## OUTLINE

Describing Motion
Measuring Motion
Speed
Velocity
Acceleration
A Closer Look: Above It All
Forces
Horizontal Motion on Land A Closer Look: A Bicycle Racer's Edge Falling Objects
A Closer Look: Free Fall
Compound Motion
Vertical Projectiles
Horizontal Projectiles
Three Laws of Motion
Newton's First Law of Motion
Newton's Second Law of Motion
Weight and Mass
Newton's Third Law of Motion
Momentum
Conservation of Momentum Impulse
Forces and Circular Motion
Newton's Law of Gravitation
A Closer Look: Space Station Weightlessness People Behind the Science: Isaac Newton


Information about the mass of a hot air balloon and forces on the balloon will enable you to predict if it is going to move up, down, or drift across the river. This chapter is about such relationships among force, mass, and changes in motion.

## СН A PTER



## Motion

In chapter 1, you learned some "tools and rules" and some techniques for finding order in your physical surroundings. Order is often found in the form of patterns, or relationships between quantities that are expressed as equations. Recall that equations can be used to (1) describe properties, (2) define concepts, and (3) describe how quantities change relative to each other. In all three uses, patterns are quantified, conceptualized, and used to gain a general understanding about what is happening in nature.

In the study of physical science, certain parts of nature are often considered and studied together for convenience. One of the more obvious groupings involves movement. Most objects around you spend a great deal of time sitting quietly without motion. Buildings, rocks, utility poles, and trees rarely, if ever, move from one place to another. Even things that do move from time to time sit still for a great deal of time. This includes you, automobiles, and bicycles (Figure 2.1). On the other hand, the sun, the moon, and starry heavens seem to always move, never standing still. Why do things stand still? Why do things move?

Questions about motion have captured the attention of people for thousands of years. But the ancient people answered questions about motion with stories of mysticism and spirits that lived in objects. It was during the classic Greek culture, between 600 b.c. and 300 B.C., that people began to look beyond magic and spirits. One particular Greek philosopher, Aristotle, wrote a theory about the universe that offered not only explanations about things such as motion but also offered a sense of beauty, order, and perfection. The theory seemed to fit with other ideas that people had and was held to be correct for nearly two thousand years after it was written. It was not until the work of Galileo and Newton during the 1600s that a new, correct understanding about motion was developed. The development of ideas about motion is an amazing and absorbing story. You will learn in this chapter how to describe and use some properties of motion. This will provide some basic understandings about motion and will be very helpful in understanding some important aspects of astronomy and the earth sciences, as well as the movement of living things.

## Describing Motion

Motion is one of the more common events in your surroundings. You can see motion in natural events such as clouds moving, rain and snow falling, and streams of water moving, all in a neverending cycle. Motion can also be seen in the activities of people who walk, jog, or drive various machines from place to place. Motion is so common that you would think everyone would intuitively understand the concepts of motion, but history indicates that it was only during the past three hundred years or so that people began to understand motion correctly. Perhaps the correct concepts are subtle and contrary to common sense, requiring a search for simple, clear concepts in an otherwise complex situation. The process of finding such order in a multitude of sensory impressions by taking measurable data, and then inventing a concept to describe what is happening, is the activity called science. We will now apply this process to motion.

What is motion? Consider a ball that you notice one morning in the middle of a lawn. Later in the afternoon, you notice that the ball is at the edge of the lawn, against a fence, and you wonder if the wind or some person moved the ball. You do not know if the wind blew it at a steady rate, if many gusts of wind moved it, or even if some children kicked it all over the yard. All you know for sure is that the ball has been moved because it is in a different position after some time passed. These are the two important aspects of motion: (1) a change of position and (2) the passage of time.

If you did happen to see the ball rolling across the lawn in the wind, you would see more than the ball at just two locations. You would see the ball moving continuously. You could consider, however, the ball in continuous motion to be a series of individual locations with very small time intervals. Moving involves a change of position during some time period. Motion is the act or process of something changing position.

The motion of an object is usually described with respect to something else that is considered to be not moving. (Such a stationary object is said to be "at rest.") Imagine that you are traveling in an automobile with another person. You know that you are moving across the land outside the car since your location on the highway changes from one moment to another. Observing your fellow passenger, however, reveals no change of position. You are in motion relative to the highway outside the car. You are not in motion relative to your fellow passenger. Your motion, and the motion of any other object or body, is the process of a change in position relative to some reference object or location. Thus motion can be defined as the act or process of changing position relative to some reference during a period of time.

## Measuring Motion

You have learned that objects can be described by measuring certain fundamental properties such as mass and length. Since motion involves (1) a change of position, and (2) the passage of


FIGURE 2.1
The motion of this windsurfer, and of other moving objects, can be described in terms of the distance covered during a certain time period.
time, the motion of objects can be described by using combinations of the fundamental properties of length and time. These combinations of measurement describe three properties of motion: speed, velocity, and acceleration.

## Speed

Suppose you are in a car that is moving over a straight road. How could you describe your motion? You need at least two measurements, (1) the distance you have traveled, and (2) the time that has elapsed while you covered this distance. Such a distance and time can be expressed as a ratio that describes your motion. This ratio is a property of motion called speed, which is a measure of how fast you are moving. Speed is defined as distance per unit of time, or

$$
\text { speed }=\frac{\text { distance }}{\text { time }}
$$



FIGURE 2.2
If you know the value of any two of the three variables of distance, time, and speed, you can find the third. What is the average speed of this car? Two ways of finding the answer are in Figure 2.3.

The units used to describe speed are usually miles/hour (mi/h), kilometers/hour ( $\mathrm{km} / \mathrm{h}$ ), or meters/second ( $\mathrm{m} / \mathrm{s}$ ).

Let's go back to your car that is moving over a straight highway and imagine you are driving to cover equal distances in equal periods of time. If you use a stopwatch to measure the time required to cover the distance between highway mile markers (those little signs with numbers along major highways), the time intervals will all be equal. You might find, for example, that one minute lapses between each mile marker. Such a uniform straight-line motion that covers equal distances in equal periods of time is the simplest kind of motion.

If your car were moving over equal distances in equal periods of time, it would have a constant speed (Figure 2.2). This means that the car is neither speeding up nor slowing down. It is usually difficult to maintain a constant speed. Other cars and distractions such as interesting scenery cause you to reduce your speed. At other times you increase your speed. If you calculate your speed over an entire trip, you are considering a large distance between two places and the total time that elapsed. The increases and decreases in speed would be averaged. Therefore, most speed calculations are for an average speed. The speed at any specific instant is called the instantaneous speed. To calculate the instantaneous speed, you would need to consider a very short time interval-one that approaches zero. An easier way would be to use the speedometer, which shows the speed at any instant.

Constant, instantaneous, or average speeds can be measured with any distance and time units. Common units in the English system are miles/hour and feet/second. Metric units for speed are commonly kilometers/hour and meters/second. The ratio of any distance to time is usually read as distance per time, such as miles per hour. The "per" means "for each."

It is easier to study the relationships between quantities if you use symbols instead of writing out the whole word. The letter $v$ can be used to stand for speed, the letter $d$ can be used to stand for distance, and the letter $t$, to stand for time. A bar over the $v(\bar{v})$ is a
symbol that means average (it is read " $v$-bar" or " $v$-average"). The relationship between average speed, distance, and time is therefore

$$
\bar{v}=\frac{d}{t}
$$

equation 2.1
This is one of the three types of equations that were discussed earlier, and in this case the equation defines a motion property. You can use this relationship to find average speed. For example, suppose a car travels 150 mi in 3 h . What was the average speed? Since $d=150 \mathrm{mi}$, and $t=3 \mathrm{~h}$, then

$$
\begin{aligned}
\bar{v} & =\frac{150 \mathrm{mi}}{3 \mathrm{~h}} \\
& =50 \frac{\mathrm{mi}}{\mathrm{~h}}
\end{aligned}
$$

As with other equations, you can mathematically solve the equation for any term as long as two variables are known (Figure 2.3). For example, suppose you know the speed and the time but want to find the distance traveled. You can solve this by first writing the relationship

$$
\bar{v}=\frac{d}{t}
$$

and then multiplying both sides of the equation by $t$ (to get $d$ on one side by itself),

$$
(\bar{v})(t)=\frac{(d)(t)}{t}
$$

and the t's on the right cancel, leaving

$$
\bar{v} t=d \quad \text { or } \quad d=\bar{v} t
$$



FIGURE 2.3
Speed is distance per unit of time, which can be calculated from the equation or by finding the slope of a distance-versus-time graph. This shows both ways of finding the speed of the car shown in Figure 2.2.

If the $\bar{v}$ is $50 \mathrm{mi} / \mathrm{h}$ and the time traveled is 2 h , then

$$
\begin{aligned}
d & =\left(50 \frac{\mathrm{mi}}{\mathrm{~h}}\right)(2 \mathrm{~h}) \\
& =(50)(2)\left(\frac{\mathrm{mi}}{\mathrm{~h}}\right)(\mathrm{h}) \\
& =100 \frac{(\mathrm{mi})(\not 2)}{\not h} \\
& =100 \mathrm{mi}
\end{aligned}
$$

Notice how both the numerical values and the units were treated mathematically. See "Problem Solving" in chapter 1 for more information.

## example 2.1

The driver of a car moving at $72.0 \mathrm{~km} / \mathrm{h}$ drops a road map on the floor. It takes him 3.00 seconds to locate and pick up the map. How far did he travel during this time?

## Solution

The car has a speed of $72.0 \mathrm{~km} / \mathrm{h}$ and the time factor is 3.00 s , so $\mathrm{km} / \mathrm{h}$ must be converted to $\mathrm{m} / \mathrm{s}$. From inside the front cover of this book, the conversion factor is $1 \mathrm{~km} / \mathrm{h}=0.2778 \mathrm{~m} / \mathrm{s}$, so

$$
\begin{aligned}
\bar{v} & =\frac{0.2778 \frac{\mathrm{~m}}{\mathrm{~s}}}{\frac{\mathrm{~km}}{\mathrm{~h}}} \times 72.0 \frac{\mathrm{~km}}{\mathrm{~h}} \\
& =(0.2778)(72.0) \frac{\mathrm{m}}{\mathrm{~s}} \times \frac{\mathrm{h}}{\mathrm{~km}} \times \frac{\mathrm{km}}{\mathrm{~h}} \\
& =20.0 \frac{\mathrm{~m}}{\mathrm{~s}}
\end{aligned}
$$

The relationship between the three variables, $\bar{v}, t$, and $d$, is found in equation 2.1: $\bar{v}=d / t$.

$$
\begin{array}{rlrl}
\bar{v} & =20.0 \frac{\mathrm{~m}}{\mathrm{~s}} & \bar{v} & =\frac{d}{t} \\
t & =3.00 \mathrm{~s} & \bar{v} t & =\frac{d t}{t} \\
d & =? & d & =\bar{v} t \\
& =\left(20.0 \frac{\mathrm{~m}}{\mathrm{~s}}\right)(3.00 \mathrm{~s}) \\
& =(20.0)(3.00) \frac{\mathrm{m}}{\mathrm{~s}} \times \frac{\mathrm{s}}{1} \\
& =60.0 \mathrm{~m}
\end{array}
$$



A bicycle has an average speed of $8.00 \mathrm{~km} / \mathrm{h}$. How far will it travel in 10.0 seconds? (Answer: 22.2 m)

## Goncents Applied

## Style Speeds

Observe how many different styles of walking you can identify in students walking across the campus. Identify each style with a descriptive word or phrase.

Is there any relationship between any particular style of walking and the speed of walking? You could find the speed of walking by measuring a distance, such as the distance between two trees, then measuring the time required for a student to walk the distance. Find the average speed for each identified style of walking by averaging the walking speeds of ten people.

Report any relationships you find between styles of walking and the average speed of people with each style. Include any problems you found in measuring, collecting data, and reaching conclusions.

## Goncepts Applied

## How Fast Is a Stream?

A stream is a moving body of water. How could you measure the speed of a stream? Would timing how long it takes a floating leaf to move a measured distance help?

What kind of relationship, if any, would you predict for the speed of a stream and a recent rainfall? Would you predict a direct relationship? Make some measurements of stream speeds and compare your findings to recent rainfall amounts.

## Velocity

The word "velocity" is sometimes used interchangeably with the word "speed," but there is a difference. Velocity describes the speed and direction of a moving object. For example, a speed might be described as $60 \mathrm{~km} / \mathrm{h}$. A velocity might be described as


FIGURE 2.4
Here are three different velocities represented by three different arrows. The length of each arrow is proportional to the speed, and the arrowhead shows the direction of travel.
$60 \mathrm{~km} / \mathrm{h}$ to the west. To produce a change in velocity, either the speed or the direction is changed (or both are changed). A satellite moving with a constant speed in a circular orbit around the earth does not have a constant velocity since its direction of movement is constantly changing. Velocity can be represented graphically with arrows. The lengths of the arrows are proportional to the magnitude, and the arrowheads indicate the direction (Figure 2.4).

## Acceleration

Motion can be changed in three different ways: (1) by changing the speed, (2) by changing the direction of travel, or (3) combining both of these by changing both the speed and direction of travel at the same time. Since velocity describes both the speed and the direction of travel, any of these three changes will result in a change of velocity. You need at least one additional measurement to describe a change of motion, which is how much time elapsed while the change was taking place. The change of velocity and time can be combined to define the rate at which the motion was changed. This rate is called acceleration. Acceleration is defined as a change of velocity per unit time, or

$$
\text { acceleration }=\frac{\text { change of velocity }}{\text { time elapsed }}
$$

Another way of saying "change in velocity" is the final velocity minus the initial velocity, so the relationship can also be written as

$$
\text { acceleration }=\frac{\text { final velocity }- \text { initial velocity }}{\text { time elapsed }}
$$

Acceleration due to a change in speed only can be calculated as follows. Consider a car that is moving with a constant, straight-line velocity of $60 \mathrm{~km} / \mathrm{h}$ when the driver accelerates to $80 \mathrm{~km} / \mathrm{h}$. Suppose it takes 4 s to increase the velocity of $60 \mathrm{~km} / \mathrm{h}$ to $80 \mathrm{~km} / \mathrm{h}$.

The change in velocity is therefore $80 \mathrm{~km} / \mathrm{h}$ minus $60 \mathrm{~km} / \mathrm{h}$, or $20 \mathrm{~km} / \mathrm{h}$. The acceleration was

$$
\begin{aligned}
\text { acceleration } & =\frac{80 \frac{\mathrm{~km}}{\mathrm{~h}}-60 \frac{\mathrm{~km}}{\mathrm{~h}}}{4 \mathrm{~s}} \\
& =\frac{20 \frac{\mathrm{~km}}{\mathrm{~h}}}{4 \mathrm{~s}} \\
& =5 \frac{\mathrm{~km} / \mathrm{h}}{\mathrm{~s}} \text { or } \\
& =5 \mathrm{~km} / \mathrm{h} / \mathrm{s}
\end{aligned}
$$

The average acceleration of the car was $5 \mathrm{~km} / \mathrm{h}$ for each ("per") second. This is another way of saying that the velocity increases an average of $5 \mathrm{~km} / \mathrm{h}$ in each second. The velocity of the car was $60 \mathrm{~km} / \mathrm{h}$ when the acceleration began (initial velocity). At the end of 1 s , the velocity was $65 \mathrm{~km} / \mathrm{h}$. At the end of 2 s , it was $70 \mathrm{~km} / \mathrm{h}$; at the end of $3 \mathrm{~s}, 75 \mathrm{~km} / \mathrm{h}$; and at the end of 4 s (total time elapsed), the velocity was $80 \mathrm{~km} / \mathrm{h}$ (final velocity). Note how fast the velocity is changing with time. In summary,

| start (initial velocity) | $60 \mathrm{~km} / \mathrm{h}$ |
| :--- | :--- |
| first second | $65 \mathrm{~km} / \mathrm{h}$ |
| second second | $70 \mathrm{~km} / \mathrm{h}$ |
| third second | $75 \mathrm{~km} / \mathrm{h}$ |
| fourth second (final velocity) | $80 \mathrm{~km} / \mathrm{h}$ |

As you can see, acceleration is really a description of how fast the speed is changing (Figure 2.5); in this case, it is increasing $5 \mathrm{~km} / \mathrm{h}$ each second.

Usually, you would want all the units to be the same, so you would convert $\mathrm{km} / \mathrm{h}$ to $\mathrm{m} / \mathrm{s}$. A change in velocity of $5.0 \mathrm{~km} / \mathrm{h}$ converts to $1.4 \mathrm{~m} / \mathrm{s}$ and the acceleration would be $1.4 \mathrm{~m} / \mathrm{s} / \mathrm{s}$. The units $\mathrm{m} / \mathrm{s}$ per s mean what change of velocity $(1.4 \mathrm{~m} / \mathrm{s})$ is occurring every second. The combination $\mathrm{m} / \mathrm{s} / \mathrm{s}$ is rather cumbersome, so it is typically treated mathematically to simplify the expression (to simplify a fraction, invert the divisor and multiply, or $\mathrm{m} / \mathrm{s} \times 1 / \mathrm{s}=$ $\mathrm{m} / \mathrm{s}^{2}$ ). Remember that the expression $1.4 \mathrm{~m} / \mathrm{s}^{2}$ means the same as $1.4 \mathrm{~m} / \mathrm{s}$ per s , a change of velocity in a given time period.

The relationship among the quantities involved in acceleration can be represented with the symbols $a$ for average acceleration, $v_{\mathrm{f}}$ for final velocity, $v_{\mathrm{i}}$ for initial velocity, and $t$ for time. The relationship is

$$
a=\frac{v_{\mathrm{f}}-v_{\mathrm{i}}}{t}
$$

equation 2.2
As in other equations, any one of these quantities can be found if the others are known. For example, solving the equation for the final velocity, $v_{\mathrm{f}}$, yields:

$$
v_{\mathrm{f}}=a t+v_{\mathrm{i}}
$$

In problems where the initial velocity is equal to zero (starting from rest), the equation simplifies to

$$
v_{\mathrm{f}}=a t
$$

$$
a=\frac{v_{\mathrm{f}}-v_{\mathrm{i}}}{t}=\frac{70 \mathrm{~km} / \mathrm{h}-70 \mathrm{~km} / \mathrm{h}}{4 \mathrm{~s}}=0 \frac{\mathrm{~km} / \mathrm{h}}{\mathrm{~s}}
$$

A


$$
a=\frac{v_{\mathrm{f}}-v_{\mathrm{i}}}{t}=\frac{80 \mathrm{~km} / \mathrm{h}-60 \mathrm{~km} / \mathrm{h}}{4 \mathrm{~s}}=5 \frac{\mathrm{~km} / \mathrm{h}}{\mathrm{~s}}
$$



FIGURE 2.5
(A) This graph shows how the speed changes per unit of time while driving at a constant $70 \mathrm{~km} / \mathrm{h}$ in a straight line. As you can see, the speed is constant, and for straight-line motion the acceleration is 0 . (B) This graph shows the speed increasing from $60 \mathrm{~km} / \mathrm{h}$ to $80 \mathrm{~km} / \mathrm{h}$ for 5 s . The acceleration, or change of velocity per unit of time, can be calculated either from the equation for acceleration or by calculating the slope of the straight-line graph. Both will tell you how fast the motion is changing with time.

Recall from chapter 1 that the symbol $\Delta$ means "the change in" a value. Therefore, equation 2.1 for speed could be written

$$
\bar{v}=\frac{\Delta d}{t}
$$

and equation 2.2 for acceleration could be written

$$
a=\frac{\Delta v}{t}
$$

This shows that both equations are a time rate of change. Speed is a time rate change of distance. Acceleration is a time rate change of velocity. The time rate of change of something is an important concept that you will meet again in the next chapter.

## A Closer Look

## Above It All

The super-speed magnetic levitation (maglev) train is a completely new technology based on magnetically suspending a train 3 to 10 cm (about 1 to 4 in) above a monorail, then moving it along with a magnetic field that travels along the monorail guides. The maglev train does not have friction between wheels and the rails since it
does not have wheels. This lack of resistance at the easily manipulated magnetic fields makes very short acceleration distances possible. For example, a German maglev train can accelerate from 0 to $300 \mathrm{~km} / \mathrm{h}$ (about $185 \mathrm{mi} / \mathrm{h}$ ) over a distance of just 5 km (about 3 mi ). A conventional train with wheels requires about 30 km (about 19 mi )
to reach the same speed from a standing start. The maglev is attractive for short runs because of its superior acceleration and braking abilities. It is also attractive for longer runs because of its high top speedup to about $500 \mathrm{~km} / \mathrm{h}$ (about $310 \mathrm{mi} / \mathrm{h}$ ). Today, only an aircraft can match such a speed.

## example 2.3

A bicycle moves from rest to $5 \mathrm{~m} / \mathrm{s}$ in 5 s . What was the acceleration?

## Solution

$$
\begin{array}{rlrl}
v_{\mathrm{i}} & =0 \mathrm{~m} / \mathrm{s} & & \\
v_{\mathrm{f}} & =5 \mathrm{~m} / \mathrm{s} \\
t & =5 \mathrm{~s} & a & =\frac{v_{\mathrm{f}}-v_{\mathrm{i}}}{t} \\
a & =? & & =\frac{5 \mathrm{~m} / \mathrm{s}-0 \mathrm{~m} / \mathrm{s}}{5 \mathrm{~s}} \\
& & =\frac{5}{5} \frac{\mathrm{~m} / \mathrm{s}}{\mathrm{~s}} \\
& & =1\left(\frac{\mathrm{~m}}{\mathrm{~s}}\right)\left(\frac{1}{\mathrm{~s}}\right) \\
& & =1 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}
\end{array}
$$

## example 2.4

An automobile uniformly accelerates from rest at $15 \mathrm{ft} / \mathrm{s}^{2}$ for 6 s . What is the final velocity in $\mathrm{ft} / \mathrm{s}$ ? (Answer: $90 \mathrm{ft} / \mathrm{s}$ )

So far, you have learned only about straight-line, uniform acceleration that results in an increased velocity. There are also other changes in the motion of an object that are associated with acceleration. One of the more obvious is a change that results in a decreased velocity. Your car's brakes, for example, can slow your car or bring it to a complete stop. This is negative acceleration, which is sometimes called deceleration. Another change in the motion of an object is a change of direction. Velocity encompasses both the rate of motion and direction, so
a change of direction is an acceleration. The satellite moving with a constant speed in a circular orbit around the earth is constantly changing its direction of movement. It is therefore constantly accelerating because of this constant change in its motion. Your automobile has three devices that could change the state of its motion. Your automobile therefore has three accelerators-the gas pedal (which can increase magnitude of velocity), the brakes (which can decrease magnitude of velocity), and the steering wheel (which can change direction of velocity). (See Figure 2.6.) The important thing to remember is that acceleration results from any change in the motion of an object.

The final velocity $\left(v_{\mathrm{f}}\right)$ and the initial velocity $\left(v_{\mathrm{i}}\right)$ are different variables than the average velocity $(\bar{v})$. You cannot use an initial or final velocity for an average velocity. You may, however, calculate an average velocity $(\bar{v})$ from the other two variables as long as the acceleration taking place between the initial and final velocities is uniform. An example of such a uniform change would be an automobile during a constant, straight-line acceleration. To find an average velocity during a uniform


FURE 2.6
Four different ways $(A-D)$ to accelerate a car.
acceleration, you add the initial velocity and the final velocity and divide by 2 . This averaging can be done for a uniform acceleration that is increasing the velocity or for one that is decreasing the velocity. In symbols,

$$
\bar{v}=\frac{v_{\mathrm{f}}+v_{\mathrm{i}}}{2}
$$

equation 2.3

## example 2.5

An automobile moving at $88 \mathrm{ft} / \mathrm{s}$ comes to a stop in 10.0 s when the driver slams on the brakes. How far did the car travel while stopping?

## Solution

The car has an initial velocity of $88 \mathrm{ft} / \mathrm{s}\left(v_{\mathrm{i}}\right)$ and the final velocity of $0 \mathrm{ft} / \mathrm{s}\left(v_{f}\right)$ is implied. The time of $10.0 \mathrm{~s}(t)$ is given. The problem asked for the distance ( $d$ ). The relationship given between $\bar{v}, t$, and $d$ is given in equation $2.1, \bar{v}=d / t$, which can be solved for $d$. The average velocity $(\bar{v})$, however, is not given but can be found from equation 2.3 ,

$$
\begin{array}{rlrl}
\bar{v} & =\frac{v_{\mathrm{f}}+v_{\mathrm{i}}}{2} & & \bar{v} \\
v_{\mathrm{i}} & =88 \mathrm{ft} / \mathrm{s} & \therefore d=\bar{v} \cdot t \\
v_{\mathrm{f}} & =0 \mathrm{ft} / \mathrm{s} & & \text { Since } \bar{v}=\frac{v_{\mathrm{f}}+v_{\mathrm{i}}}{2}, \\
t & =10.0 \mathrm{~s} & & \text { you can substitute }\left(\frac{v_{\mathrm{f}}+v_{\mathrm{i}}}{2}\right) \text { for } \bar{v}, \text { and } \\
\bar{v} & =? & & d \\
d & =? & \left(\frac{v_{\mathrm{f}}+v_{\mathrm{i}}}{2}\right)(t) \\
& =\left(\frac{0 \frac{\mathrm{ft}}{\mathrm{~s}}+88 \frac{\mathrm{ft}}{\mathrm{~s}}}{2}\right)(10.0 \mathrm{~s}) \\
& =44 \times 10.0 \frac{\mathrm{ft}}{\mathrm{~s}} \times \mathrm{s} \\
& & =440 \frac{\mathrm{ft} \cdot \mathrm{~s}}{\mathrm{~s}} \\
& & & 440 \mathrm{ft}
\end{array}
$$

## example 2.6

What was the deceleration of the automobile in example 2.5? (Answer: - $8.8 \mathrm{ft} / \mathrm{s}^{2}$ )

## Goncents Applied

## Acceleration Patterns

Suppose the radiator in your car has a leak and drops fall constantly, one every second. What pattern would the drops make on the pavement when you accelerate the car from a stoplight? What pattern would they make when you drive at a constant speed? What pattern would you observe as the car comes to a stop? Use a marker to make dots on a sheet of paper that illustrate (1) acceleration, (2) constant speed, and (3) negative acceleration. Use words to describe the acceleration in each situation.

## Forces

The Greek philosopher Aristotle considered some of the first ideas about the causes of motion back in the fourth century b.C. However, he had it all wrong when he reportedly stated that a dropped object falls at a constant speed that is determined by its weight. He also incorrectly thought that an object moving across the earth's surface requires a continuously applied force in order to continue moving. These ideas were based on observing and thinking, not measurement, and no one checked to see if they were correct. It would take about two thousand years before people began to correctly understand motion.

Aristotle did recognize an association between force and motion, and this much was acceptable. It is partly correct because a force is closely associated with any change of motion, as you will see. This section introduces the concept of a force, which will be developed more fully when the relationship between forces and motion is considered.

A force is a push or a pull that is capable of changing the state of motion of an object. Consider, for example, the movement of a ship from the pushing of two tugboats (Figure 2.7). Tugboats can vary the strength of the force exerted on a ship, but they can also push in different directions. What effect does direction have on two forces acting on an object? If the tugboats were side by side, pushing in the same direction, the overall force is the sum of the two forces. If they act in exactly opposite directions, one pushing on each side of the ship, the overall force is the difference between the strength of the two forces. If they have the same strength, the overall effect is to cancel each other without producing any motion. The net force is the sum of all the forces acting on an object. Net force means "final," after the forces are added (Figure 2.8).

When two parallel forces act in the same direction, they can be simply added. In this case, there is a net force that is equivalent to the sum of the two forces. When two parallel forces act in opposite directions, the net force is the difference in the direction of the larger force. When two forces act neither in a way that is exactly together nor exactly opposite each other, the result will be like a new, different net force having a new direction and strength.


FIGURE 2.7
The rate of movement and the direction of movement of this ship are determined by a combination of direction and size of force from each of the tugboats. Which direction are the two tugboats pushing? What evidence would indicate that one tugboat is pushing with a greater force? If the tugboat by the numbers is pushing with a greater force and the back tugboat is keeping the ship from moving, what will happen?

Forces Applied
Net Force


FIGURE 2.8
(A) When two parallel forces are acting on the ship in the same direction, the net force is the two forces added together. (B) When two forces are opposite and of equal size, the net force is zero. (C) When two parallel forces are not of equal size, the net force is the difference in the direction of the larger force.

Forces have a strength and direction that can be represented by force arrows. The tail of the arrow is placed on the object that feels the force, and the arrowhead points in the direction in which the force is exerted. The length of the arrow is proportional to the strength of the force. The use of force arrows helps you visualize and understand all the forces and how they contribute to the net force.

## Horizontal Motion on Land

Everyday experience seems to indicate that Aristotle's idea about horizontal motion on the earth's surface is correct. After all, moving objects that are not pushed or pulled do come to rest in a short period of time. It would seem that an object keeps moving only if a force continues to push it. A moving automobile will slow and come to rest if you turn off the ignition. Likewise, a ball that you roll along the floor will slow until it comes to rest. Is the natural state of an object to be at rest, and is a force necessary to keep an object in motion? This is exactly what people thought until Galileo published his book Two New Sciences in 1638, which described his findings about motion. The book had three parts that dealt with uniform motion, accelerated motion, and projectile motion. Galileo described details of simple experiments, measurements, calculations, and thought experiments as he developed definitions and concepts of motion. In one of his thought experiments, Galileo presented an argument against Aristotle's view that a force is needed to keep an object in motion. Galileo imagined an object (such as a ball) moving over a horizontal surface without the force of friction. He concluded that the object would move forever with a constant velocity as long as there was no unbalanced force acting to change the motion.

Why does a rolling ball slow to a stop? You know that a ball will roll farther across a smooth, waxed floor such as a bowling lane than it will across a floor covered with carpet. The rough carpet offers more resistance to the rolling ball. The resistance of the floor friction is shown by a force arrow, $F_{\text {floor }}$, in Figure 2.9. This force, along with the force arrow for air resistance, $F_{\text {air }}$, opposes the forward movement of the ball. Notice the dashed line arrow in part A of Figure 2.9. There is no other force applied to the ball, so the rolling speed decreases until the ball finally comes to a complete stop. Now imagine what force you would need to exert by pushing with your hand, moving along with the ball to keep it rolling at a uniform rate. An examination of the forces in part B of Figure 2.9 can help you determine the amount of force. The force you apply, $F_{\text {applied }}$, must counteract the resistance forces. It opposes the forces that are slowing down the ball as illustrated by the direction of the arrows. To determine how much force you should apply, look at the arrow equation. $F_{\text {applied }}$ has the same length as the sum of the two resistance forces, but it is in the opposite direction of the resistance forces. Therefore, the overall force, $F_{\text {net }}$, is zero. The ball continues to roll at a uniform rate when you balance the force opposing its motion. It is reasonable, then, that if there were no opposing forces, you would


FIGURE 2.9
(A)This ball is rolling to your left with no forces in the direction of motion. The sum of the force of floor friction ( $F_{\text {floor }}$ ) and the force of air friction $\left(F_{\text {air }}\right)$ results in a net force opposing the motion, so the ball slows to a stop. (B) A force is applied to the moving ball, perhaps by a hand that moves along with the ball. The force applied ( $F_{\text {applied }}$ ) equals the sum of the forces opposing the motion, so the ball continues to move with a constant velocity.
not need to apply a force to keep it rolling. This was the kind of reasoning that Galileo did when he discredited the Aristotelian view that a force was necessary to keep an object moving. Galileo concluded that a moving object would continue moving with a constant velocity if no unbalanced forces were applied, that is, if the net force were zero.

It could be argued that the difference in Aristotle's and Galileo's views of forced motion is really a degree of analysis. After all, moving objects on the earth do come to rest unless continuously pushed or pulled. But Galileo's conclusion describes why they must be pushed or pulled and reveals the true nature of the motion of objects. Aristotle argued that the natural state of objects is to be at rest, and he tried to explain why objects move. Galileo, on the other hand, argued that it is just as natural for objects to be moving, and he tried to explain why they come to rest. Galileo called the behavior of matter that causes it to persist in its state of motion inertia. Inertia is the tendency of an object to remain in unchanging motion or at rest in the absence of an unbalanced force (friction, gravity, or whatever). The development of this concept changed the way people viewed the natural state of an object and opened the way for further understandings about motion. Today, it is understood that a satellite moving through free space will continue to do so with no unbalanced forces acting on it (Figure 2.10A). An unbalanced force is needed to slow the satellite (Figure 2.10B), increase its speed (Figure 2.10C), or change its direction of travel (Figure 2.10D).


FIGURE 2.10
Examine the four illustrations and explain how together they illustrate inertia.

## Falling Objects

Did you ever wonder what happens to a falling rock during its fall? Aristotle reportedly thought that a rock falls at a uniform speed that is proportional to its weight. Thus, a heavy rock would fall at a faster uniform speed than a lighter rock. As stated in a popular story, Galileo discredited Aristotle's conclusion by dropping a solid iron ball and a solid wooden ball simultaneously from the top of the Leaning Tower of Pisa (Figure 2.11). Both balls, according to the story, hit the ground nearly at the same time. To do this, they would have to fall with the same velocity. In other words, the velocity of a falling object does not depend on its weight. Any difference in freely falling bodies is explainable by air resistance. Soon after the time of Galileo the air pump was invented. The air pump could be used to remove the air from a glass tube. The effect of air resistance on falling objects could then be demonstrated by comparing how objects fall in the air with how they fall in an evacuated glass tube. You know that a coin falls faster than a feather when both are dropped in the air. A feather and heavy coin will fall together in the near vacuum of an evacuated glass tube because the effect of air resistance on the feather has been removed. When objects fall toward the earth without considering air resistance, they are said to be in free fall. Free fall considers only gravity and neglects air resistance.

## Goncents Applied

## Falling Bodies

Galileo concluded that all objects fall together, with the same acceleration, when the upward force of air resistance is removed. It would be most difficult to remove air from the room, but it is possible to do some experiments that provide some evidence of how air influences falling objects.

1. Take a sheet of paper and your textbook and drop them side by side from the same height. Note the result.
2. Place the sheet of paper on top of the book and drop them at the same time. Do they fall together?
3. Crumple the sheet of paper into a loose ball and drop the ball and book side by side from the same height.
4. Crumple a sheet of paper into a very tight ball and again drop the ball and book side by side from the same height.

Explain any evidence you found concerning how objects fall.

Galileo concluded that light and heavy objects fall together in free fall, but he also wanted to know the details of what was going on while they fell. He now knew that the velocity of an object in free fall was not proportional to the weight of the object. He observed that the velocity of an object in free fall increased as the object fell and reasoned from this that the velocity of the falling

Galileo was one of the first to recognize the role of friction in opposing motion. As shown in Figure 2.9, friction with the surface and air friction combine to produce a net force that works against anything that is moving on the surface. This article is about air friction and some techniques that bike riders use to reduce that opposing force-perhaps giving them an edge in a close race.

The bike riders in Box Figure 2.1 are forming a single-file line, called a paceline, because the slipstream reduces the air resistance for a closely trailing rider. Cyclists say that riding in the slipstream of another cyclist will save much of their energy. They can move up to $5 \mathrm{mi} / \mathrm{h}$ faster than they would expending the same energy riding alone.

In a sense, riding in a slipstream means that you do not have to push as much air out of your way. It has been estimated that at $20 \mathrm{mi} / \mathrm{h}$ a cyclist must move a little less than half a ton of air out of the way every minute. One of the earliest demonstrations of how a slipstream can help a cyclist was done back about the turn of the century. Charles Murphy had a special bicycle trail built down the middle of a railroad track. Riding very close behind a special train caboose, Murphy was


BOX FIGURE 2.1
The object of the race is to be in the front, to finish first. If this is true, why are these racers forming a single file line?
able to reach a speed of over $60 \mathrm{mi} / \mathrm{h}$ for a one-mile course. More recently, cyclists have reached over $125 \mathrm{mi} / \mathrm{h}$ by following close, in the slipstream of a race car.

Along with the problem of moving about a half-ton of air out of the way every minute, there are two basic factors related to air resistance. These are (1) a turbulent versus a smooth flow of air, and (2) the problem of frictional drag. A turbulent flow of air contributes to air resistance because it causes the air to separate slightly on the back side, which increases the pressure on the
front of the moving object. This is why racing cars, airplanes, boats, and other racing vehicles are streamlined to a teardrop-like shape. This shape is not as likely to have the lower-pressure-producing air turbulence behind (and resulting greater pressure in front) because it smoothes, or streamlines the air flow.

The frictional drag of air is similar to the frictional drag that occurs when you push a book across a rough tabletop. You know that smoothing the rough tabletop will reduce the frictional drag on the book. Likewise, the smoothing of a surface exposed to moving air will reduce air friction. Cyclists accomplish this "smoothing" by wearing smooth lycra clothing, and by shaving hair from arm and leg surfaces that are exposed to moving air. Each hair contributes to the overall frictional drag, and removal of the arm and leg hair can thus result in seconds saved. This might provide enough of an edge to win a close race. Shaving legs and arms, together with the wearing of lycra or some other tight, smooth-fitting garments, are just a few of the things a cyclist can do to gain an edge. Perhaps you will be able to think of more ways to reduce the forces that oppose motion.


FIGURE 2.11
According to a widespread story, Galileo dropped two objects with different weights from the Leaning Tower of Pisa. They reportedly hit the ground at about the same time, discrediting Aristotle's view that the speed during the fall is proportional to weight.
object would have to be (1) somehow proportional to the time of fall and (2) somehow proportional to the distance the object fell. If the time and distance were both related to the velocity of a falling object, how were they related to one another? To answer this question, Galileo made calculations involving distance, velocity, and time and, in fact, introduced the concept of acceleration. The relationships between these variables are found in the same three equations that you have already learned. Let's see how the equations can be rearranged to incorporate acceleration, distance, and time for an object in free fall.

Step 1: Equation 2.1 gives a relationship between average velocity $(\bar{v})$, distance $(d)$, and time $(t)$. Solving this equation for distance gives

$$
d=\bar{v} t
$$

Step 2: An object in free fall should have uniformly accelerated motion, so the average velocity could be calculated from equation 2.3,

$$
\bar{v}=\frac{v_{\mathrm{f}}+v_{\mathrm{i}}}{2}
$$



FIGURE 2.12
An object dropped from a tall building covers increasing distances with every successive second of falling. The distance covered is proportional to the square of the time of falling $\left(d \propto t^{2}\right)$.

Substituting this equation in the rearranged equation 2.1 , the distance relationship becomes

$$
d=\left(\frac{v_{\mathrm{f}}+v_{\mathrm{i}}}{2}\right)(t)
$$

Step 3: The initial velocity of a falling object is always zero just as it is dropped, so the $v_{\mathrm{i}}$ can be eliminated,

$$
d=\left(\frac{v_{\mathrm{f}}}{2}\right)(t)
$$

Step 4: Now you want to get acceleration into the equation in place of velocity. This can be done by solving equation 2.2 for the final velocity $\left(v_{\mathrm{f}}\right)$, then substituting. The initial velocity $\left(v_{\mathrm{i}}\right)$ is again eliminated because it equals zero.

$$
\begin{aligned}
& a=\frac{v_{\mathrm{f}}-v_{\mathrm{i}}}{t} \\
& v_{\mathrm{f}}=a t \\
& d=\left(\frac{a t}{2}\right)(t)
\end{aligned}
$$

Step 5: Simplifying, the equation becomes

$$
d=\frac{1}{2} a t^{2}
$$

equation 2.4
Thus, Galileo reasoned that a freely falling object should cover a distance proportional to the square of the time of the fall ( $d \propto t^{2}$ ).


FIGURE 2.13
The velocity of a falling object increases at a constant rate, $9.8 \mathrm{~m} / \mathrm{s}^{2}$.

In other words the object should fall 4 times as far in 2 s as in 1 s $\left(2^{2}=4\right), 9$ times as far in $3 \mathrm{~s}\left(3^{2}=9\right)$, and so on. Compare this prediction with Figure 2.12.

Galileo checked this calculation by rolling balls on an inclined board with a smooth groove in it. He used the inclined board to slow the motion of descent in order to measure the distance and time relationships, a necessary requirement since he lacked the accurate timing devices that exist today. He found, as predicted, that the falling balls moved through a distance proportional to the square of the time of falling. This also means that the velocity of the falling object increased at a constant rate, as shown in Figure 2.13. Recall that a change of velocity during some time period is called acceleration. In other words, a falling object accelerates toward the surface of the earth.

Since the velocity of a falling object increases at a constant rate, this must mean that falling objects are uniformly accelerated by the force of gravity. All objects in free fall experience a constant acceleration. During each second of fall, the object gains $9.8 \mathrm{~m} / \mathrm{s}(32 \mathrm{ft} / \mathrm{s})$ in velocity. This gain is the acceleration of the falling object, $9.8 \mathrm{~m} / \mathrm{s}^{2}\left(32 \mathrm{ft} / \mathrm{s}^{2}\right)$.

The acceleration of objects falling toward the earth varies slightly from place to place on the earth's surface because of the earth's shape and spin. The acceleration of falling objects decreases from the poles to the equator and also varies from place to place because the earth's mass is not distributed equally. The value of $9.8 \mathrm{~m} / \mathrm{s}^{2}\left(32 \mathrm{ft} / \mathrm{s}^{2}\right)$ is an approximation that is fairly close to, but not exactly, the acceleration due to gravity in any particular location. The acceleration due to gravity is important in a number of situations, so the acceleration from this force is given a special symbol, $g$.

## A Closer Look

There are two different meanings for the term "free fall." In physics, "free fall" means the unconstrained motion of a body in a gravitational field, without considering air resistance. Without air resistance all objects are assumed to accelerate toward the surface at $9.8 \mathrm{~m} / \mathrm{s}^{2}$.

In the sport of skydiving, "free fall" means falling within the atmosphere without a drag-producing device such as a parachute. Air provides a resisting force that opposes the motion of a falling object, and the net force is the difference between the downward force (weight) and the upward force of air resistance. The weight of the falling object depends on the mass and acceleration from gravity, and this is the force downward. The resisting force
is determined by at least two variables, (1) the area of the object exposed to the airstream, and (2) the speed of the falling object. Other variables such as streamlining, air temperature, and turbulence play a role, but the greatest effect seems to be from exposed area and the increased resistance as speed increases.

A skydiver's weight is constant, so the downward force is constant. Modern skydivers typically free-fall from about $3,650 \mathrm{~m}$ (about $12,000 \mathrm{ft}$ ) above the ground until about 750 m (about 2,500 ft), where they open their parachutes. After jumping from the plane, the diver at first accelerates toward the surface, reaching speeds up to about 185 to $210 \mathrm{~km} / \mathrm{h}$ (about 115 to 130 $\mathrm{mi} / \mathrm{h}$ ). The air resistance increases with
increased speed and the net force becomes less and less. Eventually, the downward weight force will be balanced by the upward air resistance force, and the net force becomes zero. The person now falls at a constant speed and we say the terminal velocity has been reached. It is possible to change your body position to vary your rate of fall up or down to $32 \mathrm{~km} / \mathrm{h}$ (about $20 \mathrm{mi} / \mathrm{h}$ ). However, by diving or "standing up" in free fall, experienced skydivers can learn to reach speeds of up to $290 \mathrm{~km} / \mathrm{h}$ (about 180 $\mathrm{mi} / \mathrm{h}$ ). The record free fall speed, done without any special equipment, is $517 \mathrm{~km} / \mathrm{h}$ (about $321 \mathrm{mi} / \mathrm{h}$ ). Once the parachute opens, a descent rate of about $16 \mathrm{~km} / \mathrm{h}$ (about $10 \mathrm{mi} / \mathrm{h}$ ) is typical.

## example 2.7

A rock that is dropped into a well hits the water in 3.0 s. Ignoring air resistance, how far is it to the water?

## Solution 1

The problem concerns a rock in free fall. The time of fall ( $t$ ) is given, and the problem asks for a distance ( $d$ ). Since the rock is in free fall, the acceleration due to the force of gravity $(\mathrm{g})$ is implied. The metric value and unit for $g$ is $9.8 \mathrm{~m} / \mathrm{s}^{2}$, and the English value and unit is $32 \mathrm{ft} / \mathrm{s}^{2}$. You would use the metric $g$ to obtain an answer in meters and the English unit to obtain an answer in feet. Equation 2.4, $d=$ $1 / 2 a t^{2}$, gives a relationship between distance ( $d$ ), time ( $t$ ), and average acceleration (a). The acceleration in this case is the acceleration due to gravity (g), so

$$
\begin{array}{rlrl}
t=3.0 \mathrm{~s} & d & =\frac{1}{2} g t^{2}\left(a=g=9.8 \mathrm{~m} / \mathrm{s}^{2}\right) \\
g=9.8 \mathrm{~m} / \mathrm{s}^{2} & d & =\frac{1}{2}\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)(3.0 \mathrm{~s})^{2} \\
d=? & & =\left(4.9 \mathrm{~m} / \mathrm{s}^{2}\right)\left(9.0 \mathrm{~s}^{2}\right) \\
& =44 \frac{\mathrm{~m} \cdot \mathrm{~s}^{2}}{\$^{2}} \\
& =44 \mathrm{~m}
\end{array}
$$

## Solution 2

You could do each step separately. Check this solution by a threestep procedure:

1. find the final velocity, $v_{f}$ of the rock from $\bar{v}_{f}=a t$;
2. calculate the average velocity $(v)$ from the final velocity

$$
\bar{v}=\frac{v_{f}+v_{i}}{2}
$$

then;
3. use the average velocity ( $\bar{v}$ ) and the time ( $t$ ) to find distance $\bar{v}(d), d=\bar{v} t$.

Note that the one-step procedure is preferred over the threestep procedure because fewer steps mean fewer possibilities for mistakes.

## Compound Motion

So far we have considered two types of motion: (1) the horizontal, straight-line motion of objects moving on the surface of the earth and (2) the vertical motion of dropped objects that accelerate toward the surface of the earth. A third type of motion occurs when an object is thrown, or projected, into the air. Essentially, such a projectile (rock, football, bullet, golf ball, or


FIGURE 2.14
High-speed multiflash photograph of a freely falling billiard ball.
whatever) could be directed straight upward as a vertical projection, directed straight out as a horizontal projection, or directed at some angle between the vertical and the horizontal. Basic to understanding such compound motion is the observation that (1) gravity acts on objects at all times, no matter where they are, and (2) the acceleration due to gravity $(g)$ is independent of any motion that an object may have.

## Vertical Projectiles

Consider first a ball that you throw straight upward, a vertical projection. The ball has an initial velocity but then reaches a maximum height, stops for an instant, then accelerates back toward the earth. Gravity is acting on the ball throughout its climb, stop, and fall. As it is climbing, the force of gravity is accelerating it back to the earth. The overall effect during the climb is deceleration, which continues to slow the ball until the instantaneous stop. The ball then accelerates back to the surface just like a ball that has been dropped (Figure 2.14). If it were not for air resistance, the ball would return with the same speed in the opposite direction that it had initially. The velocity arrows for a ball thrown straight up are shown in Figure 2.15.

## Horizontal Projectiles

Horizontal projections are easier to understand if you split the complete motion into vertical and horizontal parts. Con-


FIGURE 2.15
On its way up, a vertical projectile is slowed by the force of gravity until an instantaneous stop; then it accelerates back to the surface, just as another ball does when dropped from the same height. The straight up and down moving ball has been moved to the side in the sketch so we can see more clearly what is happening. Note that the falling ball has the same speed in the opposite direction that it had on the way up.
sider, for example, an arrow shot horizontally from a bow. The force of gravity accelerates the arrow downward, giving it an increasing downward velocity as it moves through the air. This increasing downward velocity is shown in Figure 2.16 as increasingly longer velocity arrows ( $v_{\mathrm{v}}$ ). There are no forces in the horizontal direction if you ignore air resistance, so the horizontal velocity of the arrow remains the same as shown by the $v_{\mathrm{h}}$ velocity arrows. The combination of the increasing vertical $\left(v_{\mathrm{v}}\right)$ motion and the unchanging horizontal ( $v_{\mathrm{h}}$ ) motion causes the arrow to follow a curved path until it hits the ground.

An interesting prediction that can be made from the shot arrow analysis is that an arrow shot horizontally from a bow will hit the ground at the same time as a second arrow that is simply dropped from the same height (Figure 2.16). Would this be true of a bullet dropped at the same time as one fired horizontally from a rifle? The answer is yes; both bullets would hit the


FIGURE 2.16
A horizontal projectile has the same horizontal velocity throughout the fall as it accelerates toward the surface, with the combined effect resulting in a curved path. Neglecting air resistance, an arrow shot horizontally will strike the ground at the same time as one dropped from the same height above the ground, as shown here by the increasing vertical velocity arrows.
ground at the same time. Indeed, if you ignore air resistance, all the bullets and arrows should hit the ground at the same time if dropped or shot from the same height.

Golf balls, footballs, and baseballs are usually projected upward at some angle to the horizon. The horizontal motion of these projectiles is constant as before because there are no horizontal forces involved. The vertical motion is the same as that of a ball projected directly upward. The combination of these two motions causes the projectile to follow a curved path called a parabola, as shown in Figure 2.17. The next time you have the opportunity, observe the path of a ball that has been projected at some angle. Note that the second half of the path is almost a reverse copy of the first half. If it were not for air resistance, the two values of the path would be exactly the same. Also note the distance that the ball travels as compared
to the angle of projection. An angle of projection of $45^{\circ}$ results in the maximum distance of travel if air resistance is ignored and if the launch point and the landing are at the same elevation.

## Three Laws of Motion

In the previous sections you learned how to describe motion in terms of distance, time, velocity, and acceleration. In addition, you learned about different kinds of motion, such as straightline motion, the motion of falling objects, and the compound motion of objects projected up from the surface of the earth. You were also introduced, in general, to two concepts closely associated with motion: (1) that objects have inertia, a tendency


FIGURE 2.17
A football is thrown at some angle to the horizon when it is passed downfield. Neglecting air resistance, the horizontal velocity is a constant, and the vertical velocity decreases, then increases, just as in the case of a vertical projectile. The combined motion produces a parabolic path. Contrary to statements by sportscasters about the abilities of certain professional quarterbacks, it is impossible to throw a football with a"flat trajectory" because it begins to accelerate toward the surface as soon as it leaves the quarterback's hand.
to resist a change in motion, and (2) that forces are involved in a change of motion.

The relationship between forces and a change of motion is obvious in many everyday situations (Figure 2.18). When a car, bus, or plane starts moving, you feel a force on your back. Likewise, you feel a force on the bottoms of your feet when an elevator starts moving upward. On the other hand, you seem to be forced toward the dashboard if a car stops quickly, and it feels as if the floor pulls away from your feet when an elevator drops rapidly. These examples all involve patterns between forces and motion, patterns that can be quantified, conceptualized, and used to answer questions about why things move or stand still. These patterns are the subject of Newton's three laws of motion.

## Newton's First Law of Motion

Newton's first law of motion is also known as the law of inertia and is very similar to one of Galileo's findings about motion. Recall that Galileo used the term inertia to describe the tendency of an object to resist changes in motion. Newton's first law describes this tendency more directly. In modern terms (not Newton's words), the first law of motion is as follows:

Every object retains its state of rest or its state of uniform straight-line motion unless acted upon by an unbalanced force.

This means that an object at rest will remain at rest unless it is put into motion by an unbalanced force; that is, the net force must be greater than zero. Likewise, an object moving with uniform straight-line motion will retain that motion unless a net force causes it to speed up, slow down, or change its direction of travel. Thus, Newton's first law describes the tendency of an object to resist any change in its state of motion.


FIGURE 2.18
In a moving airplane, you feel forces in many directions when the plane changes its motion. You cannot help but notice the forces involved when there is a change of motion.

Think of Newton's first law of motion when you ride standing in the aisle of a bus. The bus begins to move, and you, being an independent mass, tend to remain at rest. You take a few steps back as you tend to maintain your position relative to the ground outside. You reach for a seat back or some part of the bus. Once you have a hold on some part of the bus it supplies the forces needed to give you the same motion as the bus and you no longer find it necessary to step backward. You now have the same motion as the bus, and no forces are involved, at least until the bus goes around a curve. You now feel a tendency to move to the side of the bus. The bus has changed its straight-line motion, but you, again being an independent mass, tend to move straight ahead. The side of the seat forces you into following the curved motion of the bus. The forces you feel when the bus starts moving or turning are a result of your tendency to remain at rest or follow a straight path until forces correct your motion so that it is the same as that of the bus (Figure 2.19).

## Goncents Applied

## First Law Experiment

Place a small ball on a flat part of the floor in a car, SUV, or pickup truck. First, predict what will happen to the ball in each of the following situations: (1) The vehicle moves forward from a stopped position. (2) The vehicle is moving at a constant speed. (3) The vehicle is moving at a constant speed, then turns to the right. (4) The vehicle is moving at a constant speed, then comes to a stop. Now, test your predictions, and then explain each finding in terms of Newton's first law of motion.


FIGURE 2.19
Top view of a person standing in the aisle of a bus. (A) The bus is at rest, and then starts to move forward. Inertia causes the person to remain in the original position, appearing to fall backward. (B) The bus turns to the right, but inertia causes the person to retain the original straight-line motion until forced in a new direction by the side of the bus.

## Newton's Second Law of Motion

Newton had successfully used Galileo's ideas to describe the nature of motion. Newton's first law of motion explains that any object, once started in motion, will continue with a constant velocity in a straight line unless a force acts on the moving object. This law not only describes motion but establishes the role of a force as well. A change of motion is therefore evidence of the action of net force. The association of forces and a change of motion is common in your everyday experience. You have felt forces on your back in an accelerating automobile, and you have felt other forces as the automobile turns or stops. You have also learned about gravitational forces that accelerate objects toward the surface of the earth. Unbalanced forces and acceleration are involved in any change of motion. Newton's second law of motion is a relationship between net force, acceleration, and mass that describes the cause of a change of motion.

Consider the motion of you and a bicycle you are riding. Suppose you are riding your bicycle over level ground in a straight line at 10 miles per hour. Newton's first law tells you that you will continue with a constant velocity in a straight line as long as no external, unbalanced force acts on you and the bicycle. The force that you are exerting on the pedals seems to equal some external force that moves you and the bicycle along (more on this later). The force exerted as you move along is needed to balance the resisting forces of tire friction and air


FIGURE 2.20
At a constant velocity the force of tire friction $\left(F_{1}\right)$ and the force of air resistance $\left(F_{2}\right)$ have a sum that equals the force applied $\left(F_{\mathrm{a}}\right)$. The net force is therefore 0 .
resistance. If these resisting forces were removed you would not need to exert any force at all to continue moving at a constant velocity. The net force is thus the force you are applying minus the forces from tire friction and air resistance. The net force is therefore zero when you move at a constant speed in a straight line (Figure 2.20).

If you now apply a greater force on the pedals the extra force you apply is unbalanced by friction and air resistance. Hence there will be a net force greater than zero, and you will accelerate. You will accelerate during, and only during, the time that the net force is greater than zero. Likewise, you will slow down if you apply a force to the brakes, another kind of resisting friction. A third way to change your velocity is to apply a force on the handlebars, changing the direction of your velocity. Thus, unbalanced forces on you and your bicycle produce an acceleration.

Starting a bicycle from rest suggests a relationship between force and acceleration. You observe that the harder you push on the pedals, the greater your acceleration. Recall that when quantities increase or decrease together in the same ratio, they are said to be directly proportional. The acceleration is therefore directly proportional to the net force applied.

Suppose that your bicycle has two seats, and you have a friend who will ride with you. Suppose also that the addition of your friend on the bicycle will double the mass of the bike and riders. If you use the same net force as before, the bicycle will undergo a much smaller acceleration. In fact, with all other factors equal, doubling the mass and applying the same extra force will produce an acceleration of only half as much (Figure 2.21). An even more massive friend would reduce the acceleration even more. Recall that when a relationship between two quantities shows that one quantity increases as another decreases, in the same ratio, the quantities are said to be


FIGURE 2.21
More mass results in less acceleration when the same force is applied. With the same force applied, the riders and bike with twice the mass will have half the acceleration, with all other factors constant. Note that the second rider is not pedaling.
inversely proportional. The acceleration of an object is therefore inversely proportional to its mass.

If we express force in appropriate units we can combine these relationships as an equation,

$$
a=\frac{F}{m}
$$

By solving for $F$ we rearrange the equation into the form in which it is most often expressed,

$$
F=m a
$$

equation 2.5
In the metric system you can see that the units for force will be the units for mass $(m)$ times acceleration (a). The unit for mass is kg and the unit for acceleration is $\mathrm{m} / \mathrm{s}^{2}$. The combination of these units, $(\mathrm{kg})\left(\mathrm{m} / \mathrm{s}^{2}\right)$, is a unit of force called the newton (N) in honor of Isaac Newton. So,

$$
1 \text { newton }=1 \mathrm{~N}=1 \frac{\mathrm{~kg} \cdot \mathrm{~m}}{\mathrm{~s}^{2}}
$$

Newton's second law of motion is the essential idea of his work on motion. According to this law there is always a relationship
between the acceleration, a net force, and the mass of an object. Implicit in this statement are two understandings, (1) that we are talking about the net force, meaning total external force acting on an object, and (2) that the motion statement is concerned with acceleration, not velocity.

The acceleration of an object depends on both the net force applied and the mass of the object. The second law of motion is as follows:

The acceleration of an object is directly proportional to the net force acting on it and inversely proportional to the mass of the object.

Until now, equations were used to describe properties of matter such as density, velocity, and acceleration. This is your first example of an equation that is used to define a concept, specifically the concept of what is meant by a force. Since the concept is defined by specifying a measurement procedure, it is also an example of an operational definition. You are told not only what a newton of force is but also how to go about measuring it. Notice that the newton is defined in terms of mass measured in kg and acceleration measured in $\mathrm{m} / \mathrm{s}^{2}$. Any other units must be converted to kg and $\mathrm{m} / \mathrm{s}^{2}$ before a problem can be solved for newtons of force.

## example 2.8

A 60 kg bicycle and rider accelerate at $0.5 \mathrm{~m} / \mathrm{s}^{2}$. How much extra force was applied?

## Solution

The mass ( m ) of 60 kg and the acceleration ( $a$ ) of $0.5 \mathrm{~m} / \mathrm{s}^{2}$ are given. The problem asked for the extra force ( $F$ ) needed to give the mass the acquired acceleration. The relationship is found in equation 2.5, $F=m a$.

$$
\begin{array}{rlrl}
m=60 \mathrm{~kg} & F & =m a \\
a & =0.5 \frac{\mathrm{~m}}{\mathrm{~s}^{2}} & & =(60 \mathrm{~kg})\left(0.5 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right) \\
F=? & & =(60)(0.5)(\mathrm{kg})\left(\frac{\mathrm{m}}{\mathrm{~s}^{2}}\right) \\
& & =30 \frac{\mathrm{~kg} \cdot \mathrm{~m}}{\mathrm{~s}^{2}} \\
& & =30 \mathrm{~N}
\end{array}
$$

An extra force of 30 N beyond that required to maintain constant speed must be applied to the pedals for the bike and rider to maintain an acceleration of $0.5 \mathrm{~m} / \mathrm{s}^{2}$. (Note that the units $\mathrm{kg} \cdot \mathrm{m} / \mathrm{s}^{2}$ form the definition of a newton of force, so the symbol N is used.)

## Example 2.9

What is the acceleration of a 20 kg cart if the net force on it is 40 N ? (Answer: $2 \mathrm{~m} / \mathrm{s}^{2}$ )

## Goncepts Applied

## Second Law Experiment

Tie one end of a string to a book and the other end to a large rubber band. With your index finger curled in the loop of the rubber band, pull the book across a smooth tabletop. How much the rubber band stretches will provide a rough estimate of the force you are applying. (1) Pull the book with a constant velocity across the tabletop. Compare the force required for different constant velocities. (2) Accelerate the book at different rates. Compare the force required to maintain the different accelerations.
(3) Use a different book with a greater mass and again accelerate the book at different rates. How does more mass change the results?

Based on your observations, can you infer a relationship between force, acceleration, and mass?

## Weight and Mass

What is the meaning of weight-is it the same concept as mass? Weight is a familiar concept to most people, and in everyday language the word is often used as having the same meaning as mass. In physics, however, there is a basic difference between weight and mass, and this difference is very important in Newton's explanation of motion and the causes of motion.

Mass is defined as the property that determines how much an object resists a change in its motion. The greater the mass the greater the inertia, or resistance to change in motion. Consider, for example, that it is easier to push a small car into motion than to push a large truck into motion. The truck has more mass, and therefore more inertia. Newton originally defined mass as the "quantity of matter" in an object, and this definition is intuitively appealing. However, Newton needed to measure inertia because of its obvious role in motion, and he redefined mass as a measure of inertia.

You could use Newton's second law to measure a mass by exerting a force on the mass and measuring the resulting acceleration. This is not very convenient, so masses are usually measured on a balance by comparing the force of gravity acting on a standard mass compared to the force of gravity acting on the unknown mass.

The force of gravity acting on a mass is the weight of an object. Weight is a force and has different units ( N ) than mass $(\mathrm{kg})$. Since weight is a measure of the force of gravity acting on an object, the force can be calculated from Newton's second law of motion,

$$
F=m a
$$

or

$$
\text { downward force }=(\text { mass })(\text { acceleration due to gravity })
$$

or

$$
\begin{aligned}
& \text { weight }=(\text { mass })(g) \\
& \text { or } \quad w=m g
\end{aligned}
$$

equation 2.6
You learned in a previous section that $g$ is the symbol used to represent acceleration due to gravity. Near the earth's surface, $g$ has an approximate value of $9.8 \mathrm{~m} / \mathrm{s}^{2}$. To understand how $g$ is applied to an object that is not moving, consider a ball you are holding in your hand. By supporting the weight of the ball you hold it stationary, so the upward force of your hand and the downward force of the ball (its weight) must add to a net force of zero. When you let go of the ball the gravitational force is the only force acting on the ball. The ball's weight is then the net force that accelerates it at $g$, the acceleration due to gravity. Thus, $F_{\text {net }}=w=m a=m g$. The weight of the ball never changes in a given location, so its weight is always equal to $w=m g$, even if the ball is not accelerating.

In the metric system, mass is measured in kilograms. The acceleration due to gravity, $g$, is $9.8 \mathrm{~m} / \mathrm{s}^{2}$. According to equation 2.6, weight is mass times acceleration. A kilogram multiplied by an acceleration measured in $\mathrm{m} / \mathrm{s}^{2}$ results in $\mathrm{kg} \cdot \mathrm{m} / \mathrm{s}^{2}$, a unit you now recognize as a force called a newton. The unit of weight in the metric system is therefore the newton $(N)$.

In the English system the pound is the unit of force. The acceleration due to gravity, $g$, is $32 \mathrm{ft} / \mathrm{s}^{2}$. The force unit of a pound is defined as the force required to accelerate a unit of mass called the slug. Specifically, a force of 1.0 lb will give a 1.0 slug mass an acceleration of $1.0 \mathrm{ft} / \mathrm{s}^{2}$.

The important thing to remember is that pounds and newtons are units of force (Table 2.1). A kilogram, on the other hand, is a measure of mass. Thus the English unit of 1.0 lb is comparable to

| TABIF 2, 1 |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Units of mass and weight in the metric and English systems of measurement |  |  |  |  |  |
|  | Mass | $\times$ | Acceleration | $=$ | Force |
| Metric System | kg | X | $\frac{\mathrm{m}}{\mathrm{~s}^{2}}$ |  | $\frac{\mathrm{kg} \cdot \mathrm{~m}}{\mathrm{~s}^{2}}$ <br> (newton) |
| English System | $\left(\frac{\mathrm{lb}}{\mathrm{ft} / \mathrm{s}^{2}}\right)$ | X | $\frac{\mathrm{ft}}{\mathrm{s}^{2}}$ | $=$ | lb (pound) |

the metric unit of 4.5 N (or 0.22 lb is equivalent to 1.0 N ). Conversion tables sometimes show how to convert from pounds (a unit of weight) to kilograms (a unit of mass). This is possible because weight and mass are proportional in a given location on the surface of the earth. Using conversion factors from inside the front cover of this book, see if you can express your weight in pounds and newtons and your mass in kg.

## example 2.10

What is the weight of a 60.0 kg person on the surface of the earth?

## Solution

A mass ( $m$ ) of 60.0 kg is given, and the acceleration due to gravity (g) $9.8 \mathrm{~m} / \mathrm{s}^{2}$ is implied. The problem asked for the weight ( $w$ ). The relationship is found in equation $2.6, w=m g$, which is a form of $F=m a$.

$$
\begin{aligned}
m & =60.0 \mathrm{~kg} \\
g & =9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}} \\
w & =?
\end{aligned}
$$

$$
\begin{aligned}
w & =m g \\
& =(60.0 \mathrm{~kg})\left(9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right) \\
& =(60.0)(9.8) \quad(\mathrm{kg})\left(\frac{\mathrm{m}}{\mathrm{~s}^{2}}\right) \\
& =588 \frac{\mathrm{~kg} \cdot \mathrm{~m}}{\mathrm{~s}^{2}} \\
& =590 \mathrm{~N}
\end{aligned}
$$

## example 2.11

A 60.0 kg person weighs 100.0 N on the moon. What is the value of $g$ on the moon? (Answer: $1.67 \mathrm{~m} / \mathrm{s}^{2}$ )

## Goncents Applied

## Apparent Weightlessness

Use a sharp pencil to make a small hole in the bottom of a Styrofoam cup. The hole should be large enough for a thin stream of water to flow from the cup, but small enough for the flow to continue for 3 or 4 seconds. Test the water flow over a sink.

Hold a finger over the hole in the cup as you fill it with water. Stand on a ladder or outside stairwell as you hold the cup out at arm's length. Move your finger, allowing a stream of water to flow from the cup, and at the same time drop the cup. Observe what happens to the stream of water as the cup is falling. Explain your observations. Also predict what you would see if you were falling with the cup.

## Newton's Third Law of Motion

Newton's first law of motion states that an object retains its state of motion when the net force is zero. The second law states what happens when the net force is not zero, describing how an object with a known mass moves when a given force is applied. The two laws give one aspect of the concept of a force; that is, if you observe that an object starts moving, speeds up, slows down, or changes its direction of travel, you can conclude that an unbalanced force is acting on the object. Thus, any change in the state of motion of an object is evidence that an unbalanced force has been applied.

Newton's third law of motion is also concerned with forces and considers how a force is produced. First, consider where a force comes from. A force is always produced by the interaction of two or more objects. There is always a second object pushing or pulling on the first object to produce a force. To simplify the many interactions that occur on the earth, consider a satellite in space. According to Newton's second law $(F=m a)$, a force must be applied to change the state of motion of the satellite. What is a possible source of such a force? Perhaps an astronaut pushes on the satellite for 1 second. The satellite would accelerate during the application of the force, then move away from the original position at some constant velocity. The astronaut would also move away from the original position, but in the opposite direction (Figure 2.22). A single force does not exist by itself. There is always a matched and opposite force that occurs at the same time. Thus, the astronaut exerted a momentary force on the satellite, but the satellite evidently exerted a momentary force back on the astronaut as well, for the astronaut moved away from the original position in the opposite direction. Newton did not have astronauts and satellites to think about, but this is the kind of reasoning he did when he concluded that forces always occur in matched pairs that are equal and opposite. Thus the third law of motion is as follows:

Whenever two objects interact, the force exerted on one object is equal in size and opposite in direction to the force exerted on the other object.


FIGURE 2.22
Forces occur in matched pairs that are equal in magnitude and opposite in direction.


FIGURE 2.23
The football player's foot is pushing against the ground, but it is the ground pushing against the foot that accelerates the player forward to catch a pass.

The third law states that forces always occur in matched pairs that act in opposite directions and on two different bodies. You could express this law with symbols as

$$
F_{\mathrm{A} \text { due to } \mathrm{B}}=F_{\mathrm{B} \text { due to } \mathrm{A}}
$$

equation 2.7
where the force on the astronaut, for example, would be "A due to $B$," and the force on the satellite would be "B due to A."

Sometimes the third law of motion is expressed as follows: "For every action there is an equal and opposite reaction," but this can be misleading. Neither force is the cause of the other. The forces are at every instant the cause of each other and they appear and disappear at the same time. If you are going to describe the force exerted on a satellite by an astronaut, then you must realize that there is a simultaneous force exerted on the astronaut by the satellite. The forces (astronaut on satellite and satellite on astronaut) are equal in magnitude but opposite in direction.

Perhaps it would be more common to move a satellite with a small rocket. A satellite is maneuvered in space by firing a rocket in the direction opposite to the direction someone wants to move the satellite. Exhaust gases (or compressed gases) are accelerated in one direction and exert an equal but opposite force on the satellite that accelerates it in the opposite direction. This is another example of the third law.

Consider how the pairs of forces work on the earth's surface. You walk by pushing your feet against the ground (Figure 2.23). Of course you could not do this if it were not for friction. You would slide as on slippery ice without friction. But since friction does exist, you exert a backward horizontal force on the ground, and, as the third law explains, the ground exerts an equal and opposite force on you. You accelerate forward from the net force as explained by the second law. If the earth had the same mass as you, however, it would accelerate backward at the same rate that you were accelerated forward. The earth is much more massive than you, however, so any acceleration of the earth is a vanishingly small amount. The overall effect is that you are accelerated forward by the force the ground exerts on you.

Return now to the example of riding a bicycle that was discussed previously. What is the source of the external force that accelerates you and the bike? Pushing against the pedals is not external to you and the bike, so that force will not accelerate you and the bicycle forward. This force is transmitted through the bike mechanism to the rear tire, which pushes against the ground. It is the ground exerting an equal and opposite force against the system of you and the bike that accelerates you forward. You must consider the forces that act on the system of the bike and you before you can apply $F=m a$. The only forces that will affect the forward motion of the bike system are the force of the ground pushing it forward and the frictional forces that oppose the forward motion. This is another example of the third law.

## example 2.12

A 60.0 kg astronaut is freely floating in space and pushes on a freely floating 120.0 kg satellite with a force of 30.0 N for 1.50 s .
(a) Compare the forces exerted on the astronaut and the satellite, and (b) compare the acceleration of the astronaut to the acceleration of the satellite.

## Solution

(a) According to Newton's third law of motion (equation 2.7),

$$
\begin{aligned}
F_{A \text { due to } B} & =F_{B \text { due to } A} \\
30.0 \mathrm{~N} & =30.0 \mathrm{~N}
\end{aligned}
$$

Both feel a 30.0 N force for 1.50 s but in opposite directions.
(b) Newton's second law describes a relationship between force, mass, and acceleration, $F=m a$.

For the astronaut:

$$
\begin{array}{ll}
m=60.0 \mathrm{~kg} & F=m a \therefore a=\frac{F}{m} \\
F=30.0 \mathrm{~N} & \\
a=? & a=\frac{30.0 \frac{\mathrm{~kg} \cdot \mathrm{~m}}{\mathrm{~s}^{2}}}{60.0 \mathrm{~kg}} \\
& =\frac{30.0}{60.0}\left(\frac{\mathrm{~kg} \cdot \mathrm{~m}}{\mathrm{~s}^{2}}\right)\left(\frac{1}{\mathrm{~kg}}\right) \\
& =0.500 \frac{\mathrm{~kg} \cdot \mathrm{~m}}{\mathrm{~kg} \cdot \mathrm{~s}^{2}}=0.500 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}
\end{array}
$$

For the satellite:

$$
\begin{aligned}
m & =120.0 \mathrm{~kg} \\
F & =30.0 \mathrm{~N} \\
a & =?
\end{aligned}
$$

$$
\begin{aligned}
a & =\frac{30.0 \frac{\mathrm{~kg} \cdot \mathrm{~m}}{\mathrm{~s}^{2}}}{120.0 \mathrm{~kg}} \\
& =\frac{30.0}{120.0}\left(\frac{\mathrm{~kg} \cdot \mathrm{~m}}{\mathrm{~s}^{2}}\right)\left(\frac{1}{\mathrm{~kg}}\right) \\
& =0.250 \frac{\mathrm{~kg} \cdot \mathrm{~m}}{\mathrm{~kg} \cdot \mathrm{~s}^{2}}=0.250 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}
\end{aligned}
$$

## example 2.13

After the interaction and acceleration between the astronaut and satellite described previously, they both move away from their original positions. What is the new speed for each? (Answer: Astronaut $v_{f}=0.750 \mathrm{~m} / \mathrm{s}$. Satellite $\left.v_{\mathrm{f}}=0.375 \mathrm{~m} / \mathrm{s}\right)\left(\right.$ Hint: $\left.v_{\mathrm{f}}=a t+v_{\mathrm{i}}\right)$

## Goncepts Applied

## Laws of Motion in the Kitchen

Relationships between variables involved in the second and third laws of motion can be studied with some common materials. Obtain a plastic 35 mm film container, an Alka-Seltzer tablet, and a large metal cookie sheet.

Procedure: Add enough cool water to about half fill a plastic 35 mm film container. Place 1/4 Alka-Seltzer tablet in the water and quickly snap on the lid. Now quickly place the container on its side at the center of a large metal cookie sheet.

Analysis: Explain the result in terms of the third law of motion. Compare the mass and evidence of the acceleration of the lid and container, explaining the result in terms of the second law of motion.

## Momentum

Sportscasters often refer to the momentum of a team, and newscasters sometimes refer to an election where one of the candidates has momentum. Both situations describe a competition where one side is moving toward victory and it is difficult to stop. It seems appropriate to borrow this term from the physical sciences because momentum is a property of movement. It takes a longer time to stop something from moving when it has a lot of momentum. The physical science concept of momentum is closely related to Newton's laws of motion. Momentum $(p)$ is defined as the product of the mass $(m)$ of an object and its velocity ( $v$ ),

$$
\text { momentum }=\text { mass } \times \text { velocity }
$$

or

$$
p=m v
$$

equation 2.8
The astronaut in example 2.12 had a mass of 60.0 kg and a velocity of $0.750 \mathrm{~m} / \mathrm{s}$ as a result of the interaction with the satellite. The resulting momentum was therefore ( 60.0 kg ) $(0.750 \mathrm{~m} / \mathrm{s})$, or 45.0 $\mathrm{kg} \cdot \mathrm{m} / \mathrm{s}$. As you can see, the momentum would be greater if the astronaut had acquired a greater velocity or if the astronaut had a greater mass and acquired the same velocity. Momentum involves both the inertia and the velocity of a moving object.

## Conservation of Momentum

Notice that the momentum acquired by the satellite in example 2.12 is also $45.0 \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}$. The astronaut gained a certain momentum in one direction, and the satellite gained the very same momentum in the opposite direction. Newton originally defined the second law in terms of a rate of change of momentum being proportional to the net force acting on an object. Since the third law explains that the forces exerted on both the astronaut and satellite were equal and opposite, you would expect both objects to acquire equal momentum in the opposite direction. This result is observed any time objects in a system interact and the only forces involved are those between the interacting objects (Figure 2.24). This statement leads to a particular kind of relationship called a law of conservation. In this case, the law applies to momentum and is called the law of conservation of momentum:

The total momentum of a group of interacting objects remains the same in the absence of external forces.

Conservation of momentum, energy, and charge are among examples of conservation laws that apply to everyday situations. These situations always illustrate two understandings, that (1) each conservation law is an expression of symmetry that describes a physical principle that can be observed; and, (2) each law holds regardless of the details of an interaction or how it took place. Since the conservation laws express symmetry that always occurs, they tell us what might be expected to happen, and what might be expected not to happen in a given situation. The symmetry also allows unknown quantities to be found by analysis.


FIGURE 2.24
Both the astronaut and the satellite received a force of 30.0 N for 1.50 s when they pushed on each other. Both then have a momentum of $45.0 \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}$ in the opposite direction. This is an example of the law of conservation of momentum.

The law of conservation of momentum, for example, is useful in analyzing motion in simple systems of collisions such as those of billiard balls, automobiles, or railroad cars. It is also useful in measuring action and reaction interactions, as in rocket propulsion, where the backward momentum of the exhaust gases equals the momentum given to the rocket in the opposite direction. When this is done, momentum is always found to be conserved.

The firing of a bullet from a rifle and the concurrent "kick" or recoil of the rifle is often used as an example of conservation of momentum where the interaction between objects results in momentum in opposite directions (Figure 2.25). When the rifle is fired, the exploding gunpowder propels the bullet with forward momentum. At the same time, the force from the exploding gunpowder pushes the rifle backward with a momentum opposite that of the bullet. The bullet moves forward with a momentum of $(m v)_{\mathrm{b}}$ and the rifle moves in an opposite direction to the bullet, so its momentum is $-(m v)_{\mathrm{r}}$. According to the law of conservation of momentum, the momentum of the bullet $(m v)_{\mathrm{b}}$ must equal the momentum of the rifle $-(m v)_{\mathrm{r}}$ in the opposite direction. If the bullet and rifle had the same mass, they would each move with equal velocities when the rifle was fired. The rifle is much more massive than a bullet, however, so the bullet has a much greater velocity than the rifle. The momentum of the rifle is nonetheless equal to the momentum of the bullet, and the recoil can be significant if the rifle is not held firmly against the shoulder. When held firmly against the shoulder, the rifle and the person's body are one object. The increased mass results in a proportionally smaller recoil velocity.


FIGURE 2.25
A rifle and bullet provide an example of conservation of momentum. Before being fired, a rifle and bullet have a total momentum $(p=m v)$ of zero since there is no motion. When fired, the bullet is then propelled in one direction with a forward momentum $(m v)_{b}$. At the same time, the rifle is pushed backward with a momentum opposite to that of the bullet, so its momentum is shown with a minus sign, or $-(m v)_{\mathrm{r}}$. Since $(m v)_{\mathrm{b}}$ plus $-(m v)_{\mathrm{r}}$ equals zero, the total momentum of the rifle and bullet is zero after as well as before the rifle is fired.

## example 2.14

A 20,000 kg railroad car is coasting at $3 \mathrm{~m} / \mathrm{s}$ when it collides and couples with a second, identical car at rest. What is the resulting speed of the combined cars?

## Solution

$$
\begin{array}{lll}
\text { Moving car } & \rightarrow m_{1}=20,000 \mathrm{~kg}, & v_{1}=3 \mathrm{~m} / \mathrm{s} \\
\text { Second car } & \rightarrow m_{2}=20,000 \mathrm{~kg}, & v_{2}=0 \\
\text { Combined cars } \rightarrow v_{182}=? \mathrm{~m} / \mathrm{s} &
\end{array}
$$

Since momentum is conserved, the total momentum of the cars should be the same before and after the collision. Thus

$$
\begin{aligned}
\text { momentum before } & =\text { momentum after } \\
\text { car } 1+\text { car } 2 & =\text { coupled cars } \\
m_{1} v_{1}+m_{2} v_{2} & =\left(m_{1}+m_{2}\right) v_{1 \& 2} \\
v_{1 \& 2} & =\frac{m_{1} v_{1}}{\left(m_{1}+m_{2}\right)} \\
v_{1 \& 2} & =\frac{(20,000 \mathrm{~kg})\left(3 \frac{\mathrm{~m}}{\mathrm{~s}}\right)}{(20,000 \mathrm{~kg})+(20,000 \mathrm{~kg})} \\
& =\frac{20,000 \mathrm{~kg} \cdot 3 \frac{\mathrm{~m}}{\mathrm{~s}}}{40,000 \mathrm{~kg}} \\
& =0.5 \times 3 \frac{\mathrm{~kg} \cdot \mathrm{~m}}{\mathrm{~s}} \times \frac{1}{\mathrm{~kg}} \\
& =1.5 \frac{\mathrm{~m}}{\mathrm{~s}} \\
& =2 \frac{\mathrm{~m}}{\mathrm{~s}}
\end{aligned}
$$

(Answer rounded to one significant figure.)

Car 2 had no momentum with a velocity of zero, so $m_{2} v_{2}$ on the left side of the equation equals zero. When the cars couple the mass is doubled $(m+m)$, and the velocity of the coupled cars will be $2 \mathrm{~m} / \mathrm{s}$.

## example 2.15

A student and her rowboat have a combined mass of 100.0 kg . Standing in the motionless boat in calm water, she tosses a 5.0 kg rock out the back of the boat with a velocity of $5.0 \mathrm{~m} / \mathrm{s}$. What will be the resulting speed of the boat? (Answer: $0.25 \mathrm{~m} / \mathrm{s}$ )

## Impulse

Have you ever heard that you should "follow through" when hitting a ball? When you follow through, the bat is in contact with the ball for a longer period of time. The force of the hit is important, of course, but both the force and how long the force is applied determine the result. The product of the force and the time of application is called impulse. This quantity can be expressed as

$$
\text { impulse }=F t
$$

where $F$ is the force applied during the time of contact $t$. The impulse you give the ball determines how fast the ball will move, and thus how far it will travel.

Impulse is related to the change of motion of a ball of a given mass, so the change of momentum ( $m v$ ) is brought about by the impulse. This can be expressed as

$$
\begin{gathered}
\text { change of momentum }=(\text { applied force })(\text { time of contact }) \\
\Delta p=F t
\end{gathered}
$$

equation 2.9
where $\Delta p$ is a change of momentum. You "follow through" while hitting a ball in order to increase the contact time. If the same force is used, a longer contact time will result in a greater impulse. A greater impulse means a greater change of momentum, and since the mass of the ball does not change, the overall result is a moving ball with a greater velocity. This means following through will result in more distance from hitting the ball with the same force. That's why it is important to follow through when you hit the ball.

Now consider bringing a moving object to a stop by catching it. In this case the mass and the velocity of the object are fixed at the time you catch it, and there is nothing you can do about these quantities. The change of momentum is equal to the impulse, and the force and time of force application can be manipulated. For example, consider how you would catch a raw egg that is tossed to you. You would probably move your hands with the egg as you caught it, increasing the contact time.

Increasing the contact time has the effect of reducing the force since $\Delta p=F t$. You change the force applied by increasing the contact time, and hopefully you reduce the force sufficiently so the egg does not break.

Contact time is also important in safety. Automobile airbags, the padding in elbow and knee pads, and the plastic barrels off the highway in front of overpass supports are examples of designs intended to increase the contact time. Again, increasing the contact time reduces the force since $\Delta p=F t$. The impact force is reduced and so are the injuries. Think about this the next time you see a car that was crumpled and bent by a collision. The driver and passengers were probably saved from more serious injuries since more time was involved in stopping the car that crumpled. A car that crumples is a safer car in a collision.

## Goncents Applied

## Momentum Experiment

The popular novelty item of a frame with five steel balls hanging from strings can be used to observe momentum exchanges during elastic collisions. When one ball is pulled back and released, it stops as it transfers its momentum to the ball it strikes and the momentum is transferred ball to ball until the end ball swings out. Make some predictions, then do the experiment for the following. What will happened when: (1) Two balls are released together on one side. (2) One ball on each side is released at the same time. (3) Two balls on one side are released together as two balls are simultaneously released together on the other side. (4) Two balls on one side are released together as a single ball is simultaneously released on the other side. Analyze the momentum transfers down the line for each situation.

As an alternative to the use of the swinging balls, consider a similar experiment using a line of marbles in contact with each other in a grooved ruler. Here, you could also vary the mass of marbles in collisions.

## Forces and Circular Motion

Consider a communications satellite that is moving at a uniform speed around the earth in a circular orbit. According to the first law of motion there must be forces acting on the satellite, since it does not move off in a straight line. The second law of motion also indicates forces, since an unbalanced force is required to change the motion of an object.

Recall that acceleration is defined as a change in velocity, and that velocity has both magnitude and direction. The velocity is changed by a change in speed, direction, or both speed and direction. The satellite in a circular orbit is continuously being accelerated. This means that there is a continuously acting


FIGURE 2.26
Centripetal force on the ball causes it to change direction continuously, or accelerate into a circular path. Without the unbalanced force acting on it, the ball would continue in a straight line.
unbalanced force on the satellite that pulls it out of a straightline path.

The force that pulls an object out of its straight-line path and into a circular path is called a centripetal (center-seeking) force. Perhaps you have swung a ball on the end of a string in a horizontal circle over your head. Once you have the ball moving, the only unbalanced force (other than gravity) acting on the ball is the centripetal force your hand exerts on the ball through the string. This centripetal force pulls the ball from its natural straight-line path into a circular path. There are no outward forces acting on the ball. The force that you feel on the string is a consequence of the third law; the ball exerts an equal and opposite force on your hand. If you were to release the string, the ball would move away from the circular path in a straight line that has a right angle to the radius at the point of release (Figure 2.26). When you release the string, the centripetal force ceases, and the ball then follows its natural straight-line motion. If other forces were involved, it would follow some other path. Nonetheless, the apparent outward force has been given a name just as if it were a real force. The outward tug is called a centrifugal force.

The magnitude of the centripetal force required to keep an object in a circular path depends on the inertia, or mass, of the object and the acceleration of the object, just as you learned in the second law of motion. The acceleration of an object moving in a circle can be shown by geometry or calculus to be directly proportional to the square of the speed around the circle $\left(v^{2}\right)$ and inversely proportional to the radius of the circle $(r)$. (A smaller radius requires a greater acceleration.) Therefore, the acceleration of an object moving in uniform circular motion $\left(a_{\mathrm{c}}\right)$ is

$$
a_{c}=\frac{v^{2}}{r}
$$

equation 2.10

The magnitude of the centripetal force of an object with a mass $(m)$ that is moving with a velocity $(v)$ in a circular orbit of a radius $(r)$ can be found by substituting equation 3.5 in $F=m a$, or

$$
F=\frac{m v^{2}}{r}
$$

equation 2.11

## ExAMPLE 2.16

A 0.25 kg ball is attached to the end of a 0.5 m string and moved in a horizontal circle at $2.0 \mathrm{~m} / \mathrm{s}$. What net force is needed to keep the ball in its circular path?

## Solution

$$
\begin{array}{rlrl}
m & =0.25 \mathrm{~kg} \\
r & =0.5 \mathrm{~m} \\
v & =2.0 \mathrm{~m} / \mathrm{s} & F & =\frac{m v^{2}}{r} \\
F & =? & & =\frac{(0.25 \mathrm{~kg})(2.0 \mathrm{~m} / \mathrm{s})^{2}}{0.5 \mathrm{~m}} \\
& =\frac{(0.25 \mathrm{~kg})\left(4.0 \mathrm{~m}^{2} / \mathrm{s}^{2}\right)}{0.5 \mathrm{~m}} \\
& =\frac{(0.25)(4.0)}{0.5} \frac{\mathrm{~kg} \cdot \mathrm{~m}^{2}}{\mathrm{~s}^{2}} \times \frac{1}{\mathrm{~m}} \\
& =2 \frac{\mathrm{~kg} \cdot \mathrm{~m}^{2}}{\mathrm{~m} \cdot \mathrm{~s}^{2}} \\
& =2 \frac{\mathrm{~kg} \cdot \mathrm{~m}}{\mathrm{~s}^{2}} \\
& =2 \mathrm{~N}
\end{array}
$$

## example 2.17

Suppose you make the string in example 2.16 half as long, 0.25 m . What force is now needed? (Answer:4.0 N)

## Newton's Law of Gravitation

You know that if you drop an object, it always falls to the floor. You define down as the direction of the object's movement and $u p$ as the opposite direction. Objects fall because of the force of gravity, which accelerates objects at $g=9.8 \mathrm{~m} / \mathrm{s}^{2}\left(32 \mathrm{ft} / \mathrm{s}^{2}\right)$ and gives them weight, $w=m g$.

Gravity is an attractive force, a pull that exists between all objects in the universe. It is a mutual force that, just like all other forces, comes in matched pairs. Since the earth attracts you with a certain force, you must attract the earth with an exact opposite force. The magnitude of this force of mutual


FIGURE 2.27
The variables involved in gravitational attraction. The force of attraction ( $F$ ) is proportional to the product of the masses $\left(m_{1}, m_{2}\right)$ and inversely proportional to the square of the distance (d) between the centers of the two masses.
attraction depends on several variables. These variables were first described by Newton in Principia, his famous book on motion that was printed in 1687. Newton had, however, worked out his ideas much earlier, by the age of twenty-four, along with ideas about his laws of motion and the formula for centripetal acceleration. In a biography written by a friend in 1752, Newton stated that the notion of gravitation came to mind during a time of thinking that "was occasioned by the fall of an apple." He was thinking about why the Moon stays in orbit around Earth rather than moving off in a straight line as would be predicted by the first law of motion. Perhaps the same force that attracts the moon toward the earth, he thought, attracts the apple to the earth. Newton developed a theoretical equation for gravitational force that explained not only the motion of the moon but the motion of the whole solar system. Today, this relationship is known as the universal law of gravitation:

Every object in the universe is attracted to every other object with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distances between them.

In symbols, $m_{1}$ and $m_{2}$ can be used to represent the masses of two objects, $d$ the distance between their centers, and $G$ a constant of proportionality. The equation for the law of universal gravitation is therefore

$$
F=G \frac{m_{1} m_{2}}{d^{2}}
$$

equation 2.12
This equation gives the magnitude of the attractive force that each object exerts on the other. The two forces are oppositely directed. The constant $G$ is a universal constant, since the law applies to all objects in the universe. It was first measured
experimentally by Henry Cavendish in 1798 . The accepted value today is $G=6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}$. Do not confuse $G$, the universal constant, with $g$, the acceleration due to gravity on the surface of the earth.

Thus, the magnitude of the force of gravitational attraction is determined by the mass of the two objects and the distance between them (Figure 2.27). The law also states that every object is attracted to every other object. You are attracted to all the objects around you-chairs, tables, other people, and so forth. Why don't you notice the forces between you and other objects? The answer is in example 2.18.

## EXAMPLE

What is the force of gravitational attraction between two 60.0 kg $(132 \mathrm{lb})$ students who are standing 1.00 m apart?

## Solution

$$
\begin{aligned}
& G=6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2} \\
& m_{1}=60.0 \mathrm{~kg} \\
& m_{2}=60.0 \mathrm{~kg} \\
& d=1.00 \mathrm{~m} \\
& F=\text { ? } \\
& F=G \frac{m_{1} m_{2}}{d^{2}} \\
& =\frac{\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)(60.0 \mathrm{~kg})(60.0 \mathrm{~kg})}{(1.00 \mathrm{~m})^{2}} \\
& =\left(6.67 \times 10^{-11}\right)\left(3.60 \times 10^{3}\right) \frac{\frac{\mathrm{N} \cdot \mathrm{~m}^{2} \cdot \mathrm{Kg}^{2}}{\mathrm{~kg}}}{\mathrm{~m}^{2}} \\
& =2.40 \times 10^{-7}\left(\mathrm{~N} \cdot \mathrm{~m}^{2}\right)\left(\frac{1}{\mathrm{~m}^{2}}\right) \\
& =2.40 \times 10^{-7} \frac{\mathrm{~N} \cdot \mathrm{~m}^{2}}{\mathrm{~m}^{2}} \\
& =2.40 \times 10^{-7} \mathrm{~N}
\end{aligned}
$$

(Note: A force of $2.40 \times 10^{-7}(0.00000024) \mathrm{N}$ is equivalent to a force of $5.40 \times 10^{-8} \mathrm{lb}(0.00000005 \mathrm{lb})$, a force that you would not notice. In fact, it would be difficult to measure such a small force.)

## example 2.19

What would be the value of $g$ if the earth were less dense, with the same mass and double the radius? (Answer: $g=2.45 \mathrm{~m} / \mathrm{s}^{2}$ )

As you can see in example 2.18, one or both of the interacting objects must be quite massive before a noticeable force results from the interaction. That is why you do not notice the force of gravitational attraction between you and objects that are not very massive compared to the earth. The attraction between you and the earth overwhelmingly predominates, and that is all you notice.

Newton was able to show that the distance used in the equation is the distance from the center of one object to the center of the second object. This does not mean that the force originates at the center, but that the overall effect is the same as if you considered all the mass to be concentrated at a center point. The weight of an object, for example, can be calculated by using a form of Newton's second law, $F=m a$. This general law shows a relationship between any force acting on a body, the mass of a body, and the resulting acceleration. When the acceleration is due to gravity, the equation becomes $F=m g$. The law of gravitation deals specifically with the force of gravity and how it varies with distance and mass. Since weight is a force, then $F=m g$. You can write the two equations together,

$$
m g=G \frac{m m_{e}}{d^{2}}
$$

where $m$ is the mass of some object on earth, $m_{\mathrm{e}}$ is the mass of the earth, $g$ is the acceleration due to gravity, and $d$ is the distance between the centers of the masses. Canceling the $m$ 's in the equation leaves

$$
g=G \frac{m_{\mathrm{e}}}{d^{2}}
$$

which tells you that on the surface of the earth the acceleration due to gravity, $9.8 \mathrm{~m} / \mathrm{s}^{2}$, is a constant because the other two variables (mass of the earth and the distance to center of earth) are constant. Since the $m$ 's canceled, you also know that the mass of an object does not affect the rate of free fall; all objects fall at the same rate, with the same acceleration, no matter what their masses are.

Example 2.20 shows that the acceleration due to gravity, $g$, is about $9.8 \mathrm{~m} / \mathrm{s}^{2}$ and is practically a constant for relatively short distances above the surface. Notice, however, that Newton's law of gravitation is an inverse square law. This means if you double the distance, the force is $1 /(2)^{2}$ or $1 / 4$ as great. If you triple the distance, the force is $1 /(3)^{2}$ or $1 / 9$ as great. In other words, the force of gravitational attraction and $g$ decrease inversely with the square of the distance from the earth's center. The weight of an object and the value of $g$ are shown for several distances in Figure 2.28. If you have the time, a good calculator, and the inclination, you could check the values given in Figure 2.28 for a 70.0 kg person by doing problems similar to example 2.20. In fact, you could even calculate the mass of the earth, since you already have the value of $g$.

Using reasoning similar to that found in example 2.20, Newton was able to calculate the acceleration of the Moon toward Earth, about $0.0027 \mathrm{~m} / \mathrm{s}^{2}$. The Moon "falls" toward Earth because it is accelerated by the force of gravitational attraction. This attraction acts as a centripetal force that keeps the Moon from following a straight-line path as would be predicted from the first law. Thus, the acceleration of the


FIGURE 2.28
The force of gravitational attraction decreases inversely with the square of the distance from the earth's center. Note the weight of a 70.0 kg person at various distances above the earth's surface.

Moon keeps it in a somewhat circular orbit around Earth. Figure 2.29 shows that the Moon would be in position A if it followed a straight-line path instead of "falling" to position B as it does. The Moon thus "falls" around Earth. Newton was


FIGURE 2.29
Gravitational attraction acts as a centripetal force that keeps the Moon from following the straight-line path shown by the dashed line to position A. It was pulled to position B by gravity ( 0.0027 $\mathrm{m} / \mathrm{s}^{2}$ ) and thus "fell" toward Earth the distance from the dashed line to B , resulting in a somewhat circular path.

## A Closer Look

## Space Station Weightlessness

When do astronauts experience weightlessness, or "zero gravity"? Theoretically, the gravitational field of Earth extends to the whole universe. You know that it extends to the Moon, and indeed, even to the Sun some 93 million miles away. There is a distance, however, at which the gravitational force must become immeasurably small. But even at an altitude of 20,000 miles above the surface of Earth, gravity is measurable. At 20,000 miles, the value of $g$ is about $1 \mathrm{ft} / \mathrm{s}^{2}$ $\left(0.3 \mathrm{~m} / \mathrm{s}^{2}\right)$ compared to $32 \mathrm{ft} / \mathrm{s}^{2}\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)$ on the surface. Since gravity does exist at these distances, how can an astronaut experience "zero gravity"?

Gravity does act on astronauts in spacecraft that are in orbit around Earth. The spacecraft stays in orbit, in fact, because of the gravitational attraction and because it has the correct speed. If the speed were less than $5 \mathrm{mi} / \mathrm{s}$, the spacecraft would return to Earth. Astronauts fire their retro-rockets, which slow the speed, causing the spacecraft to fall down to the earth. If the speed were more than $7 \mathrm{mi} / \mathrm{s}$, the spacecraft would fly off into space. The spacecraft stays in orbit because it has the right speed to continu-
ously "fall" around and around the earth. Gravity provides the necessary centripetal force that causes the spacecraft to fall out of its natural straight-line motion.

Since gravity is acting on the astronaut and spacecraft, the term zero gravity is not an accurate description of what is happening. The astronaut, spacecraft, and everything in it are experiencing apparent weightlessness because they are continuously falling toward the earth. Everything seems to float because everything is falling together. But, strictly speaking, everything still has weight, because weight is defined as a gravitational force acting on an object ( $w=m g$ ).

Whether weightlessness is apparent or real, however, the effects on people are the same. Long-term orbital flights have provided evidence that the human body changes from the effect of weightlessness. Bones lose calcium and other minerals, the heart shrinks to a much smaller size, and leg muscles shrink so much on prolonged flights that astronauts cannot walk when they return to the earth. These changes occur because on Earth humans are constantly subjected to the force of gravity. The nature
of the skeleton and the strength of the muscles are determined by how the body reacts to this force. Metabolic pathways and physiological processes that maintain strong bones and muscles evolved having to cope with a specific gravitational force. When we are suddenly subjected to a place where gravity is significantly different, these processes result in weakened systems. If we had evolved on a planet with a different gravitational force, we would have muscles and bones that were adapted to the gravity on that planet. All organisms have evolved in a world with gravity. Many kinds of organisms have been used in experiments in space to try to develop a better understanding of how their systems work.

The problems related to prolonged weightlessness must be worked out before long-term weightless flights can take place. One solution to these problems might be a large, uniformly spinning spacecraft. The astronauts tend to move in a straight line, and the side of the turning spacecraft (now the "floor") exerts a force on them to make them go in a curved path. This force would act as an artificial gravity.
able to analyze the motion of the Moon quantitatively as evidence that it is gravitational force that keeps the Moon in its orbit. The law of gravitation was extended to the Sun, other planets, and eventually the universe. The quantitative predictions of observed relationships among the planets were strong evidence that all objects obey the same law of gravitation. In addition, the law provided a means to calculate the mass of Earth, the Moon, the planets, and the Sun. Newton's law of gravitation, laws of motion, and work with mathematics formed the basis of most physics and technology for the next two centuries, as well as accurately describing the world of everyday experience.

## example 2.20

The surface of the earth is approximately $6,400 \mathrm{~km}$ from its center. If the mass of the earth is $6.0 \times 10^{24} \mathrm{~kg}$, what is the acceleration due to gravity, $g$, near the surface?

$$
\begin{aligned}
& G= 6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2} \\
& m_{\mathrm{e}}= 6.0 \times 10^{24} \mathrm{~kg} \\
& d= 6,400 \mathrm{~km}\left(6.4 \times 10^{6} \mathrm{~m}\right) \\
& g= ? \\
& g=\frac{G m_{\mathrm{e}}}{d^{2}} \\
&=\frac{\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left(6.0 \times 10^{24} \mathrm{~kg}\right)}{\left(6.4 \times 10^{6} \mathrm{~m}\right)^{2}} \\
&=\frac{\left(6.67 \times 10^{-11}\right)\left(6.0 \times 10^{24}\right)}{4.1 \times 10^{13}} \frac{\mathrm{~N} \cdot \mathrm{~m}^{2} \cdot \mathrm{~kg}}{\mathrm{~kg}^{2}} \\
& \mathrm{~m}^{2} \\
&=\frac{4.0 \times 10^{14} \frac{\mathrm{~kg} \cdot \mathrm{~m}}{\mathrm{~s}^{2}}}{4.1 \times 10^{13}} \frac{\mathrm{~kg}}{9.8 \mathrm{~m} / \mathrm{s}^{2}}
\end{aligned}
$$

(Note:In the unit calculation, remember that a newton is a $\mathrm{kg} \cdot \mathrm{m} / \mathrm{s}^{2}$.)

## People Behind the Science

## Isaac Newton (1642-1727)

Isaac Newton was a British physicist and mathematician who is regarded as one of the greatest scientists ever to have lived. In physics, he discovered the three laws of motion that bear his name and was the first to explain gravitation, clearly defining the nature of mass, weight, force, inertia, and acceleration. In his honor, the SI unit of force is called the newton. Newton also made fundamental discoveries in optics, finding that white light is composed of a spectrum of colors and inventing the reflecting telescope. In mathematics, Newton's principal contribution was to formulate the calculus and the binomial theorem.

Newton was born at Woolsthorpe, Lincolnshire, on December 25, 1642 by the old Julian calendar, but on January 4, 1643 by modern reckoning. His birthplace, Woolsthorpe Manor, is now preserved. Newton's was an inauspicious beginning for he was a premature, sickly baby born after his father's death, and his survival was not expected. When he was three, his mother remarried and the young Newton was left in his grandmother's care. He soon began to take refuge in things mechanical, reputedly making water-clocks, kites bearing fiery lanterns aloft, and a model mill powered by a mouse, as well as innumerable drawings and diagrams. When Newton was 12, he began to attend the King's School, Grantham, but his schooling was not to last. His mother, widowed again, returned to Woolsthorpe in 1658 and withdrew him from school with the intention of making him into a farmer. Fortunately, his uncle recognized Newton's ability and managed to get him back to school to prepare for university entrance. This Newton achieved in 1661, when he went to Trinity College, Cambridge, and began to delve widely and deeply into the scholarship of the day.

In 1665 , the year that he became a Bachelor of Arts, the university was closed because of the plague and Newton spent eighteen months at Woolsthorpe, with only the occa-
sional visit to Cambridge. Such seclusion was a prominent feature of Newton's creative life and, during this period, he laid the foundations of his work in mathematics, optics, dynamics, and celestial mechanics, performing his first prism experiments and reflecting on motion and gravitation.

Newton returned to Cambridge in 1666 and became a minor Fellow of Trinity in 1667 and a major Fellow the following year. He also received his Master of Arts degree in 1668 and became Lucasian Professor of Mathematics-at the age of only 26. It is said that the previous incumbent, Isaac Barrow (1630-1677), resigned the post to make way for Newton. Newton remained at Cambridge almost 30 years, studying alone for the most part, though in frequent contact with other leading scientists by letter and through the Royal Society in London, which elected him a Fellow in 1672. These were Newton's most fertile years. He labored day and night in his chemical laboratory, at his calculations, or immersed in theological and mystical speculations. In Cambridge, he completed what may be described as his greatest single work, the Philosophae Naturalis Principia Mathematica (Mathematical Principles of Natural Philosophy). This was presented to the Royal Society in 1686, who subsequently withdrew from publishing it through shortage of funds. The astronomer Edmund Halley (1656-1742), a wealthy man and friend of Newton, paid for the publication of the Principia in 1687 . In it, Newton revealed his laws of motion and the law of universal gravitation.

Newton presented his conclusions on dynamics in the Principia. Although he had already developed the calculus, he did not use it in the Principia, preferring to prove all his results geometrically. In this great work, Newton's plan was first to develop the subject of general dynamics from a mathematical point of view and then to apply the results in the solution of important astronomical and physical problems. It included
a synthesis of Kepler's laws of planetary motion and Galileo's laws of falling bodies, developing the system of mechanics we know today, including the three famous laws of motion. The first law states that every body remains at rest or in constant motion in a straight line unless it is acted upon by a force. This defines inertia, finally disproving the idea which had been prevalent since Aristotle (384-322 B.C.) had mooted it, that force is required to keep anything moving. The second law states that a force accelerates a body by an amount proportional to its mass. This was the first clear definition of force and it also distinguished mass from weight. The third law states that action and reaction are equal and opposite, which showed how things could be made to move.

Newton also developed his general theory of gravitation as a universal law of attraction between any two objects, stating that the force of gravity is proportional to the masses of the objects and decreases in proportion to the square of the distance between the two bodies. Though, in the years before, there had been considerable correspondence between Newton, Hooke, Halley, and Kepler on the mathematical formulation of these laws, Newton did not complete the work until the writing of the Principia.
"I was in the prime of my age for invention" said Newton of those two years 1665 and 1666, and it was in that period that he performed his fundamental work in optics. Again it should be pointed out that the study of Newton's optics has been limited to his published letters and the Opticks of 1704, its publication delayed until after Hooke's death to avoid yet another controversy over originality. No adequate edition or full translation of the voluminous Lectiones Opticae exists. Newton began those first, crucial experiments by passing sunlight through a prism, finding that it dispersed the white light into a spectrum of colors. He then took a second
-Continued top of next page

## Continued-

prism and showed that it could combine the colors in the spectrum and form white light again. In this way, Newton proved that the colors are a property of light and not of the prism. An interesting by-product of these early speculations was the development of the reflecting telescope. Newton held the erroneous opinion that optical dispersion was independent of the medium through which the light was refracted and, therefore, that nothing could be done to correct the chromatic aberration caused by lenses. He therefore set about building a telescope in which the objective lens is replaced by a curved mirror, in which aberration could not occur. In 1668 Newton succeeded in making the first reflecting telescope, a tiny instrument only $15 \mathrm{~cm} / 6$ in long, but the direct ancestor of today's huge astronomical reflecting telescopes. In this invention, Newton was anticipated to some degree by James Gregory (1638-1675) who had produced a design for a reflecting telescope five years earlier but had not succeeded in constructing one.

Other scientists, Hooke especially, were critical of Newton's early reports, seeing too little connection between experimental result and theory, so that, in the course of a debate lasting several years, Newton was forced to refine his theories with considerable subtlety. He performed further experiments in which he investigated many other optical phenomena, including thin film interference effects, one of which, "Newton's rings," is named for him.

The Opticks presented a highly systematized and organized account of Newton's work and his theory of the nature of light and
the effects that light produces. In fact, although he held that light rays were corpuscular in nature, he integrated into his ideas the concept of periodicity, holding that "ether waves" were associated with light corpuscles, a remarkable conceptual leap, for Hooke and Huygens, the founder of the wave theory, both denied periodicity to light waves. The corpuscle concept lent itself to an analysis by forces and established an analogy between the action of gross bodies and that of light, reinforcing the universalizing tendency of the Principia.

However, Newton's prestige was such that the corpuscular theory held sway for much longer than it deserved, not being finally overthrown until early in the 1800s. Ironically, it was the investigation of interference effects by Thomas Young (1773-1829) that led to the establishment of the wave theory of light.

Although comparatively little is known of the bulk of Newton's complete writings in chemistry and physics, we know even less about his chemistry and alchemy, chronology, prophecy, and theology. The vast number of documents he wrote on these matters have never yet been properly analyzed, but what is certain is that he took great interest in alchemy, performing many chemical experiments in his own laboratory and being in contact with Robert Boyle (1627-1691). He also wrote much on ancient chronology and the authenticity of certain biblical texts.

Newton's greatest achievement was to demonstrate that scientific principles are of universal application. In the Principia Mathematica, he built logically and analytically from mathematical premises and the evi-
dence of experiment and observation to develop a model of the universe that is still of general validity. "If I have seen further than other men," he once said with perhaps assumed modesty, "it is because I have stood on the shoulders of giants"; and Newton was certainly able to bring together the knowledge of his forebears in a brilliant synthesis. Newton's life marked the first great flowering of the scientific method, which had been evolving in fits and starts since the time of the ancient Greeks. But Newton really established it, completing a scientific revolution in Europe that had begun with Nicolaus Copernicus (1473-1543) and ushering in the Age of Reason, in which the scientific method was expected to yield complete knowledge by the elucidation of the basic laws that govern the universe. No knowledge can ever be total, but Newton's example brought about an explosion of investigation and discovery that has never really abated. He perhaps foresaw this when he remarked "To myself, I seem to have been only like a boy playing on the seashore, and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me."

With his extraordinary insight into the workings of nature and rare tenacity in wresting its secrets and revealing them in as fundamental and concise a way as possible, Newton stands as a colossus of science. In physics, only Archimedes (287-212 в.c.) and Albert Einstein (1879-1955), who also possessed these qualities, may be compared to him.

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## Summary

Motion can be measured by speed, velocity, and acceleration. Speed is a measure of how fast something is moving. It is a ratio of the distance covered between two locations to the time that elapsed while moving between the two locations. The average speed considers the distance covered during some period of time, while the instantaneous speed is the speed at some specific instant. Velocity is a measure of the speed and direction of a moving object. Acceleration is the change of velocity during some period of time.

A force is a push or a pull that can change the motion of an object. The net force is the sum of all the forces acting on an object.

Galileo determined that a continuously applied force is not necessary for motion and defined the concept of inertia: an object remains in unchanging motion in the absence of a net force. Galileo also determined that falling objects accelerate toward the earth's surface independent of the weight of the object. He found the acceleration due to gravity, $g$, to be $9.8 \mathrm{~m} / \mathrm{s}^{2}\left(32 \mathrm{ft} / \mathrm{s}^{2}\right)$, and the distance an object falls is proportional to the square of the time of free fall $\left(d \propto t^{2}\right)$.

Compound motion occurs when an object is projected into the air. Compound motion can be described by splitting the motion into vertical and horizontal parts. The acceleration due to gravity, $g$, is a constant that is acting at all times and acts independently of any
motion that an object has. The path of an object that is projected at some angle to the horizon is therefore a parabola.

Newton's first law of motion is concerned with the motion of an object and the lack of a net force. Also known as the law of inertia, the first law states that an object will retain its state of straight-line motion (or state of rest) unless a net force acts on it.

The second law of motion describes a relationship between net force, mass, and acceleration. A newton of force is the force needed to give a 1.0 kg mass an acceleration of $1.0 \mathrm{~m} / \mathrm{s}^{2}$.

Weight is the downward force that results from the earth's gravity acting on the mass of an object. Weight is measured in newtons in the metric system and pounds in the English system.

Newton's third law of motion states that forces are produced by the interaction of two different objects. These forces always occur in matched pairs that are equal in size and opposite in direction.

Momentum is the product of the mass of an object and its velocity. In the absence of external forces, the momentum of a group of interacting objects always remains the same. This relationship is the law of conservation of momentum. Impulse is a change of momentum equal to a force times the time of application.

An object moving in a circular path must have a force acting on it, since it does not move in a straight line. The force that pulls an object out of its straight-line path is called a centripetal force. The centripetal force needed to keep an object in a circular path depends on the mass of the object, its velocity, and the radius of the circle.

The universal law of gravitation is a relationship between the masses of two objects, the distance between the objects, and a proportionality constant. Newton was able to use this relationship to show that gravitational attraction provides the centripetal force that keeps the moon in its orbit.

## Summary of Equations

2.1

$$
\begin{aligned}
\text { average speed } & =\frac{\text { distance }}{\text { time }} \\
\bar{v} & =\frac{d}{t}
\end{aligned}
$$

2.2

$$
\begin{aligned}
\text { acceleration } & =\frac{\text { change of velocity }}{\text { time }} \\
& =\frac{\text { final velocity }- \text { initial velocity }}{\text { time }} \\
a & =\frac{v_{\mathrm{f}}-v_{\mathrm{i}}}{t}
\end{aligned}
$$

2.3

$$
\text { average velocity }=\frac{\text { final velocity }+ \text { initial velocity }}{2}
$$

$$
\bar{v}=\frac{v_{\mathrm{f}}+v_{\mathrm{i}}}{2}
$$

2.4

$$
\begin{aligned}
\text { distance } & =\frac{1}{2}(\text { acceleration })(\text { time })^{2} \\
d & =\frac{1}{2} a t^{2}
\end{aligned}
$$

$$
\begin{aligned}
\text { force } & =\text { mass } \times \text { acceleration } \\
F & =m a
\end{aligned}
$$

$$
\text { force on object } \mathrm{A}=\text { force on object } \mathrm{B}
$$

$$
F_{\mathrm{A} \text { due to } \mathrm{B}}=F_{\mathrm{B} \text { due to } \mathrm{A}}
$$

$$
\begin{aligned}
\text { momentum } & =\text { mass } \times \text { velocity } \\
p & =m v
\end{aligned}
$$

$$
\text { change of momentum }=\text { force } \times \text { time }
$$

$$
\Delta p=F t
$$

$$
\begin{aligned}
\text { centripetal acceleration } & =\frac{\text { velocity squared }}{\text { radius of circle }} \\
a_{\mathrm{c}} & =\frac{v^{2}}{r}
\end{aligned}
$$

$$
\begin{align*}
\text { centripetal force } & =\frac{m a s s \times \text { velocity squared }}{\text { radius of circle }} \\
F & =\frac{m v^{2}}{r}
\end{align*}
$$

$$
\begin{aligned}
& \text { gravitational force }=\text { constant } \times \frac{\text { one mass } \times \text { another mass }}{\text { distance squared }} \\
& \qquad F=G \frac{m_{1} m_{2}}{d^{2}}
\end{aligned}
$$

## KEY TERMS

acceleration (p. 29)
centrifugal force (p. 50)
centripetal force (p.50)
first law of motion (p.41)
force (p. 32)
free fall (p.35)
$g$ (p.37)
impulse (p. 49)
inertia (p. 34)
law of conservation of momentum (p. 47)
mass (p. 44)
momentum (p. 47)
net force (p. 32)
newton (p. 43)
second law of motion (p. 43)
speed (p. 27)
third law of motion (p. 45)
universal law of
gravitation (p. 51)
velocity (p. 29)

## APPLYING THE CONGEPTS

1. A quantity of $5 \mathrm{~m} / \mathrm{s}^{2}$ is a measure of
a. metric area.
b. acceleration.
c. speed.
d. velocity.
2. An automobile has how many different devices that can cause it to undergo acceleration?
a. none
b. one
c. two
d. three or more
3. Ignoring air resistance, an object falling toward the surface of the earth has a velocity that is
a. constant.
b. increasing.
c. decreasing.
d. acquired instantaneously, but dependent on the weight of the object.
4. Ignoring air resistance, an object falling near the surface of the earth has an acceleration that is
a. constant.
b. increasing.
c. decreasing.
d. dependent on the weight of the object.
5. Two objects are released from the same height at the same time, and one has twice the weight of the other. Ignoring air resistance,
a. the heavier object hits the ground first.
b. the lighter object hits the ground first.
c. they both hit at the same time.
d. whichever hits first depends on the distance dropped.
6. A ball rolling across the floor slows to a stop because
a. there is a net force acting on it.
b. the force that started it moving wears out.
c. the forces are balanced.
d. the net force equals zero.
7. Considering the forces on the system of you and a bicycle as you pedal the bike at a constant velocity in a horizontal straight line,
a. the force you are exerting on the pedal is greater than the resisting forces.
b. all forces are in balance, with the net force equal to zero.
c. the resisting forces of air and tire friction are less than the force you are exerting.
d. the resisting forces are greater than the force you are exerting.
8. If you double the unbalanced force on an object of a given mass, the acceleration will be
a. doubled.
b. increased fourfold.
c. increased by one-half.
d. increased by one-fourth.
9. If you double the mass of a cart while it is undergoing a constant unbalanced force, the acceleration will be
a. doubled.
b. increased fourfold.
c. half as much.
d. one-fourth as much.
10. Doubling the distance between the center of an orbiting satellite and the center of the earth will result in what change in the gravitational attraction of the earth for the satellite?
a. one-half as much
b. one-fourth as much
c. twice as much
d. four times as much
11. If a ball swinging in a circle on a string is moved twice as fast, the force on the string will be
a. twice as great.
b. four times as great.
c. one-half as much.
d. one-fourth as much.
12. A ball is swinging in a circle on a string when the string length is doubled. At the same velocity, the force on the string will be
a. twice as great.
b. four times as great.
c. one-half as much.
d. one-fourth as much.

## Answers

1.b 2. d 3.b 4. a 5. c 6. a 7. b 8. a 9. c 10.b 11. b 12. c

## QUESTIONS FOR THOUGHT

1. An insect inside a bus flies from the back toward the front at $5.0 \mathrm{mi} / \mathrm{h}$. The bus is moving in a straight line at $50.0 \mathrm{mi} / \mathrm{h}$. What is the speed of the insect?
2. Disregarding air friction, describe all the forces acting on a bullet shot from a rifle into the air.
3. Can gravity act in a vacuum? Explain.
4. Is it possible for a small car to have the same momentum as a large truck? Explain.
5. What net force is needed to maintain the constant velocity of a car moving in a straight line? Explain.
6. How can there ever be an unbalanced force on an object if every action has an equal and opposite reaction?
7. Why should you bend your knees as you hit the ground after jumping from a roof?
8. Is it possible for your weight to change as your mass remains constant? Explain.
9. What maintains the speed of the earth as it moves in its orbit around the sun?
10. Suppose you are standing on the ice of a frozen lake and there is no friction whatsoever. How can you get off the ice? (Hint: Friction is necessary to crawl or walk, so that will not get you off the ice.)
11. A rocket blasts off from a platform on a space station. An identical rocket blasts off from free space. Considering everything else to be equal, will the two rockets have the same acceleration? Explain.
12. An astronaut leaves a spaceship that is moving through free space to adjust an antenna. Will the spaceship move off and leave the astronaut behind? Explain.

## PARALHEL_EXERGISES

The exercises in groups A and B cover the same concepts. Solutions to group A exercises are located in appendix D.
Note: Neglect all frictional forces in all exercises.

## Group A

1. What is the average speed, in $\mathrm{km} / \mathrm{h}$, for a car that travels 22 km in exactly 15 min ?
2. Suppose a radio signal travels from earth and through space at a speed of $3.0 \times 10^{8} \mathrm{~m} / \mathrm{s}$. How far into space did the signal travel during the first 20.0 minutes?
3. How far away was a lightning strike if thunder is heard 5.00 seconds after seeing the flash? Assume that sound traveled at $350.0 \mathrm{~m} / \mathrm{s}$ during the storm.
4. A car is driven at an average speed of $100.0 \mathrm{~km} / \mathrm{h}$ for two hours, then at an average speed of $50.0 \mathrm{~km} / \mathrm{h}$ for the next hour. What was the average speed for the three-hour trip?
5. What is the acceleration of a car that moves from rest to $15.0 \mathrm{~m} / \mathrm{s}$ in 10.0 s ?
6. How long will be required for a car to go from a speed of 20.0 $\mathrm{m} / \mathrm{s}$ to a speed of $25.0 \mathrm{~m} / \mathrm{s}$ if the acceleration is $3.0 \mathrm{~m} / \mathrm{s}^{2}$ ?
7. A bullet leaves a rifle with a speed of $2,360 \mathrm{ft} / \mathrm{s}$. How much time elapses before it strikes a target 1 mile ( $5,280 \mathrm{ft}$ ) away?
8. A pitcher throws a ball at $40.0 \mathrm{~m} / \mathrm{s}$, and the ball is electronically timed to arrive at home plate 0.4625 s later. What is the distance from the pitcher to the home plate?
9. The Sun is $1.50 \times 10^{8} \mathrm{~km}$ from Earth, and the speed of light is $3.00 \times 10^{8} \mathrm{~m} / \mathrm{s}$. How many minutes elapse as light travels from the Sun to Earth?
10. An archer shoots an arrow straight up with an initial velocity magnitude of $100.0 \mathrm{~m} / \mathrm{s}$. After 5.00 s , the velocity is $51.0 \mathrm{~m} / \mathrm{s}$. At what rate is the arrow decelerated?
11. A ball thrown straight up climbs for 3.0 s before falling. Neglecting air resistance, with what velocity was the ball thrown?
12. A ball dropped from a building falls for 4.00 s before it hits the ground. (a) What was its final velocity just as it hit the ground? (b) What was the average velocity during the fall? (c) How high was the building?
13. You drop a rock from a cliff, and 5.00 s later you see it hit the ground. How high is the cliff?
14. What is the resulting acceleration when an unbalanced force of 100 N is applied to a 5 kg object?
15. What is the momentum of a 100 kg football player who is moving at $6 \mathrm{~m} / \mathrm{s}$ ?
16. A car weighing $13,720 \mathrm{~N}$ is speeding down a highway with a velocity of $91 \mathrm{~km} / \mathrm{h}$. What is the momentum of this car?

## Group B

1. A boat moves 15.0 km across a lake in 45 min . What was the average speed of the boat in kilometers per hour?
2. If the Sun is a distance of $1.5 \times 10^{8} \mathrm{~km}$ from Earth, how long does it take sunlight to reach Earth if it moves at $3.0 \times 10^{8} \mathrm{~m} / \mathrm{s}$ ?
3. How many meters away is a cliff if an echo is heard 0.500 s after the original sound? Assume that sound traveled at $343 \mathrm{~m} / \mathrm{s}$ on that day.
4. A car has an average speed of $80.0 \mathrm{~km} / \mathrm{h}$ for one hour, then an average speed of $90.0 \mathrm{~km} / \mathrm{h}$ for two hours during a three-hour trip. What was the average speed for the three-hour trip?
5. What is the acceleration of a car that moves from a speed of 5.0 $\mathrm{m} / \mathrm{s}$ to a speed of $15 \mathrm{~m} / \mathrm{s}$ during a time of 6.0 s ?
6. How much time is needed for a car to accelerate from $8.0 \mathrm{~m} / \mathrm{s}$ to a speed of $22 \mathrm{~m} / \mathrm{s}$ if the acceleration is $3.0 \mathrm{~m} / \mathrm{s}^{2}$ ?
7. A rocket moves through outer space at $11,000 \mathrm{~m} / \mathrm{s}$. At this rate, how much time would be required to travel the distance from Earth to the Moon, which is $380,000 \mathrm{~km}$ ?
8. Sound travels at $1,140 \mathrm{ft} / \mathrm{s}$ in the warm air surrounding a thunderstorm. How far away was the place of discharge if thunder is heard 4.63 s after a lightning flash?
9. How many hours are required for a radio signal from a space probe near the planet Pluto, $6.00 \times 10^{9} \mathrm{~km}$ away, to reach Earth? Assume that the radio signal travels at the speed of light, $3.00 \times$ $10^{8} \mathrm{~m} / \mathrm{s}$.
10. A rifle is fired straight up, and the bullet leaves the rifle with an initial velocity magnitude of $724 \mathrm{~m} / \mathrm{s}$. After 5.00 s the velocity is $675 \mathrm{~m} / \mathrm{s}$. At what rate is the bullet decelerated?
11. A rock thrown straight up climbs for 2.50 s , then falls to the ground. Neglecting air resistance, with what velocity did the rock strike the ground?
12. An object is observed to fall from a bridge, striking the water below 2.50 s later. (a) With what velocity did it strike the water? (b) What was its average velocity during the fall? (c) How high is the bridge?
13. A ball dropped from a window strikes the ground 2.00 seconds later. How high is the window above the ground?
14. Find the resulting acceleration from a 300 N force that acts on an object with a mass of $3,000 \mathrm{~kg}$.
15. What is the momentum of a 30.0 kg shell fired from a cannon with a velocity of $500 \mathrm{~m} / \mathrm{s}$ ?
16. What is the momentum of a 39.2 N bowling ball with a velocity of $7.00 \mathrm{~m} / \mathrm{s}$ ?
17. A 15 g bullet is fired with a velocity of $200 \mathrm{~m} / \mathrm{s}$ from a 6 kg rifle. What is the recoil velocity of the rifle?
18. An astronaut and equipment weigh $2,156 \mathrm{~N}$ on Earth. Weightless in space, the astronaut throws away a 5.0 kg wrench with a velocity of $5.0 \mathrm{~m} / \mathrm{s}$. What is the resulting velocity of the astronaut in the opposite direction?
19. (a) What is the weight of a 1.25 kg book? (b) What is the acceleration when a net force of 10.0 N is applied to the book?
20. What net force is needed to accelerate a 1.25 kg book $5.00 \mathrm{~m} / \mathrm{s}^{2}$ ?
21. What net force does the road exert on a 70.0 kg bicycle and rider to give them an acceleration of $2.0 \mathrm{~m} / \mathrm{s}^{2}$ ?
22. A $1,500 \mathrm{~kg}$ car accelerates uniformly from $44.0 \mathrm{~km} / \mathrm{h}$ to 80.0 $\mathrm{km} / \mathrm{h}$ in 10.0 s . What was the net force exerted on the car?
23. A net force of $5,000.0 \mathrm{~N}$ accelerates a car from rest to $90.0 \mathrm{~km} / \mathrm{h}$ in 5.0 s . (a) What is the mass of the car? (b) What is the weight of the car?
24. What is the weight of a 70.0 kg person?
25. How much centripetal force is needed to keep a 0.20 kg ball on a 1.50 m string moving in a circular path with a speed of $3.0 \mathrm{~m} / \mathrm{s}$ ?
26. On Earth, an astronaut and equipment weigh $1,960.0 \mathrm{~N}$. While weightless in space, the astronaut fires a 100 N rocket backpack for 2.0 s . What is the resulting velocity of the astronaut and equipment?
27. A 30.0 kg shell is fired from a $2,000 \mathrm{~kg}$ cannon with a velocity of $500 \mathrm{~m} / \mathrm{s}$. What is the resulting velocity of the cannon?
28. An 80.0 kg man is standing on a frictionless ice surface when he throws a 4.00 kg book at $20.0 \mathrm{~m} / \mathrm{s}$. With what velocity does the man move across the ice?
29. (a) What is the weight of a 5.00 kg backpack? (b) What is the acceleration of the backpack if a net force of 10.0 N is applied?
30. What net force is required to accelerate a 20.0 kg object to $10.0 \mathrm{~m} / \mathrm{s}^{2}$ ?
31. What forward force must the ground apply to the foot of a 60.0 kg person to result in an acceleration of $1.00 \mathrm{~m} / \mathrm{s}^{2}$ ?
32. A $1,000.0 \mathrm{~kg}$ car accelerates uniformly to double its speed from $36.0 \mathrm{~km} / \mathrm{h}$ in 5.00 s . What net force acted on this car?
33. A net force of $3,000.0 \mathrm{~N}$ accelerates a car from rest to $36.0 \mathrm{~km} / \mathrm{h}$ in 5.00 s . (a) What is the mass of the car? (b) What is the weight of the car?
34. How much does a 60.0 kg person weigh?
35. What tension must a 50.0 cm length of string support in order to whirl an attached $1,000.0 \mathrm{~g}$ stone in a circular path at $5.00 \mathrm{~m} / \mathrm{s}$ ?
36. A 200.0 kg astronaut and equipment move with a velocity of 2.00 $\mathrm{m} / \mathrm{s}$ toward an orbiting spacecraft. How long will the astronaut need to fire a 100.0 N rocket backpack to stop the motion relative to the spacecraft?
