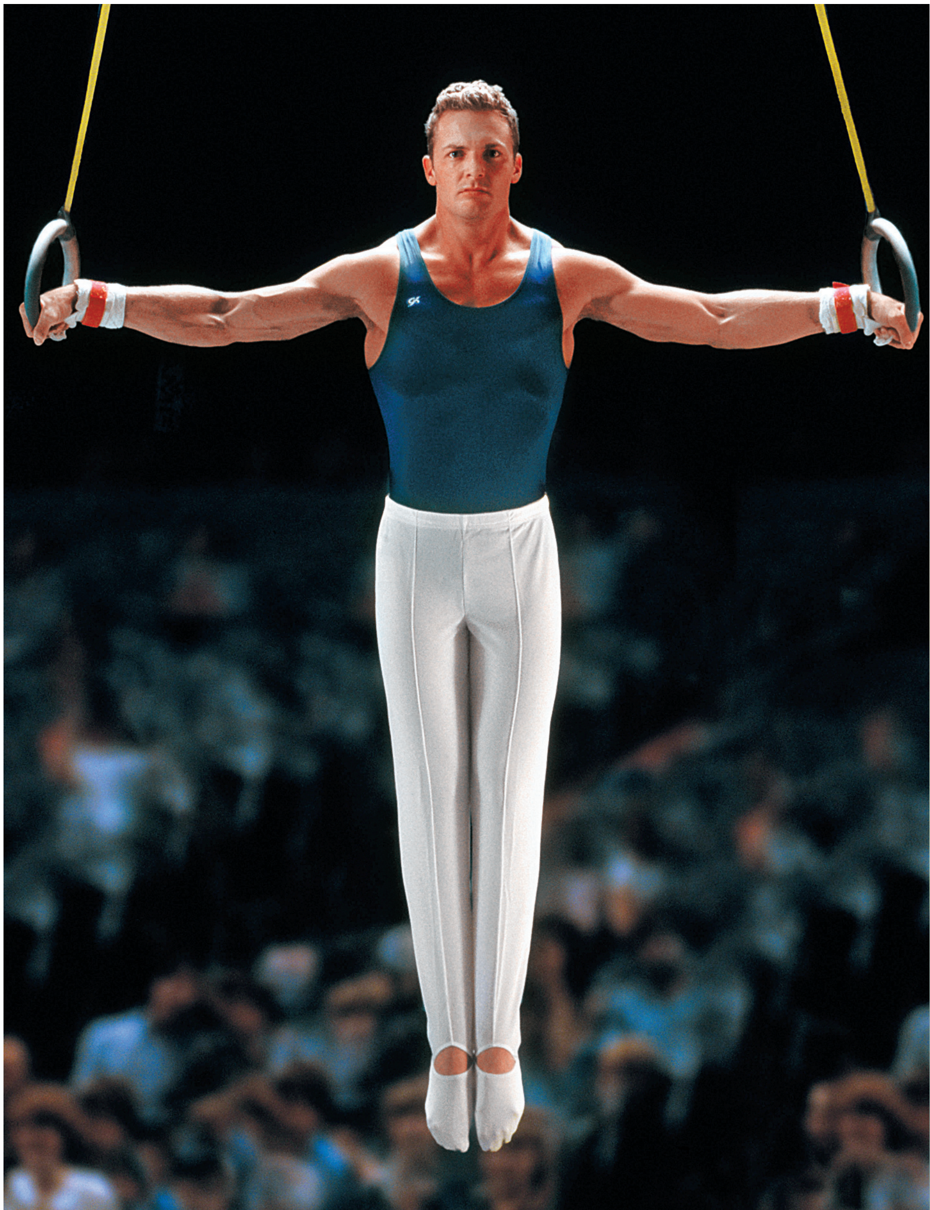
Chapter 8

Torque and Angular Momentum



In gymnastics, the iron cross is a notoriously difficult feat that requires incredible strength. Why does it require such great strength? To perform the iron cross, the forces exerted by the gymnast's muscles must be much greater in magnitude than the gymnast's weight. How does the design of the human body make such large forces necessary? (See p. 277 for the answer.)

Concepts & Skills to Review

- translational equilibrium (Section 2.3)
- uniform circular motion and circular orbits (Sections 5.1 and 5.4)
- angular acceleration (Section 5.6)
- conservation of energy (Section 6.1)
- center of mass and its motion (Sections 7.5 and 7.6)

8.1 ROTATIONAL KINETIC ENERGY AND ROTATIONAL INERTIA

When a rigid object is rotating in place, it has kinetic energy because each particle of the object is moving in a circle around the axis of rotation. In principle, we can calculate the kinetic energy of rotation by summing the kinetic energy of each particle. To say the least, that sounds like a laborious task. We need a simpler way to express the rotational kinetic energy of such an object so that we don't have to calculate this sum over and over. Our simpler expression exploits the fact that the speed of each particle is proportional to the angular speed of rotation ω.

If a rigid object consists of N particles, the sum of the kinetic energies of the particles can be written mathematically using a subscript to label the mass and speed of each particle:

$$K_{\text{rot}} = \tfrac{1}{2}m_1 v_1^2 + \tfrac{1}{2}m_2 v_2^2 + \cdots + \tfrac{1}{2}m_N v_N^2 = \sum_{i=1}^{N} \tfrac{1}{2}m_i v_i^2$$

The symbol Σ stands for a sum. $\sum_{i=1}^{N}$ means the sum for $i = 1, 2, \ldots, N$.

The speed of each particle is related to its distance from the axis of rotation. Particles that are farther from the axis move faster. In Section 5.1, we found that the speed of a particle moving in a circle is

$$v = r\omega \qquad \textbf{(5-7)}$$

where ω is the angular speed and r is the distance between the rotation axis and the particle. By substitution, the rotational kinetic energy can be written

$$K_{\text{rot}} = \sum_{i=1}^{N} \tfrac{1}{2}m_i r_i^2 \omega^2$$

The entire object rotates at the same angular velocity ω, so the constants $\frac{1}{2}$ and ω^2 can be factored out of each term of the sum:

$$K_{\text{rot}} = \tfrac{1}{2}\left(\sum_{i=1}^{N} m_i r_i^2\right)\omega^2$$

The quantity in the parentheses *cannot change* since the distance between each particle and the rotation axis stays the same if the object is rigid and doesn't change shape. However difficult it may be to compute the sum in the parentheses, we only need to do it *once* for any given mass distribution and axis of rotation.

Let's give the quantity in the brackets the symbol I. In Chapter 5, we found it useful to draw analogies between translational variables and their rotational equivalents. By using the symbol I, we can see that translational and rotational kinetic energy have similar forms: translational kinetic energy is

$$K_{\text{tr}} = \tfrac{1}{2}mv^2$$

and **rotational kinetic energy** is

Since $v = r\omega$ was used to derive Eq. (8-1), ω must be expressed in radians per unit time (normally rad/s).

Rotational kinetic energy:

$$K_{\text{rot}} = \tfrac{1}{2}I\omega^2 \qquad \textbf{(8-1)}$$

The quantity I is called the **rotational inertia**:

Rotational inertia:

$$I = \sum_{i=1}^{N} m_i r_i^2 \tag{8-2}$$

(SI unit: $\text{kg}\cdot\text{m}^2$)

Comparing the expressions for translational and rotational kinetic energy, we see that angular speed ω takes the place of speed v and rotational inertia I takes the place of mass m. Mass is a measure of the inertia of an object, or, in other words, how difficult it is to change the object's velocity. Similarly, for a rigid rotating object, I is a measure of its rotational inertia—how hard it is to change its angular velocity. That is why the quantity I is called the rotational inertia; it is also sometimes called the **moment of inertia**.

When a problem requires you to find a rotational inertia, there are four principles to follow.

Finding the Rotational Inertia

1. If the object consists of a *small* number of particles, calculate the sum $I = \sum_{i=1}^{N} m_i r_i^2$ directly.
2. For symmetric objects with simple geometric shapes, advanced mathematical methods can be used to perform the sum in Eq. (8-2). Table 8.1 lists the results of these calculations for the shapes most commonly encountered.
3. Since the rotational inertia is a sum, you can always mentally decompose the object into several parts, find the rotational inertia of each part, and then add them. This is an example of the *divide-and-conquer* problem-solving technique.
4. Since the rotational inertia involves distances from the axis of rotation, you can move mass parallel to the rotation axis without changing I. For example, you might want to calculate the rotational inertia of a door about the axis through its hinges. Mentally "compress" the door vertically (parallel to the axis) into a horizontal rod with the same mass, as in Fig. 8.1. The rotational inertia is unchanged by the compression, since every particle maintains the same distance from the axis of rotation. Thus, the formula for the rotational inertia of a rod (listed in Table 8.1) can be used for the door.

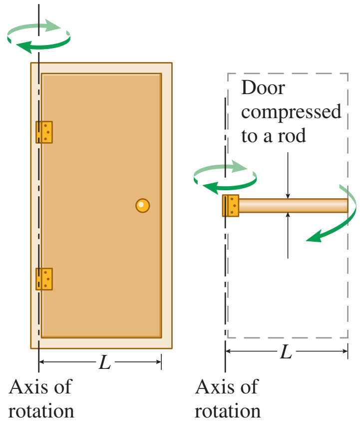


Figure 8.1 The rotational inertias of a door and a rod are both given by $\frac{1}{3}ML^2$.

Keep in mind that the rotational inertia of an object depends on the location of the rotation axis. For instance, imagine taking the hinges off the side of a door and putting them on the top so that the door swings about a horizontal axis like a cat flap door (Fig. 8.2b). The door now has a considerably larger rotational inertia than before the hinges

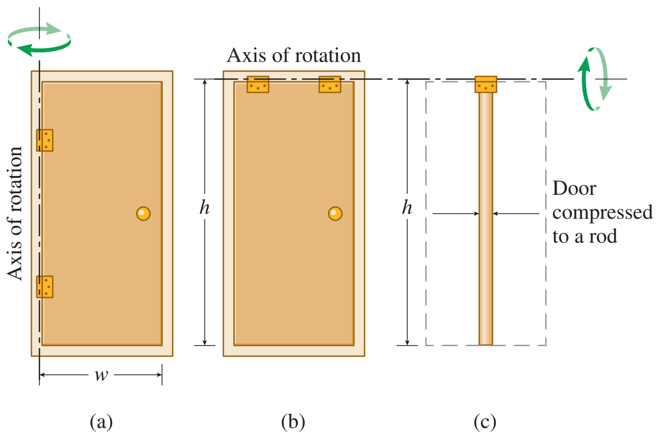
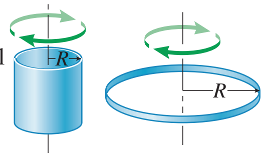
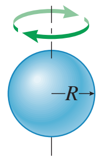
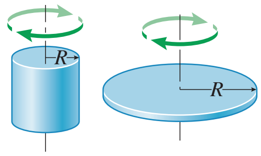
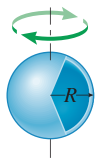
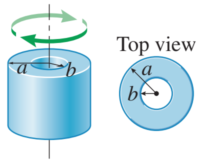
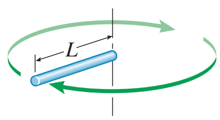
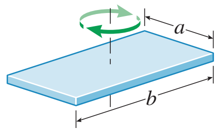
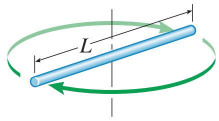


Figure 8.2 The rotational inertia of a door depends on the rotation axis. (a) The door with hinges at the side has a smaller rotational inertia, $I = \frac{1}{3}Mw^2$, than (b) the rotational inertia, $I = \frac{1}{3}Mh^2$, of the same door with hinges at the top. (c) The door can be mentally compressed horizontally into a rod.

Table 8.1

Rotational Inertia for Uniform Objects with Various Geometrical Shapes

Shape		Axis of Rotation	Rotational Inertia	Shape		Axis of Rotation	Rotational Inertia
Thin hollow cylindrical shell (or hoop)		Central axis of cylinder	MR^2	Solid sphere		Through center	$\frac{2}{5}MR^2$
Solid cylinder (or disk)		Central axis of cylinder	$\frac{1}{2}MR^2$	Thin hollow spherical shell		Through center	$\frac{2}{3}MR^2$
Hollow cylindrical shell or disk		Central axis of cylinder	$\frac{1}{2}M(a^2+b^2)$	Thin rod		Perpendicular to rod through end	$\frac{1}{3}ML^2$
Rectangular plate		Perpendicular to plate through center	$\frac{1}{12}M(a^2+b^2)$	Thin rod		Perpendicular to rod through center	$\frac{1}{12}ML^2$

were moved. The door has the same mass as before, but its mass now lies on average much farther from the axis of rotation than that of the door in Fig. 8.2a. In applying Eq. (8-2) to find the rotational inertia of the door, the values of r_i range from 0 to the height of the door (h), whereas with the hinges in the normal position the values of r_i range from 0 only to the width of the door (w). If we think of compressing the door with hinges on top into a rod, the door would have to be compressed *horizontally* into a rod of length h (Fig. 8.2c).

Example 8.1

Rotational Inertia of a Rod About Its Midpoint

Table 8.1 gives the rotational inertia of a thin rod with the axis of rotation perpendicular to its length and passing through one end ($I = \frac{1}{3}ML^2$). From this expression, derive the rotational inertia of a rod with mass M and length L that rotates about a perpendicular axis through its *midpoint* (Fig. 8.3).

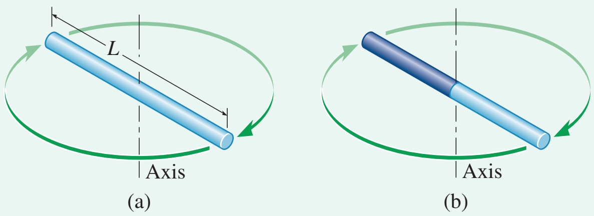


Figure 8.3

(a) A rod rotating about a vertical axis through its center. (b) The same rod, viewed as two rods, each half as long, rotating about an axis through an end.

Strategy In general, the same object rotating about a different axis has a different rotational inertia. With the axis at the midpoint, the rotational inertia is smaller than for the axis at the end, since the mass is closer to the axis, on average. Imagine performing the sum $\sum_{i=1}^{N} m_i r_i^2$; for the

Continued on next page

Example 8.1 Continued

axis at the end, the values of r_i range from 0 to L, whereas with the axis at the midpoint, r_i is never larger than $\frac{1}{2}L$.

Imagine cutting the rod in half; then there are two rods, each with its axis of rotation at one of its *ends*. Then, since rotational inertia is additive, the rotational inertia for two such rods is just twice the value for one rod.

Solution Each of the halves has mass $\frac{1}{2}M$ and length $\frac{1}{2}L$ and rotates about an axis at its endpoint. Table 8.1 gives $I = \frac{1}{3}ML^2$ for a rod with mass M and length L rotating about its end, so each of the halves has

$$I_{\text{half}} = \tfrac{1}{3} \times \text{mass of half} \times (\text{length of half})^2$$

$$= \tfrac{1}{3} \times (\tfrac{1}{2}M) \times (\tfrac{1}{2}L)^2 = \tfrac{1}{3} \times \tfrac{1}{2} \times \tfrac{1}{4} \times ML^2 = \tfrac{1}{24}ML^2$$

Since there are two such halves, the total rotational inertia is twice that:

$$I = I_{\text{half}} + I_{\text{half}} = \tfrac{1}{12}ML^2$$

Discussion The rotational inertia is less than $\frac{1}{3}ML^2$, as expected. That it is $\frac{1}{4}$ as much is a result of the distances r_i being *squared* in the definition of rotational inertia. The various particles that compose the rod are at distances that range from 0 to $\frac{1}{2}L$ from the rotation axis, instead of from 0 to L. Think of it as if the rod were compressed to half its length, still pivoting about the endpoint. All the distances r_i are half as much as before; since each r_i is squared in the sum, the rotational inertia is $(\frac{1}{2})^2 = \frac{1}{4}$ its former value.

Practice Problem 8.1 Playground Merry-Go-Round

A playground merry-go-round is essentially a uniform disk that rotates about a vertical axis through its center (Fig. 8.4). Suppose the disk has a radius of 2.0 m and a mass of 160 kg; a child weighing 180 N sits at the edge of the merry-go-round. What is the merry-go-round's rotational inertia, including the contribution due to the child? [*Hint:* Treat the child as a point mass at the edge of the disk.]

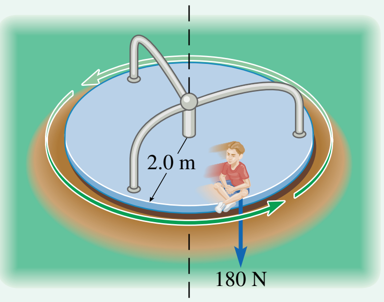


Figure 8.4 Child on a merry-go-round.

When applying conservation of energy to objects that rotate, the rotational kinetic energy is included in the mechanical energy. In Eq. (6-12),

$$W_{\text{nc}} = \Delta K + \Delta U = \Delta E_{\text{mech}} \tag{6-12}$$

just as U stands for the sum of the elastic and gravitational potential energies, K stands for the sum of the translational and rotational kinetic energies:

$$K = K_{\text{tr}} + K_{\text{rot}}$$

PHYSICS AT HOME

The change in rotational inertia of a rod as the rotation axis changes can be easily felt. Hold a baseball bat in the usual way, with your hands gripping the bottom of the bat. Swing the bat a few times. Now "choke up" on the bat—move your hands up the bat—and swing a few times. The bat is easier to swing because it now has a smaller rotational inertia. Children often choke up on a bat that is too massive for them. Even major league baseball players occasionally choke up on the bat when they want more control over their swing to place a hit in a certain spot (Fig. 8.5). On the other hand, choking up on the bat makes it impossible to hit a home run. To hit a long fly ball, you want the pitched baseball to encounter a bat that is swinging with a lot of rotational inertia.

Figure 8.5 Hank Aaron choking up on the bat.

Example 8.2

Atwood's Machine

Atwood's machine consists of a cord around a pulley of rotational inertia I, radius R, and mass M, with two blocks (masses m_1 and m_2) hanging from the ends of the cord as in Fig. 8.6. (Note that in Example 3.11 we analyzed Atwood's machine for the special case of a massless pulley; for a massless pulley $I = 0$.) Assume that the pulley is free to turn without friction and that the cord does not slip. Ignore air resistance. If the masses are released from rest, find how fast they are moving after they have moved a distance h (one up, the other down).

Strategy Neglecting both air resistance and friction means that no nonconservative forces act on the system; therefore, its mechanical energy is conserved. Gravitational potential energy is converted into the translational kinetic energies of the two blocks and the rotational kinetic energy of the pulley.

Solution For our convenience, we assume that $m_1 > m_2$. Mass m_1, therefore, moves down and m_2 moves up. After the masses have each moved a distance h, the changes in gravitational potential energy are

$$\Delta U_1 = -m_1 gh$$

$$\Delta U_2 = +m_2 gh$$

Since mechanical energy is conserved,

$$\Delta U + \Delta K = 0$$

The mechanical energy of the system includes the kinetic energies of three objects: the two masses and the pulley. All start with zero kinetic energy, so

$$\Delta K = \tfrac{1}{2}(m_1 + m_2)v^2 + \tfrac{1}{2}I\omega^2$$

The speed v of the masses is the same since the cord's length is fixed. The speed v and the angular speed of the pulley ω are related if the cord does not slip: the tangential speed of the pulley must equal the speed at which the cord moves. The tangential speed of the pulley is its angular speed times its radius:

$$v = \omega R$$

After substituting v/R for ω, the energy conservation equation becomes

$$\Delta U + \Delta K = [-m_1 gh + m_2 gh] + \left[\frac{1}{2}(m_1 + m_2)v^2 + \frac{1}{2}I\left(\frac{v}{R}\right)^2\right] = 0$$

or

$$\left[\frac{1}{2}(m_1 + m_2) + \frac{1}{2}\frac{I}{R^2}\right]v^2 = (m_1 - m_2)gh$$

Solving this equation for v yields

$$v = \sqrt{\frac{2(m_1 - m_2)gh}{m_1 + m_2 + I/R^2}}$$

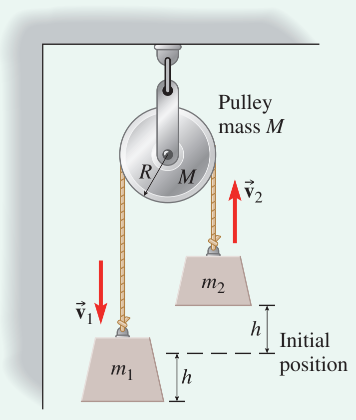


Figure 8.6
Atwood's machine.

Discussion This answer is rich in information, in the sense that we can ask many "What if?" questions. Not only do these questions provide checks as to whether the answer is reasonable, they also enable us to perform thought experiments, which could then be checked by constructing an Atwood's machine and comparing the results.

For instance: What if m_1 is only slightly greater than m_2? Then the final speed v is small—as m_2 approaches m_1, v approaches 0. This makes intuitive sense: a small imbalance in weights produces a small acceleration. You should practice this kind of reasoning by making other such checks.

It is also enlightening to look at terms in an algebraic solution and connect them with physical interpretations. The quantity $(m_1 - m_2)g$ is the imbalance in the gravitational forces pulling on the two sides. The denominator $(m_1 + m_2 + I/R^2)$ is a measure of the total inertia of the system—the sum of the two masses plus an inertial contribution due to the pulley. The pulley's contribution is *not* simply equal to its mass. If, for example, the pulley is a uniform disk with $I = \tfrac{1}{2}MR^2$, the term I/R^2 would be equal to *half* the mass of the pulley.

The same principles used to analyze Atwood's machine have many applications in the real world. One such application is in elevators, where one of the hanging masses is the elevator and the other is the counterweight. Of course, the elevator and counterweight are not allowed to hang freely from a pulley—we must also consider the energy supplied by the motor.

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Example 8.2 Continued

Practice Problem 8.2 Modified Atwood's Machine

Figure 8.7 shows a modified form of Atwood's machine where one of the blocks slides on a table instead of hanging from the pulley. The coefficient of kinetic friction between the sliding mass and the table is μ_k. The blocks are released from rest. Find the speed of the blocks after they have moved a distance h in terms of μ_k, m_1, m_2, I, R, and h.

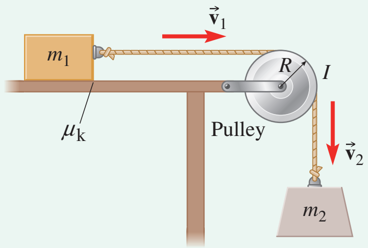

Figure 8.7 Modified Atwood's machine.

8.2 TORQUE

Suppose you place a bicycle upside down, as if you're going to repair it. First, you give one of the wheels a spin. If everything is working as it should, the wheel spins for quite a while; its angular acceleration is small. If the wheel doesn't spin for very long, then its angular velocity changes rapidly and the angular acceleration is large in magnitude; there must be excessive friction somewhere. Perhaps the brakes are rubbing on the rim or the bearings need to be repacked.

If we could eliminate *all* the frictional forces acting on the wheel, including air resistance, then we would expect the wheel to keep spinning without diminishing angular speed. In that case, its angular acceleration would be zero. The situation is reminiscent of Newton's first law: a body with no external interactions, or at least no net force acting on it, moves with constant velocity. We can state a "Newton's first law for rotation": a rotating body with no external interactions, and whose rotational inertia doesn't change, keeps rotating at constant angular velocity.

Of course, the hypothetical frictionless bicycle wheel does have external interactions. The Earth's gravitational field exerts a downward force and the axle exerts an upward force to keep the wheel from falling. Then is it true that, as long as there is no net external force, the angular acceleration is zero? No; it is easy to give the wheel an angular acceleration while keeping the net force zero. Imagine bringing the wheel to rest by pressing two hands against the tire on opposite sides. On one side, the motion of the rim of the tire is downward and the kinetic frictional force is upward (see Fig. 8.8). On the other side, the tire moves upward and the frictional force is downward. In a similar way, we could apply equal and opposite forces to the opposite sides of a wheel at rest to make it start spinning. In either case, we exert equal magnitude forces, so that the net force is zero, and still give the wheel an angular acceleration.

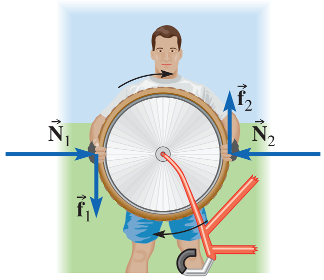

Figure 8.8 A spinning bicycle wheel slowed to a stop by friction. Each hand exerts a normal force and a frictional force on the tire. The two normal forces add to zero and the two frictional forces add to zero.

A quantity related to force, called **torque**, plays the role in rotation that force itself plays in translation. A torque is not separate from a force; it is impossible to exert a torque without exerting a force. Torque is a measure of how effective a given force is at twisting or turning something. For something rotating about a fixed axis such as the bicycle wheel, a torque can *change* the rotational motion either by making it rotate faster or by slowing it down.

When stopping the bicycle wheel with two equal and opposite forces, as in Fig. 8.8, the net applied force is zero and, thus, the wheel is in translational equilibrium; but the net torque is not zero, so it is not in rotational equilibrium. Both forces tend to give the wheel the same sign of angular acceleration; they are both making the wheel slow down. The two torques are in fact equal, with the same sign.

The *radial* direction is directly toward or away from the axis of rotation. The *perpendicular* or *tangential* direction is perpendicular to both the radial direction and the axis of rotation; it is tangent to the circular path followed by a point as the object rotates.

What determines the torque produced by a particular force? Imagine trying to push open a massive bank vault door. Certainly you would push as hard as you can; the torque is proportional to the magnitude of the force. It also matters where and in what direction the force is applied. For maximum effectiveness, you would push perpendicularly to the

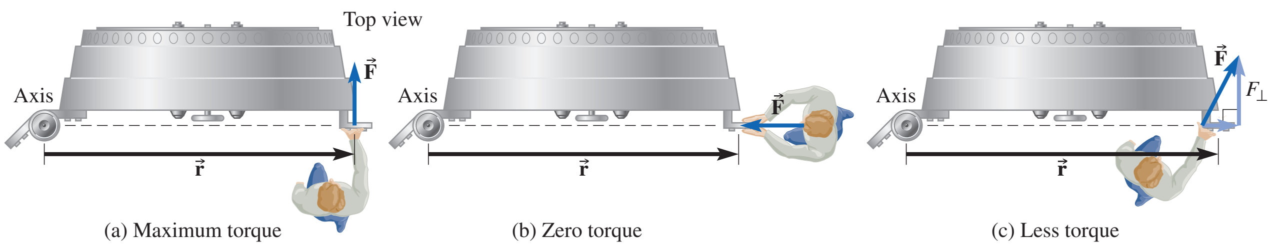

Figure 8.9 Torque on a bank vault door depends on the direction of the applied force. (a) Pushing perpendicularly gives the maximum torque. (b) Pushing radially inward with the same magnitude force gives zero torque. (c) The torque is proportional to the perpendicular component of the force ($F_\perp$).

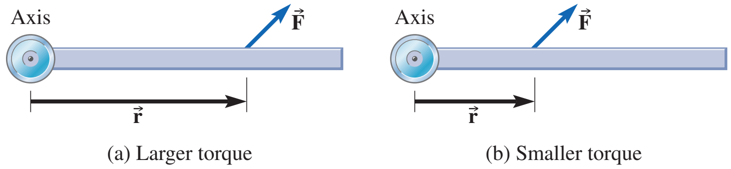

Figure 8.10 Torques; the same force at different distances from the axis.

door (Fig. 8.9a). If you pushed radially, straight in toward the axis of rotation that passes through the hinges, the door wouldn't rotate, no matter how hard you push (Fig. 8.9b). A force acting in any other direction could be decomposed into radial and perpendicular components, with the radial component contributing nothing to the torque (Fig. 8.9c). Only the perpendicular component of the force ($F_\perp$) produces a torque.

The symbol $\perp$ stands for *perpendicular*; $\|$ stands for *parallel.*

Furthermore, *where* you apply the force is critical (Fig. 8.10). Instinctively, you would push at the outer edge, as far from the rotation axis as possible. If you pushed close to the axis, it would be difficult to open the door. Torque is proportional to the distance between the rotation axis and the **point of application** of the force (the point at which the force is applied).

To satisfy the requirements of the previous paragraphs, we define the magnitude of the torque as the product of the distance between the rotation axis and the point of application of the force (r) with the perpendicular component of the force ($F_\perp$):

Definition of torque:

$$\tau = \pm rF_\perp \tag{8-3}$$

where r is the shortest distance between the rotation axis and the point of application of the force and $F_\perp$ is the perpendicular component of the force.

The symbol for torque is τ, the Greek letter tau. The SI unit of torque is the N·m. The SI unit of *energy,* the joule, is equivalent to N·m, but we do not write torque in joules. Even though both energy and torque can be written using the same SI base units, the two quantities have different meanings; torque is not a form of energy. To help maintain the distinction, the joule is used for energy but *not* for torque.

The sign of the torque indicates the direction of the angular acceleration that torque would cause *by itself.* Recall from Section 5.1 that by convention a positive angular velocity ω means counterclockwise (CCW) rotation and a negative angular velocity ω means clockwise (CW) rotation. A positive angular acceleration α either increases the rate of CCW rotation (increases the magnitude of a positive ω) or decreases the rate of CW rotation (decreases the magnitude of a negative ω).

In a more general treatment of torque, torque is a vector quantity defined as the cross product $\vec{\tau} = \vec{r} \times \vec{F}$. See Appendix A.8 for the definition of the cross product. For an object rotating about a fixed axis, Eq. (8-3) gives the component of $\vec{\tau}$ along the axis of rotation.

We use the same sign convention for torque. A force whose perpendicular component tends to cause rotation in the CCW direction gives rise to a positive torque; if it is the only torque acting, it would cause a positive angular acceleration α (see Fig. 8.11). A force whose perpendicular component tends to cause rotation in the CW direction

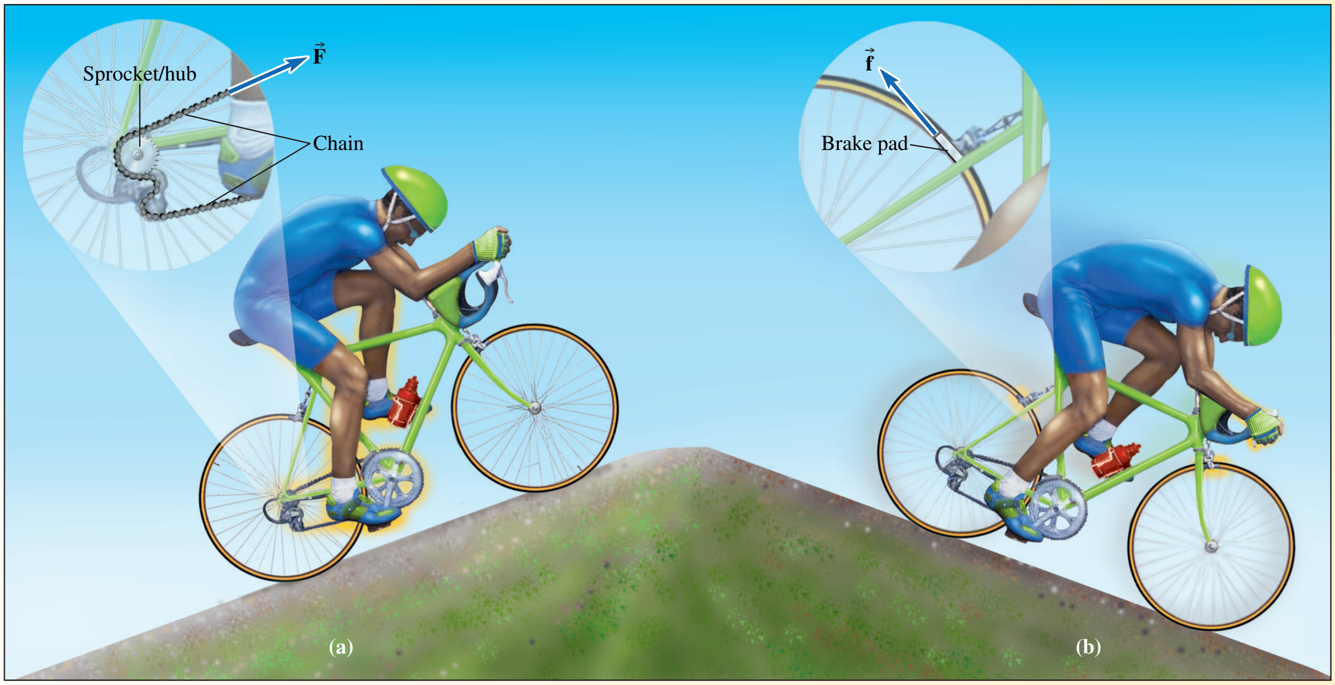

Figure 8.11 (a) When the cyclist climbs a hill, the top half of the chain exerts a large force $\vec{\mathbf{F}}$ on the sprocket attached to the rear wheel. As viewed here, the torque about the axis of rotation (the axle) due to this force is clockwise. By convention, we call this a negative torque. (b) When the brakes are applied, the brake pads are pressed onto the rim, giving rise to frictional forces on the rim. As viewed here, the frictional force $\vec{\mathbf{f}}$ causes a counterclockwise (positive) torque on the wheel about the axle. (A viewer on the other side of the bike would draw opposite conclusions about the signs of the torques.)

gives rise to a negative torque. The symbol ± in Eq. (8-3) reminds us to assign the appropriate algebraic sign each time we calculate a torque.

The sign of the torque is *not* determined by the sign of the angular velocity (in other words, whether the wheel is spinning CCW or CW); rather, it is determined by the sign of the angular *acceleration* the torque would cause if acting alone. To determine the sign of a torque, imagine which way the torque would make the object begin to spin if it is initially not rotating.

Example 8.3

A Spinning Bicycle Wheel

To stop a spinning bicycle wheel, suppose you push radially inward on opposite sides of the wheel, as shown in Fig. 8.8, with equal forces of magnitude 10.0 N. The radius of the wheel is 32 cm and the coefficient of kinetic friction between the tire and your hand is 0.75. The wheel is spinning in the clockwise sense. What is the net torque on the wheel?

Strategy The 10.0-N forces are directed radially toward the rotation axis, so they produce no torques themselves; only perpendicular components of forces give rise to torques. The forces of kinetic friction between the hands and the tire are tangent to the tire, so they do produce torques. The normal force applied to the tire is 10.0 N on each side; using the coefficient of friction, we can find the frictional forces.

Solution The frictional force exerted by each hand on the tire has magnitude

$$f = \mu_k N = 0.75 \times 10.0\ \text{N} = 7.5\ \text{N}$$

The frictional force is tangent to the wheel, so $f_\perp = f$. Then the magnitude of each torque is

$$|\tau| = rf_\perp = 0.32\ \text{m} \times 7.5\ \text{N} = 2.4\ \text{N·m}$$

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Example 8.3 Continued

The two torques have the same sign, since they are both tending to slow down the rotation of the wheel. Is the torque positive or negative? The angular velocity of the wheel is negative since it rotates clockwise. The angular acceleration has the opposite sign because the angular speed is decreasing. Since $\alpha > 0$, the net torque is also positive. Therefore,

$$\sum\tau = +4.8\ \text{N}\cdot\text{m}$$

Discussion The trickiest part of calculating torques is determining the sign. To check, look at the frictional forces in Fig. 8.8. Imagine which way the forces would make the wheel begin to rotate if the wheel were not originally rotating. The frictional forces point in a direction that would tend to cause a counterclockwise rotation, so the torques are positive.

Practice Problem 8.3 Disc Brakes

In the disc brakes that slow down a car, a pair of brake pads squeeze a spinning rotor; friction between the pads and the rotor provides the torque that slows down the car. If the normal force that each pad exerts on a rotor is 85 N and the coefficient of friction is 0.62, what is the frictional force on the rotor due to each of the pads? If this force acts 8.0 cm from the rotation axis, what is the magnitude of the torque on the rotor due to the pair of brake pads?

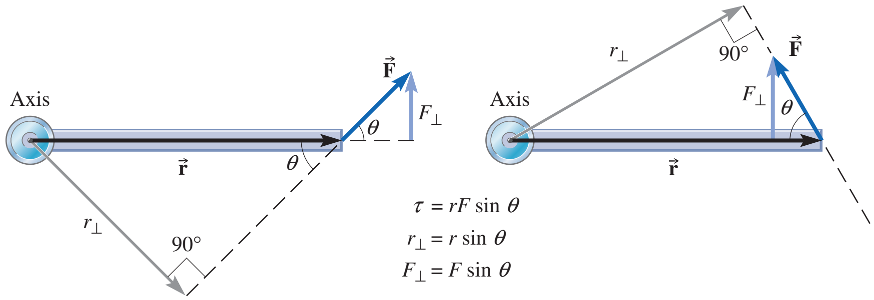

Figure 8.12 Finding the magnitude of a torque using the lever arm.

Lever Arms

There is another, completely equivalent, way to calculate torques that is often more convenient than finding the perpendicular component of the force. Figure 8.12 shows a force $\vec{\mathbf{F}}$ acting at a distance r from an axis. The distance r is the length of a line perpendicular to the axis that runs from the axis to the force's point of application. The force makes an angle θ with that line. The torque is then

$$\tau = \pm rF_\perp = \pm r(F \sin\theta)$$

The factor $\sin\theta$ could be grouped with r instead of with F. Then $\tau = \pm(r\sin\theta)F$, or

$$\tau = \pm r_\perp F \tag{8-4}$$

The distance $r_\perp$ is called the **lever arm** (or **moment arm**). The magnitude of the torque is, therefore, the magnitude of the force times the lever arm.

Finding Torques Using the Lever Arm

1. Draw a line parallel to the force through the force's point of application; this line (dashed in Fig. 8.12) is called the force's **line of action**.
2. Draw a line from the rotation axis to the line of action. This line must be perpendicular to both the axis and the line of action. The distance from the axis to the line of action along this perpendicular line is the lever arm ($r_\perp$). If the line of action of the force goes through the rotation axis, the lever arm and the torque are both zero (Fig. 8.9b).
3. The magnitude of the torque is the magnitude of the force times the lever arm:

$$\tau = \pm r_\perp F$$

4. Determine the algebraic sign of the torque as before.

Example 8.4

Screen Door Closer

An automatic screen door closer attaches to a door 47 cm away from the hinges and pulls on the door with a force of 25 N, making an angle of 15° with the door (Fig. 8.13). Find the magnitude of the torque exerted on the door due to this force about the rotation axis through the hinges using (a) the perpendicular component of the force and (b) the lever arm. (c) What is the sign of this torque as viewed from above?

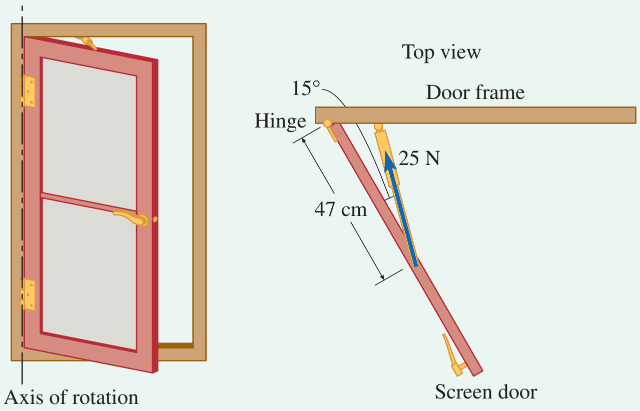

Figure 8.13
Screen door with automatic closing mechanism.

Strategy For method (a), we must find the component of the 25-N force perpendicular to the radial direction. Then this component is multiplied by the length of the radial line. For method (b), we draw in the line of action of the force. Then the lever arm is the perpendicular distance from the line of action to the rotation axis. The torque is the magnitude of the force times the lever arm. We must be careful not to combine the two methods: the torque is *not* equal to the perpendicular force component times the lever arm. For (c), we determine whether this torque would tend to make the door rotate CCW or CW.

Solution (a) As shown in Fig. 8.14a, the radial component of the force ($F_{\parallel}$) passes through the rotation axis. The angle labeled 15° would actually be a bit larger than 15°, but since the thickness of the door is much less than 47 cm, we approximate it as 15°. The perpendicular component is

$$F_{\perp} = F \sin 15°$$

The magnitude of the torque is

$$|\tau| = rF_{\perp} = 0.47\ \text{m} \times 25\ \text{N} \times \sin 15° = 3.0\ \text{N}\cdot\text{m}$$

(b) Figure 8.14b shows the line of action of the force, drawn parallel to the force and passing through the point of application. The lever arm is the perpendicular distance between the rotation axis and the line of action. The distance r is approximately 47 cm (again neglecting the thickness of the door). Then the lever arm is

$$r_{\perp} = r \sin 15°$$

and the magnitude of the torque is

$$|\tau| = r_{\perp}F = 0.47\ \text{m} \times \sin 15° \times 25\ \text{N} = 3.0\ \text{N}\cdot\text{m}$$

(c) Using the top view of Fig. 8.13, the torque tends to close the door by making it rotate counterclockwise (assuming the door is initially at rest and no other torques act). The torque is therefore positive as viewed from above.

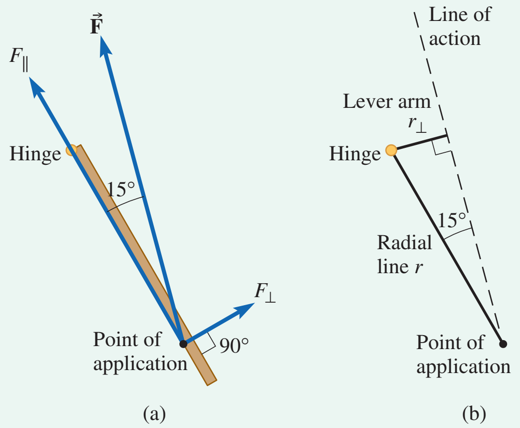

Figure 8.14
(a) Finding the perpendicular component of the force.
(b) Finding the lever arm.

Discussion The most common mistake to make in either solution method would be to use cosine instead of sine (or, equivalently, to use the complementary angle 75° instead of 15°). A check is a good idea. If the automatic closer were more nearly parallel to the door, the angle would be less than 15°. The torque would be smaller because the force is more nearly pulling straight in toward the axis. Since the sine function gets smaller for angles closer to zero, the expression checks out correctly.

It might seem silly for a door closer to pull at such an angle that the perpendicular component is relatively small. The reason it's done that way is so the door closer does not get in the way. A closer that pulled in a perpendicular direction would stick straight out from the door. As discussed in Section 8.5, the situation is much the same in our bodies. In order to not inhibit the motion of our limbs, our tendons and muscles are nearly parallel to the bones. As a result, the forces they exert must be much larger than we might expect.

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Example 8.4 Continued

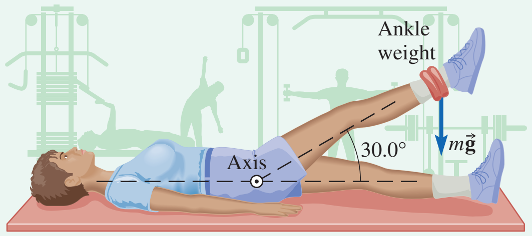

Figure 8.15
Exercise leg lifts.

Practice Problem 8.4 Exercise Is Good for You

A person is lying on an exercise mat and lifts one leg at an angle of 30.0° from the horizontal with an 89-N (20-lb) weight attached to the ankle (Fig. 8.15). The distance between the ankle weight and the hip joint (which is the rotation axis for the leg) is 84 cm. What is the torque due to the ankle weight on the leg?

Center of Gravity

We have seen that the torque produced by a force depends on the point of application of the force. What about gravity? The gravitational force on a body is not exerted at a single point, but is distributed throughout the volume of the body. When we talk of "the" force of gravity on something, we really mean the total force of gravity acting on each particle making up the system.

When calculating the torque due to gravity, consider the entire gravitational force to act at the center of gravity.

Fortunately, when we need to find the total torque due to the forces of gravity acting on an object, the total force of gravity can be considered to act at a single point. This point is called the **center of gravity**. The torque found this way is the same as finding all the torques due to the forces of gravity acting at every point in the body and adding them together. As you can verify in Problem 94, if the gravitational field is uniform in magnitude and direction, then the center of gravity of an object is located at the object's center of mass.

8.3 WORK DONE BY A TORQUE

Torques can do work, as anyone who has started a lawnmower with a pull cord can verify. Actually, it is the force that does the work, but in rotational problems it is often simpler to calculate the work done from the torque. Just as the work done by a constant force is the product of force and the parallel component of displacement, work done by a constant torque can also be calculated as the torque times the *angular* displacement.

Imagine a torque acting on a wheel that spins through an angular displacement $\Delta\theta$ while the torque is applied. The work done by the force that gives rise to the torque is the product of the perpendicular component of the force ($F_\perp$) with the arc length s through which the point of application of the force moves (see Fig. 8.16). We use the perpendicular force component because that is the component parallel to the *displacement,* which is instantaneously tangent to the arc of the circle. Thus,

$$W = F_\perp s \tag{8-5}$$

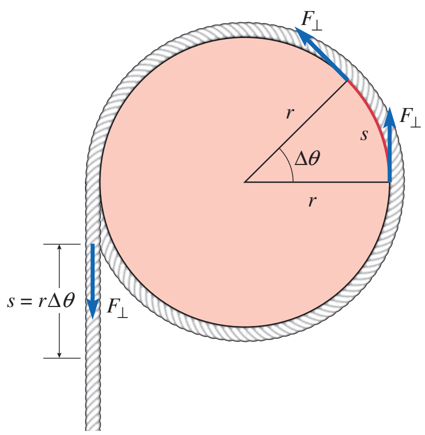

Figure 8.16 Work done by torque.

To write the work in terms of torque, note that $\tau = rF_\perp$ and $s = r\,\Delta\theta$; then

$$W = F_\perp s = \frac{\tau}{r} \times r\,\Delta\theta = \tau\,\Delta\theta$$

$$W = \tau\,\Delta\theta \quad (\Delta\theta \text{ in radians}) \qquad \text{(8-6)}$$

Work is indeed the product of torque and the angular displacement. If τ and $\Delta\theta$ have the same sign, the work done is positive; if they have opposite signs, the work done is negative. The *power* due to a constant torque—the rate at which the torque does work—is

$$P = \tau\omega \qquad \text{(8-7)}$$

Example 8.5

Work Done on a Potter's Wheel

A potter's wheel is a heavy stone disk upon which the pottery is shaped. Potter's wheels were once driven by the potter pushing on a foot treadle; today most potter's wheels are driven by electric motors. (a) If the potter's wheel is a uniform disk of mass 40.0 kg and diameter 0.50 m, how much work must be done by the motor to bring the wheel from rest to 80.0 rpm? (b) If the motor delivers a constant torque of 8.2 N·m during this time, through how many revolutions does the wheel turn in coming up to speed?

Strategy Work is an energy transfer. In this case, the motor is increasing the rotational kinetic energy of the potter's wheel. Thus, the work done by the motor is equal to the change in rotational kinetic energy of the wheel, ignoring frictional losses. In the expression for rotational kinetic energy, we must express ω in rad/s; we cannot substitute 80.0 rpm for ω. Once we know the work done, we use the torque to find the angular displacement.

Solution (a) The change in rotational kinetic energy of the wheel is

$$\Delta K = \tfrac{1}{2}I(\omega_f^2 - \omega_i^2) = \tfrac{1}{2}I\omega_f^2$$

Initially the wheel is at rest, so the initial angular velocity ω_i is zero. From Table 8.1, the rotational inertia of a uniform disk is

$$I = \tfrac{1}{2}MR^2$$

Substituting this for I,

$$\Delta K = \tfrac{1}{4}MR^2\omega_f^2$$

Before substituting numerical values, we convert 80.0 rpm to rad/s:

$$\omega_f = 80.0\,\frac{\text{rev}}{\text{min}} \times 2\pi\,\frac{\text{rad}}{\text{rev}} \times \frac{1}{60}\,\frac{\text{min}}{\text{s}} = 8.38 \text{ rad/s}$$

Substituting the known values for mass and radius,

$$\Delta K = \tfrac{1}{4} \times 40.0 \text{ kg} \times \left(\tfrac{0.50}{2} \text{ m}\right)^2 \times (8.38 \text{ rad/s})^2 = 43.9 \text{ J}$$

Therefore, the work done by the motor, rounded to two significant figures, is 44 J.

(b) The work done by a constant torque is

$$W = \tau\,\Delta\theta$$

Solving for the angular displacement $\Delta\theta$ gives

$$\Delta\theta = \frac{W}{\tau} = \frac{43.9 \text{ J}}{8.2 \text{ N·m}} = 5.35 \text{ rad}$$

Since 2π rad = 1 revolution,

$$\Delta\theta = 5.35 \text{ rad} \times \frac{1 \text{ rev}}{2\pi \text{ rad}} = 0.85 \text{ rev}$$

Discussion As always, work is an energy transfer. In this problem, the work done by the motor is the means by which the potter's wheel acquires its rotational kinetic energy. But work done by a torque does not *always* appear as a change in rotational kinetic energy. For instance, when you wind up a mechanical clock or a windup toy, the work done by the torque you apply is stored as elastic potential energy in some sort of spring.

Practice Problem 8.5 Work Done on an Air Conditioner

A belt wraps around a pulley of radius 7.3 cm that drives the compressor of an automobile air conditioner. The tension in the belt on one side of the pulley is 45 N and on the other side of the pulley it is 27 N (Fig. 8.17). How much work is done by the belt on the compressor during one revolution of the pulley?

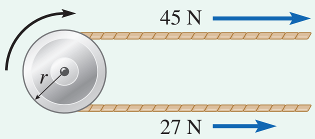

Figure 8.17

8.4 EQUILIBRIUM REVISITED

In Chapter 2, we said that an object is in equilibrium when the net force acting on it is zero. That statement is true but incomplete. It is quite possible for the net force acting to be zero, while the net torque is nonzero; the object would then have a nonzero angular acceleration. When designing a bridge or a new house, it would be unacceptable for any of the parts to have nonzero angular acceleration! Zero net force is sufficient to ensure *translational* equilibrium; if an object is also in *rotational* equilibrium, then the net torque acting on it must also be zero.

Conditions for equilibrium:

$$\sum \vec{\mathbf{F}} = 0 \quad \text{and} \quad \sum \tau = 0 \tag{8-8}$$

Before tackling equilibrium problems, we must resolve a conundrum: if something is not rotating, then where is the axis of rotation? How can we calculate torques without knowing where the axis of rotation is? In some cases, perhaps involving axles or hinges, there may be a clear axis about which the object would rotate if the balance of forces and torques is disturbed. In many cases, though, it is not clear what the rotation axis would be, and in general it depends on exactly how the equilibrium is upset. Fortunately, the axis can be chosen *arbitrarily* when calculating torques *in equilibrium problems*.

In equilibrium, the net torque about *any* rotation axis must be zero. Does that mean that we have to write down an infinite number of torque equations, one for each possible axis of rotation? Fortunately, no. Although the proof is complicated, it can be shown that if the net force acting on an object is zero and the net torque about one rotation axis is zero, then the net torque about every other axis parallel to that axis must also be zero. Therefore, one torque equation is all we need.

Since the torque can be calculated about any desired axis, a judicious choice can greatly simplify the solution of the problem. The best place to choose the axis is usually at the point of application of an unknown force so that the unknown force does not appear in the torque equation.

Example 8.6

Carrying a 6 × 6 Beam

Two carpenters are carrying a uniform 6 × 6 beam. The beam is 8.00 ft (2.44 m) long and weighs 425 N (95.5 lb). One of the carpenters, being a bit stronger than the other, agrees to carry the beam 1.00 m in from the end; the other carries the beam at its opposite end. What is the upward force exerted on the beam by each carpenter?

Strategy The conditions for equilibrium are that the net external force equal zero and the net external torque equal zero. Should we start with forces or with torques? In this problem, it is easiest to start with torques. If we choose the axis of rotation where one of the unknown forces acts, then that force has a lever arm of zero and its torque is zero. The torque equation can be solved for the other unknown force. Then with only one force still unknown, we set the sum of the y-components of the forces equal to zero.

Solution The first step is to draw a force diagram (Fig. 8.18). Each force is drawn at the point where it acts. Known distances are labeled.

We choose a rotation axis perpendicular to the xy-plane and passing through the point of application of $\vec{\mathbf{F}}_2$. The simplest way to find the torques for this example is to multiply each force by its lever arm. The lever arm for $\vec{\mathbf{F}}_1$ is

$$2.44\ \text{m} - 1.00\ \text{m} = 1.44\ \text{m}$$

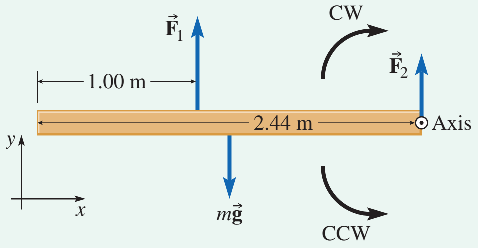

Figure 8.18
Diagram of the beam with rotation axis, forces, and distances shown.

and the magnitude of the torque due to this force is

$$|\tau| = Fr_\perp = F_1 \times 1.44\ \text{m}$$

Since the beam is uniform, its center of gravity is at its midpoint. We imagine the entire gravitational force to act at this point. Then the lever arm for the gravitational force is

$$\tfrac{1}{2} \times 2.44\ \text{m} = 1.22\ \text{m}$$

and the torque due to gravity has magnitude

$$|\tau| = Fr_\perp = 425\ \text{N} \times 1.22\ \text{m} = 518.5\ \text{N·m}$$

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Example 8.6 Continued

The torque due to $\vec{\mathbf{F}}_1$ is negative since, if it were the only torque, it would make the beam start to rotate clockwise about our chosen axis of rotation. The torque due to gravity is positive since, if it were the only torque, it would make the beam start to rotate counterclockwise. Therefore,

$$\sum \tau = -F_1 \times 1.44\ \text{m} + 518.5\ \text{N·m} = 0$$

Solving for F_1,

$$F_1 = \frac{518.5\ \text{N·m}}{1.44\ \text{m}} = 360\ \text{N}$$

Since another condition for equilibrium is that the net force be zero,

$$\sum F_y = F_1 + F_2 - mg = 0$$

Solving for F_2,

$$F_2 = 425\ \text{N} - 360\ \text{N} = 65\ \text{N}$$

Discussion A good way to check this result is to make sure that the net torque about a *different axis* is zero—for an object in equilibrium, the net torque about any axis must be zero. Suppose we choose an axis through the point of application of $\vec{\mathbf{F}}_1$. Then the lever arm for $m\vec{\mathbf{g}}$ is 1.22 m − 1.00 m = 0.22 m and the lever arm for $\vec{\mathbf{F}}_2$ is 2.44 m − 1.00 m = 1.44 m. Setting the net torque equal to zero:

$$\sum \tau = -425\ \text{N} \times 0.22\ \text{m} + F_2 \times 1.44\ \text{m} = 0$$

Solving for F_2 gives

$$F_2 = \frac{425\ \text{N} \times 0.22\ \text{m}}{1.44\ \text{m}} = 65\ \text{N}$$

which agrees with the value calculated before. We could have used this second torque equation to find F_2 instead of setting $\sum F_y$ equal to zero.

Practice Problem 8.6 A Diving Board

A uniform diving board of length 5.0 m is supported at two points; one support is located 3.4 m from the end of the board and the second is at 4.6 m from the end (Fig. 8.19). The supports exert vertical forces on the diving board. A diver stands at the end of the board over the water. Determine the directions of the support forces. [*Hint:* In this problem, consider torques about different rotation axes.]

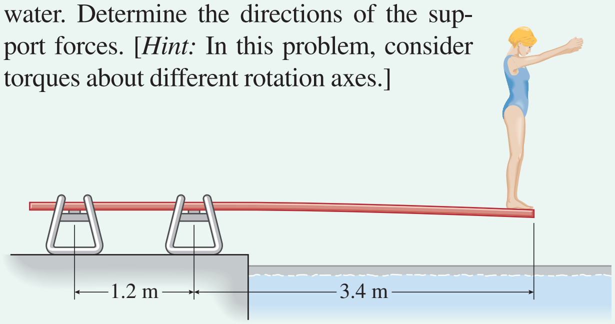

Figure 8.19
Diving board.

A diving board is an example of a cantilever—a beam or pole that extends beyond its support. The forces exerted by the supports on a diving board are considerably larger than if the same board were supported at both ends (see Problem 32). The advantage is that the far end of the board is left free to vibrate; as it does, the support forces adjust themselves to keep the board from tipping over. The architect Frank Lloyd Wright was fond of using cantilever construction to open up the sides and corners of a building, allowing corner windows that give buildings a lighter and more spacious feel (Fig. 8.20).

Making the Connection: cantilever building construction

Figure 8.20 The cantilevered master bedroom in the north wing of Wingspread by Frank Lloyd Wright juts well out over its brick foundation. The cypress trellis extending even farther beyond the bedroom balcony filters the natural light and serves to emphasize the free-floating nature of the structure with views of the landscape below.

Example 8.7
The Slipping Ladder

A 15.0-kg uniform ladder leans against a wall in the atrium of a large hotel (Fig. 8.21a). The ladder is 8.00 m long; it makes an angle $\theta = 60.0°$ with the floor. The coefficient of static friction between the floor and the ladder is μ_s = 0.45. How far along the ladder can a 60.0-kg person climb before the ladder starts to slip? Assume that the wall is frictionless.

Strategy Normal forces act on the ladder due to the wall ($\vec{\mathbf{N}}_w$) and the floor ($\vec{\mathbf{N}}_f$). A frictional force acts on the base of the ladder due to the floor ($\vec{\mathbf{f}}$), but no frictional force acts on the top of the ladder since the wall is frictionless. Gravitational forces act on the ladder and on the person climbing it. Consider the ladder and the climber as a single system. Until the ladder starts to slip, this system is in equilibrium. Therefore, the net external force and the net external torque acting on the system are both equal to zero. As the person ascends the ladder, the frictional force $\vec{\mathbf{f}}$ has to increase to keep the ladder in equilibrium. The ladder begins to slip when the frictional force required to maintain equilibrium is larger than its maximum possible value $\mu_s N_f$. The ladder is about to slip when $f = \mu_s N_f$.

Solution The first step is to make a careful drawing of the ladder and label all distances and forces (Fig. 8.21b). Instead of cluttering the diagram with numerical values, we use L for the length of the ladder, d for the unknown distance from the bottom of the ladder to the point where the person stands, and M and m for the masses of the person and ladder, respectively. The weight of the ladder acts at the ladder's center of gravity, which is the ladder's midpoint since it is uniform.

The conditions for equilibrium are

$$\sum F_x = 0, \quad \sum F_y = 0, \quad \text{and} \sum \tau = 0$$

Starting with $\sum F_x = 0$, we find

$$N_w - f = 0$$

where, if the climber is at the highest point possible, the frictional force must have its maximum possible magnitude:

$$f = \mu_s N_f$$

Combining these two equations, we obtain a relationship between the magnitudes of the two normal forces:

$$N_w = \mu_s N_f$$

Next we use the condition $\sum F_y = 0$, which gives

$$N_f - Mg - mg = 0$$

The only unknown quantity in this equation is N_f, so we can solve for it:

$$Mg = 60.0 \text{ kg} \times 9.80 \text{ m/s}^2 = 588 \text{ N}$$

$$mg = 15.0 \text{ kg} \times 9.80 \text{ m/s}^2 = 147 \text{ N}$$

$$N_f = Mg + mg = 588 \text{ N} + 147 \text{ N} = 735 \text{ N}$$

Now we can find the other normal force, N_w:

$$N_w = \mu_s N_f = 0.45 \times 735 \text{ N} = 331 \text{ N}$$

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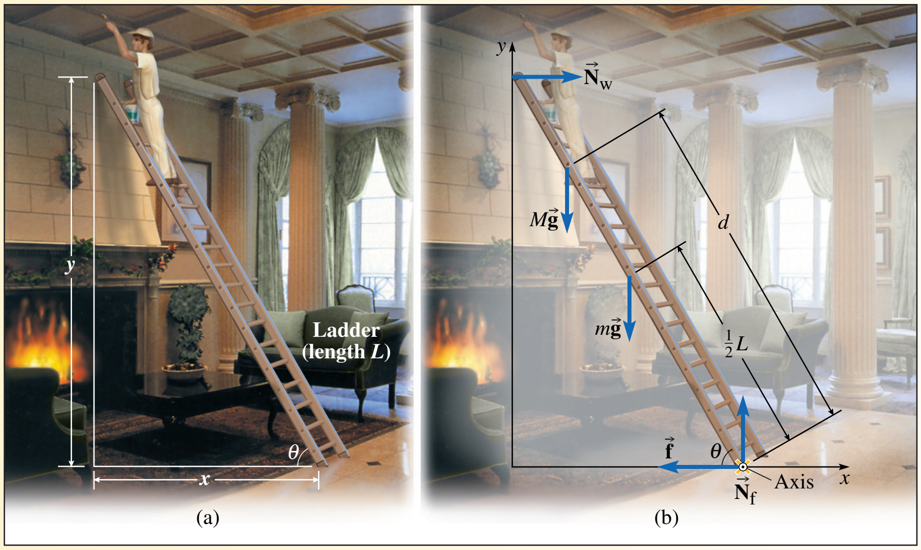

Figure 8.21 (a) A ladder and (b) forces acting on the ladder.

Example 8.7 Continued

At this point, we know the magnitudes of all the forces. We do not know the distance d, which is the goal of the problem. To find d we must set the net torque equal to zero.

First we choose a rotation axis. The most convenient choice is an axis perpendicular to the plane of Fig. 8.21 and passing through the bottom of the ladder. Since two of the five forces ($\vec{\mathbf{N}}_f$ and $\vec{\mathbf{f}}$) act at the bottom of the ladder, these two forces have zero lever arms and, thus, produce zero torque. Another reason why this is a convenient choice of axis is that the distance d is measured from the bottom of the ladder.

In this situation, with the forces either vertical or horizontal, it is probably easiest to use lever arms to find the torques. In three diagrams (Fig. 8.22), we first draw the line of action for each force; then the lever arm is the perpendicular distance between the axis and the line of action.

Using the usual convention that counterclockwise torques are positive, the torque due to $\vec{\mathbf{N}}_w$ is negative while the torques due to gravity are positive. The magnitude of each torque is the magnitude of the force times its lever arm:

$$\tau = Fr_\perp$$

Setting the net torque equal to zero yields

$$-N_w L \sin\theta + mg\left(\tfrac{1}{2}L\cos\theta\right) + Mgd\cos\theta = 0$$

Substituting known values,

$$-331\ \text{N} \times 8.00\ \text{m} \times \sin 60.0° + 147\ \text{N} \times 4.00\ \text{m} \times \cos 60.0° + 588\ \text{N} \times d\cos 60.0° = 0$$

$$-2293\ \text{N·m} + 294\ \text{N·m} + 294\ \text{N} \times d = 0$$

$$d = \frac{2293\ \text{N·m} - 294\ \text{N·m}}{294\ \text{N}} = 6.8\ \text{m}$$

The person can climb 6.8 m up the ladder without having it slip. This is the distance *along the ladder,* not the height above the ground, which is

$$h = 6.8\ \text{m} \times \sin 60.0° = 5.9\ \text{m}$$

Discussion If the person goes any higher, then his weight produces a larger CCW torque about our chosen rotation axis. To stay in equilibrium, the total CW torque would have to get larger. The only force providing a CW torque is the normal force due to the wall, which pushes to the right. However, if this force were to get larger, the frictional force would have to get larger to keep the net horizontal force equal to zero. Since friction already has its maximum magnitude, there is no way for the ladder to be in equilibrium if the person climbs any higher.

Practice Problem 8.7 Another Ladder Leaning on a Wall

A uniform ladder of mass 10.0 kg and length 3.2 m leans against a frictionless wall with its base located 1.5 m from the wall. If the ladder is not to slip, what must be the minimum coefficient of static friction between the bottom of the ladder and the ground? Assume the wall is frictionless.

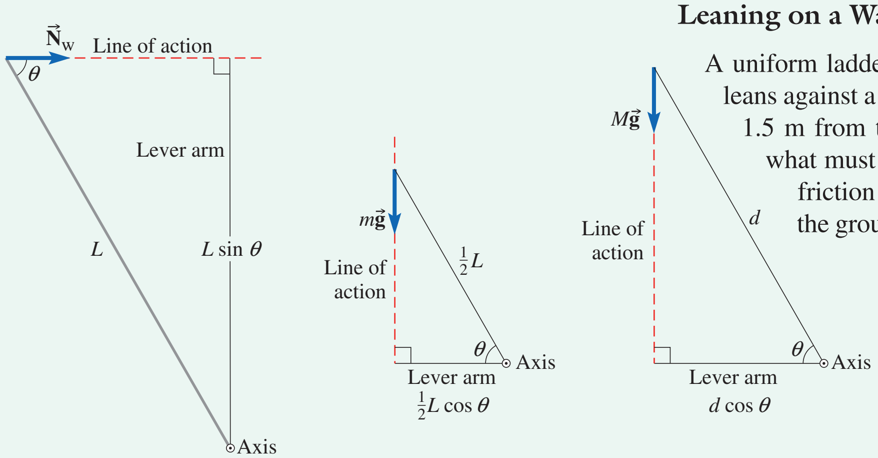

Figure 8.22
Finding the lever arm for each force.

PHYSICS AT HOME

Take a dumbbell and wrap some string around the center of its axle. (An alternative: slide two spools of thread onto a pencil near its center with a small gap between the spools. Wrap some thread around the pencil between the two spools.) Place the dumbbell on a table (or on the floor). Unwind a short length of string and try pulling perpendicularly to the axle at different angles to the horizontal (see Fig. 8.23). Depending on the direction of your pull, the dumbbell can roll in either direction. Try to find the angle at which the rolling changes direction; at this angle the dumbbell does not roll at all. (If using the pencil and spools of thread, pull gently and try to find the angle at which the whole thing *slides* along the table without any rotation.)

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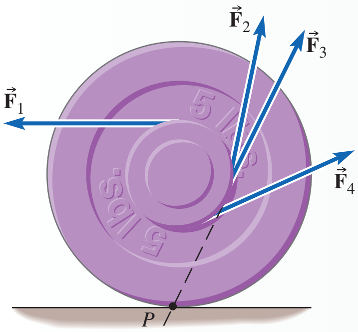

Figure 8.23 Forces $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ make the dumbbell roll to the left; $\vec{\mathbf{F}}_4$ makes it roll to the right; $\vec{\mathbf{F}}_3$ does not make it roll.

What is special about this angle? Since the dumbbell is in equilibrium when pulling at this angle, we can analyze the torques using any rotation axis we choose. A convenient choice is the axis that passes through point *P*, the point of contact with the table. Then the contact force between the table and the dumbbell acts at the rotation axis and its torque is zero. The torque due to gravity is also zero, since the line of action passes through point *P*. The dumbbell can only be in equilibrium if the torque due to the remaining force (the tension in the string) is zero. This torque is zero if the lever arm is zero, which means the line of action passes through point *P*.

Example 8.8

The Sign and the Breaking Cord

A uniform beam of weight 196 N and of length 1.00 m is attached to a hinge on the outside wall of a restaurant. A cord is attached at the center of the beam and is attached to the wall, making an angle of 30.0° with the beam (Fig. 8.24a). The cord keeps the beam perpendicular to the wall. If the breaking tension of the cord is 620 N, how large can the mass of the sign be without breaking the cord?

Strategy The beam is in equilibrium; both the net force and the net torque acting on it must be zero. To find the maximum weight of the sign, we let the tension in the cord have its maximum value of 620 N. We do not know the force exerted by the hinge on the beam, so we choose an axis of rotation through the hinge. Then the force exerted by the hinge on the beam has a zero lever arm and does not enter the torque equation.

Before doing anything else, we draw a diagram showing each force acting on the beam and the chosen rotation axis. The free-body diagrams in previous chapters often placed all the force vectors starting from a single point. Now we draw each force vector starting at its point of application so that we can find the torque—either by finding the lever arm or by finding the perpendicular force component and the distance from the axis to the point of application.

Solution Figure 8.24b shows the forces acting on the beam; three of these contribute to the torque. The gravitational force on the beam can be taken to act at the midpoint of the beam since it is uniform. The force due to the cord has a perpendicular component (Fig. 8.24c) of

$$F_\perp = 620\ \text{N} \times \sin 30.0° = 310\ \text{N}$$

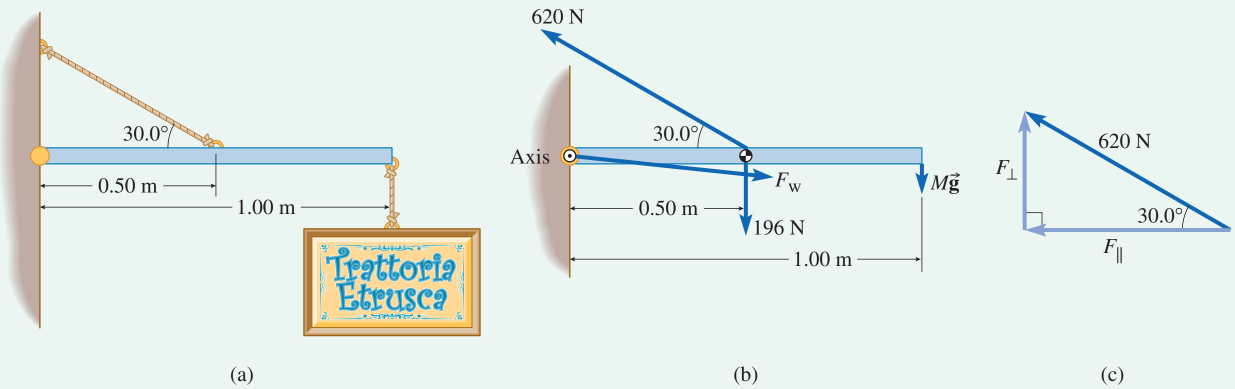

Figure 8.24
(a) A sign outside a restaurant. (b) Forces acting on the beam. (c) Finding the components of the tension in the cord.

Continued on next page

Example 8.8 Continued

The two gravitational forces tend to rotate the beam clockwise, while the tension in the cord tends to rotate it counterclockwise. As an alternative to setting the net torque equal to zero, we can set the total magnitude of the CW torques equal to the total magnitude of the CCW torques:

$$0.50\text{ m} \times 196\text{ N} + 1.00\text{ m} \times Mg = 0.50\text{ m} \times 310\text{ N}$$

or

$$1.00\text{ m} \times Mg = 0.50\text{ m} \times (310\text{ N} - 196\text{ N})$$

Now we solve for the unknown mass M:

$$M = \frac{0.50\text{ m} \times (310\text{ N} - 196\text{ N})}{1.00\text{ m} \times 9.80\text{ N/kg}} = 5.8\text{ kg}$$

Discussion In this problem, we did not have to set the net force equal to zero. By placing the axis of rotation at the hinge, we eliminated two of the three unknowns from the torque equation: the horizontal and vertical components of the hinge force (or, equivalently, its magnitude and direction). If we wanted to find the hinge force as well, setting the net force equal to zero would be necessary.

Practice Problem 8.8 Hinge Forces

Find the vertical component of the force exerted by the hinge in two different ways: (a) setting the net force equal to zero and (b) using a torque equation about a different axis.

Distributed Forces

Gravity is not the only force that is distributed rather than acting at a point. Contact forces, including both the normal component and friction, are spread over the contact surface. Just as for gravity, we can consider the contact force to act at a single point, but the location of that point is often not at all obvious. For a book sitting on a horizontal table, it seems reasonable that the normal force effectively acts at the geometric center of the book cover that touches the table. It is less clear where that effective point is if the book is on an incline or is sliding. As Example 8.9 shows, when something is about to topple over, contact is about to be lost everywhere except at the corner around which the toppling object is about to rotate. That corner then must be the location of the contact forces.

Example 8.9

The Toppling File Cabinet

A file cabinet of height a and width b is on a ramp at angle θ (Fig. 8.25a). The file cabinet is filled with papers in such a way that its center of gravity is at its geometric center. Find the largest θ for which the file cabinet does not tip over. Assume the coefficient of static friction is large enough to prevent sliding.

Strategy Until the file cabinet begins to tip over, it is in equilibrium; the net force acting on it must be zero and the total torque about any axis must also be zero. We first draw a force diagram showing the three forces (gravity, normal, friction) acting on the file cabinet. The point of application of the two contact forces (normal, friction) must be at the lower edge of the file cabinet if it is on the steepest possible incline, just about to tip over. In that case, contact has been lost over the rest of the bottom surface of the file cabinet so that only the lower edge makes good contact with the ramp.

As in all equilibrium problems, a good choice of rotation axis makes the problem easier to solve. We know that, at the maximum angle, the contact forces act at the bottom edge of the file cabinet. A good choice of rotation axis is along the bottom edge of the file cabinet, because then the normal and frictional forces have zero lever arm.

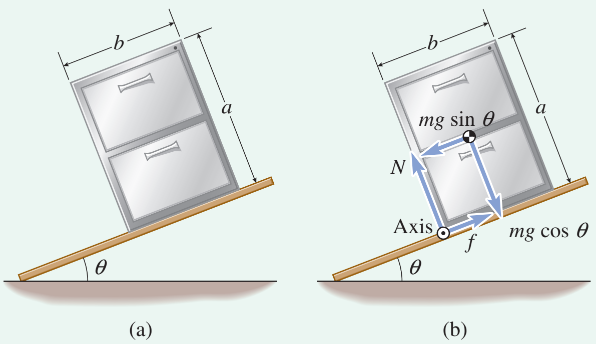

Figure 8.25

(a) File cabinet on an incline. (b) Forces acting on the file cabinet.

Solution Figure 8.25b shows the forces acting on the file cabinet at the maximum angle θ. The gravitational force is drawn at the center of gravity. Instead of drawing a single vector arrow for the gravitational force, we represent the gravitational force by its components parallel

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Example 8.9 Continued

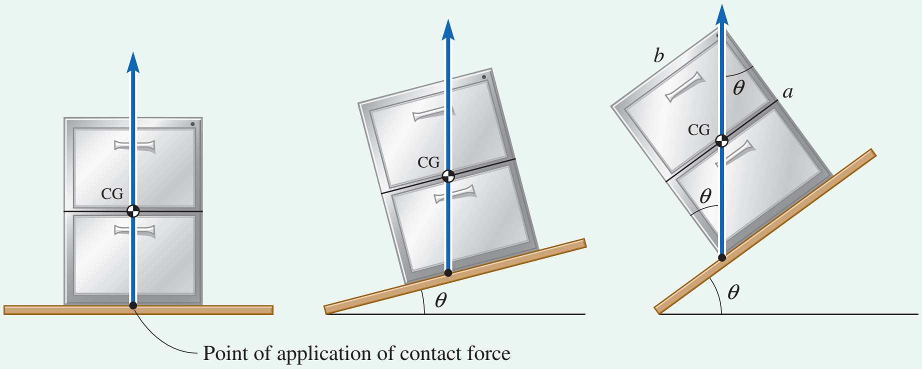


Figure 8.26
Contact force for various incline angles.

and perpendicular to the ramp. Then we find the lever arm for each of the components. The lever arm for the parallel component of the weight ($mg \sin \theta$) is $\frac{1}{2}a$ and the lever arm for the perpendicular component ($mg \cos \theta$) is $\frac{1}{2}b$. Setting the net torque equal to zero:

$$\sum \tau = -mg \cos \theta \times \tfrac{1}{2}b + mg \sin \theta \times \tfrac{1}{2}a = 0$$

After dividing out the common factors of $\frac{1}{2}mg$,

$$\cos \theta \times b = \sin \theta \times a$$

Solving for θ,

$$\theta = \tan^{-1} \frac{b}{a}$$

Discussion As a check, we can regard the normal and friction forces as two components of a single contact force. We can think of that contact force as acting at a single point—a "center of contact" analogous to the center of gravity. As the file cabinet is put on steeper and steeper surfaces, the effective point of application of the contact force moves toward the lower edge of the file cabinet (see Fig. 8.26). If we take the rotation axis through the center of gravity so there is no gravitational torque, then the torque due to the contact force must be zero. The only way that can happen is if its lever arm is zero, which means that the contact force must point directly toward the center of gravity. If the angle θ has its maximum value, the contact force acts at the lower edge and $\tan \theta = b/a$. The file cabinet is about to tip when its center of gravity is directly above the lower edge. Any object supported only by contact forces can be in equilibrium only if the point of application of the total contact force is directly below the object's center of gravity.

Conceptual Practice Problem 8.9 Gymnast Holding a Pike Position

Figure 8.27 shows a gymnast holding a pike position. What can you say about the location of the gymnast's center of gravity?

Figure 8.27
Yuri Chechi of Italy holds the pike position on the rings at the World Gymnastic Championships in Sabae, Japan.

PHYSICS AT HOME

When a person stands up straight, the body's center of gravity lies directly above a point between the feet, about 3 cm in front of the ankle joint (see Fig. 8.28a). When a person bends over to touch her toes, the center of gravity lies outside the body (Fig. 8.28b). Note that the lower half of the body must move backward to keep the center of gravity from moving out in front of the toes, which would cause the person to fall over.

An interesting experiment can be done that illustrates what happens to your balance when you shift your center of gravity. Stand against a wall with the heels of your feet touching the wall and your back pressed against the wall. Then carefully try to bend over as if to touch your toes, without bending your knees. Can you do this without falling over? Explain.

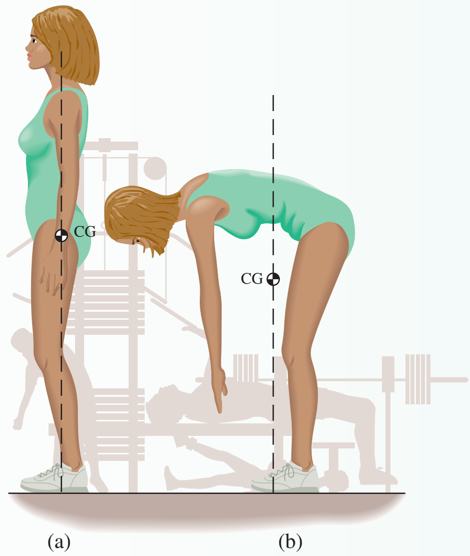


Figure 8.28 Location of the center of gravity when (a) standing and (b) reaching for the floor.

Problem-Solving Steps in Equilibrium Problems

- Identify an object or system in equilibrium. Draw a diagram showing all the forces acting on that object, each drawn at its point of application. Use the center of gravity as the point of application of any gravitational forces.
- To apply the force condition $\Sigma\vec{\mathbf{F}} = 0$, choose a convenient coordinate system and resolve each force into its x- and y-components.
- To apply the torque condition $\Sigma\tau = 0$, choose a convenient rotation axis—generally one that passes through the point of application of an unknown force. Then find the torque due to each force. Use whichever method is easier: either the lever arm times the magnitude of the force or the distance times the perpendicular component of the force. Determine the direction of each torque; then either set the sum of all the torques (with their correct signs) equal to zero or set the magnitude of the CW torques equal to the magnitude of the CCW torques.
- Not all problems require all three equations (two force component equations and one torque equation). Sometimes it is easier to use more than one torque equation, with a different axis. Before diving in and writing down all the equations, think about which approach is the easiest and most direct.

8.5 EQUILIBRIUM IN THE HUMAN BODY

Making the Connection: flexor versus extensor muscles

We can use the concepts of torque and equilibrium to understand some of how the musculoskeletal system of the human body works. A muscle has tendons at each end that connect it to two different bones across a joint (the flexible connection between the bones). When the muscle contracts, it pulls the tendons, which in turn pull on the bones. Thus, the muscle produces a pair of forces of equal magnitude, one acting on each of the two bones. The biceps muscle (Fig. 8.29) in the upper arm attaches the scapula to the forearm (radius) across the inside of the elbow joint. When the biceps contracts, the forearm is pulled toward the upper arm. The biceps is a *flexor* muscle; it moves one bone closer to another.

A muscle can pull but not push, so a flexor muscle such as the biceps cannot reverse its action to push the forearm away from the upper arm. The *extensor* muscles make bones move apart from each other. In the upper arm (Fig. 8.29), an extensor muscle—the triceps—connects the humerus to the ulna (another bone in the forearm parallel to the

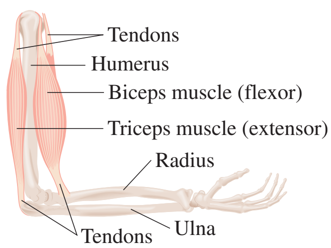


Figure 8.29 Muscles and bones in the upper arm.

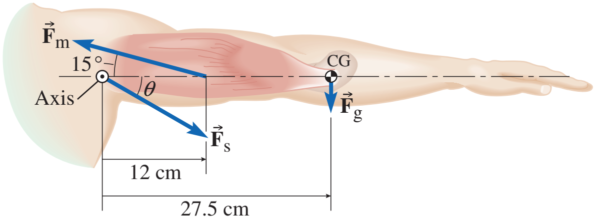


Figure 8.30 Forces exerted on an outstretched arm by the deltoid muscle ($\vec{\mathbf{F}}_m$), the scapula ($\vec{\mathbf{F}}_s$), and gravity ($\vec{\mathbf{F}}_g$).

radius) across the outside of the elbow. Since the biceps and triceps connect to the forearm on opposite sides of the elbow joint, they tend to cause rotation about the joint in opposite directions. When the triceps contracts it pulls the forearm away from the upper arm. Using flexor and extensor muscles on opposite sides of the joint, the body can produce both positive and negative torques, even though both muscles pull in the same direction.

Suppose the arm is held in a horizontal position. The deltoid muscle (the muscle shown in Fig. 8.30) exerts a force $\vec{\mathbf{F}}_m$ on the humerus at an angle of about 15° above the horizontal. This force has to do two things. The vertical component (magnitude $F_m \sin 15° \approx 0.26F_m$) supports the weight of the arm, while the horizontal component (magnitude $F_m \cos 15° \approx 0.97F_m$) stabilizes the joint by pulling the humerus in against the shoulder (scapula). In Example 8.10, we estimate the magnitude of $\vec{\mathbf{F}}_m$.

Example 8.10

Force to Hold Arm Horizontal

A person is standing with his arm outstretched in a horizontal position. The weight of the arm is 30.0 N and its center of gravity is at the elbow joint, 27.5 cm from the shoulder joint (Fig. 8.30). The deltoid pulls on the upper arm at an angle of 15° above the horizontal and at a distance of 12 cm from the joint. What is the magnitude of the force exerted by the deltoid muscle on the arm?

Strategy The arm is in equilibrium, so we can apply the conditions for equilibrium: $\Sigma\vec{\mathbf{F}} = 0$ and $\Sigma\tau = 0$. When calculating torques, we choose the rotation axis at the shoulder joint because then the unknown force $\vec{\mathbf{F}}_s$, which acts on the arm at the joint, has a zero lever arm and produces zero torque. With only one unknown in the torque equation, we can solve immediately for F_m. We do not need to apply the condition $\Sigma\vec{\mathbf{F}} = 0$ unless we want to find $\vec{\mathbf{F}}_s$.

Solution The gravitational force is perpendicular to the line between its point of application and the rotation axis. Gravity produces a clockwise torque of magnitude

$$|\tau| = Fr = 30.0 \text{ N} \times 0.275 \text{ m} = 8.25 \text{ N·m}$$

For the torque due to $\vec{\mathbf{F}}_m$, we find the component of $\vec{\mathbf{F}}_m$ that is perpendicular to the line between its point of application and the rotation axis. Since this line is horizontal, we need the vertical component of $\vec{\mathbf{F}}_m$, which is $F_m \sin 15°$. Then the magnitude of the counterclockwise torque due to $\vec{\mathbf{F}}_m$ is

$$|\tau| = F_\perp r = F_m \sin 15° \times 0.12 \text{ m}$$

These torques must be equal in magnitude:

$$\text{magnitude of CCW torque} = \text{magnitude of CW torque}$$

$$F_m \sin 15° \times 0.12 \text{ m} = 8.25 \text{ N·m}$$

Solving for F_m,

$$F_m = \frac{8.25 \text{ N·m}}{\sin 15° \times 0.12 \text{ m}} = 270 \text{ N}$$

Discussion The force exerted by the muscle is much larger than the 30.0-N weight of the arm. The muscle must exert a larger force because the lever arm is small; the point of application is less than half as far from the joint as the center of gravity [0.12 m/(0.275 m) ≈ 4/9]. Also, the muscle cannot pull straight up on the arm; the vertical component of the muscle force is only about $\frac{1}{4}$ of the magnitude of the force. These two factors together make the weight supported (30.0 N) only $\frac{4}{9} \times \frac{1}{4} = \frac{1}{9}$ as large as the force exerted by the muscle.

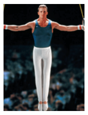
Practice Problem 8.10 Holding a Juice Carton

Find the force exerted by the same person's deltoid muscle when holding a 1-L juice carton (weight 9.9 N) with the arm outstretched and parallel to the floor (as in Fig. 8.30). Assume that the juice carton is 60.0 cm from the shoulder.

The Iron Cross

When a gymnast does the iron cross (Fig. 8.31a), the primary muscles involved are the latissimus dorsi ("lats") and pectoralis major ("pecs"). Since the rings are supporting the gymnast's weight, they exert an upward force on the gymnast's arms. Thus, the task for the muscles is not to hold the arm up, but to pull it down. The lats pull on the humerus about 3.5 cm from the shoulder joint (Fig. 8.31b). The pecs pull on the humerus about 5.5 cm from the joint (Fig. 8.31c). The other ends of these two muscles connect to bone in many places, widely distributed over the back (lats) and chest (pecs).

Making the Connection: muscle forces for the iron cross (gymnastics)



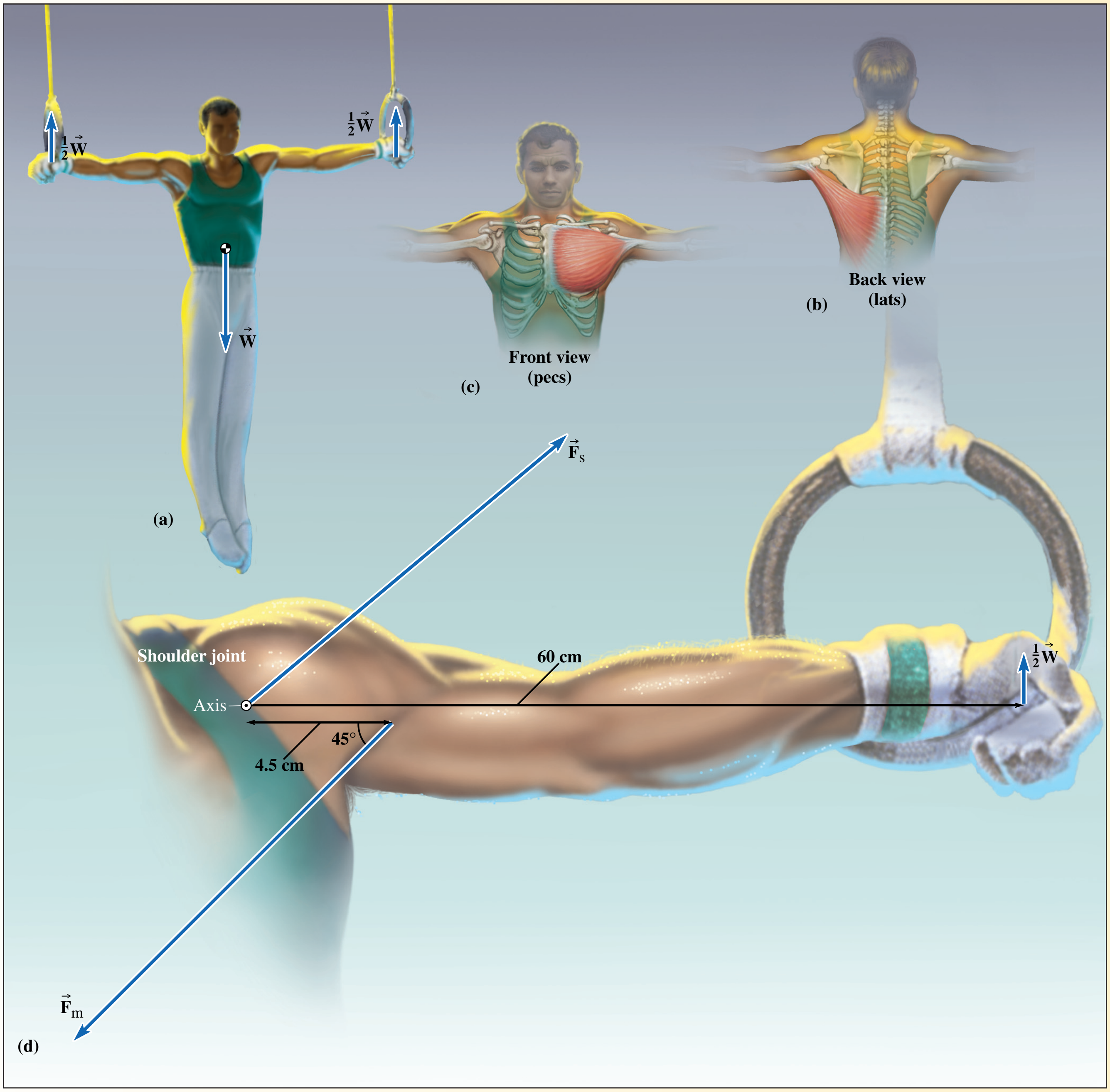


Figure 8.31 (a) Gymnast doing the iron cross. The principal muscles involved are (b) the "lats" and (c) the "pecs." (d) Simplified model of the forces acting on the arm of the gymnast.

As a reasonable simplification, we can assume that these muscles pull at a 45° angle below the horizontal in the iron cross maneuver. We also assume that the two muscles exert equal forces, so we can replace the two with a single force acting at 4.5 cm from the joint.

To determine the force exerted, we look at the entire arm as a system in equilibrium. This time we can ignore the weight of the arm itself since the force exerted on the arm by the ring is much larger—half the gymnast's weight is supported by each ring. The ring exerts an upward force that acts on the hand about 60 cm from the shoulder joint (see Fig. 8.31d). Taking torques about the shoulder, in equilibrium we have

$$|\text{CW torque}| = |\text{CCW torque}|$$

$$F_{\text{m}} \times 0.045\text{ m} \times \sin 45° = \tfrac{1}{2}W \times 0.60\text{ m}$$

$$F_{\text{m}} = \frac{\tfrac{1}{2}W \times 0.60\text{ m}}{0.045\text{ m} \times \sin 45°} = 9.4W$$

Thus, the force exerted by the lats and pecs *on one side* of the gymnast's body is more than nine times his weight.

The design of the human body makes large muscular forces necessary. Are there advantages to the design? Due to the small lever arms, the muscle forces are much larger than they would otherwise be, but the human body has traded this for a wide range of movement of the bones. The biceps and triceps muscles can move the lower arms through almost 180° while they change their lengths by only a few centimeters. The muscles also remain nearly parallel to the bones. If the biceps and triceps muscles were attached to the lower arm much farther from the elbow, there would have to be a large flap of skin to allow them to move so far away from the bones. The arrangement of our bones and muscles favors a wide range of movement.

Another advantage of the design is that it tends to minimize the rotational inertia of our limbs. For example, the muscles that control the motion of the lower arm are contained mostly within the *upper* arm. This keeps the rotational inertia of the lower arms about the elbow smaller. It also keeps the rotational inertia of the entire arm about the shoulder smaller. Smaller rotational inertia means that the energy we have to expend to move our limbs around is smaller.

The biceps muscle with its tendons is almost parallel to the humerus. One interesting observation is that the tendon connects to the radius at different points in different people. In one person this point may be 5.0 cm from the elbow joint, while in another person whose arm is the same length it may be 5.5 cm from the elbow. Thus, some people are naturally stronger than others because of their internal structure. Chimpanzees have an advantage over humans because their biceps muscle has a longer lever arm. Do not make the mistake of arm wrestling with an adult chimp; challenge the chimp to a game of chess instead.

Heavy Lifting

Making the Connection: forces on the human spine during heavy lifting

When lifting an object from the floor, our first instinct is to bend over and pick it up. This is not a good way to lift something heavy. The spine is an ineffective lever and is susceptible to damage when a heavy object is lifted with bent waist. It is much better to squat down and use the powerful leg muscles to do the lifting instead of using our back muscles. Analyzing torques in a simplified model of the back can illustrate why.

The spine can be modeled as a rod with an axis at the tailbone (the sacrum). The sacrum exerts a force, marked $\vec{\mathbf{F}}_{\text{s}}$ in Fig. 8.32, when a person bends at the waist with the back horizontal. The forces due to the complicated set of back muscles can be replaced with a single equivalent force $\vec{\mathbf{F}}_{\text{b}}$ as shown. This equivalent force makes an angle of 12° with the spine and acts about 44 cm from the sacrum. The weight of the upper body, $m\vec{\mathbf{g}}$ in Fig. 8.32, is about 65% of total body weight; its center of gravity is about 38 cm from the sacrum. By placing an axis at the sacrum we can ignore the force $\vec{\mathbf{F}}_{\text{s}}$ in our torque

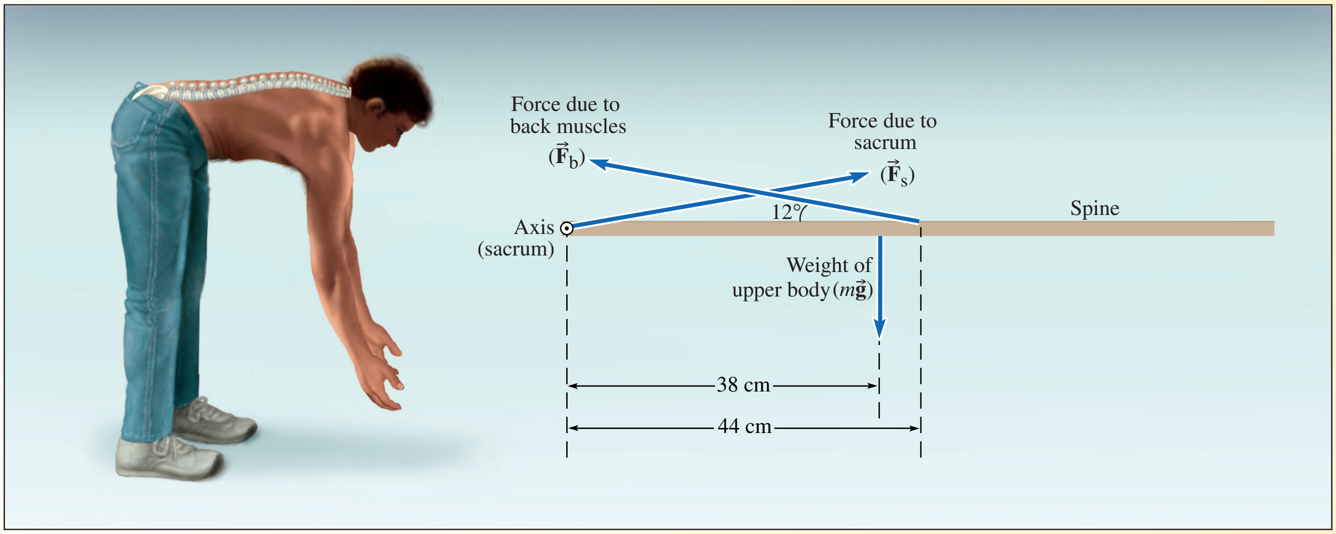


Figure 8.32 A simplified model of the human back when bent over.

equation. Since the vertical component of $\vec{\mathbf{F}}_b$ is $F_b \sin 12° \approx 0.21F_b$, only about $\frac{1}{5}$ the magnitude of the forces exerted by the back muscles is supporting the body weight. The rest, the horizontal component, is pressing the rod representing the spine into the sacrum.

If we put some numbers into this example, we can get an idea of the forces required for just supporting the upper body in this position. If the person's total weight is 710 N (160 lb), then the upper body weight is

$$mg = 0.65 \times 710 \text{ N}$$

Now we set the magnitude of the CCW torques about the axis equal to the magnitude of the CW torques:

$$F_b \times 0.44 \text{ m} \times \sin 12° = mg \times 0.38 \text{ m}$$

Substituting and solving,

$$F_b = \frac{0.65 \times 710 \text{ N} \times 0.38 \text{ m}}{0.44 \text{ m} \times \sin 12°} = 1920 \text{ N}$$

The muscular force that compresses the spine is the horizontal component of $\vec{\mathbf{F}}_b$:

$$F_b \cos 12° = 1900 \text{ N}$$

or about 430 lb. This is over four times the weight of the upper body.

Now if the person tries to lift something with his arms in this position, the lever arm for the weight of the load is even longer than for the weight of the upper body. The back muscles must supply a much larger force. The spine is now compressed with a dangerously large force. A cushioning disk called the lumbosacral disk, at the bottom of the spine, separates the last vertebra from the sacrum. This disk can be ruptured or deformed, causing great pain when the back is misused in such a fashion.

If, instead of bending over, we bend our knees and lower our body, keeping it vertically aligned as much as possible while lifting a load, the centers of gravity of the body and load are positioned more closely in a line above the sacrum, as in Fig. 8.33. Then the lever arms of these forces with respect to an axis through the sacrum are relatively small and the force on the lumbosacral disk is roughly equal to the upper body weight plus the weight being lifted.

Figure 8.33 A safer way to lift a heavy object.

8.6 ROTATIONAL FORM OF NEWTON'S SECOND LAW

The concepts of torque and rotational inertia can be used to formulate a "Newton's second law for rotation"—a law that fills the role of $\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}}$ for rotation about a fixed axis. What is that role? Newton's second law determines the translational acceleration of an object if its mass and the net force acting on it are known. For rotation, we want to determine the angular acceleration, so α takes the place of $\vec{\mathbf{a}}$. Net torque takes the place of net force. If the net torque is zero, then there is zero angular acceleration; the greater the net torque, the greater the angular acceleration. In place of mass, rotational inertia measures how difficult it is to change the angular velocity. The equation that enables us to calculate the angular acceleration of a rigid object given the applied torques takes a form analogous to $\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}}$:

When calculating the net torque, remember to assign the correct algebraic sign to each torque before adding them.

Rotational form of Newton's second law:

$$\Sigma\tau = I\alpha \tag{8-9}$$

The sum of the torques due to internal forces acting on a rigid object is always zero. Therefore, only *external* torques need be included in Eq. (8-9).

Thus, the angular acceleration of a rigid body is proportional to the net torque (more torque causes a larger α) and is inversely proportional to the rotational inertia (more inertia causes a smaller α). In equilibrium, the angular acceleration must be zero; Eq. (8-9) then requires that the net torque be zero. We used $\Sigma\tau = 0$ as one of the conditions of equilibrium in Sections 8.4 and 8.5.

Equation (8-9) is proved in Problem 58. It is subject to an important restriction. Just as $\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}}$ is valid only if the mass of the object is constant, $\Sigma\tau = I\alpha$ is valid only if the rotational inertia of the object is constant. For a rigid object rotating about a fixed axis, I cannot change, so Eq. (8-9) is always applicable.

Newton's second law for rotation explains why a tightrope walker carries a long pole to help maintain balance. If the acrobat is about to topple over sideways, the pole would have to go with him, rotating up and over the rope in a large arc. The pole has a large rotational inertia due to its length, so a large torque is required to make the pole start to rotate. The angular acceleration of the system (acrobat plus pole) due to a small gravitational torque is much smaller than it would be without the pole. The pole greatly increases the stability of the acrobat.

Example 8.11
The Grinding Wheel

A grinding wheel is a solid, uniform disk of mass 2.50 kg and radius 9.00 cm. Starting from rest, what constant torque must a motor supply so that the wheel attains a rotational speed of 126 rev/s in a time of 6.00 s?

Strategy Since the grinding wheel is a uniform disk, we can find its rotational inertia using Table 8.1. After converting the revolutions per second to radians per second, we can find the angular acceleration from the change in angular velocity over the given time interval. Once we have I and α, we can find the net torque from Newton's second law for rotation.

Solution The grinding wheel is a uniform disk, so its rotational inertia is

$$I = \tfrac{1}{2}mr^2$$

$$\tfrac{1}{2} \times 2.50\text{ kg} \times (0.0900\text{ m})^2 = 0.010125\text{ kg}\cdot\text{m}^2$$

A single rotation of the wheel is equivalent to 2π radians, so

$$\omega = 126\,\frac{\text{rev}}{\text{s}} \times 2\pi\,\frac{\text{rad}}{\text{rev}}$$

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Example 8.11 Continued

The angular acceleration is

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

Then the torque required is

$$\sum\tau = I\alpha = I\frac{\Delta\omega}{\Delta t}$$

$$= 0.010125\ \text{kg}\cdot\text{m}^2 \times \frac{126\ \text{rev/s} \times 2\pi\ \text{rad/rev}}{6.00\ \text{s}}$$

$$= 1.34\ \text{N}\cdot\text{m}$$

If there are no other torques on the wheel, the motor must supply a constant torque of 1.34 N·m.

Discussion We assumed that no other torques are exerted on the wheel. There is certain to be at least a small frictional torque on the wheel with a sign opposite to the sign of the motor's torque. Then the motor would have to supply a torque larger than 1.34 N·m. The *net* torque would still be 1.34 N·m.

Practice Problem 8.11 Another Approach

Verify the answer to Example 8.11 by: (a) finding the angular displacement of the wheel using equations for constant α; (b) finding the change in rotational kinetic energy of the wheel; and (c) finding the torque from $W = \tau\,\Delta\theta$.

8.7 THE MOTION OF ROLLING OBJECTS

A rolling object combines translational motion of the center of mass with rotation about an axis that passes through the center of mass (Section 5.1). For an object that is rolling without slipping, $v_{\text{CM}} = \omega R$. As a result, there is a specific relationship between the rolling object's translational and rotational kinetic energies. The total kinetic energy of a rolling object is the sum of its translational and rotational kinetic energies.

A wheel with mass M and radius R has a rotational inertia that is some pure number times MR^2; it couldn't be anything else and still have the right units. We can write the rotational inertia about an axis through the center of mass as $I_{\text{CM}} = \beta MR^2$ where β is a pure number that measures how far from the axis of rotation the mass is distributed. Larger β means the mass is, on average, farther from the axis. From Table 8.1, a hoop has $\beta = 1$; a disk, $\beta = \frac{1}{2}$; and a solid sphere, $\beta = \frac{2}{5}$.

Using $I_{\text{CM}} = \beta MR^2$ and $v_{\text{CM}} = \omega R$, the rotational kinetic energy for a rolling object can be written

$$K_{\text{rot}} = \frac{1}{2}I_{\text{CM}}\,\omega^2 = \frac{1}{2} \times \beta MR^2 \times \left(\frac{v_{\text{CM}}}{R}\right)^2 = \beta \times \frac{1}{2}Mv_{\text{CM}}^2$$

Since $\frac{1}{2}Mv_{\text{CM}}^2$ is the translational kinetic energy,

$$K_{\text{rot}} = \beta K_{\text{tr}} \qquad \textbf{(8-10)}$$

This is convenient since β depends only on the shape, not on the mass or radius of the object. For a given shape rolling without slipping, the ratio of its rotational to translational kinetic energy is always the same (β).

The total kinetic energy can be written

$$K = K_{\text{tr}} + K_{\text{rot}}$$

$$K = \tfrac{1}{2}Mv_{\text{CM}}^2 + \tfrac{1}{2}I_{\text{CM}}\omega^2 \qquad \textbf{(8-11)}$$

or in terms of β,

$$K = (1 + \beta)\,K_{\text{tr}}$$

$$K = (1 + \beta)\,\tfrac{1}{2}Mv_{\text{CM}}^2 \qquad \textbf{(8-12)}$$

Thus, two objects of the same mass rolling at the same translational speed do *not* necessarily have the same kinetic energy. The object with the larger value of β has more rotational kinetic energy.

Conceptual Example 8.12

Hollow and Solid Rolling Balls

Starting from rest, two balls are rolled down a hill as in Fig. 8.34. One is solid, the other hollow. Which one is moving faster when it reaches the bottom of the hill?

Strategy and Solution Energy conservation is the best way to approach this problem. As a ball rolls down the hill, its gravitational potential energy decreases as its kinetic energy increases by the same amount. The total kinetic energy is the sum of the translational and rotational contributions.

We do not know the mass or the radius of either ball and we cannot assume they are the same. Since both kinetic and potential energies are proportional to mass, mass does not affect the final speed. Also, the total kinetic energy does not depend on the radius of the ball [see Eq. (8-12)]. The final speeds of the two balls differ because a different *fraction* of their total kinetic energies is translational.

One ball is a solid sphere and the other is approximately a spherical shell. The mass of a spherical shell is all concentrated on the surface of a sphere, while a solid sphere has its mass distributed throughout the sphere's volume. Therefore, the shell has a larger β than the solid sphere. When the shell rolls, it converts a bigger fraction of the lost potential energy into rotational kinetic energy; therefore, a smaller fraction becomes translational kinetic energy. The final speed of the solid sphere is larger since it puts a larger fraction of its kinetic energy into translational motion.

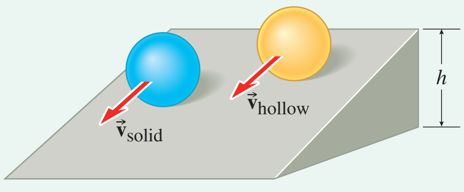


Figure 8.34 Rolling balls.

Discussion We can make this conceptual question into a quantitative one: what is the ratio of the speeds of the two balls at the bottom of the hill?

Let the height of the hill be h. Then for a ball of mass M, the loss of gravitational potential energy is Mgh. This amount of gravitational potential energy is converted into translational and rotational kinetic energy:

$$Mgh = K_{\text{tr}} + K_{\text{rot}} = (1+\beta)K_{\text{tr}} = (1+\beta)\frac{Mv_{\text{CM}}^2}{2}$$

Mass cancels out, as expected. We can solve for the final speed in terms of g, h, and β. The final speed is independent of the ball's mass and radius.

$$v_{\text{CM}} = \sqrt{\frac{2gh}{1+\beta}}$$

The ratio of the final speeds for two balls rolling down the same hill is, therefore,

$$\frac{v_1}{v_2} = \sqrt{\frac{1+\beta_2}{1+\beta_1}}$$

To evaluate the ratio, we look up the rotational inertias in Table 8.1. The solid sphere has $\beta = \frac{2}{5}$ and the spherical shell has $\beta = \frac{2}{3}$. Then

$$\frac{v_{\text{solid}}}{v_{\text{hollow}}} = \sqrt{\frac{1+\frac{2}{3}}{1+\frac{2}{5}}} \approx 1.091$$

The solid ball's final speed is, therefore, 9.1% faster than that of the hollow ball. This ratio depends neither on the masses of the balls, the radii of the balls, the height of the hill, nor the slope of the hill.

Practice Problem 8.12 Fraction of Kinetic Energy That Is Rotational Energy

What fraction of a rolling ball's kinetic energy is rotational kinetic energy? Answer both for a solid ball and a hollow one.

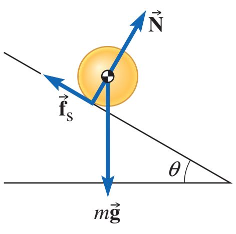


Figure 8.35 Forces acting on a ball rolling downhill.

What is the acceleration of a ball rolling down an incline? Figure 8.35 shows the forces acting on the ball. Static friction is the force that makes the ball rotate; if there were no friction, instead of rolling, the ball would just *slide* down the incline. This is true because friction is the only force acting that yields a nonzero torque about the rotation axis through the ball's center of mass. Gravity gives zero torque because it acts at the axis, so the lever arm is zero. The normal force points directly at the axis, so its lever arm is also zero.

The frictional force $\vec{\mathbf{f}}$ provides a torque

$$\tau = rf$$

where r is the ball's radius. An analysis of the forces and torques combined with Newton's second law in both forms enables us to calculate the acceleration of the ball in Example 8.13.

Example 8.13
Acceleration of a Rolling Ball

Calculate the acceleration of a solid ball rolling down a slope inclined at an angle θ to the horizontal (Fig. 8.36a).

Strategy The net torque is related to the angular acceleration by $\sum\tau = I\alpha$, Newton's second law for rotation. Similarly, the net force acting on the ball gives the acceleration of the center of mass: $\sum\vec{\mathbf{F}} = m\vec{\mathbf{a}}_{CM}$. The axis of rotation is through the ball's center of mass. As already discussed, neither gravity nor the normal force produce a torque about this axis; the net torque is $\sum\tau = rf$, where f is the magnitude of the frictional force. One problem is that the force of friction is unknown. We must resist the temptation to assume that $f = \mu_s N$; there is no reason to assume that static friction has its maximum possible magnitude. We do know that the two accelerations, translational and rotational, are related. We know that v_{CM} and ω are proportional since r is constant. To stay proportional they must change in lock step; their rates of change, a_{CM} and α, are proportional to each other by the same factor of r. Thus, $a_{CM} = \alpha r$. This connection should enable us to eliminate f and solve for the acceleration. Since the speed of a ball after rolling a certain distance was found to be independent of the mass and radius of the ball in Example 8.12, we expect the same to be true of the acceleration.

Solution Since the net torque is

$$\sum\tau = rf$$

the angular acceleration is

$$\alpha = \frac{\sum\tau}{I} = \frac{rf}{I} \tag{1}$$

where I is the ball's rotational inertia about its center of mass.

Figure 8.36b shows the forces along the incline acting on the ball. The acceleration of the center of mass is found from Newton's second law. The component of the net force acting along the incline (in the direction of the acceleration) is

$$\sum F_x = mg\sin\theta - f = ma_{CM} \tag{2}$$

Because the ball is rolling without slipping, the acceleration of the center of mass and the angular acceleration are related by

$$a_{CM} = \alpha r$$

Now we try to eliminate the unknown frictional force f from the equations above. Solving Eq. (1) for f gives

$$f = \frac{I\alpha}{r}$$

Substituting this into Eq. (2), we get

$$mg\sin\theta - \frac{I\alpha}{r} = ma_{CM}$$

Now to eliminate α, we can substitute $\alpha = a_{CM}/r$:

$$mg\sin\theta - \frac{Ia_{CM}}{r^2} = ma_{CM}$$

Solving for a_{CM},

$$a_{CM} = \frac{g\sin\theta}{1 + I/(mr^2)}$$

For a solid sphere, $I = \frac{2}{5}mr^2$, so

$$a_{CM} = \frac{g\sin\theta}{1+\frac{2}{5}} = \frac{5}{7}g\sin\theta$$

Discussion The acceleration of an object *sliding* down an incline without friction is $a = g\sin\theta$. The acceleration of the rolling ball is smaller than $g\sin\theta$ due to the frictional force directed up the incline.

We can check the answer using the result of Example 8.12. The ball's acceleration is constant. If the ball starts from rest as in Fig. 8.36a, after it has rolled a distance d, its speed v is

$$v = \sqrt{2ad} = \sqrt{2\left(\frac{g\sin\theta}{1+\beta}\right)d}$$

where $\beta = \frac{2}{5}$. The vertical drop during this time is $h = d\sin\theta$, so

$$v = \sqrt{\frac{2gh}{1+\beta}}$$

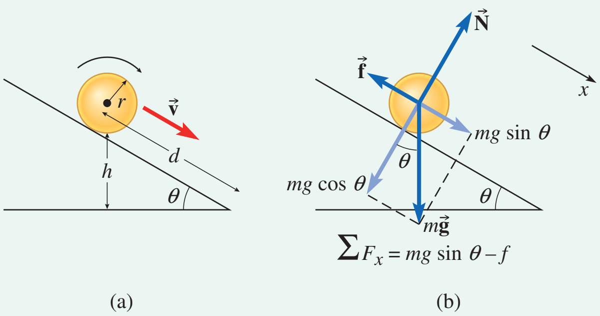


Figure 8.36
(a) A ball rolling downhill. (b) Free-body diagram for the ball, with the gravitational force resolved into components perpendicular and parallel to the incline.

Practice Problem 8.13 Acceleration of a Hollow Cylinder

Calculate the acceleration of a thin hollow cylindrical shell rolling down a slope inclined at an angle θ to the horizontal.

8.8 ANGULAR MOMENTUM

Newton's second law for translational motion can be written in two ways:

$$\sum \vec{\mathbf{F}} = \lim_{\Delta t \to 0} \frac{\Delta \vec{\mathbf{p}}}{\Delta t} \text{ (general form)} \quad \text{or} \quad \sum \vec{\mathbf{F}} = m\vec{\mathbf{a}} \text{ (constant mass)}$$

In Eq. (8-9) we wrote Newton's second law for rotation as $\sum \tau = I\alpha$, which applies only when I is constant—that is, for a rigid body rotating about a fixed axis. A more general form of Newton's second law for rotation uses the concept of **angular momentum** (symbol L).

The net external torque acting on a system is equal to the rate of change of the angular momentum of the system.

$$\sum \tau = \lim_{\Delta t \to 0} \frac{\Delta L}{\Delta t} \tag{8-13}$$

Note the analogy with $\sum \vec{\mathbf{F}} = \lim_{\Delta t \to 0} \frac{\Delta \vec{\mathbf{p}}}{\Delta t}$

The angular momentum of a rigid body rotating about a fixed axis is the rotational inertia times the angular velocity, which is analogous to the definition of linear momentum (mass times velocity):

Angular momentum:

$$L = I\omega \tag{8-14}$$

(rigid body, fixed axis)

Note the analogy with $\vec{\mathbf{p}} = m\vec{\mathbf{v}}$. See the Master the Concepts section for a complete table of these analogies.

Either Eq. (8-13) or Eq. (8-14) can be used to show that the SI units of angular momentum are $\text{kg}\cdot\text{m}^2/\text{s}$.

For a rigid body rotating around a fixed axis, angular momentum doesn't tell us anything new. The rotational inertia is constant for such a body since the distance r_i between every point on the object and the axis stays the same. Then any change in angular momentum must be due to a change in angular velocity ω:

$$\sum \tau = \lim_{\Delta t \to 0} \frac{\Delta L}{\Delta t} = \lim_{\Delta t \to 0} \frac{I\Delta \omega}{\Delta t} = I \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t} = I\alpha$$

However, Eq. (8-13) is *not* restricted to rigid objects or to fixed rotation axes. In particular, if the net external torque acting on a system is zero, then the angular momentum of the system cannot change. This is the **law of conservation of angular momentum**:

Conservation of angular momentum:

$$\text{If } \sum \tau = 0,\ L_\text{i} = L_\text{f} \tag{8-15}$$

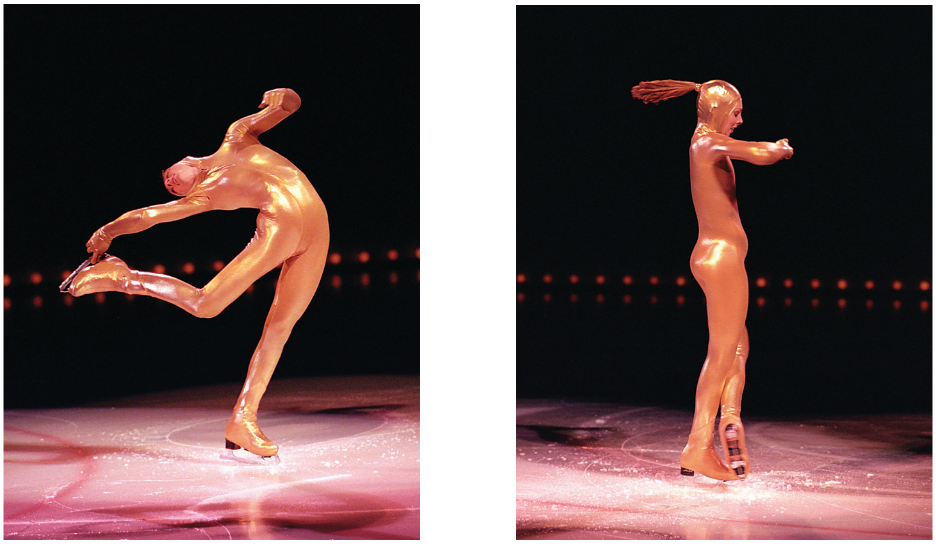
Conservation of angular momentum can be applied to any system if the net external torque on the system is zero (or negligibly small).

Here L_i and L_f represent the angular momentum of the system at two different times. Conservation of angular momentum is one of the most basic and fundamental laws of physics, along with the two other conservation laws we have studied so far (energy and linear momentum). For an isolated system, the total energy, total linear momentum, and total angular momentum of the system are each conserved. None of these quantities can change unless some external agent causes the change.

With conservation of energy, we add up the amounts of the different forms of energy (such as kinetic energy and gravitational potential energy) to find the *total* energy. The conservation law refers to the total energy. By contrast, linear momentum and angular momentum *cannot* be added to find the "total momentum." The two quantities have different dimensions, so it is impossible to add them. Conservation of linear momentum and conservation of angular momentum are *separate* laws of physics.

Making the Connection: rotational inertia of a figure skater

In this section, we restrict our consideration to cases where the axis of rotation is fixed but where the rotational inertia is not necessarily constant. One familiar example of a changing rotational inertia occurs when a figure skater spins (Fig. 8.37). To start the



(a) (b)

Figure 8.37 Figure skater Lucinda Ruh at the (a) beginning and (b) end of a spin. Her angular velocity is much higher in (b) than in (a).

spin, the skater glides along with her arms outstretched and then begins to rotate her body about a vertical axis by pushing against the ice with a skate. The push of the ice against the skate provides the external torque that gives the skater her initial angular momentum. Initially the skater's arms and the leg not in contact with the ice are extended away from her body. The mass of the arms and leg when extended contribute more to her rotational inertia than they do when held close to the body. As the skater spins, she pulls her arms and leg close and straightens her body to decrease her rotational inertia. As she does, her angular velocity increases dramatically in such a way that her angular momentum stays the same.

Many natural phenomena can be understood in terms of angular momentum. In a hurricane, circulating air is sucked inward by a low pressure region at the center of the storm (the *eye*). As the air moves closer and closer to the axis of rotation, it circulates faster and faster. An even more dramatic example is the formation of a pulsar. Under certain conditions, a star can implode under its own gravity, forming a neutron star (a collection of tightly packed neutrons). If the Sun were to collapse into a neutron star, its radius would be only about 13 km. If a star is rotating before its collapse, then as its rotational inertia decreases dramatically, its angular velocity must increase to keep its angular momentum constant. Such rapidly rotating neutron stars are called pulsars because they emit regular pulses of x-rays, at the same frequency as their rotation, that can be detected when they reach Earth. Some pulsars rotate in only a few thousandths of a second per revolution.

Example 8.14

Mouse on a Wheel

A 0.10-kg mouse is perched at point *B* on the rim of a 2.00-kg wagon wheel that rotates freely in a horizontal plane at 1.00 rev/s (Fig. 8.38). The mouse crawls to point *A* at the center. Assume the mass of the wheel is concentrated at the rim. What is the frequency of rotation in rev/s when the mouse arrives at point *A*?

Strategy Assuming that frictional torques are negligibly small, there is no external torque acting on the mouse/wheel system. Then the angular momentum of the mouse/wheel system must be conserved; it takes an external torque to change angular momentum. The mouse and wheel exert torques on each other, but these *internal* torques only transfer some angular momentum between the wheel and the mouse without changing the total angular momentum. We can think of the system as initially being a rigid body with rotational inertia I_i. When the mouse reaches the center, we think of the system as a rigid body with a different rotational inertia I_f. The mouse

Continued on next page

Example 8.14 Continued

changes the rotational inertia of the mouse/wheel system by moving from the outer rim, where its mass makes the maximum possible contribution to the rotational inertia, to the rotation axis, where its mass makes no contribution to the rotational inertia.

Solution Initially, all of the mass of the system is at a distance R from the rotation axis, where R is the radius of the wheel. Therefore,

$$I_{\mathrm{i}} = (M + m)R^2$$

where M is the mass of the wheel and m is the mass of the mouse. After the mouse moves to the center of the wheel, its mass contributes nothing to the rotational inertia of the system:

$$I_{\mathrm{f}} = MR^2$$

From conservation of angular momentum,

$$I_{\mathrm{i}}\,\omega_{\mathrm{i}} = I_{\mathrm{f}}\,\omega_{\mathrm{f}}$$

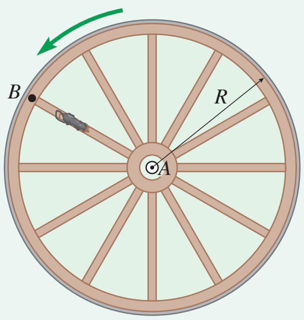


Figure 8.38
Mouse on a rotating wheel.

Substituting the rotational inertias and $\omega = 2\pi f$,

$$(M + m)R^2 \times 2\pi f_{\mathrm{i}} = MR^2 \times 2\pi f_{\mathrm{f}}$$

Factors of $2\pi R^2$ cancel from each side, leaving

$$(M + m)f_{\mathrm{i}} = Mf_{\mathrm{f}}$$

Solving for f_{f},

$$f_{\mathrm{f}} = \frac{M + m}{M} f_{\mathrm{i}} = \frac{2.10\text{ kg}}{2.00\text{ kg}}(1.00\text{ rev/s}) = 1.05\text{ rev/s}$$

Discussion Conservation laws are powerful tools. We do not need to know the details of what happens as the mouse crawls along the spoke from the outer edge of the wheel; we need only look at the initial and final conditions.

A common mistake in this sort of problem is to say that the initial rotational kinetic energy is equal to the final rotational kinetic energy. This is not true because the mouse crawling in toward the center must expend energy to do so. In other words, the mouse does work, converting some internal energy into rotational kinetic energy.

Practice Problem 8.14 Change in Rotational Kinetic Energy

What is the percentage change in the rotational kinetic energy of the mouse/wheel system?

Making the Connection:
Kepler's laws of planetary motion

Angular Momentum in Planetary Orbits

Conservation of angular momentum applies to planets orbiting the Sun in elliptical orbits. Kepler's second law says that the orbital speed varies in such a way that the planet sweeps out area at a constant rate (Fig. 8.39a). In Problem 106, you can show that Kepler's second law is a direct result of conservation of angular momentum, where the angular momentum of the planet is calculated using an axis of rotation perpendicular to the plane of the orbit and passing through the Sun. When the planet is closer to the Sun, it moves faster; when it is farther away, it moves more slowly. Conservation of angular momentum can be used to relate the orbital speeds and radii at two different points in the orbit. The same applies to satellites and moons orbiting planets.

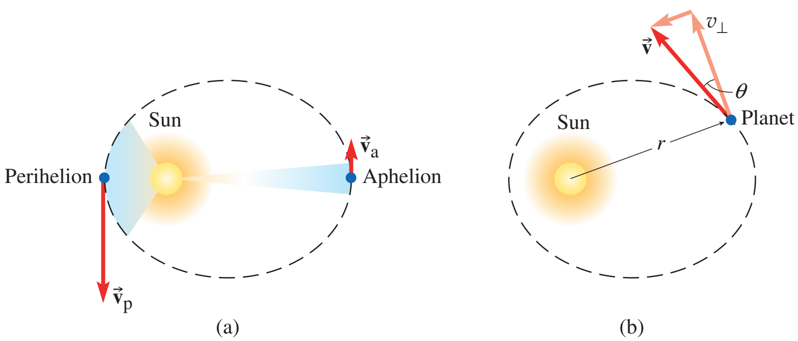


Figure 8.39 The planet's speed varies such that it sweeps out equal areas in equal time intervals. The eccentricity of the planetary orbit is exaggerated for clarity.

Example 8.15
Earth's Orbital Speed

At perihelion (closest approach to the Sun), Earth is 1.47×10^8 km from the Sun and its orbital speed is 30.3 km/s. What is Earth's orbital speed at aphelion (greatest distance from the Sun), when it is 1.52×10^8 km from the Sun? Note that at these two points Earth's velocity is perpendicular to a radial line from the Sun (see Fig. 8.39a).

Strategy We take the axis of rotation through the Sun. Then the gravitational force on Earth points directly toward the axis; with zero lever arm, the torque is zero. With no other external forces acting on the Earth, the net external torque is zero. Earth's angular momentum about the rotation axis through the Sun must therefore be conserved. To find Earth's rotational inertia, we treat it as a point particle since its radius is much less than its distance from the axis of rotation.

Solution The rotational inertia of the Earth is

$$I = mr^2$$

where m is Earth's mass and r is its distance from the Sun. The angular velocity is

$$\omega = \frac{v_\perp}{r}$$

where $v_\perp$ is the component of the velocity perpendicular to a radial line from the Sun. At the two points under consideration, $v_\perp = v$. As the distance from the Sun r varies, its speed v must vary to conserve angular momentum:

$$I_i\, \omega_i = I_f\, \omega_f$$

By substitution,

$$mr_i^2 \times \frac{v_i}{r_i} = mr_f^2 \times \frac{v_f}{r_f}$$

or

$$r_i v_i = r_f v_f \qquad (1)$$

Solving for v_f,

$$v_f = \frac{r_i}{r_f} v_i = \frac{1.47 \times 10^8 \text{ km}}{1.52 \times 10^8 \text{ km}} \times 30.3 \text{ km/s} = 29.3 \text{ km/s}$$

Discussion Earth moves slower at a point farther from the Sun. This is what we expect from energy conservation. The potential energy is greater at aphelion than at perihelion. Since the mechanical energy of the orbit is constant, the kinetic energy must be smaller at aphelion.

Equation (1) implies that the orbital speed and orbital radius are inversely proportional, but strictly speaking this equation only applies to the perihelion and aphelion. At a general point in the orbit, the *perpendicular component* $v_\perp$ is inversely proportional to r (see Fig. 8.39b). The orbits of Earth and most of the other planets are nearly circular so that $\theta \approx 0°$ and $v_\perp \approx v$.

Practice Problem 8.15 Puck on a String

A puck on a frictionless, horizontal air table is attached to a string that passes down through a hole in the table. Initially the puck moves at 12 cm/s in a circle of radius 24 cm. If the string is pulled through the hole, reducing the radius of the puck's circular motion to 18 cm, what is the new speed of the puck?

8.9 THE VECTOR NATURE OF ANGULAR MOMENTUM

Until now we have treated torque and angular momentum as scalar quantities. Such a treatment is adequate in the cases we have considered so far. However, the law of conservation of angular momentum applies to *all* systems, including rotating objects whose axis of rotation changes direction. Torque and angular momentum are actually vector quantities. Angular momentum is conserved in *both magnitude and direction* in the absence of external torques.

An important special case is that of a symmetric object rotating about an axis of symmetry, such as the spinning disk in Fig. 8.40. The magnitude of the angular momentum of such an object is $L = I\omega$. The direction of the angular momentum vector points along the axis of rotation. To find which of the two directions along the axis is correct, a **right-hand rule** is used. Align your right hand so that, as you curl your fingers in toward your palm, your fingertips follow the object's rotation; then your thumb points in the direction of $\vec{\mathbf{L}}$.

A disk with a large rotational inertia can be used as a *gyroscope*. When the gyroscope spins at a large angular velocity, it has a large angular momentum. It is then difficult to

Making the Connection: angular momentum of a gyroscope

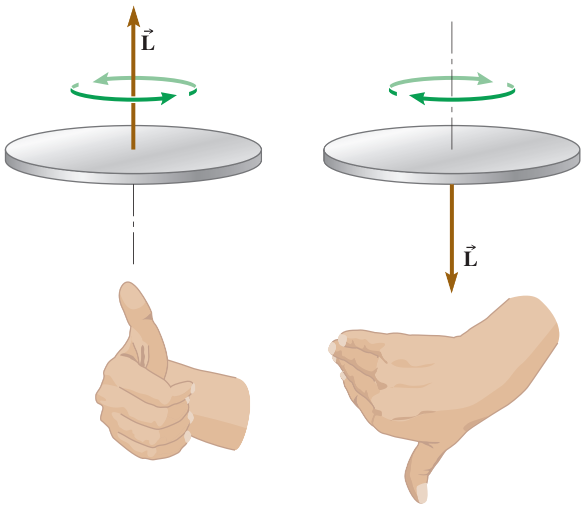


Figure 8.40 Right-hand rule for finding the direction of the angular momentum of a spinning disk.

change the orientation of the gyroscope's rotation axis, because to do so requires changing its angular momentum. To change the direction of a large angular momentum requires a correspondingly large torque. Thus, a gyroscope can be used to maintain stability. Gyroscopes are used in guidance systems in airplanes, submarines, and space vehicles to maintain a constant direction in space.

The same principle explains the great stability of rifle bullets and spinning tops. A rifle bullet is made to spin as it passes through the rifle's barrel. The spinning bullet then keeps its correct orientation—nose first—as it travels through the air. Otherwise, a small torque due to air resistance could make the bullet turn around randomly, greatly increasing air resistance and undermining accuracy. A properly thrown football is made to spin for the same reasons. A spinning top can stay balanced for a long time, while the same top falls over immediately when it is not spinning.

The Earth's rotation gives it a large angular momentum. As the Earth orbits the Sun, the axis of rotation stays in a fixed direction in space. The axis points nearly at Polaris (the North Star), so even as the Earth rotates around its axis, Polaris maintains its position in the northern sky. The fixed direction of the rotation axis gives us the regular progression of the seasons (see Fig. 8.41).

A Classic Demonstration

A demonstration often done in physics classes is for a student to hold a spinning bicycle wheel while standing on a platform that is free to rotate. The wheel's rotation axis is initially horizontal (Fig. 8.42a). Then the student repositions the wheel so that its axis of

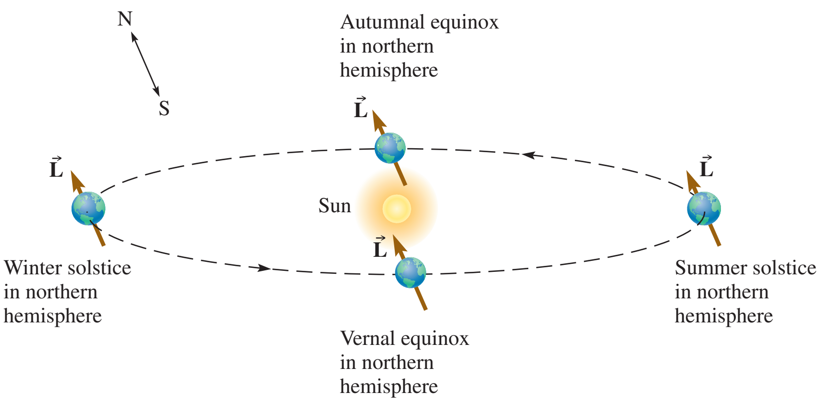


Figure 8.41 Spinning like a top, the Earth maintains the direction of its angular momentum due to rotation as it revolves around the Sun (not to scale).

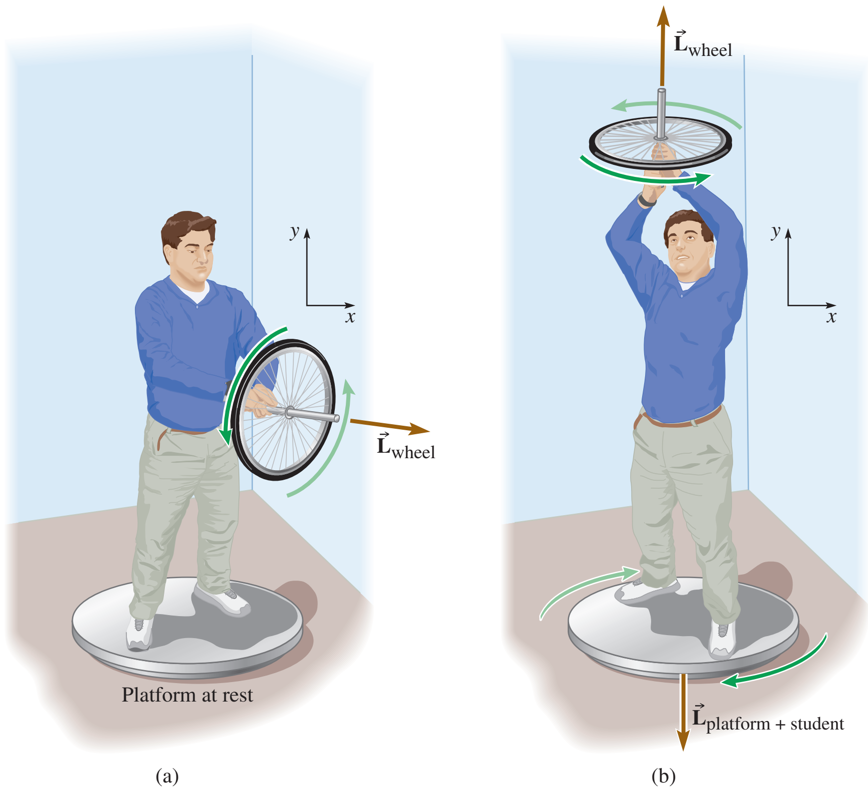


Figure 8.42 A demonstration of angular momentum conservation.

rotation is vertical (Fig. 8.42b). As he repositions the wheel, the platform begins to rotate opposite to the wheel's rotation. If we assume *no* friction acts to resist rotation of the platform, then the platform continues to rotate as long as the wheel is held with its axis vertical. If the student returns the wheel to its original orientation, the rotation of the platform stops.

The platform is free to rotate about a vertical axis. As a result, once the student steps onto the platform, *the vertical component* L_y of the angular momentum of the system (student + platform + wheel) is conserved. The horizontal components of $\vec{\mathbf{L}}$ are *not* conserved. The platform is not free to rotate about any horizontal axis since the floor can exert external torques to keep it from doing so. In vector language, we would say that only the vertical component of the external torque is zero, so only the vertical component of angular momentum is conserved.

Initially $L_y = 0$ since the student and the platform have zero angular momentum and the wheel's angular momentum is horizontal. When the wheel is repositioned so that it spins with an upward angular momentum ($L_y > 0$), the rest of the system (the student and the platform) must acquire an equal magnitude of downward angular momentum ($L_y < 0$) so that the vertical component of the total angular momentum is still zero. Thus, the platform and student rotate in the opposite sense from the rotation of the wheel. Since the platform and student have more rotational inertia than the wheel, they do not spin as fast as the wheel, but their vertical angular momentum is just as large.

The student and the wheel apply torques to each other to transfer angular momentum from one part of the system to the other. These torques are equal and opposite and they have both vertical and horizontal components. As the student lifts the wheel, he feels a strange twisting force that tends to rotate him about a horizontal axis. The platform prevents the horizontal rotation by exerting unequal normal forces on the student's feet. The horizontal component of the torque is so counterintuitive that, if the student is not expecting it, he can easily be thrown from the platform!

MASTER THE CONCEPTS

- The rotational kinetic energy of a rigid object with rotational inertia I and angular velocity ω is

$$K_{\text{rot}} = \tfrac{1}{2}I\omega^2 \qquad (8\text{-}1)$$

In this expression, ω must be measured in *radians* per unit time.

- Rotational inertia is a measure of how difficult it is to change an object's angular velocity. It is defined as:

$$I = \sum_{i=1}^{N} m_i r_i^2 \qquad (8\text{-}2)$$

where r_i is the perpendicular distance between a particle of mass m_i and the rotation axis. The rotational inertia depends on the location of the rotation axis.

- Torque measures the effectiveness of a force for twisting or turning an object. It can be calculated in two equivalent ways: either as the product of the perpendicular component of the force with the shortest distance between the rotation axis and the point of application of the force

$$\tau = \pm rF_{\perp} \qquad (8\text{-}3)$$

or as the product of the magnitude of the force with its lever arm (the perpendicular distance between the line of action of the force and the axis of rotation)

$$\tau = \pm r_{\perp}F \qquad (8\text{-}4)$$

- A force whose perpendicular component tends to cause rotation in the CCW direction gives rise to a positive torque; a force whose perpendicular component tends to cause rotation in the CW direction gives rise to a negative torque.
- The work done by a constant torque is the product of the torque and the angular displacement:

$$W = \tau\,\Delta\theta \quad (\Delta\theta \text{ in radians}) \qquad (8\text{-}6)$$

- The conditions for equilibrium are

$$\sum \vec{\mathbf{F}} = 0 \text{ and } \sum \tau = 0 \qquad (8\text{-}8)$$

The rotation axis can be chosen *arbitrarily* when calculating torques in equilibrium problems. Generally, the best place to choose the axis is at the point of application of an unknown force so that the unknown force does not appear in the torque equation.

- Newton's second law for rotation is

$$\sum \tau = I\alpha \qquad (8\text{-}9)$$

where radian measure must be used for α. A more general form is

$$\sum \tau = \lim_{\Delta t \to 0} \frac{\Delta L}{\Delta t} \qquad (8\text{-}13)$$

where L is the angular momentum of the system.

- The total kinetic energy of a body that is rolling without slipping is the sum of the rotational kinetic energy about an axis through the center of mass and the translational kinetic energy:

$$K = \tfrac{1}{2}Mv_{\text{CM}}^2 + \tfrac{1}{2}I_{\text{CM}}\omega^2 \qquad (8\text{-}11)$$

- The angular momentum of a rigid body rotating about a fixed axis is the rotational inertia times the angular velocity:

$$L = I\omega \qquad (8\text{-}14)$$

- The law of conservation of angular momentum: if the net external torque acting on a system is zero, then the angular momentum of the system cannot change.

$$\text{If } \sum \tau = 0,\ L_{\text{i}} = L_{\text{f}} \qquad (8\text{-}15)$$

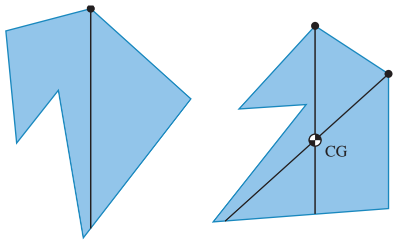
- This table summarizes the analogous quantities in translational and rotational motion.

Translation	**Rotation**
m	I
$\vec{\mathbf{F}}$	τ
$\vec{\mathbf{a}}$	α
$\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}}$	$\sum \tau = I\alpha$
Δx	$\Delta\theta$
$W = F_x\,\Delta x$	$W = \tau\,\Delta\theta$
$\vec{\mathbf{v}}$	ω
$K = \tfrac{1}{2}mv^2$	$K = \tfrac{1}{2}I\omega^2$
$\vec{\mathbf{p}} = m\vec{\mathbf{v}}$	$L = I\omega$
$\sum \vec{\mathbf{F}} = \lim_{\Delta t \to 0} \frac{\Delta \vec{\mathbf{p}}}{\Delta t}$	$\sum \tau = \lim_{\Delta t \to 0} \frac{\Delta L}{\Delta t}$
If $\sum \vec{\mathbf{F}} = 0$, $\vec{\mathbf{p}}$ is conserved	If $\sum \tau = 0$, L is conserved

Conceptual Questions

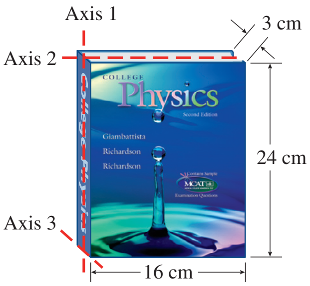
1. Explain why it is easier to drive a wood screw using a screwdriver with a large diameter handle rather than one with a thin handle.
2. One way to find the center of gravity of an irregular flat object is to suspend it from various points so that it is free to rotate. When the object hangs in equilibrium, a vertical line is drawn downward from the support point. After drawing lines from several different

support points, the center of gravity is the point where the lines all intersect. Explain how this works.



3. One of the effects of significant global warming would be the melting of part or all of the polar ice caps. This, in turn, would change the length of the day (the period of the Earth's rotation). Explain why. Would the day get longer or shorter?

4. A book measures 3 cm by 16 cm by 24 cm. About which of the axes shown in the figure is its rotational inertia smallest?



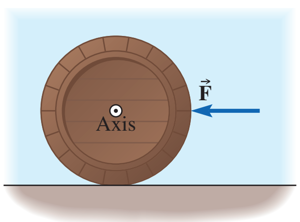
Conceptual Question 4

5. A body in equilibrium has only two forces acting on it. We found in Section 2.3 that the forces must be equal in magnitude and opposite in direction in order to give a translational net force of zero. What else must be true of the two forces for the body to be in equilibrium? [*Hint:* Consider the lines of action of the forces.]

6. Why do many helicopters have a small propeller attached to the tail that rotates in a vertical plane? Why is this attached at the tail rather than somewhere else? [*Hint:* Most of the helicopter's mass is forward, in the cab.]

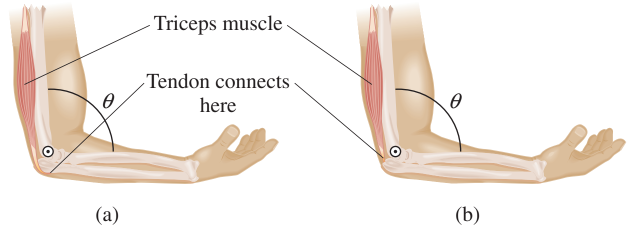
7. In the "Pinewood Derby," Cub Scouts construct cars and then race them down an incline. Some say that, everything else being equal (friction, drag coefficient, same wheels, etc.), a heavier car will win; others maintain that the weight of the car does not matter. Who is right? Explain. [*Hint:* Think about the fraction of the car's kinetic energy that is rotational.]

8. A large barrel lies on its side. In order to roll it across the floor, you apply a horizontal force, as shown in the figure. If the applied force points toward the axis of rotation, which runs down the center of the barrel through the center of mass, it produces zero torque about that axis. How then can this applied force make the barrel start to roll?

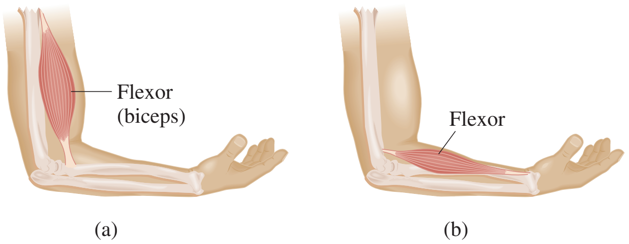


9. Animals that can run fast always have thin legs. Their leg muscles are concentrated close to the hip joint; only tendons extend into the lower leg. Using the concept of rotational inertia, explain how this helps them run fast.

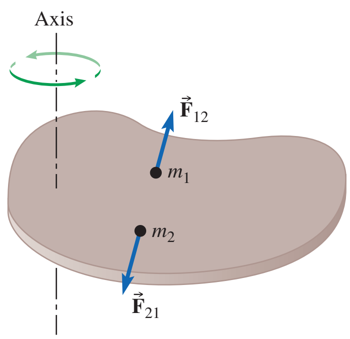
10. Figure (a) shows a simplified model of how the triceps muscle connects to the forearm. As the angle θ is changed, the tendon wraps around a nearly circular arc. Explain how this is much more effective than if the tendon is connected as in part (b) of the figure. [*Hint:* Look at the lever arm as θ changes.]



11. Part (a) of the figure for this question shows a simplified model of how the biceps muscle enables the forearm to support a load. What are the advantages of this arrangement as opposed to the alternative shown in part (b), where the flexor muscle is in the forearm instead of in the upper arm? Are the two equally effective when the forearm is horizontal? What about for other angles between the upper arm and the forearm? Consider also the rotational inertia of the forearm about the elbow and of the entire arm about the shoulder.



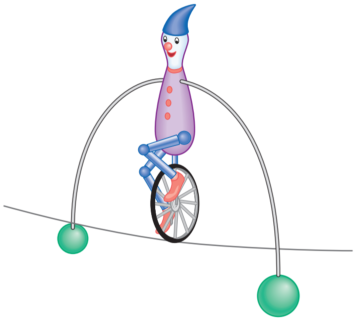
12. In Section 8.6, it was asserted that the sum of all the internal torques (that is, the torques due to internal forces) acting on a rigid object is zero. The figure shows two particles in a rigid object. The particles exert forces $\vec{\mathbf{F}}_{12}$ and $\vec{\mathbf{F}}_{21}$ on each other. These forces are directed along a line that joins the two particles. Explain why the torques due to these two forces must be equal and opposite even though the forces are applied at different points (and, therefore, possibly different distances from the axis).



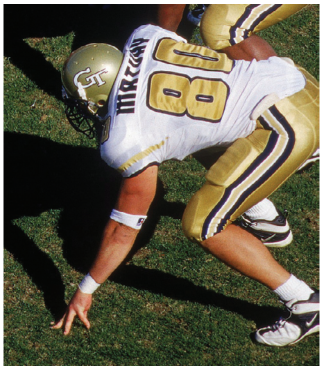
13. A playground merry-go-round (Fig. 8.4) spins with negligible friction. A child moves from the center out to

the rim of the merry-go-round platform. Let the system be the merry-go-round plus the child. Which of these quantities change: angular velocity of the system, rotational kinetic energy of the system, angular momentum of the system? Explain your answer.

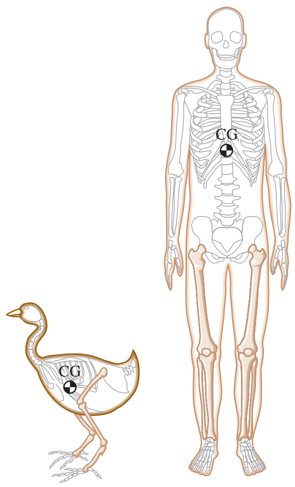
14. The figure shows a balancing toy with weights extending on either side. The toy is extremely stable. It can be pushed quite far off center one way or the other but it does not fall over. Explain why it is so stable.



15. Explain why the posture taken by defensive football linemen makes them more difficult to push out of the way. Consider both the height of the center of gravity and the size of the support base (the area on the ground bounded by the hands and feet touching the ground). In order to knock a person over, what has to happen to the center of gravity? Which do you think needs a more complex neurological system for maintaining balance: four legged animals or humans?



16. The CG of the upper body of a bird is located below the hips; in a human, the CG of the upper body is located well above the hips. Since the upper body is supported by the hips, are birds or humans more stable? Consider what happens if the upper body is displaced a little so that its CG is not directly above or below the hips. In what direction does the torque due to gravity tend to make the upper body rotate about an axis through the hips?



17. An astronaut wants to remove a bolt from a satellite in orbit. He positions himself so that he is at rest with respect to the satellite, then pulls out a wrench and attempts to remove the bolt. What is wrong with his method? How can he remove the bolt?

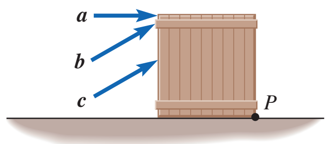
18. Your door is hinged to close automatically after being opened. Where is the best place to put a wedge shaped door stopper on a slippery floor in order to hold the door open? Should it be placed close to the hinge or far from it?

19. You are riding your bicycle and approaching a rather steep hill. Which gear should you use to go uphill, a low gear or a high gear? With a low gear the wheel rotates less than with a high gear for one rotation of the pedals.

20. Why is it easier to push open a swinging door from near the edge away from the hinges rather than in the middle of the door?

Multiple-Choice Questions

1. A heavy box is resting on the floor. You would like to push the box to tip it over on its side, using the minimum force possible. Which of the force vectors in the diagram shows the correct location and direction of the force? The forces have equal horizontal components. Assume enough friction so that the box does not slide; instead it rotates about point *P*.



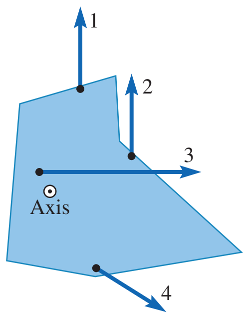
2. When both are expressed in terms of SI *base* units, torque has the same units as
(a) angular acceleration (b) angular momentum
(c) force (d) energy
(e) rotational inertia (f) angular velocity

Questions 3–4. A uniform solid cylinder rolls without slipping down an incline. At the bottom of the incline, the speed, v, of the cylinder is measured and the translational and rotational kinetic energies (K_{tr}, K_{rot}) are calculated. A hole is drilled through the cylinder along its axis and the experiment is repeated; at the bottom of the incline the cylinder now has speed v' and translational and rotational kinetic energies K'_{tr} and K'_{rot}.

3. How does the speed of the cylinder compare to its original value?
(a) $v' < v$ (b) $v' = v$ (c) $v' > v$
(d) Answer depends on the radius of the hole drilled.

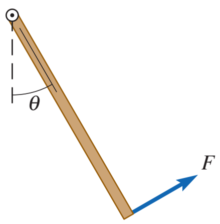
4. How does the ratio of rotational to translational kinetic energy of the cylinder compare to its original value?
(a) $\dfrac{K'_{rot}}{K'_{tr}} < \dfrac{K_{rot}}{K_{tr}}$ (b) $\dfrac{K'_{rot}}{K'_{tr}} = \dfrac{K_{rot}}{K_{tr}}$ (c) $\dfrac{K'_{rot}}{K'_{tr}} > \dfrac{K_{rot}}{K_{tr}}$
(d) Answer depends on the radius of the hole drilled.

5. The SI units of angular momentum are
(a) $\dfrac{\text{rad}}{\text{s}}$ (b) $\dfrac{\text{rad}}{\text{s}^2}$ (c) $\dfrac{\text{kg}\cdot\text{m}}{\text{s}^2}$
(d) $\dfrac{\text{kg}\cdot\text{m}^2}{\text{s}^2}$ (e) $\dfrac{\text{kg}\cdot\text{m}^2}{\text{s}}$ (f) $\dfrac{\text{kg}\cdot\text{m}}{\text{s}}$



Multiple-Choice Questions 6–8

6. Which of the forces in the figure produces the largest magnitude torque about the rotation axis indicated?
(a) 1 (b) 2 (c) 3 (d) 4
7. Which of the forces in the figure produces a clockwise torque about the rotation axis indicated?
(a) 3 only (b) 4 only (c) 1 and 2
(d) 1, 2, and 3 (e) 1, 2, and 4
8. Which pair of forces in the figure might produce equal magnitude torques with opposite signs?
(a) 2 and 3 (b) 2 and 4 (c) 1 and 2
(d) 1 and 3 (e) 1 and 4 (f) 3 and 4
9. A high diver in midair pulls her legs inward toward her chest in order to rotate faster. Doing so changes which of these quantities: her angular momentum L, her rotational inertia I, and her rotational kinetic energy K_{rot}?
(a) L only (b) I only (c) K_{rot} only
(d) L and I only (e) I and K_{rot} only (f) all three
10. A uniform bar of mass m is supported by a pivot at its top, about which the bar can swing like a pendulum. If a force F is applied perpendicularly to the lower end of the bar as in the diagram, how big must F be in order to hold the bar in equilibrium at an angle θ from the vertical?



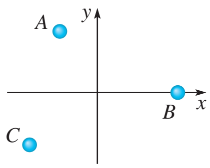
(a) $2mg$ (b) $2mg \sin \theta$
(c) $(mg/2) \sin \theta$ (d) $2mg \cos \theta$
(e) $(mg/2) \cos \theta$ (f) $mg \sin \theta$

Problems

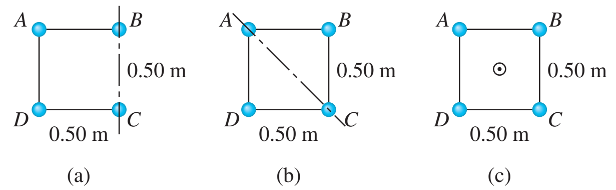
Ⓒ Combination conceptual/quantitative problem
Biological or medical application
No ✦ Easy to moderate difficulty level
✦ More challenging
✦✦ Most challenging
Blue # Detailed solution in the Student Solutions Manual
[1 2] Problems paired by concept

8.1 Rotational Kinetic Energy and Rotational Inertia

1. Verify that $\frac{1}{2}I\omega^2$ has dimensions of energy.
2. Find the rotational inertia of the system of point particles shown in the figure assuming the system rotates about the (a) x-axis, (b) y-axis, (c) z-axis. The z-axis is perpendicular to the xy-plane and points out of the page. Point particle A has a mass of 200 g and is located at $(x, y, z) = (-3.0\text{ cm}, 5.0\text{ cm}, 0)$, point particle B has a mass of 300 g and is at (6.0 cm, 0, 0), and point particle C has a mass of 500 g and is at (–5.0 cm, –4.0 cm, 0). (d) What are the x- and y-coordinates of the center of mass of the system?



3. Four point masses of 3.0 kg each are arranged in a square on massless rods. The length of a side of the square is 0.50 m. What is the rotational inertia for rotation about an axis (a) passing through masses B and C? (b) passing through masses A and C? (c) passing through the center of the square and perpendicular to the plane of the square?



4. What is the rotational inertia of a solid iron disk of mass 49 kg, with a thickness of 5.00 cm and radius of 20.0 cm, about an axis through its center and perpendicular to it?
5. A bowling ball made for a child has half the radius of an adult bowling ball. They are made of the same material (and therefore have the same mass *per unit volume*). By what factor is the (a) mass and (b) rotational inertia of the child's ball reduced compared to the adult ball?
6. How much work is done by the motor in a CD player to make a CD spin, starting from rest? The CD has a diameter of 12.0 cm and a mass of 15.8 g. The laser scans at a constant tangential velocity of 1.20 m/s. Assume that the music is first detected at a radius of 20.0 mm from the center of the disk. Ignore the small circular hole at the CD's center.
7. Find the ratio of the rotational inertia of the Earth for rotation about its own axis to its rotational inertia for rotation about the Sun.
8. A bicycle has wheels of radius 0.32 m. Each wheel has a rotational inertia of 0.080 kg·m^2 about its axle. The total mass of the bicycle including the wheels and the rider is 79 kg. When coasting at constant speed, what fraction of the total kinetic energy of the bicycle (including rider) is the rotational kinetic energy of the wheels?
Ⓒ 9. In many problems in previous chapters, cars and other objects that roll on wheels were considered to act as if

they were sliding without friction. (a) Can the same assumption be made for a wheel rolling *by itself*? Explain your answer. (b) If a moving car of total mass 1300 kg has four wheels, each with rotational inertia of $0.705\ \text{kg}\cdot\text{m}^2$ and radius of 35 cm, what fraction of the total kinetic energy is rotational?

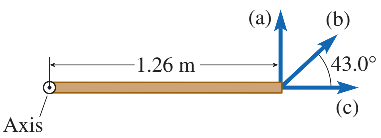
10. A centrifuge has a rotational inertia of $6.5 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$. How much energy must be supplied to bring it from rest to 420 rad/s (4000 rpm)?

8.2 Torque

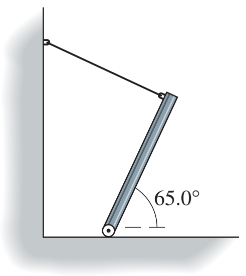
11. A mechanic turns a wrench using a force of 25 N at a distance of 16 cm from the rotation axis. The force is perpendicular to the wrench handle. What magnitude torque does she apply to the wrench?

12. The pull cord of a lawnmower engine is wound around a drum of radius 6.00 cm. While the cord is pulled with a force of 75 N to start the engine, what magnitude torque does the cord apply to the drum?

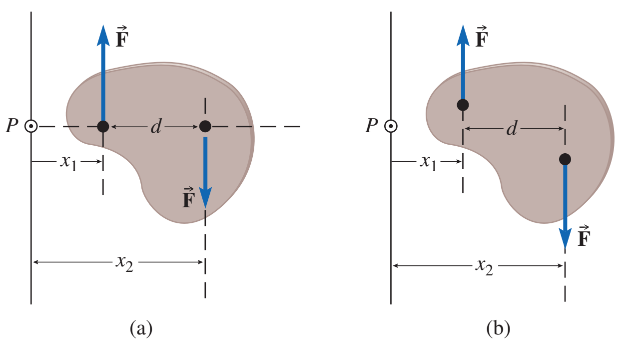
13. A 46.4-N force is applied to the outer edge of a door of width 1.26 m in such a way that it acts (a) perpendicular to the door, (b) at an angle of 43.0° with respect to the door surface, (c) so that the line of action of the force passes through the axis of the door hinges. Find the torque for these three cases.



14. A trap door, of length and width 1.65 m, is held open at an angle of 65.0° with respect to the floor. A rope is attached to the raised edge of the door and fastened to the wall behind the door in such a position that the rope pulls perpendicularly to the trap door. If the mass of the trap door is 16.8 kg, what is the torque exerted on the trap door by the rope?



15. Any pair of equal and opposite forces acting on the same object is called a *couple*. Consider the couple in part (a) of the figure. The rotation axis is perpendicular to the page and passes through point P. (a) Show that the net torque due to this couple is equal to Fd, where d is the distance between the lines of action of the two forces. Because the distance d is independent of the location of the rotation axis, this shows that the torque is the same for any rotation axis. (b) Repeat for the couple in part (b) of the figure. Show that the torque is still Fd if d is the *perpendicular* distance between the lines of action of the forces.



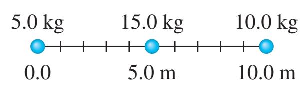
16. A uniform door weighs 50.0 N and is 1.0 m wide and 2.6 m high. What is the magnitude of the torque due to the door's own weight about a horizontal axis perpendicular to the door and passing through a corner?

17. A child of mass 40.0 kg is sitting on a horizontal seesaw at a distance of 2.0 m from the supporting axis. What is the magnitude of the torque about the axis due to the weight of the child?

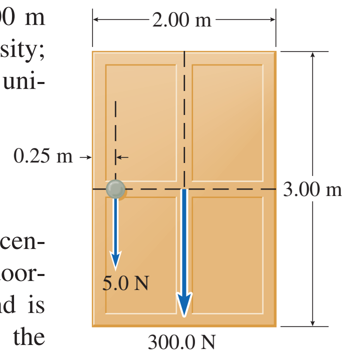
18. A 124-g mass is placed on one pan of a balance, at a point 25 cm from the support of the balance. What is the magnitude of the torque about the support exerted by the mass?

19. A tower outside the Houses of Parliament in London has a famous clock commonly referred to as Big Ben, the name of its 13-ton chiming bell. The hour hand of each clock face is 2.7 m long and has a mass of 60.0 kg. Assume the hour hand to be a uniform rod attached at one end. (a) What is the torque on the clock mechanism due to the weight of one of the four hour hands when the clock strikes noon? The axis of rotation is perpendicular to a clock face and through the center of the clock. (b) What is the torque due to the weight of one hour hand about the same axis when the clock tolls 9:00 A.M.?

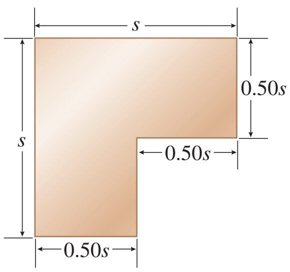
20. A weightless rod, 10.0 m long, supports three weights as shown. Where is its center of gravity?



21. A door weighing 300.0 N measures 2.00 m × 3.00 m and is of uniform density; that is, the mass is uniformly distributed throughout the volume. A doorknob is attached to the door as shown. Where is the center of gravity if the doorknob weighs 5.0 N and is located 0.25 m from the edge?



22. A plate of uniform thickness is shaped as shown. Where is the center of gravity? Assume the origin (0, 0) is located at the lower left corner of the plate; the upper left corner is at (0, s) and upper right corner is at (s, s).



8.3 Work Done by a Torque

23. The radius of a wheel is 0.500 m. A rope is wound around the outer rim of the wheel. The rope is pulled with a force of magnitude 5.00 N, unwinding the rope and making the wheel spin counterclockwise about its central axis. Ignore the mass of the rope. (a) How much rope unwinds while the wheel makes 1.00 revolution? (b) How much work is done by the rope on the wheel during this time? (c) What is the torque on the wheel due to the rope? (d) What is the angular displacement $\Delta\theta$, in radians, of the wheel during 1.00 revolution? (e) Show that the numerical value of the work done is equal to the product $\tau\Delta\theta$.

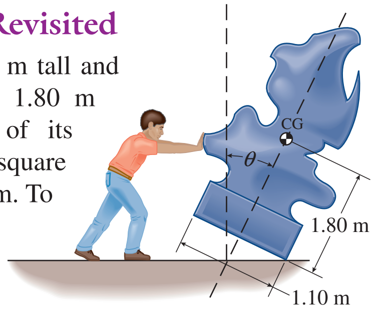
24. A stone used to grind wheat into flour is turned through 12 revolutions by a constant force of 20.0 N applied to the rim of a 10.0-cm-radius shaft connected to the wheel. How much work is done on the stone during the 12 revolutions?

✦ 25. A flywheel of mass 182 kg has an effective radius of 0.62 m (assume the mass is concentrated along a circumference located at the effective radius of the flywheel). (a) What torque is required to bring this wheel from rest to a speed of 120 rpm in a time interval of 30.0 s? (b) How much work is done during the 30.0 s?

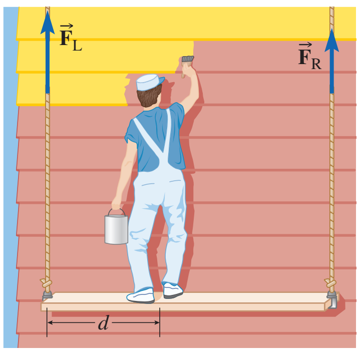
✦ 26. A Ferris wheel rotates because a motor exerts a torque on the wheel. The radius of the London Eye, a huge observation wheel on the banks of the Thames, is 67.5 m and its mass is 1.90×10^6 kg. The cruising angular speed of the wheel is 3.50×10^{-3} rad/s. (a) How much work does the motor need to do to bring the stationary wheel up to cruising speed? [*Hint:* Treat the wheel as a hoop.] (b) What is the torque (assumed constant) the motor needs to provide to the wheel if it takes 20.0 seconds to reach the cruising angular speed?

8.4 Equilibrium Revisited

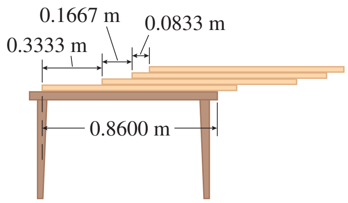
27. A sculpture is 4.00 m tall and has its CG located 1.80 m above the center of its base. The base is a square with a side of 1.10 m. To what angle θ can the sculpture be tipped before it falls over?



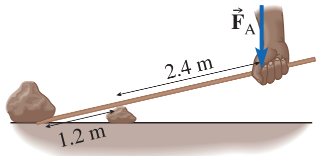
28. A house painter is standing on a uniform, horizontal platform that is held in equilibrium by two cables attached to supports on the roof. The painter has a mass of 75 kg and the mass of the platform is 20.0 kg. The distance from the left end of the platform to where the painter is standing is d = 2.0 m and the total length of the platform is 5.0 m. (a) How large is the force exerted by the left-hand cable on the platform? (b) How large is the force exerted by the right-hand cable?



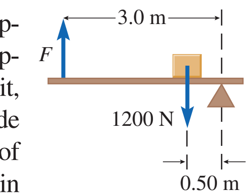
29. Four identical uniform metersticks are stacked on a table as shown. Where is the x-coordinate of the CM of the metersticks if the origin is chosen at the left end of the lowest stick? Why does the system balance?



30. A rod is being used as a lever as shown. The fulcrum is 1.2 m from the load and 2.4 m from the applied force. If the load has a mass of 20.0 kg, what force must be applied to lift the load?

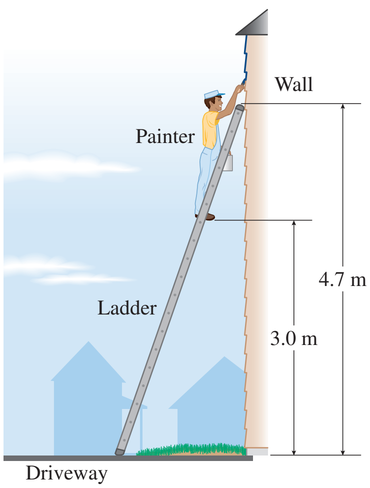


31. A weight of 1200 N rests on a lever at a point 0.50 m from a support. On the same side of the support, at a distance of 3.0 m from it, an upward force with magnitude F is applied. Neglect the weight of the board itself. If the system is in equilibrium, what is F?

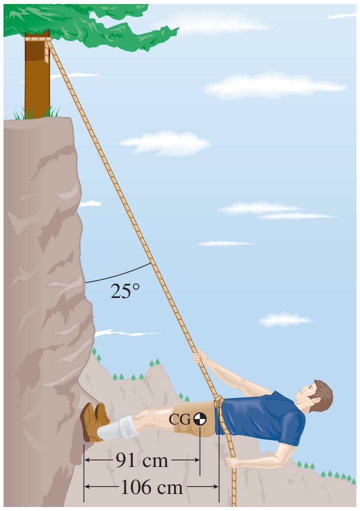


✦ 32. A uniform diving board, of length 5.0 m and mass 55 kg, is supported at two points; one support is located 3.4 m from the end of the board and the second is at 4.6 m from the end (see Fig. 8.19). What are the forces acting on the board due to the two supports when a diver of mass 65 kg stands at the end of the board over the water? Assume that these forces are vertical. [*Hint:* In this problem, consider using two different torque equations about different rotation axes. This may help you determine the directions of the two forces.]

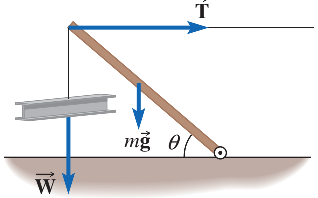
✦ 33. A house painter stands 3.0 m above the ground on a 5.0-m-long ladder that leans against the wall at a point 4.7 m above the ground. The painter weighs 680 N and the ladder weighs 120 N. Assuming no friction between the house and the upper end of the ladder, find the force of friction that the driveway exerts on the bottom of the ladder.



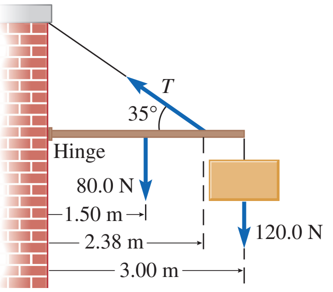
✦ 34. A mountain climber is rappelling down a vertical wall. The rope attaches to a buckle strapped to the climber's waist 15 cm to the right of his center of gravity. If the climber weighs 770 N, find (a) the tension in the rope and (b) the magnitude and direction of the contact force exerted by the wall on the climber's feet.



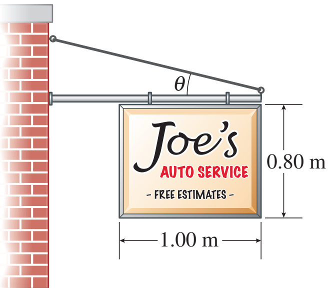
✦ 35. A boom of mass m supports a steel girder of weight W hanging from its end. One end of the boom is hinged at the floor; a cable attaches to the other end of the boom and pulls horizontally on it. The boom makes an angle θ with the horizontal. Find the tension in the cable as a function of m, W, θ, and g. Comment on the tension at $\theta = 0$ and $\theta = 90°$.



36. A sign is supported by a uniform horizontal boom of length 3.00 m and weight 80.0 N. A cable, inclined at an angle of 35° with the boom, is attached at a distance of 2.38 m from the hinge at the wall. The weight of the sign is 120.0 N. What is the tension in the cable and what are the horizontal and vertical forces F_x and F_y exerted on the boom by the hinge? Comment on the magnitude of F_y.

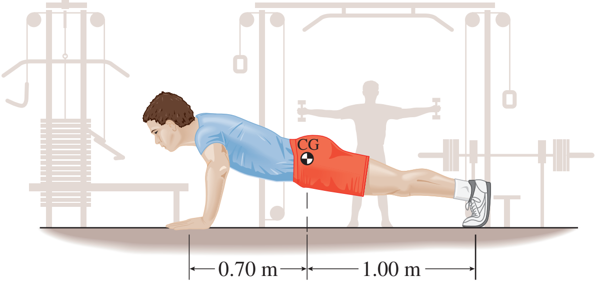


37. You are asked to hang a uniform beam and sign using a cable that has a breaking strength of 417 N. The store owner desires that it hang out over the sidewalk as shown. The sign has a weight of 200.0 N and the beam's weight is 50.0 N. The beam's length is 1.50 m and the sign's dimensions are 1.00 m horizontally × 0.80 m vertically. What is the minimum angle θ that you can have between the beam and cable?



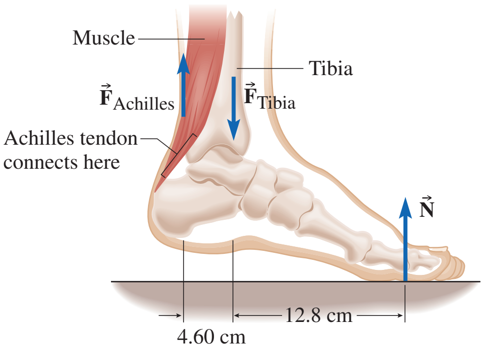
38. Refer to Problem 37. You chose an angle θ of 33.8°. An 8.7-kg cat has climbed onto the beam and is walking from the wall toward the point where the cable meets the beam. How far can the cat walk before the cable breaks?

39. A man is doing push-ups. He has a mass of 68 kg and his center of gravity is located at a horizontal distance of 0.70 m from his palms and 1.00 m from his feet. Find the forces exerted by the floor on his palms and feet.

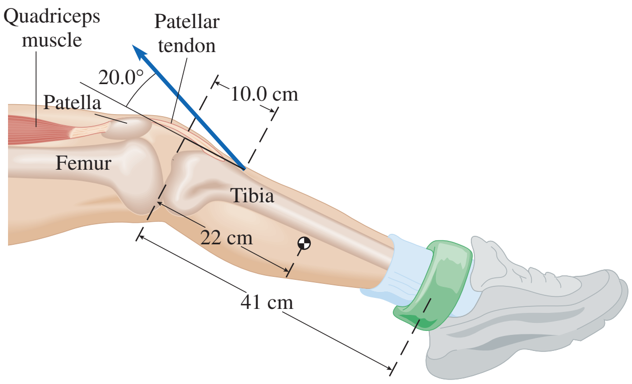


8.5 Equilibrium in the Human Body

40. Find the tension in the Achilles tendon and the force that the tibia exerts on the ankle joint when a person who weighs 750 N supports himself on the ball of one foot. The normal force $N = 750$ N pushes up on the ball of the foot on one side of the ankle joint, while the Achilles tendon pulls up on the foot on the other side of the joint.

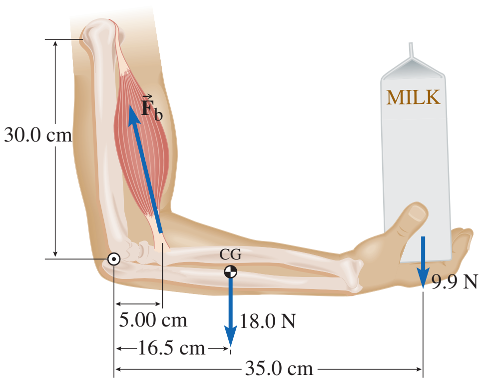


41. In the movie *Terminator*, Arnold Schwarzenegger lifts someone up by the neck and, with both arms fully extended and horizontal, holds the person off the ground. If the person being held weighs 700 N, is 60 cm from the shoulder joint, and Arnold has an anatomy analogous to that in Fig. 8.30, what force must *each* of the deltoid muscles exert to perform this task?



42. A person is doing leg lifts with 3.0-kg ankle weights. She is sitting in a chair with her legs bent at a right angle initially. The quadriceps muscles are attached to the patella via a tendon; the patella is connected to the tibia by the patellar tendon, which attaches to bone 10.0 cm below the knee joint. Assume that the tendon pulls at an angle of 20.0° *with respect to the lower leg*, regardless of the position of the lower leg. The lower leg has a mass of 5.0 kg and its center of gravity is 22 cm below the knee. The ankle weight is 41 cm from the knee. If the person lifts one leg, find the force exerted by the patellar tendon to hold the leg at an angle of (a) 30.0° and (b) 90.0° with respect to the vertical.

43. Find the force exerted by the biceps muscle in holding a 1-L milk carton (weight 9.9 N) with the forearm parallel to the floor. Assume that the hand is 35.0 cm from the elbow and that the upper arm is 30.0 cm long. The elbow is bent at a right angle and one tendon of the biceps is attached to the forearm at a position 5.00 cm from the elbow, while the other tendon is attached at 30.0 cm from the elbow. The weight of the forearm and empty hand is 18.0 N and the center of gravity of the forearm is at a distance of 16.5 cm from the elbow.



44. A friend complains that he often has pain in his lower back. One day while he is picking up a package, you notice that he bends at the waist to pick it up rather than keeping his back straight and bending his knees. You suspect that his lower back problems are due to the extreme force ($\vec{\mathbf{F}}_s$ in Fig. 8.32) on his lower vertebrae from lifting objects in this way. Assume that the back muscles exert a force $\vec{\mathbf{F}}_b$ at 44 cm from the sacrum at an angle of 12°. The mass of his upper body is $M = 55$ kg (about 65% of his total mass), which you assume has a center of mass at its geometric center (38 cm from the sacrum). Determine the horizontal component of $\vec{\mathbf{F}}_s$ when your friend is holding a package with a mass of $m = 10$ kg at a distance of 76 cm from his sacrum. Compare this with the force of 540 N from his torso alone when he stands upright. ($Mg = 55\text{ kg} \times 9.80\text{ m/s}^2 = 540\text{ N}$.) Ignore the weight of the arms.

45. Your friend from Problem 44 now picks up the same package by bending at the knees. He lifts it over his head and balances it on top of his head. Assume $\vec{\mathbf{F}}_b$ is nearly straight down. What is $\vec{\mathbf{F}}_s$ now?

46. A man is trying to lift 60.0 kg off the floor by bending at the waist (see Fig. 8.32). Assume that the man's upper body weighs 455 N and the upper body's center of gravity is 38 cm from the sacrum (tailbone). (a) If, when bent over, the hands are a horizontal distance of 76 cm from the sacrum, what torque must be exerted by the erector spinae muscles to lift 60.0 kg off the floor? (The axis of rotation passes through the sacrum, as shown in Fig. 8.32.) (b) When bent over, the erector spinae muscles are a horizontal distance of 44 cm from the sacrum and act at a 12° angle above the horizontal. What force ($\vec{\mathbf{F}}_b$ in Fig. 8.32) do the erector spinae muscles need to exert to lift the weight? (c) What is the component of this force that compresses the spinal column?

8.6 Rotational Form of Newton's Second Law

47. Verify that the units of the rotational form of Newton's second law [Eq. (8-9)] are consistent. In other words, show that the product of a rotational inertia expressed in $\text{kg}\cdot\text{m}^2$ and an angular acceleration expressed in rad/s^2 is a torque expressed in $\text{N}\cdot\text{m}$.

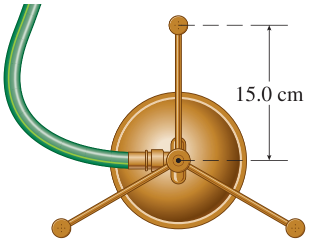
48. A spinning flywheel has rotational inertia $I = 400.0\ \text{kg}\cdot\text{m}^2$. Its angular velocity decreases from 20.0 rad/s to zero in 300.0 s due to friction. What is the frictional torque acting?

49. An LP turntable must spin at 33.3 rpm (3.49 rad/s) to play a record. How much torque must the motor deliver if the turntable is to reach its final angular speed in 2.0 revolutions, starting from rest? The turntable is a uniform disk of diameter 30.5 cm and mass 0.22 kg.

50. A chain pulls tangentially on a 40.6-kg uniform cylindrical gear with a tension of 72.5 N. The chain is attached along the outside radius of the gear at 0.650 m

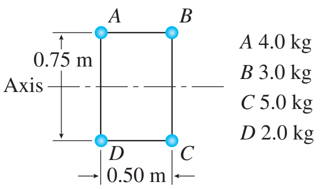
from the axis of rotation. Starting from rest, the gear takes 1.70 s to reach its rotational speed of 1.35 rev/s. What is the total frictional torque opposing the rotation of the gear?

51. A lawn sprinkler has three spouts that spray water, each 15.0 cm long. As the water is sprayed, the sprinkler turns around in a circle. The sprinkler has a total moment of inertia of 9.20×10^{-2} kg·m^2. If the sprinkler starts from rest and takes 3.20 s to reach its final speed of 2.2 rev/s, what force does each spout exert on the sprinkler?



52. Refer to Atwood's machine (Example 8.2). (a) Assuming that the cord does not slip as it passes around the pulley, what is the relationship between the angular acceleration of the pulley (α) and the magnitude of the linear acceleration of the blocks (a)? (b) What is the net torque on the pulley about its axis of rotation in terms of the tensions T_1 and T_2 in the left and right sides of the cord? (c) Explain why the tensions cannot be equal if $m_1 \neq m_2$. (d) Apply Newton's second law to each of the blocks and Newton's second law for rotation to the pulley. Use these three equations to solve for a, T_1, and T_2. (e) Since the blocks move with constant acceleration, use the result of Example 8.2 along with the constant acceleration equation $v_{fy}^2 - v_{iy}^2 = 2a_y\Delta y$ to check your answer for a.

53. Four masses are arranged as shown. They are connected by rigid, massless rods of lengths 0.75 m and 0.50 m. What torque must be applied to cause an angular acceleration of 0.75 rad/s^2 about the axis shown?



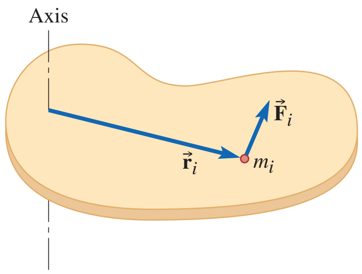
54. A grinding wheel, with a mass of 20.0 kg and a radius of 22.4 cm, is a uniform cylindrical disk. (a) Find the rotational inertia of the wheel about its central axis. (b) When the grinding wheel's motor is turned off, friction causes the wheel to slow from 1200 rpm to rest in 60.0 s. What torque must the motor provide to accelerate the wheel from rest to 1200 rpm in 4.00 s? Assume that the frictional torque is the same regardless of whether the motor is on or off.

55. A playground merry-go-round (Fig. 8.4), made in the shape of a solid disk, has a diameter of 2.50 m and a mass of 350.0 kg. Two children, each of mass 30.0 kg, sit on opposite sides at the edge of the platform. Approximate the children as point masses. (a) What torque is required to bring the merry-go-round from rest to 25 rpm in 20.0 s? (b) If two other bigger children are going to push on the merry-go-round rim to produce this acceleration, with what force magnitude must each child push?

56. Two children standing on opposite sides of a merry-go-round (Fig. 8.4) are trying to rotate it. They each push in opposite directions with forces of magnitude 10.0 N. (a) If the merry-go-round has a mass of 180 kg and a radius of 2.0 m, what is the angular acceleration of the merry-go-round? (Assume the merry-go-round is a uniform disk.) (b) How fast is the merry-go-round rotating after 4.0 s?

57. A bicycle wheel, of radius 0.30 m and mass 2 kg (concentrated on the rim), is rotating at 4.00 rev/s. After 50 s the wheel comes to a stop because of friction. What is the magnitude of the average torque due to frictional forces?

58. Derive the rotational form of Newton's second law as follows. Consider a rigid object that consists of a large number N of particles. Let F_i, m_i, and r_i represent the tangential component of the net force acting on the ith particle, the mass of that particle, and the particle's distance from the axis of rotation, respectively. (a) Use Newton's second law to find a_i, the particle's tangential acceleration. (b) Find the torque acting on this particle. (c) Replace a_i with an equivalent expression in terms of the angular acceleration α. (d) Sum the torques due to all the particles and show that



$$\sum_{i=1}^{N} \tau_i = I\alpha$$

8.7 The Motion of Rolling Objects

59. A solid sphere is rolling without slipping or sliding down a board that is tilted at an angle of 35° with respect to the horizontal. What is its acceleration?

60. A solid sphere is released from rest and allowed to roll down a board that has one end resting on the floor and is tilted at 30° with respect to the horizontal. If the sphere is released from a height of 60 cm above the floor, what is the sphere's speed when it reaches the lowest end of the board?

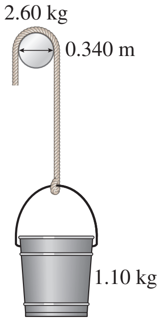
61. A hollow cylinder, a uniform solid sphere, and a uniform solid cylinder all have the same mass m. The

three objects are rolling on a horizontal surface with identical translational speeds v. Find their total kinetic energies in terms of m and v and order them from smallest to largest.

62. A solid sphere of mass 0.600 kg rolls without slipping along a horizontal surface with a translational speed of 5.00 m/s. It comes to an incline that makes an angle of 30° with the horizontal surface. Neglecting energy losses due to friction, (a) what is the total energy of the rolling sphere and (b) to what vertical height above the horizontal surface does the sphere rise on the incline?

63. A bucket of water with a mass of 2.0 kg is attached to a rope that is wound around a cylinder. The cylinder has a mass of 3.0 kg and is mounted horizontally on frictionless bearings. The bucket is released from rest. (a) Find its speed after it has fallen through a distance of 0.80 m. What are (b) the tension in the rope and (c) the acceleration of the bucket?

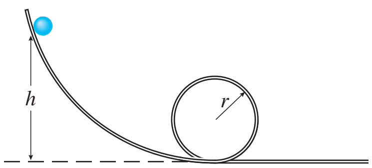
64. A 1.10-kg bucket is tied to a rope of negligible mass. The rope is wrapped around a pole that is mounted horizontally on frictionless bearings. The cylindrical pole has a diameter of 0.340 m and a mass of 2.60 kg. When the bucket is released from rest, how long will it take to fall to the bottom of the 17.0 m well?



✦ 65. A uniform solid cylinder rolls without slipping down an incline. A hole is drilled through the cylinder along its axis. The radius of the hole is 0.50 times the (outer) radius of the cylinder. (a) Does the cylinder take more or less time to roll down the incline now that the hole has been drilled? Explain. (b) By what percentage does drilling the hole change the time for the cylinder to roll down the incline?

✦✦ 66. A solid sphere of radius R and mass M *slides* without friction down a loop-the-loop track. The sphere starts from rest at a height of h above the horizontal. Assume that the radius of the sphere is small compared to the radius r of the loop. (a) Find the minimum value of h in terms of r so that the sphere remains on the track all the way around the loop. (b) Find the minimum value of h if, instead, the sphere rolls without slipping on the track.

✦ 67. A hollow cylinder, of radius R and mass M, rolls without slipping down a loop-the-loop track of radius r. The cylinder starts from rest at a height h above the horizontal section of track. What is the minimum value of h so that the cylinder remains on the track all the way around the loop?



Problems 66 and 67

68. If the hollow cylinder of Problem 67 is replaced with a solid sphere, will the minimum value of h increase, decrease, or remain the same? Once you think you know the answer and can explain why, redo the calculation to find h.

✦✦ 69. The string in a yo-yo is wound around an axle of radius 0.500 cm. The yo-yo has both rotational and translational motion, like a rolling object, and has mass 0.200 kg and outer radius 2.00 cm. Starting from rest, it rotates and falls a distance of 1.00 m (the length of the string). Assume for simplicity that the yo-yo is a uniform circular disk and that the string is thin compared to the radius of the axle. (a) What is the speed of the yo-yo when it reaches the distance of 1.00 m? (b) How long does it take to fall? [*Hint:* The translational and rotational kinetic energies are related, but the yo-yo is *not* rolling on its outer radius.]

8.8 Angular Momentum

70. A turntable of mass 5.00 kg has a radius of 0.100 m and spins with a frequency of 0.550 rev/s. What is its angular momentum? Assume the turntable is a uniform disk.

71. Assume the Earth is a uniform solid sphere with radius of 6.37×10^6 m and mass of 5.97×10^{24} kg. Find the magnitude of the angular momentum of the Earth due to rotation about its axis.

72. The mass of a flywheel is 5.6×10^4 kg. This particular flywheel has its mass concentrated at the rim of the wheel. If the radius of the wheel is 2.6 m and it is rotating at 350 rpm, what is the magnitude of its angular momentum?

73. A uniform disk with a mass of 800 g and radius 17.0 cm is rotating on frictionless bearings with an angular speed of 18.0 Hz when Jill drops a 120-g clod of clay on a point 8.00 cm from the center of the disk, where it sticks. What is the new angular speed of the disk?

74. The angular momentum of a spinning wheel is 240 kg·m^2/s. After application of a constant braking torque for 2.5 s, it slows and has a new angular momentum of 115 kg·m^2/s. What is the torque applied?

75. How long would a braking torque of 4.00 N·m have to act to just stop a spinning wheel that has an initial angular momentum of 6.40 kg·m^2/s?

76. Consider the merry-go-round of Practice Problem 8.1. The child is initially standing on the ground when the merry-go-round is rotating at 0.75 rev/s. The child then steps on the merry-go-round. How fast is the merry-go-round rotating now? By how much did the rotational kinetic energy of the merry-go-round and child change?

77. A rotating star collapses under the influence of gravitational forces to form a pulsar. The radius of the star after collapse is 1.0×10^{-4} times the radius before collapse. There is no change in mass. In both cases, the mass of the star is uniformly distributed in a spherical shape. Find the ratios of the (a) angular momentum,

(b) angular velocity, and (c) rotational kinetic energy of the star after collapse to the values before collapse. (d) If the period of the star's rotation before collapse is 1.0×10^7 s, what is its period after collapse?

78. A figure skater is spinning at a rate of 1.0 rev/s with her arms outstretched. She then draws her arms in to her chest, reducing her rotational inertia to 67% of its original value. What is her new rate of rotation?

79. A skater is initially spinning at a rate of 10.0 rad/s with a rotational inertia of 2.50 kg·m^2 when her arms are extended. What is her angular velocity after she pulls her arms in and reduces her rotational inertia to 1.60 kg·m^2?

80. A spoked wheel with a radius of 40.0 cm and a mass of 2.00 kg is mounted horizontally on frictionless bearings. JiaJun puts his 0.500-kg guinea pig on the outer edge of the wheel. The guinea pig begins to run along the edge of the wheel with a speed of 20.0 cm/s with respect to the ground. What is the angular velocity of the wheel? Assume the spokes of the wheel have negligible mass.

81. A diver can change his rotational inertia by drawing his arms and legs close to his body in the tuck position. After he leaves the diving board (with some unknown angular velocity), he pulls himself into a ball as closely as possible and makes 2.00 complete rotations in 1.33 s. If his rotational inertia decreases by a factor of 3.00 when he goes from the straight to the tuck position, what was his angular velocity when he left the diving board?

✦ 82. The rotational inertia for a diver in a pike position is about 15.5 kg·m^2; it is only 8.0 kg·m^2 in a tuck position. (a) If the diver gives himself an initial angular momentum of 106 kg·m^2/s as he jumps off the board, how many turns can he make when jumping off a 10.0-m platform in a tuck position? (b) How many in a pike position? [*Hint:* Gravity exerts no torque on the person as he falls; assume he is rotating throughout the 10.0-m dive.]

(a) (b)

Problem 82. (a) Gregory Louganis in the pike position. (b) Mark Ruiz in the tuck position.

8.9 The Vector Nature of Angular Momentum

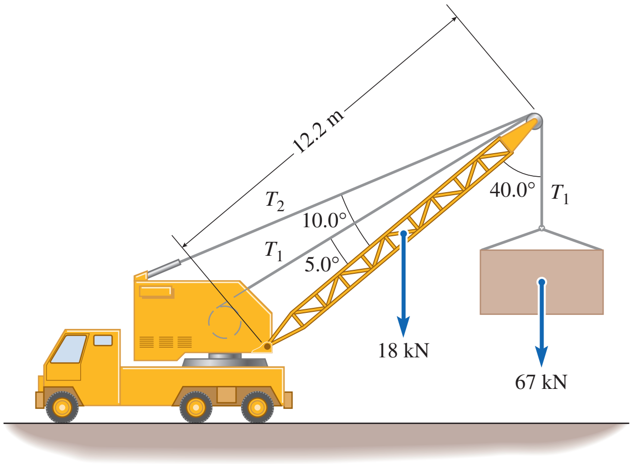
Problems 83 and 84. A solid cylindrical disk is to be used as a stabilizer in a ship. By using a massive disk rotating in the hold of the ship, the captain knows that a large torque is required to tilt its angular momentum vector. The mass of the disk to be used is 1.00×10^5 kg and it has a radius of 2.00 m.

✦ 83. If the cylinder rotates at 300.0 rpm, what is the magnitude of the average torque required to tilt its axis by 60.0° in a time of 3.00 s? [*Hint:* Draw a vector diagram of the initial and final angular momenta.]

Ⓒ 84. How should the disk be oriented to prevent rocking from side to side and from bow to stern? Does this orientation make it difficult to steer the ship? Explain.

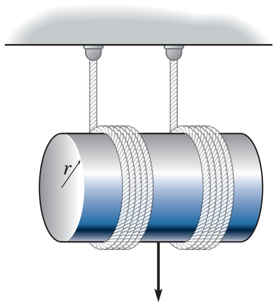
Comprehensive Problems

✦ 85. The 12.2-m crane weighs 18 kN and is lifting a 67-kN load. The hoisting cable (tension T_1) passes over a pulley at the top of the crane and attaches to an electric winch in the cab. The pendant cable (tension T_2), which supports the crane, is fixed to the top of the crane. Find the tensions in the two cables and the force $\vec{\mathbf{F}}_p$ at the pivot.

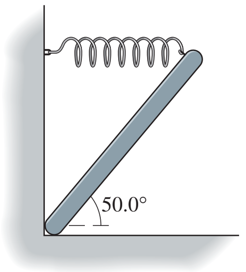


86. A collection of objects is set to rolling, without slipping, down a slope inclined at 30°. The objects are a solid sphere, a hollow sphere, a solid cylinder, and a hollow cylinder. A frictionless cube is also allowed to slide down the same incline. Which one gets to the bottom first? List the others in the order they arrive at the finish line.

87. A uniform cylinder with a radius of 15 cm has been attached to two cords and the cords are wound around it and hung from the ceiling. The cylinder is released from rest and the cords unwind as the cylinder descends. (a) What is the acceleration of the cylinder? (b) If the mass of the cylinder is 2.6 kg, what is the tension in each cord?



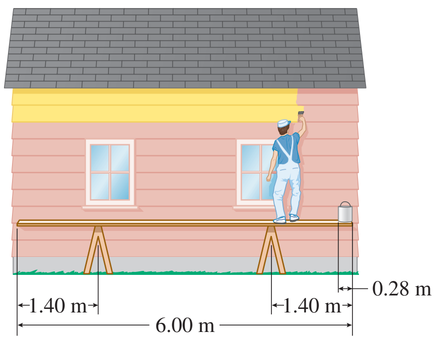
88. A modern sculpture has a large horizontal spring, with a spring constant of 275 N/m, that is attached to a 53.0-kg piece of uniform metal at its end and holds the metal at an angle of 50.0° above the horizontal direction. The other end of the metal is wedged into a corner as shown. By how much has the spring stretched?



89. A ceiling fan has four blades, each with a mass of 0.35 kg and a length of 60 cm. Model each blade as a rod connected to the fan axle at one end. When the fan is turned on, it takes 4.35 s for the fan to reach its final angular speed of 1.8 rev/s. What torque was applied to the fan by the motor? Ignore torque due to the air.

90. The Moon's distance from Earth varies between 3.56×10^5 km at perigee and 4.07×10^5 km at apogee. What is the ratio of its orbital speed around Earth at perigee to that at apogee?

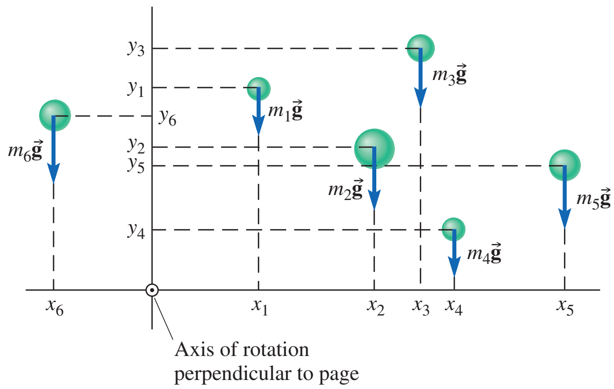
✦ 91. A painter (mass 61 kg) is walking along a trestle, consisting of a uniform plank (mass 20.0 kg, length 6.00 m) balanced on two sawhorses. Each sawhorse is placed 1.40 m from an end of the plank. A paint bucket (mass 4.0 kg, diameter 28 cm) is placed as close as possible to the right-hand edge of the plank while still having the whole bucket in contact with the plank. (a) How close to the right-hand edge of the plank can the painter walk before tipping the plank and spilling the paint? (b) How close to the left-hand edge can the same painter walk before causing the plank to tip? [*Hint:* As the painter walks toward the right-hand edge of the plank and the plank starts to tip clockwise, what is the force acting upward on the plank from the left-hand sawhorse support?]



✦ 92. An experimental flywheel, used to store energy and replace an automobile engine, is a solid disk of mass 200.0 kg and radius 0.40 m. (a) What is its rotational inertia? (b) When driving at 22.4 m/s (50 mph), the fully energized flywheel is rotating at an angular speed of 3160 rad/s. What is the initial rotational kinetic energy of the flywheel? (c) If the total mass of the car is 1000.0 kg, find the ratio of the initial rotational kinetic energy of the flywheel to the translational kinetic energy of the car. (d) If the force of air resistance on the car is 670.0 N, how far can the car travel at a speed of 22.4 m/s (50 mph) with the initial stored energy? Ignore losses of mechanical energy due to means other than air resistance.

✦✦ 93. (a) Assume the Earth is a uniform solid sphere. Find the kinetic energy of the Earth due to its rotation about its axis. (b) Suppose we could somehow extract 1.0% of the Earth's rotational kinetic energy to use for other purposes. By how much would that change the length of the day? (c) For how many years would 1.0% of the Earth's rotational kinetic energy supply the world's energy usage (assume a constant 1.0×10^{21} J per year)?

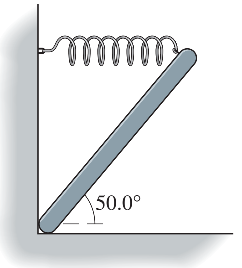
94. A flat object in the xy-plane is free to rotate about the z-axis. The gravitational field is uniform in the $-y$-direction. Think of the object as a large number of particles with masses m_i located at coordinates (x_i, y_i), as in the figure. (a) Show that the torques on the particles about the z-axis can be written $\tau_i = -x_i m_i g$. (b) Show that if the center of gravity is located at (x_{CG}, y_{CG}), the total torque due to gravity on the object must be $\sum \tau_i = -x_{CG} M g$, where M is the total mass of the object. (c) Show that $x_{CG} = x_{CM}$. (This same line of reasoning can be applied to objects that are not flat and to other axes of rotation to show that $y_{CG} = y_{CM}$ and $z_{CG} = z_{CM}$.)



95. The operation of the Princeton Tokomak Fusion Test Reactor requires large bursts of energy. The power needed exceeds the amount that can be supplied by the utility company. Prior to pulsing the reactor, energy is stored in a giant flywheel of mass 7.27×10^5 kg and rotational inertia 4.55×10^6 kg·m^2. The flywheel rotates at a maximum angular speed of 386 rpm. When the stored energy is needed to operate the reactor, the flywheel is connected to an electrical generator, which converts some of the rotational kinetic energy into electrical energy. (a) If the flywheel is a uniform disk, what is its radius? (b) If the flywheel is a hollow cylinder with its mass concentrated at the rim, what is its radius? (c) If the flywheel slows to 252 rpm in 5.00 s, what is the average power supplied by the flywheel during that time?

96. The distance from the center of the breastbone to a man's hand, with the arm outstretched and horizontal to the floor, is 1.0 m. The man is holding a 10.0-kg dumbbell, oriented vertically, in his hand, with the arm

88. A modern sculpture has a large horizontal spring, with a spring constant of 275 N/m, that is attached to a 53.0-kg piece of uniform metal at its end and holds the metal at an angle of 50.0° above the horizontal direction. The other end of the metal is wedged into a corner as shown. By how much has the spring stretched?



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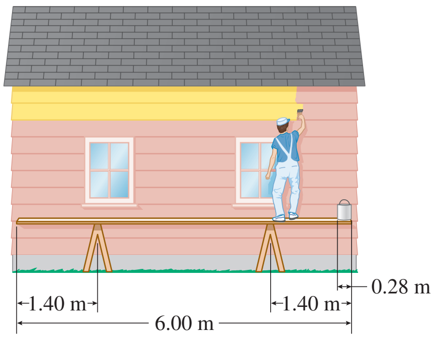

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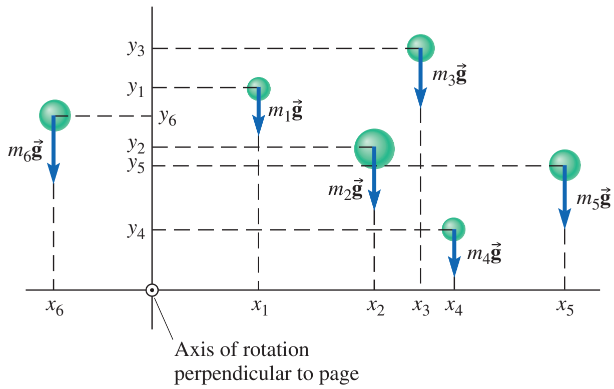

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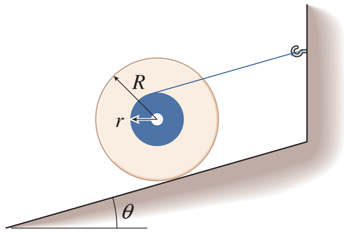
the foot a distance of 4.4 cm behind the ankle joint. If the Achilles tendon is inclined at an angle of 81° with respect to the horizontal, find the force that each calf muscle needs to exert while she is standing. [*Hint:* Consider the equilibrium of the part of the body *above* the ankle joint.]

108. A merry-go-round (radius R, rotational inertia I_i) spins with negligible friction. Its initial angular velocity is ω_i. A child (mass m) on the merry-go-round moves from the center out to the rim. (a) Calculate the angular velocity after the child moves out to the rim. (b) Calculate the rotational kinetic energy and angular momentum of the system (merry-go-round + child) before and after.

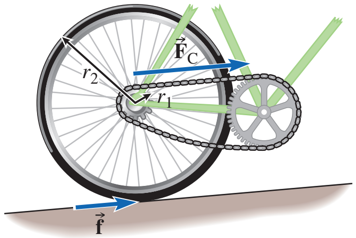
✦ 109. In a motor, a flywheel (solid disk of radius R and mass M) is rotating with angular velocity ω_i. When the clutch is released, a second disk (radius r and mass m) initially not rotating is brought into frictional contact with the flywheel. The two disks spin around the same axle with frictionless bearings. After a short time, friction between the two disks brings them to a common angular velocity. (a) Ignoring external influences, what is the final angular velocity? (b) Does the total angular momentum of the two change? If so, account for the change. If not, explain why it does not. (c) Repeat (b) for the rotational kinetic energy.

110. Since humans are generally not symmetrically shaped, the height of our center of gravity is generally not half of our height. One way to determine the location of the center of gravity is shown in the diagram. A 2.2-m-long uniform plank is supported by two bathroom scales, one at either end. Initially the scales each read 100.0 N. A 1.60-m-tall student then lies on top of the plank, with the soles of his feet directly above scale B. Now scale A reads 394.0 N and scale B reads 541.0 N. (a) What is the student's weight? (b) How far is his center of gravity from the soles of his feet? (c) When standing, how far above the floor is his center of gravity, expressed as a fraction of his height?

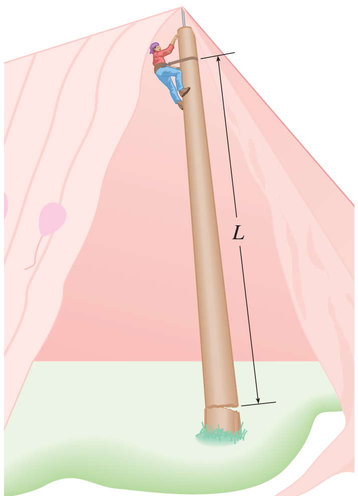
✦✦ 111. A spool of thread of mass m rests on a plane inclined at angle θ. The end of the thread is tied as shown. The outer radius of the spool is R and the inner radius (where the thread is wound) is r. The rotational inertia of the spool is I. Give all answers in terms of m, θ, R, r, I, and g. (a) If there is no friction between the spool and the incline, describe the motion of the spool and calculate its acceleration. (b) If the coefficient of friction is large enough to keep the spool from slipping, calculate the magnitude and direction of the frictional force. (c) What is the minimum possible coefficient of friction to keep the spool from slipping in part (b)?

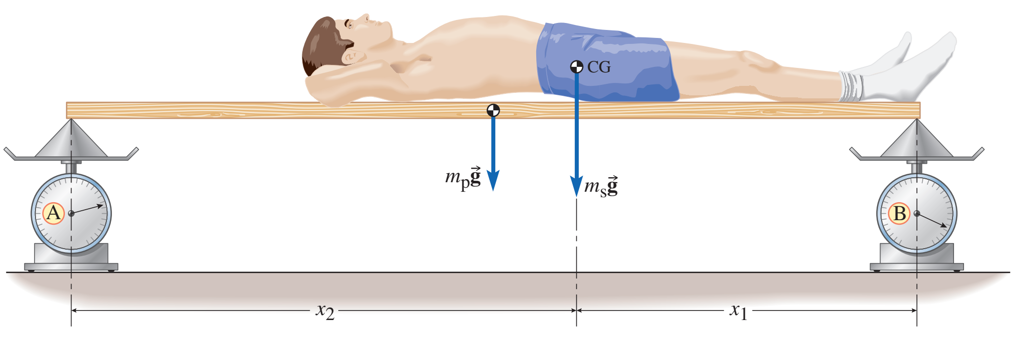


112. A bicycle travels up an incline at constant velocity. The magnitude of the frictional force due to the road on the rear wheel is $f = 3.8$ N. The upper section of chain pulls on the sprocket wheel, which is attached to the rear wheel, with a force $\vec{\mathbf{F}}_C$. The lower section of chain is slack. If the radius of the rear wheel is 6.0 times the radius of the sprocket wheel, what is the magnitude of the force $\vec{\mathbf{F}}_C$ with which the chain pulls?



✦ 113. A circus roustabout is attaching the circus tent to the top of the main support post of length L when the post suddenly breaks at the base. The worker's weight is negligible compared to that of the uniform post. What is the speed with which the roustabout reaches the ground if (a) he jumps at the instant

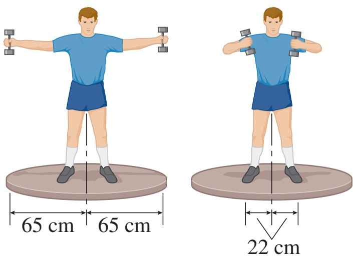




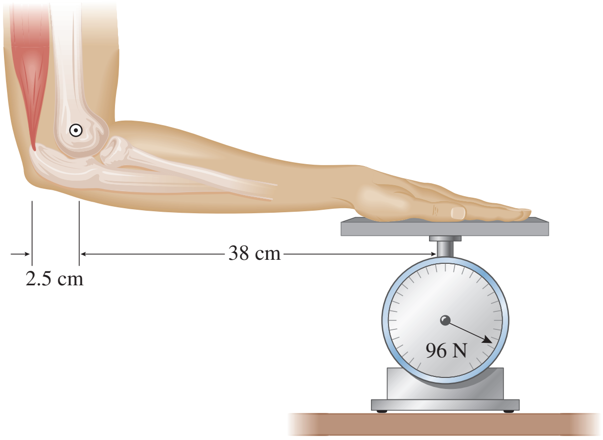
Problem 110

he hears the post crack or (b) if he clings to the post and rides to the ground with it? (c) Which is the safest course of action for the roustabout?

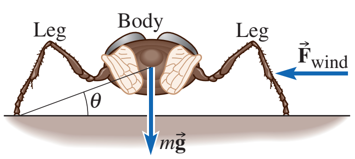
114. A student stands on a platform that is free to rotate and holds two dumbbells, each at a distance of 65 cm from his central axis. Another student gives him a push and starts the system of student, dumbbells, and platform rotating at 0.50 rev/s. The student on the platform then pulls the dumbbells in close to his chest so that they are each 22 cm from his central axis. Each dumbbell has a mass of 1.00 kg and the rotational inertia of the student, platform, and dumbbells is initially 2.40 kg·m^2. Model each arm as a uniform rod of mass 3.00 kg with one end at the central axis; the length of the arm is initially 65 cm and then is reduced to 22 cm. What is his new rate of rotation?



115. A person places his hand palm downward on a scale and pushes down on the scale until it reads 96 N. The triceps muscle is responsible for this arm extension force. Find the force exerted by the triceps muscle. The bottom of the triceps muscle is 2.5 cm to the left of the elbow joint and the palm is pushing at approximately 38 cm to the right of the elbow joint.



116. The posture of small animals may prevent them from being blown over by the wind. For example, with wind blowing from the side, a small insect stands with bent legs; the more bent the legs, the lower the body and the smaller the angle θ. The wind exerts a force on the insect, which causes a torque about the point where the downwind feet touch. The torque due to the weight of the insect must be equal and opposite to keep the insect from being blown over. For example, the drag force on a blowfly due to a sideways wind is $F_{\text{wind}} = cAv^2$, where v is the velocity of the wind, A is the cross-sectional area on which the wind is blowing, and $c \approx 1.3\ \text{N·s}^2\text{·m}^{-4}$. (a) If the blowfly has a cross-sectional side area of 0.10 cm^2, a mass of 0.070 g, and crouches such that $\theta = 30.0°$, what is the maximum wind speed in which the blowfly can stand? (Assume that the drag force acts at the center of gravity.) (b) How about if it stands such that $\theta = 80.0°$? (c) Compare to the maximum wind velocity that a dog can withstand, if the dog stands such that $\theta = 80.0°$, has a cross-sectional area of 0.030 m^2, and weighs 10.0 kg. (Assume the same value of c.)



117. (a) Redo Example 8.7 to find an algebraic solution for d in terms of M, m, μ_s, L, and θ. (b) Use this expression to show that placing the ladder at a larger angle θ (that is, more nearly vertical) enables the person to climb farther up the ladder without having it slip, all other things being equal. (c) Using the numerical values from Example 8.7, find the minimum angle θ that enables the person to climb all the way to the top of the ladder.

Answers to Practice Problems

8.1 390 kg·m^2

8.2 $v = \sqrt{\dfrac{2(m_2 - \mu_k m_1)gh}{m_1 + m_2 + I/R^2}}$

8.3 53 N; 8.4 N·m

8.4 –65 N·m

8.5 8.3 J

8.6 left support, downward; right support, upward

8.7 0.27

8.8 57 N, downward

8.9 It must lie in the same vertical plane as the two ropes holding up the rings. Otherwise, the gravitational force would have a nonzero lever arm with respect to a horizontal axis that passes through the contact points between his hands and the rings; thus, gravity would cause a net torque about that axis.

8.10 460 N

8.11 (a) 2380 rad; (b) 3.17 kJ; (c) 1.34 N·m

8.12 solid ball, $\frac{2}{7}$; hollow ball, $\frac{2}{5}$

8.13 $\frac{1}{2}g \sin \theta$

8.14 5% increase

8.15 16 cm/s

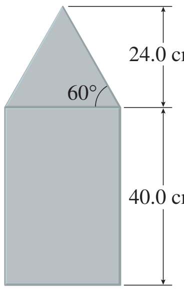
REVIEW AND SYNTHESIS: CHAPTERS 6–8

Review Exercises

1. A spring scale in a French market is calibrated to show the *mass* of vegetables in grams and kilograms. (a) If the marks on the scale are 1.0 mm apart for every 25 grams, what maximum extension of the spring is required to measure up to 5.0 kg? (b) What is the spring constant of the spring? [*Hint*: Remember that the scale really measures *force*.]
2. Plot a graph of this data for a spring resting horizontally on a table. Use your graph to find (a) the spring constant and (b) the relaxed length of the spring.

Force (N)	0.200	0.450	0.800	1.500
Spring length (cm)	13.3	15.0	17.3	22.0

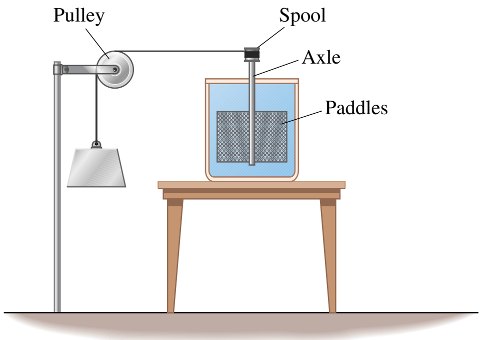
3. A pendulum consists of a bob of mass m attached to the end of a cord of length L. The pendulum is released from a point at a height of $L/2$ above the lowest point of the swing. What is the tension in the cord as the bob passes the lowest point?
4. Marvin kicks a 1.5-kg ball. His foot makes contact with the ball for 0.20 s, and he accelerates it from rest to a speed of 11 m/s. Marvin's foot loses contact with the ball when it is 40 cm above the ground. The direction of the ball's velocity is 45° at this time. Ignore air resistance. (a) What is the average acceleration Marvin gives to the ball? (b) What is the average force applied to the ball? (c) How high does the ball travel above the ground? (d) What is the horizontal distance traveled by the ball after it leaves Marvin's foot? (e) What are the ball's horizontal and vertical velocity components just before it strikes the ground?
5. How much energy is expended by an 80.0-kg person in climbing a vertical distance of 15 m? Assume that muscles have an efficiency of 22%; that is, the work done by the muscles to climb is 22% of the energy expended.
6. Ugonna stands at the top of an incline and pushes a 100-kg crate to get it started sliding down the incline. The crate slows to a halt after traveling 1.50 m along the incline. (a) If the initial speed of the crate was 2.00 m/s and the angle of inclination is 30.0°, how much energy was dissipated by friction? (b) What is the coefficient of sliding friction?
7. A packing carton slides down an inclined plane of angle 30.0° and of incline length 2.0 m. If the initial speed of the carton is 4.0 m/s directed down the incline, what is the speed at the bottom? Neglect friction.
8. A child's playground swing is supported by chains that are 4.0 m long. If the swing is 0.50 m above the ground and moving at 6.0 m/s when the chains are vertical, what is the maximum height of the swing?
9. A block slides down a plane that is inclined at an angle of 53° with respect to the horizontal. If the coefficient of kinetic friction is 0.70, what is the acceleration of the block?
10. Gerald wants to know how fast he can throw a ball, so he hangs a 2.30-kg target on a rope from a tree. He picks up a 0.50-kg ball of putty and throws it horizontally against the target. The putty sticks to the target and the putty and target swing up a vertical distance of 1.50 m from its original position. How fast did Gerald throw the ball of putty?
11. A hollow cylinder rolls without slipping or sliding along a horizontal surface toward an incline. If the cylinder's speed is 3.00 m/s at the base of the incline and the angle of inclination is 37.0°, how far along the incline does the cylinder travel before coming to a stop?
12. An 11-kg bicycle is moving with a linear speed of 7.5 m/s. Each wheel can be modeled as a thin hoop with a mass of 1.3 kg and a diameter of 70 cm. The bicycle is stopped in 4.5 s by the action of brake pads that squeeze the wheels and slow them down. The coefficient of friction between the brake pads and a wheel is 0.90. There are four brake pads altogether; assume they apply equal magnitude normal forces on the wheels. What is the normal force applied to a wheel by one of the brake pads?
13. A 0.185-kg spherical steel ball is used in a pinball machine. The ramp is 2.05 m long and tilted at an angle of 5.00°. Just after a flipper hits the ball at the bottom of the ramp, the ball has an initial speed of 2.20 m/s. What is the speed of the ball when it reaches the top of the pinball machine?
14. Carissa places a solid toy ball atop the apex of her dollhouse roof. It rolls down, then falls the 40.0-cm vertical distance to the floor. (a) What is the speed of the ball as it loses contact with the roof? (b) How far from the base of the house does it land?



15. ✦ A 0.122-kg dart is fired from a gun with a speed of 132 m/s horizontally into a 5.00-kg wooden block. The block is attached to a spring with a spring constant of 8.56 N/m. The coefficient of kinetic friction between the block and the horizontal surface it is resting on is 0.630. After the dart embeds itself into the block, the block slides along the surface and compresses the spring. What is the maximum compression of the spring?
16. A 5.60-kg uniform door is 0.760 m wide by 2.030 m high, and is hung by two hinges, one at 0.280 m from the top and one at 0.280 m from the bottom of the door. If the vertical components of the forces on each of the two

hinges are identical, find the vertical and horizontal force components acting on each hinge due to the door.

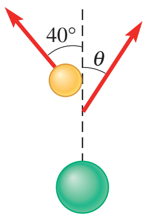
17. Consider the apparatus shown in the figure (not to scale). The pulley, which can be treated as a uniform disk, has a mass of 60.0 g and a radius of 3.00 cm. The spool also has a radius of 3.00 cm. The rotational inertia of the spool, axle, and paddles about their axis of rotation is 0.00140 kg·m^2. The block has a mass of 0.870 kg and is released from rest. After the block has fallen a distance of 2.50 m, it has a speed of 3.00 m/s. How much energy has been delivered to the fluid in the beaker?



18. It is the bottom of the ninth inning at a baseball game. The score is tied and there is a runner on second base when the batter gets a hit. The 85-kg base runner rounds third base and is heading for home with a speed of 8.0 m/s. Just before he reaches home plate, he crashes into the opposing team's catcher, and the two players slide together along the base path toward home plate. The catcher has a mass of 95 kg and the coefficient of friction between the players and the dirt on the base path is 0.70. How far do the catcher and base runner slide?

19. Pendulum bob A has half the mass of pendulum bob B. Each bob is tied to a string that is 5.1 m long. When bob A is held with its string horizontal and then released, it swings down and, once bob A's string is vertical, it collides elastically with bob B. How high do the bobs rise after the collision?

✦✦ 20. During a game of marbles, the "shooter," a marble with three times the mass of the other marbles, has a speed of 3.2 m/s just before it hits one of the marbles. The other marble bounces off the shooter in an elastic collision at an angle of 40°, as shown, and the shooter moves off at an angle θ. Determine (a) the speed of the shooter after the collision, (b) the speed of the marble after the collision, and (c) the angle θ.



21. At the beginning of a scene in an action movie, the 78.0-kg star, Indianapolis Jones, will stand on a ledge 3.70 m above the ground and the 55.0-kg heroine, Georgia Smith, will stand on the ground. Jones will swing down on a rope, grab Smith around the waist, and continue swinging until they come to rest on another ledge on the other side of the set. At what height above the ground should the second ledge be placed? Assume that Jones and Smith remain nearly upright during the swing so that their centers of mass are always the same distance above their feet.

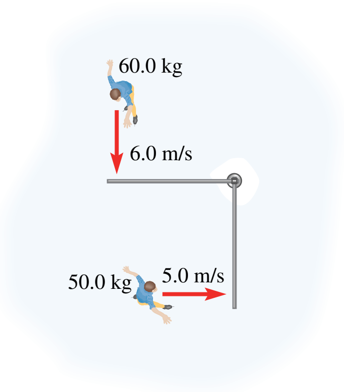
22. A uniform disk is rotated about its symmetry axis. The disk goes from rest to an angular speed of 11 rad/s in a time of 0.20 s with a constant angular acceleration. The moment of inertia and radius of the disk are 1.5 kg·m^2 and 11.5 cm, respectively. (a) What is the angular acceleration during the 0.20-s interval? (b) What is the net torque on the disk during this time? (c) After the applied torque stops, a frictional torque remains. This torque has an associated angular acceleration of 9.8 rad/s^2. Through what total angle θ (starting from time $t = 0$) does the disk rotate before coming to rest? (d) What is the speed of a point halfway between the rim of the disk and its rotation axis 0.20 s after the applied torque is removed?

23. A block is released from rest and slides down an incline. The coefficient of sliding friction is 0.38 and the angle of inclination is 60.0°. Use energy considerations to find how fast the block is sliding after it has traveled a distance of 30.0 cm along the incline.

24. A uniform solid cylinder rolls without slipping or sliding down an incline. The angle of inclination is 60.0°. Use energy considerations to find the cylinder's speed after it has traveled a distance of 30.0 cm along the incline.

25. A block of mass 2.00 kg slides eastward along a frictionless surface with a speed of 2.70 m/s. A chunk of clay with a mass of 1.50 kg slides southward on the same surface with a speed of 3.20 m/s. The two objects collide and move off together. What is their velocity after the collision?

26. An ice-skater, with a mass of 60.0 kg, glides in a circle of radius 1.4 m with a tangential speed of 6.0 m/s. A second skater, with a mass of 50.0 kg, glides on the same circular path with a tangential speed of 5.0 m/s. At an instant of time, both skaters grab the ends of a lightweight, rigid set of rods, set at 90° to each other, that can freely rotate about a pole, fixed in place on the ice. (a) If each rod is 1.4 m long, what is the tangential speed of the skaters after they grab the rods? (b) What is the direction of the angular momentum before and after the skaters "collide" with the rods?

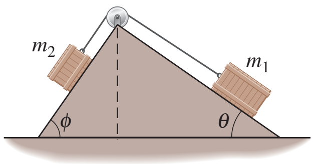


27. A child's toy is made of a 12.0-cm-radius rotating wheel that picks up 1.00-g pieces of candy in a pocket at its lowest point, brings the candy to the top, then releases it. The frequency of rotation is 1.60 Hz. (a) How far from its

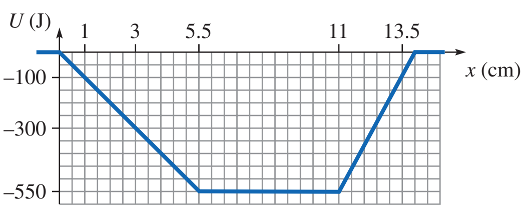
starting point does the candy land? (b) What is the radial acceleration of the candy when it is on the wheel?

28. A Vulcan spaceship has a mass of 65,000 kg and a Romulan spaceship is twice as massive. Both have engines that generate the same total force of 9.5×10^6 N. (a) If each spaceship fires its engine for the same amount of time, starting from rest, which will have the greater kinetic energy? Which will have the greater momentum? (b) If each spaceship fires its engine for the same *distance*, which will have the greater kinetic energy? Which will have the greater momentum? (c) Calculate the energy and momentum of each spaceship in parts (a) and (b), neglecting any change in mass due to whatever is expelled by the engines. In part (a), assume that the engines are fired for 100 s. In part (b), assume that the engines are fired for 100 m.

29. Two blocks of masses m_1 and m_2, resting on frictionless inclined planes, are connected by a massless rope passing over an ideal pulley. Angle $\phi = 45.0°$ and angle $\theta = 36.9°$; mass m_1 is 6.00 kg and mass m_2 is 4.00 kg. (a) Using energy conservation, find how fast the blocks are moving after they travel 2.00 m along the inclines. (b) Now solve the same problem using Newton's second law. [*Hint:* First find the acceleration of each of the blocks. Then find how fast either block is moving after it travels 2.00 m along the incline with constant acceleration.]



✦✦ 30. A particle, constrained to move along the x-axis, has a total mechanical energy of –100 J. The potential energy of the particle is shown in the graph. At time $t = 0$, the particle is located at $x = 5.5$ cm and is moving to the left. (a) What is the particle's potential energy at $t = 0$? What is its kinetic energy at this time? (b) What are the particle's total, potential, and kinetic energies when it is at $x = 1$ cm and moving to the right? (c) What is the particle's kinetic energy when it is at $x = 3$ cm and moving to the left? (d) Describe the motion of this particle starting at $t = 0$.



MCAT Review

The section that follows includes MCAT exam material and is reprinted with permission of the Association of American Medical Colleges (AAMC).

1. A projectile with a mass of 0.2 kg and a horizontal speed of 2.0 m/s hits a recycle bin (which is free to move), then rebounds at 1.0 m/s back along the same path. What is the magnitude of the horizontal momentum the bin receives?
   A. 0.2 kg·m/s B. 0.3 kg·m/s
   C. 0.5 kg·m/s D. 0.6 kg·m/s

2. A vertically oriented spring is stretched by 0.15 m when a 100-g mass is suspended from it. What is the approximate spring constant of the spring?
   A. 0.015 N/m B. 0.15 N/m
   C. 1.5 N/m D. 6.5 N/m

3. When a downward force is applied at a point 0.60 m to the left of a fulcrum, equilibrium is obtained by placing a mass of 1.0×10^{-7} kg at a point 0.40 m to the right of the fulcrum. What is the magnitude of the downward force?
   A. 1.5×10^{-7} N B. 6.5×10^{-7} N
   C. 9.8×10^{-7} N D. 1.5×10^{-6} N

4. A 0.50-kg ball accelerates from rest at 10 m/s^2 for 2.0 s. It then collides with and sticks to a 1.0-kg ball that is initially at rest. After the collision, approximately how fast are the balls going?
   A. 3.3 m/s B. 6.7 m/s
   C. 10.0 m/s D. 15.0 m/s

5. A 1000-kg car requires 10,000 W of power to travel at 15 m/s on a level highway. How much extra power in watts is required for the car to climb a 10° hill at the same speed? (Use $g = 10$ m/s^2.)
   A. $1.0 \times 10^4 \times \sin 10°$ B. $1.5 \times 10^4 \times \sin 10°$
   C. $1.0 \times 10^5 \times \sin 10°$ D. $1.5 \times 10^5 \times \sin 10°$

6. A 90-kg patient walks the treadmill at a speed of 2 m/s, and $\theta_{in} = 30°$ for 10 min (600 s). What is the total work done by the patient on the treadmill? (Use $g = 10$ m/s^2.)
   A. 1.80 kJ B. 18.0 kJ
   C. 0.54 MJ D. 1.08 MJ

7. A 100-kg patient walks the treadmill at a speed of 3 m/s, and $\theta_{in} = 30°$ for 5 min (300 s). What is the mechanical power output of the patient in watts? (Use $g = 10$ m/s^2.)
   A. 300 W B. 1500 W
   C. 3000 W D. 7500 W

Read the paragraphs and then answer the following questions:

An exercise bike has the basic construction of a bicycle with a single heavy disk wheel. In addition to friction in the bearings and the transmission system, resistance to pedaling is provided by two narrow friction pads that push with equal force on each side of the wheel. The coefficient of kinetic friction between the pads and the wheel is 0.4, and the pads provide a total retarding force of 20 N tangential to the wheel. The pads are located at a position 0.3 m from the center of the wheel. The distance, recorded on the odometer, is considered to be the distance that a point on the wheel 0.3 m from the center moves. The pedals move in a circle of 0.15 m in radius and complete one revolution, while a transmission system allows the wheel to rotate twice.

In human metabolic processes, the ratio of energy released to volume of oxygen consumed averages 20,000 J/L. A cyclist with a basal metabolic rate of 85 W (rate of internal energy conversion while awake but inactive) pedals continuously for 20 min, registering 4800 m on the odometer. During this activity, the cyclist's average metabolic rate is 535 W. The cyclist's body converts the extra energy into mechanical work output with an efficiency of 20%.

8. What is the magnitude of the force pushing each friction pad onto the wheel?
   A. 10 N
   B. 25 N
   C. 40 N
   D. 50 N
9. Which of the following is closest to the radial acceleration of the part of the wheel that passes between the friction pads?
   A. 10 m/s^2
   B. 20 m/s^2
   C. 40 m/s^2
   D. 50 m/s^2
10. If the wheel has a kinetic energy of 30 J when the cyclist stops pedaling, how many rotations will it make before coming to rest?
    A. Less than 1
    B. Between 1 and 2
    C. Between 2 and 3
    D. Between 3 and 4
11. What is the difference between the average mechanical power output of the cyclist in the passage and the power dissipated by the wheel at the friction pads?
    A. 5 W
    B. 10 W
    C. 20 W
    D. 27 W
12. Which of the following actions would most likely increase the fraction of the cyclist's mechanical power output that is dissipated by the wheel at the friction pads?
    A. Reducing the force on the friction pads and pedaling at the same rate
    B. Maintaining the same force on the friction pads and pedaling at a slower rate
    C. Maintaining the same force on the friction pads and pedaling at a faster rate
    D. Increasing the force on the friction pads and pedaling at the same rate
13. Which of the following is the best estimate of the number of liters of oxygen the cyclist in the passage would consume in the 20 min of activity?
    A. 25 L
    B. 30 L
    C. 45 L
    D. 50 L
14. During a second workout, the cyclist reduces the force on the friction pads by 50%, then pedals for two times the previous distance in $\frac{1}{2}$ the previous time. How does the amount of energy dissipated by the pads in the second workout compare with energy dissipated in the first workout?
    A. One-eighth as much
    B. One-half as much
    C. Equal
    D. Two times as much
15. What is the ratio of the distance moved by a pedal to the distance moved by a point on the wheel located at a radius of 0.3 m in the same amount of time?
    A. 0.25
    B. 0.5
    C. 1
    D. 2
16. A cyclist's average metabolic rate during a workout is 500 W. If the cyclist wishes to expend at least 300 kcal (1 kcal = 4186 J) of energy, how long must the cyclist exercise at this rate?
    A. 0.6 min
    B. 3.6 min
    C. 36.0 min
    D. 41.9 min
17. If the friction pads are moved to a location 0.4 m from the center of the wheel, how does the amount of work done on the wheel, per revolution, change?
    A. It decreases by 25%
    B. It stays the same
    C. It increases by 33%
    D. It increases by 78%