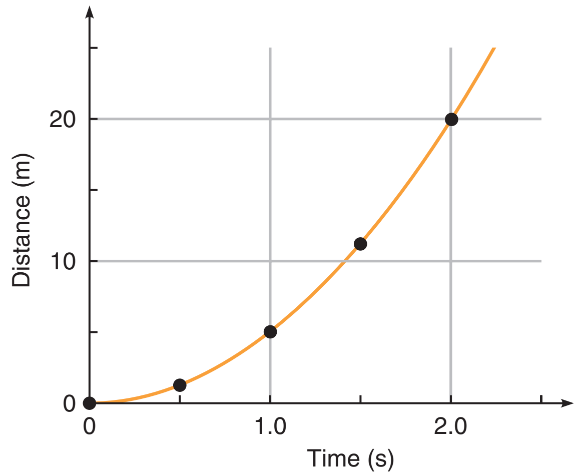


figure **3.8** A plot of distance versus time for the dropped ball.

The first term is the distance that the ball would travel if it continued to move with just its original velocity. This distance also increases with time. The second term is due to the acceleration and has the same values as shown in figures 3.7 and 3.8.

In the sample exercise in Try This Box 3.2, we calculate velocity and distance traveled during the first 2 seconds of motion for a ball thrown downward. Notice that after 2 seconds the ball has traveled a distance of 60 meters, much larger than the 20 meters when the ball is simply dropped.

try this box 3.2

Sample Exercise: Throwing a Ball Downward

A ball is thrown downward with an initial velocity of 20 m/s. Using the value 10 m/s^2 for the gravitational acceleration, find (a) the velocity and (b) the distance traveled at 1-s time intervals for the first 2 s of motion.

a. $v_0 = 20$ m/s $\quad v = v_0 + at$

$a = 10$ m/s^2 $\quad$ for $t = 1$ s

$v = ?$ $\quad v = 20 \text{ m/s} + (10 \text{ m/s}^2)(1 \text{ s})$

$$= 20 \text{ m/s} + 10 \text{ m/s}$$

$$= \mathbf{30\ m/s}$$

$t = 2$ s $\quad v = 20 \text{ m/s} + (10 \text{ m/s}^2)(2 \text{ s})$

$$= 20 \text{ m/s} + 20 \text{ m/s} = \mathbf{40\ m/s}$$

b. $d = ?$ $\quad d = v_0 t + \frac{1}{2}at^2$

$t = 1$ s $\quad d = (20 \text{ m/s})(1 \text{ s}) + \frac{1}{2}(10 \text{ m/s}^2)(1 \text{ s})^2$

$$= 20 \text{ m} + 5 \text{ m} = \mathbf{25\ m}$$

$t = 2$ s $\quad d = (20 \text{ m/s})(2 \text{ s}) + \frac{1}{2}(10 \text{ m/s}^2)(2 \text{ s})^2$

$$= 40 \text{ m} + 20 \text{ m} = \mathbf{60\ m}$$

After just 1 second the ball has already traveled 25 meters, which means that it would be near the ground if thrown from our sixth-story window.

Keep in mind, though, that we have ignored the effects of air resistance in arriving at these results. For a compact object falling just a few meters, the effects of air resistance are very small. These effects increase as the velocity increases, however, so that the farther the object falls, the greater the effects of air resistance. In chapter 4, we will discuss the role of air resistance in more depth in the context of sky diving.

When an object is dropped, its velocity increases by approximately 10 m/s every second due to the gravitational acceleration. The distance traveled increases at an ever-increasing rate because the velocity is increasing. In just a few seconds, the object is moving very rapidly and has fallen a large distance. In section 3.3, we will explore the effects of gravitational acceleration on an object thrown upward.

3.3 Beyond Free Fall: Throwing a Ball Upward

In section 3.2, we discussed what happens when a ball is dropped or thrown downward. In both of these cases, the ball gains velocity as it falls due to the gravitation acceleration. What if the ball is thrown upward instead, as in figure 3.9? How does gravitational acceleration affect the ball's motion? What goes up must come down—but when and how fast are interesting questions with everyday applications.

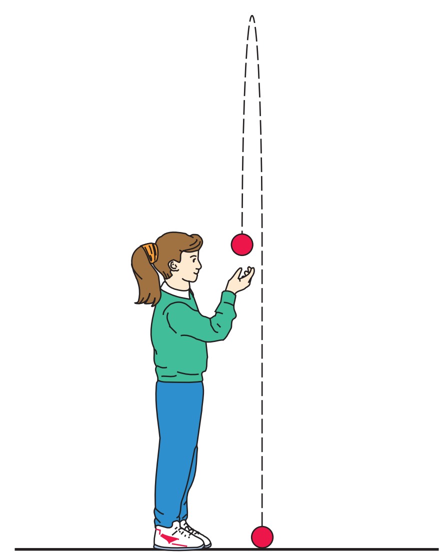


figure **3.9** A ball thrown upward returns to the ground. What are the magnitude and direction of the velocity at different points in the flight?

The directions of the acceleration and velocity vectors merit our close attention. The gravitational acceleration is always directed downward toward the center of the earth, because that is the direction of the gravitational force that produces this acceleration. This means that the acceleration is in the opposite direction to the original upward velocity.

How does the ball's velocity change?

Suppose that we throw a ball straight up with an original velocity of 20 m/s. Many of us can throw a ball at this velocity: it is approximately 45 MPH. This is a lot less than a 90-MPH fastball, but throwing a ball upward with good velocity is harder than throwing it horizontally.

Once the ball leaves our hand, the primary force acting on it is gravity, which produces a downward acceleration of 9.8 m/s^2 or approximately 10 m/s^2. Every second, there is a change in velocity of 10 m/s. This change in velocity is directed downward, however, opposite to the direction of the original velocity. It subtracts from the original velocity rather than adding to it.

Once you are aware of how important direction is in observing the ball thrown upward, finding the velocity at different times is not hard. After 1 second, the velocity of the ball has decreased by 10 m/s, so if it started at +20 m/s (*choosing the positive direction to be upward in this case*), it is now moving upward with a velocity of just +10 m/s. After 2 seconds, it loses another 10 m/s, so its velocity is then zero. It does not stop there, of course. In another second (3 seconds from the start), its velocity decreases by another 10 m/s, and it is then moving downward at −10 m/s. *The sign of the velocity indicates its direction.* All of these values can be found from the relationship $v = v_0 + at$, where $v_0 = +20$ m/s and $a = -10$ m/s^2.

Clearly, the ball has changed direction, as you might expect. Just as before, the velocity changes steadily at −10 m/s each second, due to the constant downward acceleration. After 4 seconds, the ball is moving downward with a velocity of −20 m/s and is back at its starting position. These results are illustrated in figure 3.10. The high point in the motion occurs at a time 2 seconds after the ball is thrown, where the velocity is zero. If the velocity is zero, the ball is moving neither upward nor downward, so this is the turnaround point.

An interesting question, a favorite on physics tests (and often missed by students), asks for the value of acceleration at the high point in the motion. If the velocity is zero at this point, what is the value of the acceleration? The quick, but incorrect, response given by many people is that the acceleration must also be zero at that point. The correct answer is that the acceleration is still −10 m/s^2. The gravitational acceleration is constant and does not change. The velocity of the ball is still changing at that instant, from a positive to a negative value, even though the instantaneous velocity is zero. Acceleration is the *rate of change* of velocity and is *unrelated* to the size of the velocity.

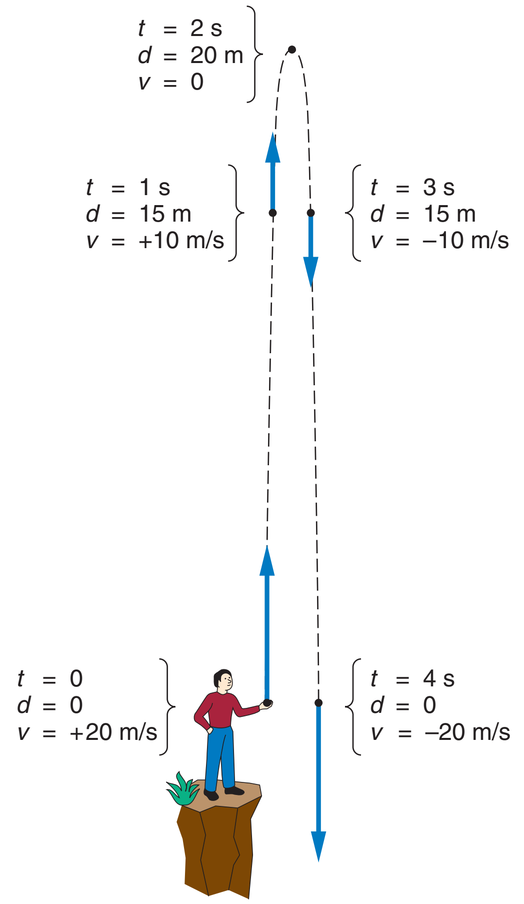


figure 3.10 The velocity vectors at different points in the flight of a ball thrown upward with a starting velocity of +20 m/s.

What would a graph of velocity plotted against time look like for the motion just described? If we make the upward direction of motion positive, the velocity starts with a value of +20 m/s and changes at a steady rate, decreasing by −10 m/s each second. This is a straight-line graph, sloping downward as in figure 3.11. The positive values of velocity represent upward motion, where the size of the velocity is decreasing, and the negative values of velocity represent downward motion. If the ball did not hit the ground, but was thrown from the edge of a cliff, it would continue to gain negative velocity as it moved downward.

How high does the ball go?

The position or height of the ball at different times can be computed using the methods in section 3.2. These distance computations involve the formula for uniform acceleration developed in section 2.5. In the sample exercise in Try This Box 3.3, we compute the height or distance traveled at 1-second intervals for the ball thrown upward at +20 m/s, using −10 m/s^2 for the gravitational acceleration.

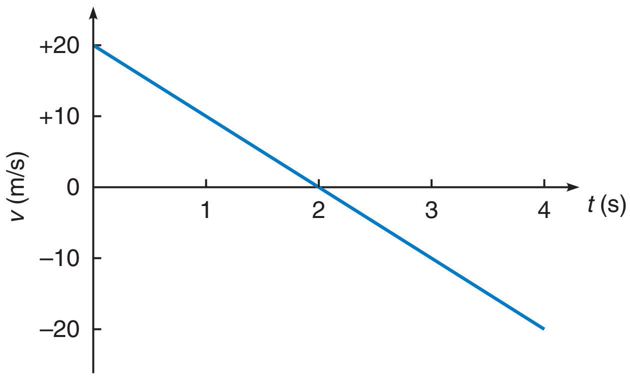


figure **3.11** A plot of the velocity versus time for a ball thrown upward with an initial velocity of +20 m/s. The negative values of velocity represent downward motion.

What should you notice about these results? First, the high point of the motion is 20 meters above the starting point. The high point is reached when the velocity is zero, and we determined earlier that this occurs at a time of 2 seconds. This time depends on how fast the ball is thrown initially. The larger the original velocity, the greater the time to reach the high point. Knowing this time, we can use the distance formula to find the height.

You should also notice that after just 1 second, the ball has reached a height of 15 meters. It covers just five additional meters in the next second of motion, and then falls back to 15 meters in the following second. The ball spends a full 2 seconds above the height of 15 meters, even though it only reaches a height of 20 meters. The ball is moving more slowly near the top of its flight than it is at lower points—this is why the ball appears to "hang" near the top of its flight.

try this box 3.3

Sample Exercise: Throwing a Ball Upward

A ball is thrown upward with an initial velocity of 20 m/s. Find its height at 1-s intervals for the first 4 s of its flight.

$d = ?$ $\quad d = v_0t + \frac{1}{2}at^2$

$t = 1\text{ s}$ $\quad = (20\text{ m/s})(1\text{ s}) + \frac{1}{2}(-10\text{ m/s}^2)(1\text{ s})^2$

$= 20\text{ m} - 5\text{ m} = \mathbf{15\ m}$

$t = 2\text{ s}$ $\quad d = (20\text{ m/s})(2\text{ s}) + \frac{1}{2}(-10\text{ m/s}^2)(2\text{ s})^2$

$= 40\text{ m} - 20\text{ m} = \mathbf{20\ m}$

$t = 3\text{ s}$ $\quad d = (20\text{ m/s})(3\text{ s}) + \frac{1}{2}(-10\text{ m/s}^2)(3\text{ s})^2$

$= 60\text{ m} - 45\text{ m} = \mathbf{15\ m}$

$t = 4\text{ s}$ $\quad d = (20\text{ m/s})(4\text{ s}) + \frac{1}{2}(-10\text{ m/s}^2)(4\text{ s})^2$

$= 80\text{ m} - 80\text{ m} = \mathbf{0\ m}$

Finally, the time taken for the ball to fall back to its starting point from the high point is equal to the time taken for the ball to reach the high point in the first place. It takes 2 seconds to reach the high point and another 2 seconds for it to return to the starting point. The total time of flight is just twice the time needed to reach the high point, in this case, 4 seconds. A larger starting velocity would produce a higher turnaround point and a greater "hang time" for the ball.

A ball thrown upward is slowed by the downward gravitational acceleration until its velocity is reduced to zero at the high point. The ball then falls from that high point accelerating downward at the same constant rate as when it was rising. The ball travels more slowly near the top of its flight, so it appears to "hang" there. It spends more time in the top few meters than it does in the rest of the flight. We will find that these features are also present when a ball is projected at an angle to the horizontal, as discussed in section 3.5.

3.4 Projectile Motion

Suppose that instead of throwing a ball straight up or down, you throw it horizontally from some distance above the ground. What happens? Does the ball go straight out until it loses all of its horizontal velocity and then starts to fall like the perplexed coyote in the *Roadrunner* cartoons (fig. 3.12)? What does the real path, or *trajectory,* look like?

Cartoons give us a misleading impression. In fact, two different things are happening at the same time: (1) the ball is accelerating downward under the influence of gravity, and (2) the ball is moving sideways with an approximately constant horizontal component of velocity. Combining these two motions gives the overall **trajectory** or path.

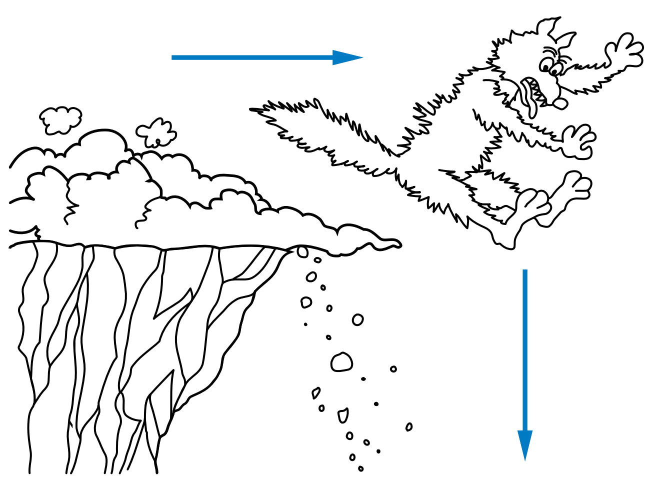


figure **3.12** A cartoon coyote falling off a cliff. Is this a realistic picture of what happens?

What does the trajectory look like?

You can perform a simple experiment to help you visualize the path that the projectile follows. Take a marble or small ball, roll it along the top of a desk or table, and let it roll off the edge. What does the path of the ball look like as it travels through the air to the floor? Is it like the coyote in figure 3.12? Roll the ball at different velocities and see how the path changes. Try to sketch the path after making these observations.

How do we go about analyzing this motion? The key lies in thinking about the horizontal and vertical components of the motion separately and then combining them to get the actual path (fig. 3.13).

The acceleration of the horizontal motion is zero, provided that air resistance is small enough to be ignored. This implies that the ball moves with a constant horizontal velocity once it has rolled off the table or has left the hand. The ball travels equal horizontal distances in equal time intervals, as shown across the top of figure 3.13. In constructing this diagram, we assumed an initial horizontal velocity of 2 m/s for the ball. Every tenth of a second, then, the ball travels a horizontal distance of 0.2 meter.

At the same time that the ball travels with constant horizontal velocity, it accelerates downward with the constant gravitational acceleration *g*. Its vertical velocity increases exactly like that of the falling ball photographed for figure 3.3. This motion is depicted along the left side of figure 3.13. In each successive time interval, the ball falls a greater distance than in the time interval before, because the vertical velocity increases with time.

Combining the horizontal and vertical motions, we get the path shown curving downward in figure 3.13. For each time shown, we draw a horizontal dashed line locating the vertical position of the ball, and a vertical dashed line for the horizontal position. The position of the ball at any time is the point where these lines intersect. The resulting trajectory (the solid curve) should look familiar if you have performed the simple experiments suggested in the first paragraph on this page.

If you understand how we obtained the path of the ball, you are well on your way to understanding **projectile motion.** The total velocity of the ball at each position pictured is in the direction of the path at that point, since this is the actual direction of the ball's motion. This total velocity is a vector sum of the horizontal and vertical components of the velocity (fig. 3.14). (See appendix C.) The horizontal velocity remains constant, because there is no acceleration in that direction. The downward (vertical) velocity gets larger and larger.

The actual shape of the path followed by the ball depends on the original horizontal velocity given the ball by throwing it or rolling it from the tabletop. If this initial horizontal velocity is small, the ball does not travel very far horizontally. Its trajectory will then be like the smallest starting velocity v_1 in figure 3.15.

The three trajectories shown in figure 3.15 have three different starting velocities. As you would expect, the ball travels greater horizontal distances when projected with a larger initial horizontal velocity.

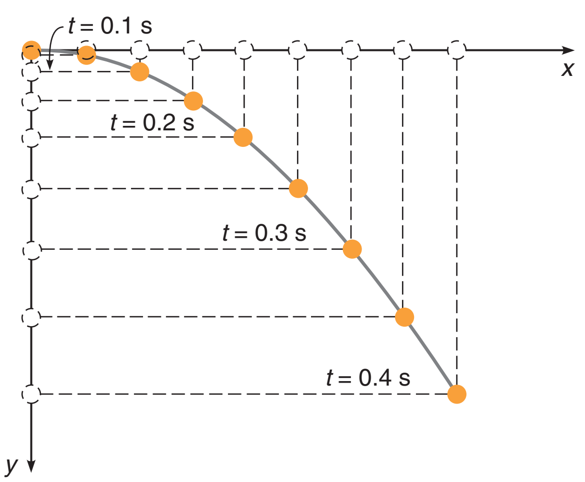


figure 3.13 The horizontal and vertical motions combine to produce the trajectory of the projected ball.

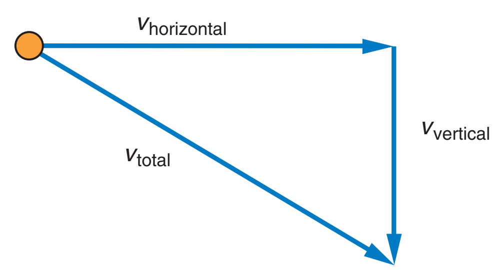


figure 3.14 The total velocity at any point is found by adding the vertical component of the velocity to the horizontal component.

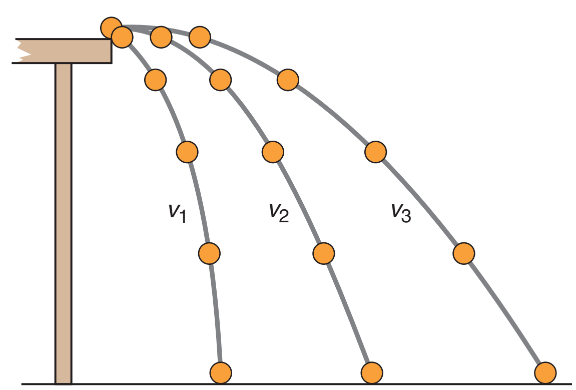


figure 3.15 Trajectories for different initial velocities of a ball rolling off a table: v_3 is larger than v_2, which in turn is larger than v_1.

What determines the time of flight?

Which of the three balls in figure 3.15 would hit the floor first if all three left the tabletop at the same time? Does the time taken for the ball to hit the floor depend on its horizontal velocity? There is a natural tendency to think that the ball that travels farther takes a longer time to reach the floor.

In fact, the three balls should all reach the floor at the same time. The reason is that they are all accelerating downward at the same rate of 9.8 m/s^2. This downward acceleration is not affected by how fast the ball travels horizontally. The time taken to reach the floor for the three balls in figure 3.15 is determined strictly by how high above the floor the tabletop is. The vertical motion is *independent* of the horizontal velocity.

This fact often surprises people. It contradicts our intuitive sense of what is going on but can be confirmed by doing simple experiments using two similar balls (fig. 3.16). If you throw one ball horizontally at the same time that you simply drop the second ball from the same height, the two balls should reach the floor at roughly the same time. They may fail to hit at the same time, most likely because it is hard to throw the first ball completely horizontally and to release both balls at the same time. A special spring gun, often used in demonstrations, will do this more precisely.

If we know how far the ball falls, we can compute the time of flight. This can then be used to determine the horizontal distance that the ball will travel, if we know the initial horizontal velocity. The sample exercise in Try This Box 3.4 shows this type of analysis. Notice that the horizontal distance traveled is determined by two factors: the time of flight and the initial velocity.

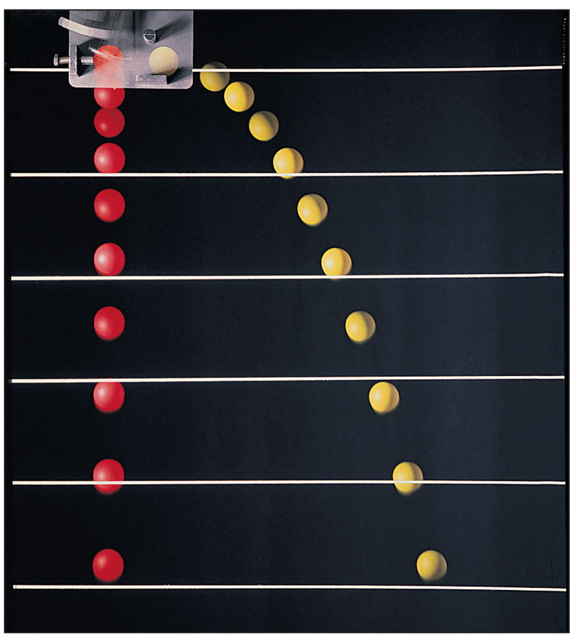


figure **3.16** A ball is dropped at the same time that a second ball is projected horizontally from the same height. Which ball reaches the floor first?

try this box **3.4**

Sample Exercise: Projectile Motion

A ball rolls off a tabletop with an initial velocity of 3 m/s. If the tabletop is 1.25 m above the floor,

a. How long does it take for the ball to hit the floor?
b. How far does the ball travel horizontally?

a. In figure 3.7, we saw that a ball will fall a distance of 1.25 m in approximately half a second. This could be found directly from

$d_{vertical} = 1.25$ m

$a = g = 10\ m/s^2$

$t = ?$

$$d_{vertical} = \frac{1}{2}at^2$$

Solving for t^2:

$$t^2 = \frac{d}{\frac{1}{2}a} = \frac{1.25\ m}{5\ m/s^2} = 0.25\ s^2$$

Taking the square root to get t:

$$t = \mathbf{0.5\ s}$$

b. Knowing the time of flight t, we can now compute the horizontal distance traveled:

$v_0 = 3$ m/s

$t = 0.5$ s

$d_{horizontal} = ?$

$$d_{horizontal} = v_0 t = (3.0\ m/s)(0.50\ s) = \mathbf{1.5\ m}$$

Treating the vertical motion independently of the horizontal motion and then combining them to find the trajectory is the secret to understanding projectile motion. A horizontal glide combines with a vertical plunge to produce a graceful curve. The downward gravitational acceleration behaves the same as for any falling object, but there is no acceleration in the horizontal direction if air resistance can be ignored. The projectile moves with constant horizontal velocity while it is accelerating downward.

3.5 Hitting a Target

As long as humans have been hunters or warriors, they have wanted to predict where a projectile such as a cannonball will land after it is fired. Being able to hit a target such as a bird in a tree or a ship at sea has obvious implications for survival. Being able to hit a catcher's mitt with a baseball thrown from center field is also a highly valued skill.

Does the bullet fall when a rifle is fired?

Imagine that you are firing a rifle at a small target some distance away, with the rifle and target at exactly the same distance above the ground (fig. 3.17). If the rifle is fired directly at the target in a horizontal direction, will the bullet hit the center of the target? If you think of the ball rolling off the table in section 3.4, you should conclude that the bullet will strike the target slightly below the center. Why? The bullet will be accelerated downward by the gravitational pull of the earth and will fall slightly as it travels to the target.

Since the time of flight is small, the bullet does not fall very far, but it falls far enough to miss the center of the target. How do you compensate for the fall of the bullet? You aim a little high. You correct your aim either through trial and error or by adjusting your rifle sight so that your aim is automatically a little above center. Rifle sights are often adjusted for some average distance to the target. For longer distances you must aim high, for shorter distances a little low.

If you aim a little high, the bullet no longer starts out in a completely horizontal direction. The bullet travels up slightly during the first part of its flight and then comes down to meet the target. This also happens when you fire a cannon or throw a ball at a distant target. The rise and fall is more obvious for the trajectory of a football, though, than it is for a bullet.

The flight of a football

Whenever you throw a ball such as a football at a somewhat distant target, the ball must be launched at an angle above the horizontal so that the ball does not fall to the ground too soon. A good athlete does this automatically as a result of practice. The harder you throw, the less you need to direct the ball upward, because a larger initial velocity causes the ball to reach the target more quickly, giving it less time to fall.

Figure 3.18 shows the flight of a football thrown at an angle of 30° above the horizontal. The vertical position of the ball is plotted on the left side of the diagram, as in figure 3.13 for the horizontally projected ball. The horizontal position of the ball is shown across the bottom of the diagram. We have assumed that air resistance is small, so the ball travels with a constant horizontal velocity. Combining these two motions yields the overall path.

As the football climbs, the vertical component of its velocity decreases because of the constant downward gravitational acceleration. At the high point, this vertical component of the velocity is zero, just as it is for a ball thrown straight upward. The velocity of the ball is completely horizontal at this high point. The ball then begins to fall, gaining downward velocity as it accelerates. Unlike the ball thrown straight upward, however, there is a constant horizontal component to the velocity throughout the flight. We need to add this horizontal motion to the up-and-down motion that we described in section 3.3.

In throwing a ball, you can vary two quantities to help you hit your target. One is the initial velocity, which is determined by how hard you throw the ball. The other is the launch angle, which can be varied to fit the circumstances.

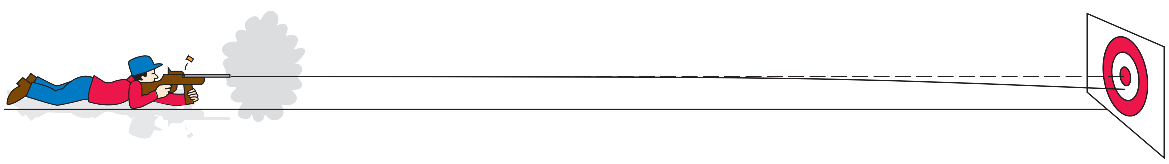


figure 3.17 A target shooter fires at a distant target. The bullet falls as it travels to the target.

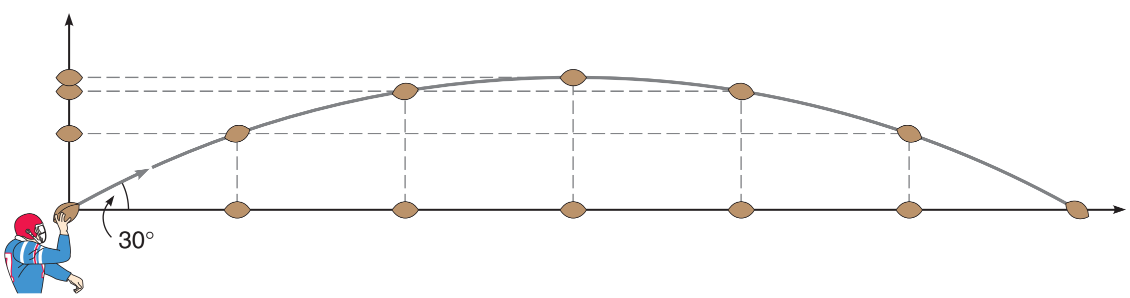


figure 3.18 The flight of a football launched at an angle of 30° to the horizontal. The vertical and horizontal positions of the ball are shown at regular time intervals.

A ball thrown with a large initial velocity does not have to be aimed as high and will reach the target more quickly. It may not clear the on-rushing linemen, however, and it might be difficult to catch because of its large velocity.

There is no time like the present to test these ideas. Take a page of scrap paper and crumple it into a compact ball. Then take your wastebasket and put it on your chair or desk. Throwing underhand, experiment with different throwing speeds and launch angles to see which is most effective in making a basket. Try to get a sense of how the launch angle and throwing speed interact to produce a successful shot. A low, flat-trajectory shot should require a greater throwing speed than a higher, arching shot. The flatter shot must also be aimed more accurately, since the effective area of the opening in the basket is smaller when the ball approaches at a flat angle. The ball "sees" a smaller opening. (This effect is discussed in Everyday Phenomenon Box 3.1.)

How can we achieve maximum distance?

In firing a rifle or cannon, the initial velocity of the projectile is usually set by the amount of gunpowder in the shell. The launch angle is then the only variable we can change in attempting to hit a target. Figure 3.19 shows three possible paths, or trajectories, for a cannonball fired at different launch angles for the same initial speed. For different launch angles, we tilt the cannon barrel by different amounts from the position shown.

Note that the greatest distance is achieved using an intermediate angle, an angle of 45° if the effects of air resistance are negligible. The same considerations are involved in the shot put in track-and-field events. The launch angle is very important and, for the greatest distance, will be near 45°. Air resistance and the fact that the shot hits the ground below the launch point are also factors, so the most effective angle is somewhat less than 45° in the shot put.

Everyday Phenomenon box 3.1

Shooting a Basketball

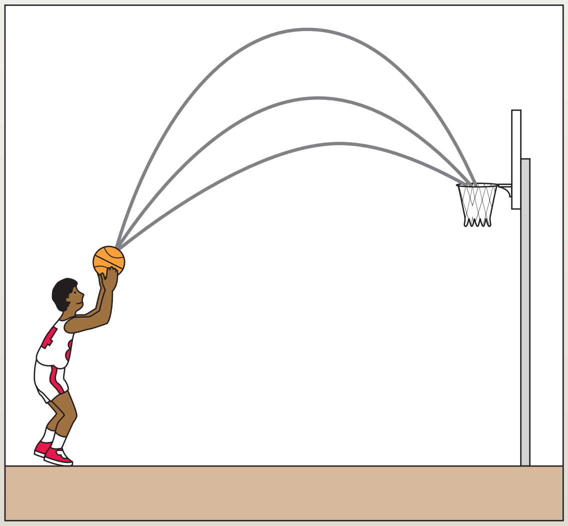
The Situation. Whenever you shoot a basketball, you unconsciously select a trajectory for the ball that you believe will have the greatest likelihood of getting the ball to pass through the basket. Your target is above the launch point (with the exception of dunk shots and sky hooks), but the ball must be on the way down for the basket to count.

What factors determine the best trajectory? When is a high, arching shot desirable, and when might a flatter trajectory be more effective? Will these factors be different for a free throw than for a shot taken when you are guarded by another player? How can our understanding of projectile motion help us to answer these questions?

The Analysis. The diameter of the basketball and the diameter of the basket opening limit the angle at which the basketball can pass cleanly through the hoop. The second drawing shows the range of possible paths for a ball coming straight down and for one coming in at a 45° angle to the basket. The shaded area in each case shows how much the center of the ball can vary from the center line if the ball is to pass through the hoop. As you can see, a wider range of paths is available when the ball is coming straight down. The diameter of the basketball is a little more than half the diameter of the basket.

This second drawing illustrates the advantage of an arched shot. There is a larger margin of error in the path that the ball can take and still pass through the hoop cleanly.

For the dimensions of a regulation basketball and basket, the angle must be at least 32° for a clean shot. As the angle gets larger, the range of possible paths increases. At smaller angles, appropriate spin on the basketball will sometimes cause the ball to rattle through, but the smaller the angle, the less the likelihood of that happening.



Different possible trajectories for a basketball free throw. Which has the greatest chance of success?

(continued)

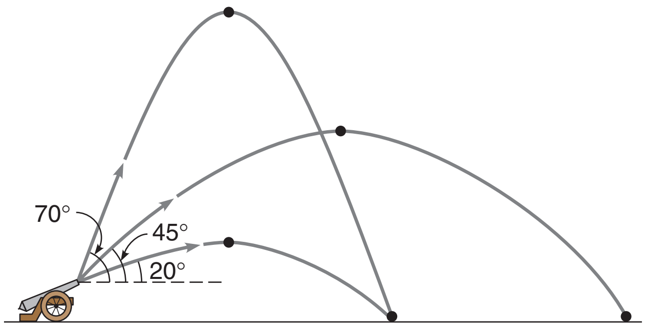
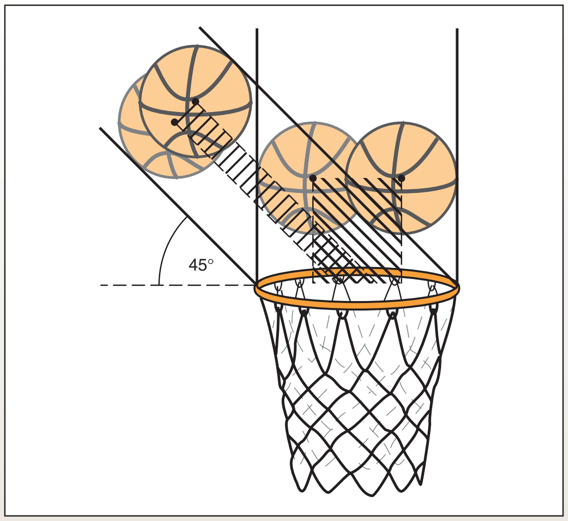


figure **3.19** Cannonball paths for different launch angles but the same initial launch speed.

Thinking about what happens to the horizontal and vertical components of the initial velocity at different launch angles will show us why the angle for maximum distance is approximately 45°. (See figure 3.20.) Velocity is a vector, and its horizontal and vertical components can be found by drawing the vector to scale and adding dashed lines to the horizontal and vertical directions (fig. 3.20). This process is described more fully in appendix C.

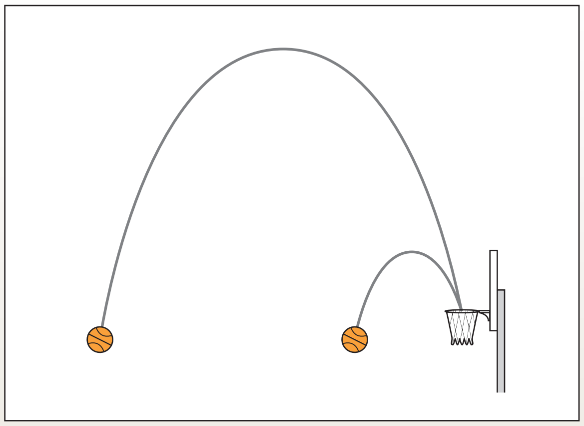
For the lowest launch angle 20°, we see that the horizontal component of the velocity is much larger than the vertical. Since the initial upward velocity is small, the ball does not go very high. Its time of flight is short, and it hits the ground sooner than in the other two cases shown. The ball gets there quickly because of its large horizontal velocity and short travel time, but it does not travel very far before hitting the ground.

The high launch angle of 70° produces a vertical component much larger than the horizontal component. The ball thus travels much higher and stays in the air for a longer time than at 20°. It does not travel very far horizontally,



Possible paths for a basketball coming straight down and for one coming in at a 45° angle. The ball coming straight down has a wider range of possible paths.

The disadvantage of the arched shot is less obvious. As you get farther away from the basket, launching conditions for an arched shot must be more precise for the ball to travel the horizontal distance to the basket. If an arched shot is launched from 30 ft, it must travel a much higher path than a shot launched at the same angle closer to the basket, as shown in the third drawing. Since the ball stays in the air for a longer time, small variations in either the release speed or angle can cause large errors in the distance traveled. This distance depends on both the time of flight and the horizontal component of the velocity.



An arched shot launched from a large distance stays in the air longer than one launched at the same angle from much closer to the basket.

A highly arched shot is more effective when you are close to the basket. You can then take advantage of the greater range of paths available to the arched shot without suffering much from the uncertainty in the horizontal distance. Away from the basket, the desirable trajectories gradually become flatter, permitting more accurate control of the shot. An arched shot is sometimes necessary from anywhere on the court, however, to avoid having the shot blocked.

The spin of the basketball, the height of the release, and other factors all play a role in the success of a shot. A fuller analysis can be found in an article by Peter J. Brancazio in the *American Journal of Physics* (April 1981) entitled "Physics of Basketball." A good understanding of projectile motion might improve the game of even an experienced player.

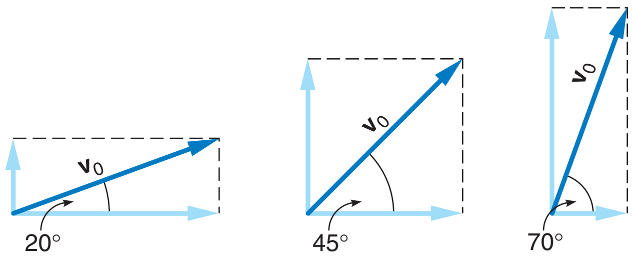


figure **3.20** Vector diagrams showing the horizontal and vertical components of the initial velocity for the three cases illustrated in figure 3.19.

however, because of its small horizontal velocity. The ball travels about the same horizontal distance as for the 20° launch, but it takes longer getting there. (If we shot it straight up, the horizontal distance covered would be zero, of course.)

The intermediate angle of 45° splits the initial velocity into equal-sized horizontal and vertical components. The ball therefore stays in the air longer than in the low-angle launch but also travels with a greater horizontal velocity than in the high-angle launch. In other words, with relatively large values for both the vertical and horizontal components of velocity, the vertical motion keeps the ball in the air long enough for the horizontal velocity to be effective. This produces the greatest distance of travel.

The time of flight and the horizontal distance traveled can be found if the launch angle and the size of the initial velocity are known. It is first necessary to find the horizontal and vertical components of the velocity to do these computations, however, and this makes the problem more complex than those discussed earlier. The ideas can be understood without doing the computations. The key is to think about the vertical and horizontal motions separately and then combine them.

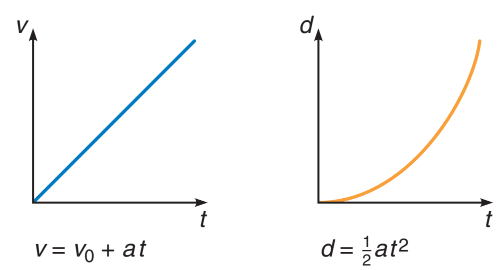
For a projectile launched at an angle, the initial velocity can be broken down into vertical and horizontal components. The vertical component determines how high the object will go and how long it stays in the air, while the horizontal component determines how far it will go in that time. The launch angle and the initial speed interact to dictate where the object will land. Through the entire flight, the constant downward gravitational acceleration is at work, but it changes only the vertical component of the velocity. Producing or viewing such trajectories is a common part of our everyday experience.

summary

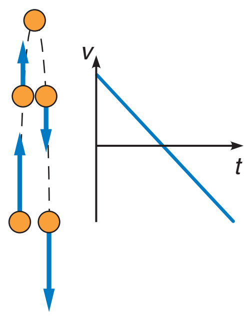
The primary aim in this chapter has been to introduce you to the gravitational acceleration for objects near the surface of the earth and to show how that acceleration affects the motion of objects dropped or launched in various ways.

1 **Acceleration due to gravity.** To find the acceleration due to gravity, we use measurements of the position of a dropped object at different times. The gravitational acceleration is 9.8 m/s^2. It does not vary with time as the object falls, and it has the same value for different objects regardless of their weight.

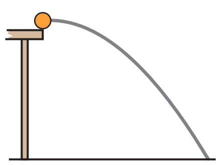
2 **Tracking a falling object.** The velocity of a falling object increases by approximately 10 m/s every second of its fall. Distance traveled increases in proportion to the square of the time, so that it increases at an ever-increasing rate. In just 1 second, a dropped ball is moving with a velocity of 10 m/s and has traveled 5 meters.



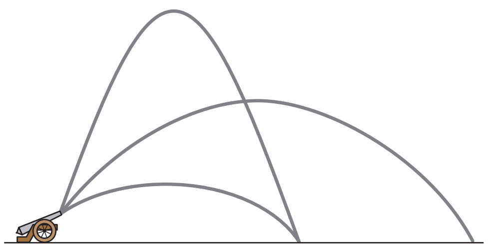
3 Throwing a ball upward. The speed of an object thrown upward first decreases due to the downward gravitational acceleration, passes through zero at the high point, and then increases as the object falls. The object spends more time near the top of its flight because it is moving more slowly there.



4 Projectile motion. If an object is launched horizontally, it moves with a constant horizontal velocity at the same time that it accelerates downward due to gravity. These two motions combine to produce the object's curved trajectory.



5 Hitting a target. There are two factors, the launch speed and the launch angle, that can be varied to determine the path of an object launched at an angle to the horizontal. Once again, the horizontal and vertical motions combine to produce the overall motion as the projectile moves toward a target.



key terms

Acceleration due to gravity, 41
Air resistance, 41
Trajectory, 46
Projectile motion, 47

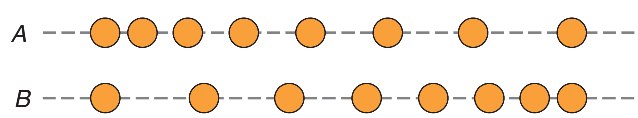
questions

*Questions identified with an asterisk are more open-ended than the others. They call for lengthier responses and are more suitable for group discussion.

Q1. A small piece of paper is dropped and flutters to the floor. Is the piece of paper accelerating at any time during this motion? Explain.

Q2. The diagram shows the positions at intervals of 0.10 seconds of a ball moving from left to right (as in a photograph taken with a stroboscope that flashes every tenth of a second). Is the ball accelerated? Explain.

Q3. The diagram shows the positions at intervals of 0.05 seconds of two balls moving from left to right. Are either or both of these balls accelerated? Explain.



Q4. A lead ball and an aluminum ball, each 1 in. in diameter, are released simultaneously and allowed to fall to the ground. Due to its greater density, the lead ball has a substantially larger mass than the aluminum ball. Which of these balls, if either, has the greater acceleration due to gravity? Explain.

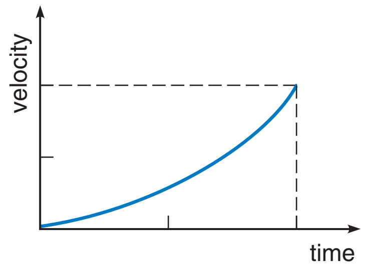
Q5. Two identical pieces of paper, one crumpled into a ball and the other left uncrumpled, are released simultaneously from the same height above the floor. Which one, if either, do you expect to reach the floor first? Explain.

*Q6. Aristotle stated that heavier objects fall faster than lighter objects. Was Aristotle wrong? In what sense could Aristotle's view be considered correct?

Q7. Two identical pieces of paper, one crumpled into a ball and the other left uncrumpled, are released simultaneously from inside the top of a large evacuated tube. Which one, if either, do you expect will reach the bottom of the tube first? Explain.

Q8. A rock is dropped from the top of a diving platform into the swimming pool below. Will the distance traveled by the rock in a 0.1-second interval near the top of its flight be the same as the distance covered in a 0.1-second interval just before it hits the water? Explain.

Q9. The graph shows the velocity plotted against time for a certain falling object. Is the acceleration of this object constant? Explain.



Q10. A ball is thrown downward with a large starting velocity.
a. Will this ball reach the ground sooner than one that is just dropped at the same time from the same height? Explain.
b. Will this ball accelerate more rapidly than one that is dropped with no initial velocity? Explain.

Q11. A ball thrown straight upward moves initially with a decreasing upward velocity. What are the directions of the velocity and acceleration vectors during this part of the motion? Does the acceleration decrease also? Explain.

Q12. A rock is thrown straight upward reaching a height of 20 meters. On its way up, does the rock spend more time in the top 5 meters of its flight than in its first 5 meters of its flight? Explain.

Q13. A ball is thrown straight upward and then returns to the earth. Choosing the positive direction to be upward, sketch a graph of the velocity of this ball against time. Where does the velocity change direction? Explain.

Q14. A ball is thrown straight upward. At the very top of its flight, the velocity of the ball is zero. Is its acceleration at this point also zero? Explain.

*Q15. A ball rolls up an inclined plane, slows to a stop, and then rolls back down. Do you expect the acceleration to be constant during this process? Is the velocity constant? Is the acceleration equal to zero at any point during this motion? Explain.

Q16. A ball is thrown straight upward and then returns to the earth. Does the acceleration change direction during this motion? Explain.

Q17. A ball rolling rapidly along a tabletop rolls off the edge and falls to the floor. At the exact instant that the first ball rolls off the edge, a second ball is dropped from the same height. Which ball, if either, reaches the floor first? Explain.

Q18. For the two balls in question 17, which, if either, has the larger total velocity when it hits the floor? Explain.

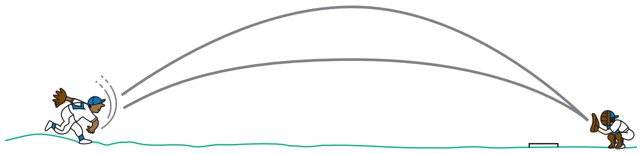
Q19. Is it possible for an object to have a horizontal component of velocity that is constant at the same time that the object is accelerating in the vertical direction? Explain by giving an example, if possible.

Q20. A ball rolls off a table with a large horizontal velocity. Does the direction of the velocity vector change as the ball moves through the air? Explain.

Q21. A ball rolls off a table with a horizontal velocity of 5 m/s. Is this velocity an important factor in determining the time that it takes for the ball to hit the floor? Explain.

Q22. An expert marksman aims a high-speed rifle directly at the center of a nearby target. Assuming that the rifle sight has been accurately adjusted for more distant targets, will the bullet hit the near target above or below the center? Explain.

Q23. In the diagram, two different trajectories are shown for a ball thrown by a center fielder to home plate in a baseball game. Which of the two trajectories (if either), the higher one or the lower one, will result in a longer time for the ball to reach home plate? Explain.



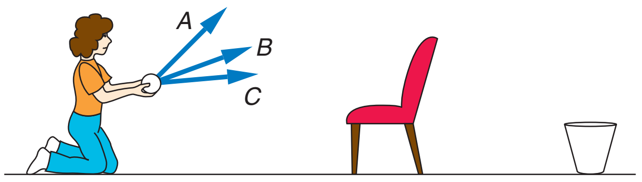
Q24. For either of the trajectories shown in the diagram for question 23, is the velocity of the ball equal to zero at the high point in the trajectory? Explain.

Q25. Assuming that the two trajectories in the diagram for question 23 represent throws by two different center fielders, which of the two is likely to have been thrown by the player with the stronger arm? Explain.

Q26. A cannonball fired at an angle of 70° to the horizontal stays in the air longer than one fired at 45° from the same cannon. Will the 70° shot travel a greater horizontal distance than the 45° shot? Explain.

Q27. Will a shot fired from a cannon at a 20° launch angle travel a longer horizontal distance than a 45° shot? Explain.

Q28. The diagram shows a wastebasket placed behind a chair. Three different directions are indicated for the velocity of a ball thrown by the kneeling woman. Which of the three directions—A, B, or C—is most likely to result in the ball landing in the basket? Explain.



Q29. In the situation pictured in question 28, is the magnitude of the velocity important to the success of the shot? Explain.

Q30. In shooting a free throw in basketball, what is the primary advantage that a high, arching shot has over one with a flatter trajectory? Explain.

Q31. In shooting a basketball from greater than free-throw range, what is the primary disadvantage of a high, arching shot? Explain.

*Q32. A football quarterback must hit a moving target while eluding onrushing linemen. Discuss the advantages and disadvantages of a hard low-trajectory throw to a higher-lofted throw.

exercises

E1. A steel ball is dropped from a diving platform (with an initial velocity of zero). Using the approximate value of $g = 10\ \text{m/s}^2$,
  a. What is the velocity of the ball 0.8 seconds after its release?
  b. What is its velocity 1.6 seconds after its release?

E2. For the ball in exercise 1:
  a. Through what distance does the ball fall in the first 0.8 seconds of its flight? (Assume $g = 10\ \text{m/s}^2$.)
  b. How far does it fall in the first 1.6 seconds of its flight?

E3. A large rock is dropped from the top of a high cliff. Assuming that air resistance can be ignored and that the acceleration has the constant value of $10\ \text{m/s}^2$, how fast would the rock be traveling 5 seconds after it is dropped? What is this speed in MPH? (See inside front cover for conversion factors.)

E4. Suppose Galileo's pulse rate was 75 beats per minute.
  a. What is the time in seconds between consecutive pulse beats?
  b. How far does an object fall in this time when dropped from rest?

E5. A ball is thrown downward with an initial velocity of 12 m/s. Using the approximate value of $g = 10\ \text{m/s}^2$, what is the velocity of the ball 1.0 seconds after it is released?

E6. A ball is dropped from a high building. Using the approximate value of $g = 10\ \text{m/s}^2$, find the *change* in velocity between the first and fourth second of its flight.

E7. A ball is thrown upward with an initial velocity of 15 m/s. Using the approximate value of $g = 10\ \text{m/s}^2$, what are the magnitude and direction of the ball's velocity:
  a. 1 second after it is thrown?
  b. 2 seconds after it is thrown?

E8. How high above the ground is the ball in exercise 7:
  a. 1 second after it is thrown?
  b. 2 seconds after it is thrown?

E9. At what time does the ball in exercise 7 reach the high point in its flight? (Use the approximate value of $g = 10\ \text{m/s}^2$, and remember that the velocity is equal to zero at the high point.)

E10. Suppose that the gravitational acceleration on a certain planet is only $2.0\ \text{m/s}^2$. A space explorer standing on this planet throws a ball straight upward with an initial velocity of 18 m/s.
  a. What is the velocity of the ball 4 seconds after it is thrown?
  b. How much time elapses before the ball reaches the high point in its flight?

E11. A bullet is fired horizontally with an initial velocity of 900 m/s at a target located 150 m from the rifle.
  a. How much time is required for the bullet to reach the target?
  b. Using the approximate value of $g = 10\text{m/s}^2$, how far does the bullet fall in this time?

E12. A ball rolls off a shelf with a horizontal velocity of 6 m/s. At what horizontal distance from the shelf does the ball land if it takes 0.4 s to reach the floor?

E13. A ball rolls off a table with a horizontal velocity of 4 m/s. If it takes 0.5 seconds for the ball to reach the floor, how high above the floor is the tabletop? (Use $g = 10\ \text{m/s}^2$.)

E14. A ball rolls off a table with a horizontal velocity of 5 m/s. If it takes 0.4 seconds for it to reach the floor:
  a. What is the vertical component of the ball's velocity just before it hits the floor? (Use $g = 10\ \text{m/s}^2$.)
  b. What is the horizontal component of the ball's velocity just before it hits the floor?

E15. A ball rolls off a platform that is 5 meters above the ground. The ball's horizontal velocity as it leaves the platform is 6 m/s.
a. How much time does it take for the ball to hit the ground? (See Try This Box 3.3, use $g = 10$ m/s^2.)
b. How far from the base of the platform does the ball hit the ground?

E16. A projectile is fired at an angle such that the vertical component of its velocity and the horizontal component of its velocity are both equal to 30 m/s.
a. Using the approximate value of $g = 10$m/s^2, how long does it take for the ball to reach its high point?
b. What horizontal distance does the ball travel in this time?

challenge problems

CP1. A ball is thrown straight upward with an initial velocity of 16 m/s. Use $g = 10$ m/s^2 for computations listed here.
a. What is its velocity at the high point in its motion?
b. How much time is required to reach the high point?
c. How high above its starting point is the ball at its high point?
d. How high above its starting point is the ball 2 seconds after it is released?
e. Is the ball moving up or down 2 seconds after it is released?

CP2. Two balls are released simultaneously from the top of a tall building. Ball A is simply dropped with no initial velocity, and ball B is thrown downward with an initial velocity of 12 m/s.
a. What are the velocities of the two balls 1.5 seconds after they are released?
b. How far has each ball dropped in 1.5 seconds?
c. Does the difference in the velocities of the two balls change at any time after their release? Explain.

CP3. Two balls are rolled off a tabletop that is 0.8 m above the floor. Ball A has a horizontal velocity of 3 m/s and that of ball B is 5 m/s.
a. Assuming $g = 10$ m/s^2, how long does it take each ball to reach the floor after it rolls off the edge?
b. How far does each ball travel horizontally before hitting the floor?
c. If the two balls started rolling at the same time at a point 1.2 m behind the edge of the table, will they reach the floor at the same time? Explain.

CP4. A cannon is fired over level ground at an angle of 30° to the horizontal. The initial velocity of the cannonball is 400 m/s, but because the cannon is fired at an angle, the vertical component of the velocity is 200 m/s and the horizontal component is 346 m/s.
a. How long is the cannonball in the air? (Use $g = 10$ m/s^2 and the fact that the total time of flight is twice the time required to reach the high point.)
b. How far does the cannonball travel horizontally?
c. Repeat these calculations, assuming that the cannon was fired at a 60° angle to the horizontal, resulting in a vertical component of velocity of 346 m/s and a horizontal component of 200 m/s. How does the distance traveled compare to the earlier result?

CP5. A good pitcher can throw a baseball at a speed of 90 MPH. The pitcher's mound is approximately 60 ft from home plate.
a. What is the speed in m/s?
b. What is the distance to home plate in m?
c. How much time is required for the ball to reach home plate?
d. If the ball is launched horizontally, how far does the ball drop in this time, ignoring the effects of spin?

home experiments and observations

HE1. Gather numerous small objects and drop them from equal heights, two at a time. Record which objects fall significantly more slowly than a compact dense object such as a marble or similar object. Rank order these slower objects by their time of descent. What factors seem to be important in determining this time?

HE2. Try dropping a ball from one hand at the same time that you throw a second ball with your other hand. At first, try to throw the second ball horizontally, with no upward or downward component to its initial velocity. (It may take some practice.)
a. Do the balls reach the floor at the same time? (It helps to enlist a friend for making this judgment.)
b. If the second ball is thrown slightly upward from the horizontal, which ball reaches the ground first?
c. If the second ball is thrown slightly downward from the horizontal, which ball reaches the ground first?

HE3. Take a ball outside and throw it straight up in the air as hard as you can. By counting seconds yourself, or by enlisting a friend with a watch, estimate the time that the ball remains in the air. From this information, can you find the initial velocity that you gave to the ball? (The time required for the ball to reach the high point is just half the total time of flight.)

HE4. Take a stopwatch to a football game and estimate the hang time of several punts. Also note how far (in yards) each punt travels horizontally. Do the highest punts have the longest hang times? Do they travel the greatest distances horizontally?

HE5. Using rubber bands and a plastic rule or other suitable support, design and build a marble launcher. By pulling the rubber band back by the same amount each time, you should be able to launch the marble with approximately the same speed each time.

a. Produce a careful drawing of your launcher and note the design features that you used. (Prizes may be available for the best design.)
b. Placing your launcher at a number of different angles to the horizontal, launch marbles over a level surface and measure the distance that they travel from the point of launch. Which angle yields the greatest distance?
c. Fire the marbles at different angles from the edge of a desk or table. Which angle yields the greatest horizontal distance?

HE6. Try throwing a ball or a wadded piece of paper into a wastebasket placed a few meters from your launch point.

a. Which is most effective, an overhanded or underhanded throw? (Five practice shots followed by ten attempts for each might produce a fair test.)
b. Repeat this process with a barrier such as a chair placed near the wastebasket.