## Light and Image Formation

## chapter overview

Our main goal in this chapter is to provide an understanding of the laws of reflection and refraction and how they are used to explain image formation. To do this, we will need to discuss the relationship between waves and rays and to show how rays can be traced to define and locate images. In the process of exploring image formation, we will examine the behavior of mirrors and lenses and the workings of simple optical instruments such as cameras, magnifiers, microscopes, and telescopes.


## chapter outline

## Reflection and image

formation. What is the relationship between rays and waves? What is the law of reflection, and how are images formed by a plane mirror?
2
Refraction of light. How does the law of refraction describe the bending of light rays when they pass from one medium to another? How can we explain the separation of colors by a prism?
3 Lenses and image formation. How can the law of refraction explain image formation by simple lenses? How do positive lenses differ from negative lenses?
4 Focusing light with curved mirrors. How can curved mirrors be used to focus light? How do concave mirrors differ from convex mirrors in their image-forming properties?
5 Eyeglasses, microscopes, and telescopes. How do eyeglasses help us to see better? How can lenses and mirrors be combined to produce microscopes and telescopes?

Have you ever looked in a mirror and wondered how you are able to stare yourself in the face? How does that early-morning, mussed-up face pretending to be you appear behind the glass plate (fig. 17.1)? You know that you are seeing an image that, depending on the quality of the mirror, may or may not be an accurate reflection of reality. You are free to believe what you wish.

You may also wear eyeglasses, and you probably have used optical instruments that incorporate lenses like binoculars, overhead projectors, and microscopes. Images formed by mirrors involve the reflection of light. Images formed by lenses involve the refraction or bending of light. Both reflection and refraction are at work in producing rainbows.

In chapter 16, we introduced the idea that light is an electromagnetic wave and described the general features of these waves. How are such waves involved in the formation of images in a mirror, by a slide projector, or in your eye itself? What are the basic principles of image formation? Can we predict where the images will be formed and how they will appear?

Such questions lie in the realm of geometric optics, in which we describe the behavior of light waves by using rays that are perpendicular to the wavefronts. The laws of reflection and refraction are the basic principles of

figure 17.1 An early-morning look in the mirror. How does that discouraging image get there?
geometric optics. They allow us to trace the paths of light rays, and to predict how and where images will be formed. Physical optics (chapter 16) treats phenomena such as interference and diffraction, which involve the wave aspects of light more directly.

### 17.1 Reflection and Image Formation

How is your image in the bathroom mirror produced? You know that light is involved somehow, as you can easily verify by turning off the bathroom light. If the room is completely dark, the image disappears, only to reappear instantly when the light is turned back on. Light waves from the bathroom light must bounce off your face, travel from there to the mirror, and then reflect back to your eyes. How does this process create the image that we see?

## How are light rays related to wavefronts?

If we consider just one point on your face and trace what happens to the waves that are reflected from that point, we get a clearer idea of what is happening. Since the skin on your face is somewhat rough (at least on a microscopic scale), light that reaches your face is reflected or scattered in all directions from any given point. The tip of your nose, for example, behaves as though it were a source of light waves that spread out uniformly from that point (fig. 17.2). These waves are like the ripples that spread on a pond when a rock is dropped into the water.

The light waves scattered from your face are electromagnetic waves, not waves of water, but they have crests (where the electric and magnetic fields are the strongest) that move outward from the source point just as water waves do. If we connect the points on the wave that are all

figure 17.2 Any point on your face acts as a secondary source of light rays that are reflected in all directions from that point.
at the same point in their cycle, we define a wavefront. We often choose the crest of the wave (the point of maximum positive amplitude) for this purpose, since it is clearly visible in water waves. The next wavefront behind the leading one is the next point at which the waves are at their crest, and it is separated from the previous wavefront by a distance of one wavelength, as in figure 17.3. For light waves, these wavefronts move away from the source point at the speed of light.

We could describe almost everything that happens to these waves by tracing what happens to the wavefronts. It is easier, however, to examine their behavior using rays perpendicular to the wavefronts. If the waves are traveling in the same medium (air, for example), the wavefronts move forward uniformly and the resulting rays are straight lines (fig. 17.3). These rays are easier to draw than the curved wavefronts and can be traced more readily.

Since each point on your face scatters light in all directions, these points act as sources of diverging light rays, as in figure 17.2. The light rays travel from your face to the mirror, where they are reflected in a predictable manner because of the smoothness of the mirror. Your eyes are receiving light from the light bulb, but that light has been reflected by both your face and the mirror before getting back to your eyes.

## What is the law of reflection?

What happens when light rays and wavefronts strike a smooth reflecting surface like a flat mirror? The waves are reflected, and after reflection, they travel away from the mirror with the same speed that they had before reflection. Figure 17.4 depicts this process for plane wavefronts (no curvature) that are approaching the plane mirror at an angle rather than head-on.

Since the wavefronts approach the mirror at an angle, some parts of the wavefront are reflected sooner than others

figure 17.3 Light rays are drawn so that they are everywhere perpendicular to the wavefronts. If the waves travel with uniform speed, the rays are straight lines.

figure 17.4 Plane light waves approaching a mirror at an angle travel with the same speed both before and after striking the mirror. The angle of reflection equals the angle of incidence.
(fig. 17.4). These wavefronts now travel away from the mirror with the same spacing and speed but in a new direction. The angle between the wavefront and the mirror is the same, however, for the emerging wave. Because the outgoing wavefronts travel at the same speed and cover the same distance in a given time as the incoming waves, this produces equal angles between the wavefronts and the surface of the mirror.

This result is usually stated using rays. The angle that a ray makes to a line drawn perpendicular to the surface is the same angle that the wavefront makes to the surface of the mirror. Using the word normal to mean perpendicular (as in chapter 4 when we discussed normal forces), we call the line drawn perpendicular to the surface of the mirror the surface normal. The equal angles the wavefronts make to the mirror dictate that the angle the reflected ray makes to the surface normal is equal to the angle that the incoming, or incident, ray makes to the surface normal (fig 17.4).

What we have just described is the law of reflection, which can be stated concisely as

When light is reflected from a smooth reflecting surface, the angle the reflected ray makes with the surface normal is equal to the angle the incident ray makes with the surface normal.

In other words, the angle of reflection ( $\theta_{r}$ in figure 17.4) is equal to the angle of incidence $\left(\theta_{i}\right)$. The reflected ray also lies within the plane defined by the incident ray and the surface normal. It does not deviate in or out of the plane of the page in our diagram.

## How are images formed by a plane mirror?

How can the law of reflection help us to explain how an image is formed in a plane mirror? What happens to the light rays that are scattered from your nose? Tracing these rays from their origin at the nose and following them through reflection from the mirror using the law of reflection shows what happens to individual rays (fig. 17.5).

If we extend the reflected rays backward from the mirror, they intersect at a point behind the mirror, as shown. As your eye collects a small bundle of reflected rays, it perceives an image that appears to lie at this point of intersection. In other words, as far as your eye can tell, these light rays are coming from that point. You see the tip of your nose as lying behind the mirror. The same argument holds for any other point on your face-they all seem to lie behind the mirror.

Using simple geometry, you can see that the distance of this image behind the mirror (measured from the mirror surface) is equal to the distance of the original object from the front of the mirror. These equal distances follow from the law of reflection, as figure 17.6 illustrates. Here we have taken just two rays coming from the top of a candle and traced them as before. The ray that comes into the mirror in the horizontal direction and perpendicular to the mirror is reflected back along the same line. The angle of incidence for this ray is zero-so is the angle of reflection.

The other ray is shown as being reflected from a point on the mirror even with the base of the candle. This ray is reflected at an angle equal to its angle of incidence. When these two rays are extended backward, their intersection locates the image position. Any other rays traced from the top of the candle would also appear to come from this point. The two rays shown form identical triangles on either side of the mirror (fig. 17.6). Since the angles are equal, and the short side of both triangles is equal to the height of the candle, the long sides of these identical trian-

figure 17.5 A few rays are traced from the tip of the nose to show their reflection from the mirror. They diverge after reflection as though they were coming from a point behind the mirror.

figure 17.6 Two light rays coming from the top of the candle appear to diverge after reflection from a point as far behind the mirror as the candle is in front of the mirror.
gles must also be equal. The image is therefore located behind the mirror at an image distance, $i$, equal to the object distance, $o$, of the candle from the front of the mirror.

Since we all have access to plane mirrors, you can verify some of these ideas by observing your own image and images of other objects as you move before the mirror. The mirror does not have to be as tall as you, for example, for you to see your entire height. Do you see more of you as you move toward the mirror or away from the mirror? What about other objects? Where must you be positioned to see various other objects in the room? Can you explain these observations by the law of reflection and by which rays reach your eyes?

The image formed by a plane mirror is called a virtual image because the light never actually passes through the point where the image is located. In fact, the light never gets behind the mirror at all-it just appears to come from points behind the mirror as it is reflected. The image can also be characterized as being upright (right side up) and as having the same size as the object (not magnified). There is a reversal, however, of right and left: what appears to be the right hand of your mirror image is actually the image of your left hand, and vice versa. Take another look. You see such images every day, but you probably have not given them much thought.

Light rays are drawn perpendicular to the wavefronts. Tracing these rays often provides an easier way of seeing what happens to light than following the wavefronts themselves. The law of reflection states that when light is reflected from a smooth reflecting surface, the angle of reflection equals the angle of incidence (both measured from the surface normal). The image formed by a plane mirror lies at the point from which the light rays appear to be diverging after reflection, which is as far behind the mirror as the object is in front of the mirror.

### 17.2 Refraction of Light

The most familiar images are those formed by plane mirrors. Images can be formed in other ways, though, with prisms, lenses, or maybe just a tank of water. These examples all involve substances that are transparent to visible light. What happens to light rays when they encounter the surface of a transparent object? Why do we get misleading impressions about the location of underwater objects? The law of refraction helps us answer these questions.

## What is the law of refraction?

Suppose that light waves encounter a plane surface of a piece of glass after traveling initially through air. What happens to these waves as they pass into the glass and continue traveling through the glass? Experimental measurements have shown that the speed of light in glass or water is less than the speed of light in a vacuum or air. (The speed of light in air is very close to its speed in a vacuum.) The distance between wavefronts (the wavelength) will be shorter in glass or water than in air (fig. 17.7), since the waves travel a smaller distance in one cycle given their smaller speed.

The difference in the speed of light in different substances is usually described by a quantity called the index of refraction, represented by the symbol $n$. The index of refraction is defined as the ratio of the speed of light $c$ in a vacuum to the speed of light $v$ in some substance, $n=c / v$. The speed of light $v$ in the substance is then related to the speed of light $c$ in a vacuum by

$$
\nu=\frac{c}{n} .
$$


figure 17.7 The wavelength of light waves traveling from air into glass is shorter in the glass than in air because of the smaller speed of light waves in glass.

In other words, to find the speed of light in some transparent material, we divide the speed of light in a vacuum ( $c=3 \times 10^{8} \mathrm{~m} / \mathrm{s}$ ) by the index of refraction of that material. Typical values for the index of refraction of glass are between 1.5 and 1.6 , so the speed of light in glass is approximately two-thirds the speed of light in air.

What effect does this reduction in speed and wavelength have on the direction of light rays as they pass into glass? If we consider wavefronts and their corresponding rays approaching the surface at an angle, as in figure 17.8, we can see that the rays will bend as the waves pass from air to glass. The bending occurs because the wavefronts do not travel as far in one cycle in the glass as they do in air. As the diagram shows, the wavefront halfway into the glass travels a smaller distance in glass than it does in air, causing it to bend in the middle. Thus, the ray, which is perpendicular to the wavefront, is also bent. The situation is like a marching band marching onto a muddy field at an angle to the edge of the field. The rows bend as their speed is reduced by the mud.

The amount of bending depends on the angle of incidence and on the indices of refraction of the materials involved, which determine the change in speed. A larger difference in speed will produce a greater difference in how far the wavefronts travel in the two substances. A larger difference in indices of refraction of the two substances therefore produces a larger bend in the wavefront and ray.

figure 17.8 Wavefronts approaching a glass surface at an angle to the surface are bent as they pass into the glass. The angle of refraction, $\theta_{2}$, is less than the angle of incidence, $\theta_{1}$. $\left(n_{2}\right.$ is larger than $n_{1}$.)

The bending described by the law of refraction can be stated in qualitative* terms as:

When light passes from one transparent medium to another, the rays are bent toward the surface normal (the axis drawn perpendicular to the surface) if the speed of light is smaller in the second medium than in the first. The rays are bent away from this axis if the speed of light in the second medium is greater than in the first.

If the angles are small, the quantitative statement of the law of refraction takes a simple form. For small angles, the sine function is proportional to the angle itself, so

$$
n_{1} \theta_{1}=n_{2} \theta_{2}
$$

The product of the index of refraction of the first medium times the angle of incidence is approximately equal to the product of the index of refraction of the second medium times the angle of refraction. As the index of refraction of the second medium increases, the angle of refraction must decrease, which means that the ray is bent closer to the axis (the surface normal) for larger indices of refraction.

For light waves traveling from glass to air, the bending is in the opposite direction: the rays are bent away from the surface normal, according to the law of refraction. Simply reversing the directions of the rays and wavefronts in figure 17.8 will make this clear. The increase in speed as the wave travels from glass to air causes the ray to bend away from the axis.

## Why do underwater objects appear to be closer than they are?

The bending of light rays at the interface of two transparent substances is responsible for some deceptive appearances. Suppose, for example, that you are standing on a bridge over a stream looking down at a fish. Water has an index of refraction of about 1.33, and air has an index of refraction of approximately 1 . Light traveling from the fish to your eyes is bent away from the surface normal, as in figure 17.9.

This bending of the light rays coming from the fish causes them to diverge more strongly in air than when they were traveling in the water. If we extend these rays backwards, we see that they now appear to come from a point closer to the surface of the water than their actual point of origin (fig. 17.9). Since this is true for any point on the fish, the fish appears to be closer to the surface than it

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figure 17.9 Light rays coming from the fish are bent as they pass from the water to air, so that the rays appear to diverge from a point closer to the surface.
actually is. If you were attempting to shoot the fish (illegal in most places), you would likely miss unless you were shooting straight down.

The apparent distance of the fish beneath the surface can be predicted from the law of refraction if we know the actual distance. The argument involves some geometry and the assumption that the angles of incidence and refraction are small. We find that the apparent distance as seen from the air (the image distance $i$ ) is related to the actual distance under water (the object distance $o$ ) and the indices of refraction by

$$
i=o\left(\frac{n_{2}}{n_{1}}\right)
$$

where the index of refraction of the second medium $n_{2}$ is that of air $\left(n_{a}=1\right)$ and the index of the first medium $n_{1}$ is that of water $\left(n_{w}=1.33\right)$. The image distance is therefore less than the actual distance, as figure 17.9 clearly illustrates. If the fish is actually 1 m below the surface, its apparent distance below the surface will be

$$
i=1 \mathrm{~m}\left(\frac{1}{1.33}\right)=0.75 \mathrm{~m}
$$

This apparent location of the fish is the position of the image of the fish. Light rays scattered from the fish seem to come from this point rather than from the actual position of the fish. We see a virtual image, like the image seen in a mirror, since the light rays do not actually pass through the image position. They only appear to come from that point. If instead we view an object in air from underwater, the

## example box 17.1

## Sample Exercise: Viewing an Object from Underwater

A girl swimming underwater views a dragonfly hovering over her in the air. The dragonfly appears to be about 60 cm above the water surface. What is the actual height of the dragonfly above the water?

Since the object (the dragonfly) is in air, the light is traveling initially in air, so:

$$
\begin{array}{rlrl}
n_{1} & =1.00 & i & =o \frac{n_{2}}{n_{1}} \\
n_{2} & =1.33 & & \\
i & =60 \mathrm{~cm} & & =i \frac{n_{1}}{n_{2}} \\
& =60 \mathrm{~cm} \frac{1}{1.33} \\
& =\mathbf{4 5} \mathbf{c m}
\end{array}
$$

(The object is actually closer to the water surface than it appears to be.)
image appears farther away from the water surface than the actual object position (example box 17.1).

The misleading position of objects viewed underwater is something we observe daily but often fail to notice simply because the experience is so common. A straight stick or rod seems to bend or break if part of it is above water and the rest below. When viewed from the top, each point of the underwater object appears to lie closer to the surface than its actual distance. A straw or spoon in a glass of water or other beverage likewise appears to bend, as in figure 17.10 . We are used to the deception and seldom give it a second thought.

figure 17.10 When viewed from above, a straw appears to bend when part of it is above water and the rest below. Viewed from the side, the straw is magnified.

## Total internal reflection

Another interesting phenomenon occurs when light rays travel from either water or glass to air. As we have already indicated, the light rays bend away from the axis as they pass into the medium with the lower index of refraction (air in these examples). What happens, though, if the rays are bent so much that the angle of refraction is $90^{\circ}$ ? The angle of refraction cannot be any larger and still result in rays passing into the second medium.

This situation is depicted in figure 17.11. As the angle of incidence for rays traveling in the glass gets larger, so does the angle of refraction (ray 1). Ultimately, we reach the point at which the angle of refraction is $90^{\circ}$ (ray 2 ). At this point, the refracted ray would just skim the surface as it emerged from the glass. For any angle of incidence

figure 17.11 When light travels from glass to air, the angle of incidence that would produce an angle of refraction of $90^{\circ}$ (ray 2 ) is called the critical angle $\theta_{c}$. Rays incident at equal or greater angles than $\theta_{c}$ are totally reflected.
inside the glass larger than this angle (ray 3), the ray does not escape the glass at all-it is reflected instead. The angle of incidence for which the angle of refraction is $90^{\circ}$ is called the critical angle $\theta_{c}$.* Rays incident at angles greater than the critical angle are reflected back inside the glass and obey the law of reflection rather than the law of refraction.

This phenomenon is called total internal reflection. For angles equal to or greater than the critical angle, $100 \%$ of the light is reflected inside the material with the larger index of refraction. Under these conditions, the glass-air interface makes an excellent mirror. For glass with an index of refraction of 1.5 , the critical angle is approximately $42^{\circ}$. Glasses with larger indices of refraction have even smaller critical angles. A prism cut with two $45^{\circ}$ angles, as in figure 17.12, can be used as a reflector. Light incident perpendicular to

[^1]
figure 17.12 A prism cut with two $45^{\circ}$ angles can be used as a mirror, since the light is totally reflected.

## everyday phenomenon

## box 17.1

## Rainbows

The Situation. Figure 1.1 in chapter 1 is a photograph of a rainbow. We have all seen such sights and been awed by their beauty. We know that rainbows occur when the sun is shining and it is raining nearby. If conditions are right, we can see an entire semicircular arc of color with red on the outside and violet on the inside. Sometimes we can also see a fainter bow of color forming a secondary arc outside of the primary one. The colors in the secondary rainbow are in reverse order of those in the primary rainbow, as the photo shows.

How is a rainbow formed? What conditions are necessary for observing a rainbow? Where should we look? Can the laws of reflection and refraction be used to explain this


The primary rainbow has red on the outside and violet on the inside. The secondary rainbow, sometimes visible, has the colors reversed.
phenomenon? We can now address these questions, some of which were first raised in chapter 1.

The Analysis. The secret to understanding the rainbow lies in considering what happens to light rays when they enter a raindrop, as in the first drawing. When a light ray strikes the first surface of the raindrop, some of it is refracted into the drop. Since the amount of bending depends on the index of refraction, which depends on the wavelength of the light, blue light is bent more than red light at this first surface. This effect is the same as the dispersion that occurs when light passes through a prism.


Light rays entering a raindrop are refracted by different amounts at the first surface, reflected at the back surface, and refracted again as they leave.
the first surface strikes the long surface at an angle of incidence of $45^{\circ}$, which is greater than the critical angle. It is totally reflected at this surface, so the surface acts as a plane mirror.

## How do prisms bend light, and what is dispersion?

You know that white light can be separated into different colors by a prism, producing an effect like a rainbow. How do we get the colors of the rainbow when we start with ordinary white light?

The index of refraction of a material varies with the wavelength of light: different wavelengths are bent by different amounts. The wavelength, in turn, is associated with the color that we perceive as discussed in chapter 16. Red light, at one end of the visible spectrum, has longer wavelengths and lower frequencies than violet light, at the opposite end of the spectrum. The index of refraction for violet or blue light is greater than for red light for most types of glass and other transparent substances like water. Blue
light is bent more than red light, and the intermediate wavelengths associated with the colors green, yellow, and orange are bent by intermediate amounts.

When light passes through a prism at an angle, the light is bent as it enters the prism and again as it leaves. The minimum deflection of the light occurs when it passes through the prism symmetrically, as in figure 17.13. The rays bend toward the surface normal as they enter the prism and away from the normal at the second surface, consistent with the law of refraction. Since the index of refraction varies for different wavelengths, the violet and blue rays are bent the most and lie at the bottom of the resulting spectrum of colors, as figure 17.14 shows.

This variation of index of refraction with wavelength is called dispersion, and it exists for all transparent materials including water, glass, and clear plastics. Dispersion is responsible for the colors that you see when light passes through a fish tank or around the edges of a lens, as in an overhead projector. It is also responsible for the beautiful displays of color seen in rainbows, as explained in everyday phenomenon box 17.1.

After being refracted at the first surface, the light rays travel through the drop and strike its back surface, where they are partially reflected. Some of the light passes out of the drop, and some is reflected back toward the front surface as shown. At the front surface, the rays are refracted again, causing more dispersion as the rays leave the drop.

It may seem paradoxical that red light, which is bent the least in the refractions, is actually diverted through a larger angle than blue light in its overall path through and back out of the raindrop. The reason can be understood from the first drawing. The smaller bending at the first surface causes the red rays to strike the back surface of the drop at a greater angle of incidence than for the violet rays. The red rays are reflected through a greater angle, according to the law of reflection. This larger angle of reflection dominates in determining the overall deflection of the ray.

When we view a rainbow, the sun must be behind us, since we are observing reflected rays. We see different colors at different points in the sky because, for a given color, the raindrops must be at the appropriate height for the light rays to reach our eyes. Since red rays are deviated the most, we see red light reflected from raindrops at the top of the rainbow or at the greatest angle from the center of the arc. Violet light comes to us from raindrops lying at smaller angles. The other colors lie in between, producing the colorful arc that we see.

The secondary rainbow, which is usually much fainter than the primary rainbow, is produced by a double reflection inside the raindrops, as the second drawing shows. Light rays entering near the bottom of the first surface of the drop may strike the back surface at a large enough angle of incidence to be


Light rays entering near the bottom of the first surface of a raindrop may be reflected twice before reemerging. These rays produce the secondary rainbow.
reflected twice before getting out of the drop. The light rays cross over in this case and are deviated through larger overall angles than any of the colors in the primary bow. For this reason, the secondary bow is seen outside the arc of the primary bow. Blue light is bent through a larger angle than red light so blue light lies at the top of the secondary rainbow.

The sun must be reasonably low in the sky for either the primary or secondary rainbow to be observed. During the summer, a late afternoon shower usually provides the best opportunity for viewing a rainbow. Rainbows can be seen at almost any time during the day from a high vantage point such as an airplane, however, and sometimes make a complete circle rather than just an arc. If you can understand why this is so, you have mastered the explanations provided here.

figure 17.13 Light rays passing through a prism are bent at both surfaces, with blue light being bent more strongly than red.

figure 17.14 White light incident on one side of the prism emerges as a spectrum of different colors because of dispersion.

Light rays are bent when they pass from one transparent substance to another because the speed of light changes at the boundary between the two substances. The law of refraction describes how much bending occurs and whether the bending is toward or away from the surface normal. Because of this bending, the image of an underwater object appears to lie closer to the surface of the water than the actual position of the object. For light traveling initially inside glass or water, there is a critical angle of incidence beyond which the light is totally reflected rather than being refracted. The index of refraction varies with wavelength, producing the dispersion of colors that we see when light passes through a prism.

### 17.3 Lenses and Image Formation

We encounter the images formed by mirrors daily and seldom give them a second thought. We may be even less aware of the images formed by lenses, although many of
us have lenses hanging on our noses or sitting on our eyeballs in the form of corrective eyeglasses or contact lenses. We also encounter lenses in cameras, overhead projectors, opera glasses, and simple magnifying glasses.

How do lenses form images? Lenses are usually made of glass or plastic, so the law of refraction governs their behavior. The bending of light rays as they pass through a lens is responsible for the size and nature of the images formed. We can understand the basics of this process by tracing what happens to just a few of these rays.

## Tracing rays through a positive lens

A lens shaped so that both sides are spherical surfaces with the convex sides facing out is pictured in figure 17.15. According to the law of refraction, the light rays are bent toward the surface normal at the first surface (going from air to glass) and away from the surface normal at the second surface (going from glass to air). If both surfaces are convex, as shown, each of these refractions bends the ray toward the axis (a line passing through the center of the lens and perpendicular to the lens). Such a lens causes light rays to converge. A converging lens is called a positive lens.

The easiest way to see that the light will bend as pictured in figure 17.15 is to imagine that each section of the lens behaves like a prism. The prism angle (the angle between the two sides) becomes larger toward the top of the lens so that light coming through the lens near the top is bent more than light passing through near the middle. Because the prism effect gets stronger farther from the axis, light rays coming in parallel to the axis are bent by different amounts as they pass through the lens. This causes them to all pass approximately through a single point $F$ on the opposite side of the lens, which we call the focal point. The focal point is the point where rays traveling parallel to the axis when they enter the lens are focused after leaving the lens.
The distance from the center of the lens to the focal point is called the focal length $f$. Focal length is a property

figure 17.15 Parallel light rays passing through a simple convex lens are bent toward the axis so that they all pass approximately through the focal point $F$. A lens has two focal points, one on either side.
of the lens that depends on how strongly the surfaces are curved and on the index of refraction of the lens material. There is also a focal point a distance $f$ on the other side of the lens associated with parallel light rays coming from the opposite direction. Since the paths of rays are reversible, rays that diverge from either focal point and pass through the lens will emerge parallel to the axis.

How can we use ray-tracing techniques to show how images are formed by such a lens? The process is illustrated in figure 17.16 for an object lying beyond the focal point of the lens. Three rays (labeled on the diagram) are traced, taking advantage of the properties of the focal points:

1. A ray coming from the top of the object traveling parallel to the axis is bent so that it passes through the focal point on the far side of the lens.
2. A ray coming through the focal point on the near side emerges parallel to the axis.
3. A ray coming in through the center of the lens is undeviated and passes through the lens without being bent.

The sides of the lens near the center are parallel (like a window pane), which is why ray 3 is not bent. When tracing rays, we usually show the bending taking place at a vertical line through the center of the lens.

The image lies on the opposite side of the lens from the object in figure 17.16 and is a real image, since the light rays pass through the image point. Light rays diverging from the object are converged by the lens to this image point. If you placed a screen at that point, you would see the upside-down (inverted) image on the screen. When a slide projector is used, the slides must be inserted upside down to produce upright images on the screen.

## How is the image distance related to the object distance?

Can we predict where an image will be found for a given location of an object? One way to do so is to carefully trace the rays, as we have already illustrated. Using triangle relationships and the law of refraction, we can also

figure 17.16 Three rays can be traced from the top of an object through a positive lens to locate the image. If the object is beyond the focal point, an inverted real image is formed on the opposite side of the lens.
develop a quantitative relationship between the object distance $o$, the image distance $i$, and the focal length $f$ of a lens. (These distances are all measured from the center of the lens.) The relationship involves the reciprocals of these distances-the reciprocal of the object distance plus the reciprocal of the image distance is equal to the reciprocal of the focal length. Stated in symbols,

$$
\frac{1}{o}+\frac{1}{i}=\frac{1}{f}
$$

In the case in figure 17.16, these distances are all positive quantities. In the most frequently used sign convention, object and image distances for real objects or images are positive, but the image distance for a virtual image is negative. The focal length is positive for a converging (positive) lens, one that bends the rays toward the axis, and is negative for a diverging (negative) lens, one that bends the rays away from the axis.

The geometry of figure 17.16 can also be used to find a relationship between the magnification of the image $m$ and the object and image distances. Magnification is defined as the ratio of the image height $h_{i}$ to the object height $h_{o}$, or

$$
m=\frac{h_{i}}{h_{o}}=-\frac{i}{o}
$$

The sign in this equation indicates whether the image is upright or inverted. A negative magnification represents an inverted image, as is the case when the image and object distances are both positive (fig. 17.16). Depending on the object and image distances, the image can be either magnified or reduced in size.

If the object lies "inside" the focal point (closer to the lens) of a positive lens, we can get a virtual image with a positive magnification, as in figure 17.17. In this case, the image distance is a negative quantity, since it is associated with a virtual image. Light rays appear to diverge from the

figure 17.17 A magnified virtual image is formed when the object lies inside the focal point of a positive lens. The emerging light rays appear to diverge from a point behind the object.

## example box 17.2

## Sample Exercise: A Virtual Magnified Image

An object 2 cm in height lies 10 cm to the left of a positive lens with a focal length of 20 cm .
a. Where is the image located?
b. What is its magnification?
a. $f=+20 \mathrm{~cm}$

$$
o=+10 \mathrm{~cm}
$$

$$
i=\text { ? }
$$

$$
\begin{aligned}
\frac{1}{o}+\frac{1}{i} & =\frac{1}{f} \\
\frac{1}{i} & =\frac{1}{f}-\frac{1}{o} \\
& =\frac{1}{20 \mathrm{~cm}}-\frac{1}{10 \mathrm{~cm}} \\
& =\frac{1}{20 \mathrm{~cm}}-\frac{2}{20 \mathrm{~cm}} \\
& =-\frac{1}{20 \mathrm{~cm}} \\
i & =-20 \mathrm{~cm}
\end{aligned}
$$

The image lies 20 cm to the left of the lens, on the same side as the object (fig. 17.17).
b. $m=? \quad m=-\frac{i}{o}$

$$
\begin{aligned}
& =-\frac{-20 \mathrm{~cm}}{10 \mathrm{~cm}} \\
& =+\mathbf{2}
\end{aligned}
$$

The image is magnified to twice the height of the object as shown in figure 17.17. It is upright, since the magnification is positive.
image point but do not actually pass through this point. A virtual image lies on the side of the lens from which the light is coming, the left side in this case. This situation is dealt with in example box 17.2.

When we view the image of an object placed inside the focal point of a positive lens, we are using the lens as a magnifying glass. The image is magnified, but it also lies behind the object, farther from our eyes. This greater distance makes it easier for us to focus on the image than on the object itself. This is an advantage, particularly for older people who have lost the ability to accommodate their focus to view objects that are close to their eyes.

## Tracing rays through negative lenses

As we have seen, a simple convex lens is a positive, converging lens-it bends light rays toward the axis. What happens if we change the direction of the curvature of the lens surfaces so that they are concave rather than convex?

If we imagine that each section of the lens behaves like a prism (as discussed on page 356), these prism sections are upside down compared to the convex lens. As you can see in figure 17.18, these prism sections bend light rays away from the axis rather than toward it. The lens is therefore a diverging or negative lens.

Light rays coming in parallel to the axis are bent away from the axis by the negative lens so that they all appear to be diverging from a common point $F$, the focal point (fig. 17.18). This point is one of two focal points of the negative lens. The other lies on the opposite side at the same distance from the center of the lens as the first one. Light coming in toward the focal point on the far side of the lens is bent so that it comes out parallel to the axis. As mentioned earlier, the focal length $f$ of a negative lens is defined as a negative quantity.

We can trace the same three rays that we traced for the positive lens to locate an image for the negative lens, as in figure 17.19. The ray coming from the top of the object parallel to the axis (ray 1) is bent away from the axis in this case, so that it appears to come from the focal point on the near side of the lens. A ray coming in toward the focal point on the far side (ray 2) is bent to come out parallel to the axis. Ray 3 passes through the center of the lens undeviated, as before.

The resulting image lies on the same side of the lens as the object and is upright and reduced in size (fig. 17.19). This can be verified by using the object-image distance formula, treating the focal length as a negative quantity. The image is virtual because the rays appear to come from the image point but do not (except for the undeviated ray) actually pass through that point. This is true regardless of the object distance-a negative lens used by itself always forms a virtual image smaller than the object.

figure 17.18 Light rays traveling parallel to the axis are bent away from the axis by a negative lens so that they appear to diverge from a common focal point $F$.

figure 17.19 Three rays are traced from the top of an object to locate the image formed by a negative lens. The virtual, upright image lies on the same side of the lens as the object and is reduced in size.

If you hold a negative lens near a printed page and view the page through the lens, the letters will appear smaller than their actual size. Distinguishing a negative lens from a positive lens is easy: one makes the print smaller and the other (the positive lens) magnifies the print when held close to the page. If you move the positive lens away from the page, the image disappears when you reach a distance equal to the focal length and then reappears upside down.

> A positive lens converges light toward the axis. If the incoming rays are parallel to the axis, they converge approximately to a single point, the focal point. The properties of the focal points can be used to trace rays to locate and characterize the images formed. The distance of the focal points from a thin lens is called the focal length, which can be used to find the image position for any given object position. A negative lens diverges light rays and always forms a reduced-in-size virtual image of a real object. The object-image distance formula and the associated ray-tracing techniques can be used to find and describe the images formed by both positive and negative lenses.

### 17.4 Focusing Light with Curved Mirrors

Most of us have had the experience of using a shaving or makeup mirror that magnifies features on your face. This experience can be even more disconcerting early in the morning than that provided by an ordinary mirror. What is going on here? How is the magnification accomplished?

Magnifying mirrors involve curved surfaces rather than plane surfaces. The curvature is usually spherical in nature-
the surface of the mirror is a portion of a sphere. A spherical reflecting surface has the ability to focus light rays in a manner similar to a lens. Simple ray-tracing techniques can provide an understanding of the resulting images.

## Ray tracing with a concave mirror

Mirrors that produce magnification are concave mirrors, which means that light is being reflected from the inside of a spherical surface. Their focusing properties can be understood by following rays that approach the mirror parallel to the axis, as shown in figure 17.20. The center of curvature of the spherical surface lies on the axis, as shown, and the law of reflection dictates where each ray will go.

Each ray that we have traced in figure 17.20 obeys the law of reflection-the angle of reflection equals the angle of incidence. The surface normal for each ray is found by drawing a line from the center of curvature of the sphere to the reflecting surface. A radius of a sphere is always perpendicular to its surface.

As you can see from the diagram, each ray is reflected so that it crosses the axis at approximately the same point as all the other rays. This point of intersection is the focal point, labeled with the letter $F$. Like the focal point of a lens, it is the point where rays coming in parallel to the axis are focused. Since any of these rays could be reversed in direction and still obey the law of reflection, rays that pass through the focal point on their way to the mirror will emerge parallel to the axis of the mirror.

We can also see from the diagram in figure 17.20 that the distance of the focal point $F$ from the mirror is approximately half the distance of the center of curvature $C$ from the mirror. These two points, $F$ and $C$, can be used in tracing

figure 17.20 Light rays approaching a spherical concave mirror traveling parallel to the axis are reflected so that they all pass approximately through a common focal point $F$. The focal length $f$ is half the radius of curvature $R$.
rays to locate and describe images formed by the mirror. When you use such a mirror to examine your face, you generally place your face "inside" the focal point (closer to the mirror). This produces the magnified image that you normally observe.

Figure 17.21 shows how rays can be traced to find the image of an object placed inside the focal point of the mirror. There are three rays that we can easily trace, but any two of these rays would be sufficient to locate the image. The three rays are:

1. A ray coming from the top of the object parallel to the axis and reflected through the focal point.
2. A ray coming in through the focal point and reflected parallel to the axis.
3. A ray coming in along a line passing through the center of curvature and reflected back along itself.

The third ray comes in and out along the same line because it strikes the mirror perpendicular to its surface. The angles of incidence and reflection are both zero in this case.

When these three rays are extended backward, we see that they all appear to come from a point behind the mirror. This point defines the position of the top of the image. The bottom of the image lies on the axis. We see the image as lying behind the mirror and, as is obvious from the diagram, magnified.

The resulting image is upright, magnified, and virtual. It is a virtual image because, just as in the case of the plane mirror, the light rays do not actually pass through the image. The rays never get behind the mirror, but the image appears to lie behind the mirror.

It is also possible to form real images with a concave mirror. This occurs when the object is located beyond the focal point of the mirror, as is illustrated in figure 17.22. In this case, when the three rays are traced, we see that they

figure 17.21 Three rays are traced from the top of the candle placed in front of the mirror. Extending the reflected rays backward locates the top of the image behind the mirror.

figure 17.22 Light rays coming from an object located beyond the focal point of a concave mirror converge to intersect in front of the mirror forming an inverted real image.
converge rather than diverge as they leave the mirror. The rays coming from the top of the object intersect at a point on the same side of the mirror as the object and then diverge again from this point. If our eyes collect these rays, we see an image that lies in front of the mirror.

Since we are used to looking at images that lie behind a mirror, it is a little harder for us to focus on one lying in front. This image can be observed, however, using a curved makeup or shaving mirror. As you move the mirror away from your face, the original magnified image becomes larger and finally disappears. It is replaced by an upsidedown image that grows smaller as you continue to move away from the mirror. This image is a real image because the light rays do pass through the image and then diverge again from these points. As is also clear from the diagram, the image is inverted (upside down).

## Object and image distances

Can we predict where an image will be found for a given location of an object? One way of doing this is simply to carefully trace the rays as already illustrated. Using triangle relationships and the law of reflection, it is also possible to develop a quantitative relationship between the object and image distances. The relationship turns out to be the same as that stated earlier for a thin lens.

$$
\frac{1}{o}+\frac{1}{i}=\frac{1}{f}
$$

The object distance $o$, the image distance $i$, and the focal length $f$ are all measured from the vertex, the point where the axis meets the mirror. As shown in figure 17.20, the focal length is half the radius of curvature.

In the case shown in figure 17.22 these distances are all positive quantities. In general, distances are positive if they lie on the same side of the mirror as the light rays themselves. If they lie behind the mirror, the distances are negative. The exercise in example box 17.3 demonstrates the use of the relationship between object and image distances.

The triangles in figures 17.21 and 17.22 can also be used to find a relationship between the image height and object height-or, in other words, the magnification, $m$. This relationship is also the same as that obtained earlier for a thin lens,

$$
m=-\frac{i}{o}
$$

In example box 17.3 , where the image distance is -10 cm and the object distance +5 cm , the magnification is +2.0 . In other words, the image height is twice that of the object. The fact that this magnification is positive indicates that the image in this case is upright.

## example box 17.3

## Sample Exercise: Finding an Image for a Concave Mirror

An object lies 5 cm to the left of a concave mirror with a focal length of +10 cm . Where is the image? Is it real or virtual?
$o=5 \mathrm{~cm} \quad \frac{1}{o}+\frac{1}{i}=\frac{1}{f}$
$f=10 \mathrm{~cm}$

$$
\frac{1}{i}=\frac{1}{f}-\frac{1}{o}
$$

$$
\frac{1}{i}=\frac{1}{10 \mathrm{~cm}}-\frac{1}{5 \mathrm{~cm}}
$$

$$
\frac{1}{i}=\frac{1}{10 \mathrm{~cm}}-\frac{2}{10 \mathrm{~cm}}
$$

$$
\frac{1}{i}=-\frac{1}{10 \mathrm{~cm}}
$$

$$
i=-10 \mathrm{~cm}
$$

Since the image distance is negative, the image lies 10 cm behind the mirror and is virtual. The situation is much like that pictured in figure 17.21.

## Convex mirrors

Up to this point, we have been considering concave mirrors, which curve inward toward the viewer. What happens if the mirror curvature is in the opposite direction? Convex mirrors, for which light is reflected from the outside of the spherical surface, are used as wide-angle mirrors in store aisles or as the side-view mirror on the passenger side of a car. These mirrors produce a reduced-in-size image, but a large field of view. It is this wide-angle view that makes them useful.

Figure 17.23 pictures rays approaching a convex mirror traveling parallel to the axis of the mirror. The center of curvature $C$ lies behind the mirror in this case. Lines drawn from the center of curvature are perpendicular to the surface of the mirror, as before. The law of reflection dictates that the parallel rays will be reflected away from the axis as shown. When the reflected rays are extended backward, they all appear to have come from the same point $F$, the focal point.

A convex mirror is therefore a diverging or negative mirror. Parallel light rays diverge as they leave the mirror rather than converging as they do with a concave mirror. We can use the same ray-tracing techniques, however, to locate an image. Figure 17.24 illustrates this process. The ray coming from the top of the object traveling parallel to the axis (1) is reflected as though it came from the focal

figure 17.23 Light rays traveling parallel to the axis of a convex mirror are reflected so that they appear to come from a focal point, $F$, located behind the mirror.

figure 17.24 Three rays are traced to locate the virtual image of an object placed in front of a convex mirror. The reflected rays diverge as though coming from the image point behind the mirror.
point. A ray coming in toward the focal point (2) is reflected parallel to the axis, and the ray coming in toward the center of curvature (3) is reflected back along itself since it is perpendicular to the surface.

When extended backward, these three rays all appear to come from a point behind the mirror, thus locating the top of the image. The image is virtual since it lies behind the mirror, and it is obviously upright and reduced in size. Check it out the next time you are in a store that has one of these mirrors mounted above an aisle. The images are small but cover a broad area.

The object-image distance formula introduced for concave mirrors and lenses can also be used to locate images for a convex mirror. The one difference is that the focal length of a convex mirror must be treated as a negative quantity, like that for a negative lens. The image formed by a convex mirror always lies behind the mirror and therefore the image distance will always be negative for any object distance that you choose.

The side mirror on the passenger side of a car is usually a convex mirror. It produces a wide-angle view of the traffic lane to the right. Since it is a negative mirror, the image is reduced in size and lies just behind the mirror as in figure 17.24 . However, there is usually a warning written on the mirror saying: "OBJECTS IN MIRROR ARE CLOSER THAN THEY APPEAR." How can this be if the image being viewed is very close to the mirror itself?

The answer lies in the fact that our brains use many different cues to determine distance. In this case, since the size of the image viewed in the mirror is small, our brains interpret this as meaning that the vehicles must be farther away than they actually are. We know the actual size of that truck or car, and our brains use this size information to determine distance. If you viewed some object of unknown size, your binocular depth perception might place the image of this object at the actual image location behind the mirror.


#### Abstract

Mirrors with curved surfaces can be used to focus light rays and form images. We can locate and describe these images by tracing rays associated with the focal point of the mirror and its center of curvature. The same formula used for lenses can be used to relate object and image distances for mirrors. A concave mirror produces a magnified virtual image when the object is inside the focal point and real images when the object is beyond the focal point. A convex mirror produces a reduced-in-size image for any object position, but provides a wide angle of view.


### 17.5 Eyeglasses, Microscopes, and Telescopes

Lensmaking was an art that developed during the Renaissance. Before then, it was not possible to correct visual problems such as nearsightedness or farsightedness or to magnify objects with a magnifying glass. Once lenses became common, though, it did not take long for people to discover that they could be combined to make optical instruments like microscopes and telescopes. Both were invented in Holland in the early 1600s.

Correction of visual problems is still the most familiar use of lenses. Most of us will wear eyeglasses at some time in our lives, and many of us have worn them since adolescence or even earlier. What goes wrong with our vision that requires corrective lenses? To answer that question, we need to explore the optics of the eye itself.

## How do our eyes work?

Our eyes contain positive lenses that focus light rays on the back surface of the eyeball when working properly. As shown in figure 17.25 , the eye actually contains two positive lenses-the cornea, which is the curved membrane forming the front surface of the eye, and the accommodating lens attached to muscles inside the eye. Most of the bending of light occurs at the cornea. The accommodating lens is more for fine-tuning.

There is a good analogy between the eye and a camera. A camera uses a compound positive lens system to focus light rays coming from objects being photographed onto the film at the back of the camera. The lens system in a camera can be moved back and forth to focus on objects at different distances from the camera. In the eye, the distance between the lens system and the back surface of the eye is fixed so that we need a variable focal-length lens, the accommodating lens, to focus on objects at different distances.

The positive lenses form an inverted real image on the retina, the layer of receptor cells on the back inside surface of the eye. The retina plays the role of the film in a camera and is the sensor that detects the image. The light reaching the cells in the retina initiates nerve impulses that are

figure 17.25 Light rays entering the eye from a distant object are focused on the back surface of the eye (the retina) by the cornea and the accommodating lens, much as light rays are focused on the film by a camera lens.
carried to our brain. The brain processes the nerve impulses received from both eyes and interprets the image according to its experience. Most of the time this interpretation is straightforward, and what we see is what we get, but at times, the brain's interpretation can produce misleading impressions.

Even though the image on the retina is upside down, our brains interpret the scene as right side up. Interestingly, if we fit people with inverting lenses that turn the image on the retina right side up, they initially see things as being upside down. After some time, the brain makes an adjustment, and they begin to see things right side up again. Everything is fine until they take the inverting lenses off. Then everything appears to be upside down again until the brain readjusts! A great deal of processing takes place between the signal received by your eyes and what you actually perceive.

## What problems are corrected with eyeglasses?

The most common visual problem of people who do a lot of reading is nearsightedness or myopia. The eyes of a nearsighted person bend the light rays from a distant object too strongly, causing them to focus in front of the retina, as in figure 17.26a. By the time the rays reach the retina, they are diverging again and no longer form a sharp focus. Things appear fuzzy, although we sometimes fail to notice, because we grow accustomed to this indistinct view. A

figure 17.26 For a nearsighted person, parallel light rays from a distant object are focused in front of the retina. A negative lens placed in front of the eye can correct this problem.
myopic person can see near objects distinctly, because the incident light rays are diverging more strongly from a near object than from a distant object.

Negative eyeglass lenses correct for the tendency of the eye itself to converge the light rays too strongly (fig. 17.26b).

Since a negative lens diverges light rays, it compensates for the excessive convergence by the lenses in the eye and forms distinct images of distant objects on the retina. For a nearsighted person who has not worn glasses before, the difference can be striking.

A farsighted person has the opposite problem. The eye does not converge light rays strongly enough, and images of near objects are formed behind the retina. Positive lenses correct this problem. Laser refractive surgery (see everyday phenomenon box 17.2) can correct both nearsightedness and farsightedness by reshaping the cornea, eliminating the need for eyeglasses.

As we age, the accommodating lenses lose their flexibility. We gradually lose the ability to change the converging power of our eyes and cannot focus on near objects, since light rays diverge more strongly from near objects than from distant objects. At this point, we need bifocals, in which the top half of the lens has one focal length and the bottom half another. We look through the bottom half to do close work and through the top half to view distant objects.

## How does a microscope work?

How are lenses combined to form a microscope? A microscope consists of two positive lenses spaced as shown in figure 17.27 . They are usually held together by a connecting tube, which is not shown in the diagram. If you have ever used a microscope, you know that the object being viewed is placed near the first lens, called the objective lens.

The objective lens forms a real, inverted image of the object, provided that the object lies beyond the focal point of the objective lens. If the object lies just beyond this focal point, the real image has a large image distance and the image is magnified. This can be verified by tracing rays or by using the object-image distance formula.

Since light rays actually pass through a real image and diverge again from that point, this real image becomes the
object for the second lens in the microscope. The eyepiece lens, or ocular, is used like a magnifying glass to observe the real image formed by the objective lens. This real image is focused just inside the focal point of the eyepiece, which then produces the magnified virtual image that we see. The virtual image is also located farther from your eye, so that it can be focused on more readily (fig. 17.27).

Both lenses in a microscope cooperate to produce the desired magnification. The objective lens forms a magnified real image, and this image is magnified again by the eyepiece. The overall magnification of the microscope is found by multiplying these two magnifications together, sometimes achieving magnifications of several hundred times the original object size.

Since eyepiece powers have a limited range, the magnification power of a microscope is determined primarily by the power of the objective lens. A high-power objective lens has a very short focal length, and the object must be placed very close to the objective lens. Microscopes often have two or three different objective lenses of different powers mounted on a turret (fig. 17.28).

The invention of the microscope opened up a whole new world for biologists and other scientists. Microorganisms too small to be seen with the naked eye or with a simple magnifying glass became visible when viewed through a microscope. Seemingly clean pond water was revealed to be teeming with life. The structure of a fly's wing and various kinds of human tissue suddenly became apparent. The microscope is a striking example of how developments in one area of science have a dramatic impact on other areas.

## How does a telescope work?

The development of the microscope opened up the world of the very small. The earlier invention of the telescope had an equally dramatic impact in opening up the world of distant objects. Astronomy was the primary beneficiary. A simple astronomical telescope, like a microscope, can be

figure 17.27 A microscope consists of two positive lenses separated by a connecting tube (not shown). The real image formed by the first lens is viewed through the second lens. Both lenses produce magnification.

## everyday phenomenon

## box 17.2

## Laser Refractive Surgery

The Situation. Megan Evans has been nearsighted since her early teens. She has worn contact lenses for several years after first using spectacle lenses. Now in her twenties, she has heard friends talk about a new procedure called laser refractive surgery that can allow people to see well without corrective lenses. She is intrigued and wants to know more about it.

How can bombarding her eye with a laser beam improve her vision? She knows that lasers can be dangerous in other situations. Is this procedure safe? How does it work, and can it help her situation?

The Analysis. In our culture, myopia or nearsightedness is the most common visual problem. It may develop from doing a lot of near work such as reading during childhood, although there are also hereditary factors. As is described in figure 17.26, the lens system of the myopic eye is too strong, which causes light from distant objects to focus in front of the retina rather than on the retina.

Most of the optical power of the eye is produced by the front surface of the cornea. Optical power is measured in diopters, which is the reciprocal of the focal length measured in meters ( $P=1 / f$ ) when the lens is surrounded by air. The shorter the focal length, the stronger the optical power because a short focal length implies that the light rays are being strongly bent by the lens. The overall power of the lens system of the eye is about 60 diopters, but the front surface of the cornea produces 40 to 50 diopters by itself.

The optical power of the cornea (or of any lens) is determined by two things-how strongly the surface is curved and the difference in index of refraction on either side of the surface. For a nearsighted person, the surface of the cornea is too strongly curved for the length of the eyeball. It is not unusual for a person like Megan to have an optical power of the cornea that is too strong by 4 to 5 diopters. She then requires a corrective lens of -4 to -5 diopters to allow her to see distant objects clearly.

The purpose of laser refractive surgery is to reshape the cornea by vaporizing different portions of the cornea by different amounts. The most commonly used procedure is called LASIK, which is an acronym for laser assisted in situ keratomileusis. In this procedure, the surgeon cuts a circular flap of the outer layer of the cornea with a surgical scalpel and pulls this flap to the side as shown in the drawing. She then uses a pulsed excimer laser to vaporize small amounts of corneal tissue to produce a predetermined new shape for the central portion of the cornea. When finished, the flap of the outer layer is replaced.

The excimer laser used has a wavelength of 192 nm , which lies in the ultraviolet portion of the spectrum. This wavelength


In the LASIK procedure a circular flap of the outer layer of the cornea is pulled aside. Controlled pulses from the laser reshape the central region of the cornea.
is strongly absorbed by corneal tissue, so it vaporizes or ablates this tissue without heating the surrounding tissue. The laser operates in a pulsed mode, with each pulse delivering a definite amount of energy. The surgeon can then control how much tissue is ablated by the number of pulses that are delivered to each section of the cornea. This is all controlled by a computer program to achieve the desired new shape.

The LASIK procedure is done on an outpatient basis, and the cornea heals in just a few days. When successful, the reshaped cornea generally allows a person to discard their glasses or contact lenses. Sometimes a weak correction is still needed because the cornea does not heal to quite the desired power. Older people who have lost the ability to accommodate will generally still need reading glasses unless one eye is shaped to have a stronger power than the other. The LASIK procedure is most commonly used to cure myopia where the goal is to flatten the shape of the cornea. It can also be used, though, for farsightedness (hyperopia) or astigmatism. In the case of astigmatism, the cornea is not spherical and this can also be addressed by reshaping with the laser.

Is the procedure safe? The jury is still out on possible longterm effects, but most patients experience only minor problems, if any. There is always a small risk of infection or poor healing, as with any surgical procedure. People sometimes experience problems with night vision after undergoing LASIK. This is because only the central portion of the cornea is reshaped so there is then a circular boundary between the reshaped and untreated portions of the cornea. At night when light levels are low, the pupil of the eye opens more widely and some light may get through this boundary region producing blurring of the image.
constructed from two positive lenses. How does a telescope differ from a microscope in its design and function?

Distant objects, such as stars, are very large but so far away that they appear to be tiny. One obvious difference between the uses of a microscope and a telescope is that objects viewed with a telescope are much farther from the objective lens. As shown in figure 17.29, the objective lens of a telescope, like a microscope, forms a real image of the object, which is then viewed through the eyepiece. Unlike the microscope, however, the real image formed by a telescope is reduced in size rather than magnified.

figure 17.28 A laboratory microscope often has three or four objective lenses mounted on a rotatable turret. Light passes through the object slide from a source below the slide.

If the real image formed by the objective lens is smaller than the object, how can there be an advantage to using a telescope? The answer is that this image is much closer to the eye than the original object. Even though this image is smaller than the object itself, it forms a larger image on the retina of the eye when viewed through the eyepiece. Figure 17.30 shows two objects of equal height at different distances from the eye. By assuming that the images of both objects are focused on the retina and tracing just the central undeviated ray from the top of each object, we see that the nearer object forms a larger image on the retina.

When you want to see fine detail on an object, you bring the object closer to your eye to take advantage of the larger image formed on the retina. Since the size of the image on the retina is proportional to the angle that the object forms at the eye, we say that we have achieved an angular magnification by bringing the object nearer. We are limited in how close we can bring the object by the focusing power of the eye. The eyepiece of either a telescope or microscope solves this problem by forming a virtual image farther from the eye but at the same angle as the original real image.

The magnifying effect of a telescope is basically an angular magnification. Because it is closer to the eye, the image seen through the telescope forms a larger angle at the eye than the original object. This larger angle produces a larger image on the retina and allows us to see more detail on the object, even though the real image is much smaller than the actual object.

The overall angular magnification produced by a telescope is equal to the ratio of the focal lengths of the two lenses,

$$
M=(-) \frac{f_{o}}{f_{e}}
$$

where $f_{o}$ is the focal length of the objective lens, $f_{e}$ is the focal length of the eyepiece lens, and $M$ is the angular magnification. A minus sign is sometimes included in this

figure 17.29 The objective lens of a telescope forms a real, reduced image of the object, which is then viewed through the eyepiece. The real image is much closer to the eye than the original object. (Not drawn to scale.)

figure 17.30 Two objects of the same size but at different distances from the eye form different-sized images on the retina. We can see more detail on the nearer object.

figure 17.31 Opera glasses and prism binoculars employ different means of reinverting the image so that it is upright.
relationship to indicate that the image is inverted. From this relationship, we see that it is desirable to have a large focal length for the objective lens of a telescope to produce a large angular magnification. A microscope, on the other hand, uses an objective lens with a very short focal length. This is the fundamental difference in the design of telescopes and microscopes.

The large telescopes used in astronomy have concave mirrors instead of lenses for the objective lens. The objects astronomers study are often very dim, and the telescope must collect as much of their light as possible. This requires an objective lens or mirror with a large aperture, or open-
ing, for the incoming light. Since it is easier to make and physically support large mirrors than large lenses, concave mirrors are used in the telescopes at most observatories.

## Binoculars and opera glasses

The image formed by an astronomical telescope is inverted, like the image formed by a microscope. The inverted image is not a big problem for viewing stars or planets, but it can be confusing for viewing objects on land. The most familiar form of land or terrestrial telescope is a pair of prism binoculars, which use multiple reflections in the prisms to reinvert the image (fig. 17.31).

Opera glasses are a simpler form of terrestrial telescope. The two tubes are straight, and the image is reinverted by using negative instead of positive lenses for the eyepieces. Using negative lenses has the additional advantage of making the tubes shorter because the negative lenses must be placed in front of where the real image would be formed. The disadvantage of opera glasses is their narrow field of view and weak magnification. They fit into a purse or pocket more readily than prism binoculars, though.

The two tubes in binoculars and opera glasses allow us to use both eyes when viewing distant objects. Using both eyes preserves some of the three-dimensional aspects of what we see. In normal vision, your two eyes form slightly different images of what you are viewing, because each eye sees objects from a slightly different angle. Your brain interprets these differences as being produced by threedimensional features of the scene. Try closing one eye when you are viewing near objects, and then reopen that eye.

Can you see the difference? A person with just one functional eye sees a flatter world at first, although the brain can make use of head movements and other cues for judging distances.

Our eyes are similar to cameras. They use positive lenses to focus an inverted image on the retina or film. If the point of focus does not lie on the retina, we need corrective lenses. Negative lenses are used to correct nearsight-
edness and positive lenses to correct farsightedness. A microscope uses a combination of positive lenses to produce a magnified virtual image. The overall magnification is the product of the magnifications produced by each lens. A telescope produces an angular magnification of distant objects by bringing the image that we view closer to our eyes. Binoculars and opera glasses are terrestrial telescopes that reinvert the image and allow us to use both eyes.

## summary

For many purposes, the propagation of light can be studied using rays drawn perpendicular to the wavefronts. The laws of reflection and refraction are the basic principles governing these rays. Using these ideas, we can explain how images are formed by mirrors and lenses and how these elements can be combined to make optical instruments.
I Reflection and image formation. The law of reflection states that the angle that the reflected ray makes to an axis drawn perpendicular to the surface equals the angle made by the incident ray. The image formed by a plane mirror is the same distance behind the mirror as the object is from the front of the mirror. Light rays appear to diverge from this image.


2 Refraction of light. A light ray passing into glass or water from air is bent toward the axis by an amount that depends on the index of refraction $n$. Because of this bending, the image of an underwater object seems to lie closer to the surface than it actually does. The index of refraction depends on the wavelength of the light causing dispersion or different amounts of bending for different colors.


3 Lenses and image formation. Lenses can focus light rays to form either real or virtual images. A convex or positive lens converges light rays and can be used as a magnifying glass. A concave or negative lens diverges light rays and forms reduced images. Image positions can be predicted by ray tracing or by using the object-image equation.


4 Focusing light with curved mirrors. A mirror with a spherical curved surface can focus light so that incoming parallel rays pass through or appear to come from a single focal point. A concave mirror can form real images or magnified virtual images depending upon the object position. A convex mirror forms reduced virtual images with a wide angle of view.


5 Eyeglasses, microscopes, and telescopes. Lenses can be used to correct vision problems and can also be combined to make optical instruments. Negative lenses are prescribed for nearsightedness and positive lenses for farsightedness. A microscope forms a magnified real image of the object with the objective lens. This real image is then magnified again when viewed through the eyepiece. A telescope produces an angular magnification by forming an image of a distant object that is much nearer to the eye than the original object.

## key terms

Geometric optics, 348
Physical optics, 348
Wavefront, 348
Surface normal, 349
Law of reflection, 349
Index of refraction, 351

Law of refraction, 352
Dispersion, 355
Positive lens, 356
Focal point, 356
Focal length, 356
Magnification, 357

Negative lens, 358
Concave mirror, 359
Convex mirror, 361
Myopia, 363
Microscope, 364
Telescope, 364

## questions

* = more open-ended questions, requiring lengthier responses, suitable for group discussion
$\mathbf{Q}=$ sample responses are available in appendix D
$\mathbf{Q}=$ sample responses are available on the Online Learning Center
Q1. Does either the velocity or the speed of light change when a beam of light is reflected from a mirror? Explain.
Q2. Does light actually pass through the position of the image formed by a plane mirror? Explain.
Q3. How can an image lie behind a mirror hanging on a wall when no light can reach that point? Explain.
*Q4. When you view your image in a plane mirror, your right hand appears to be your left hand and vice versa. Explain how this reversal occurs.
Q5. If you want to view your full height in a plane mirror, must the mirror be as tall as you are? Explain using a ray diagram.
Q6. Can a plane mirror focus light rays to a point like a positive lens does? Explain.
Q7. Objects A, B, and C lie in the next room hidden from direct view of the person shown in the diagram. A plane mirror is placed on the wall of the passageway between the two rooms as shown. Which of the objects will the person be able to see in the mirror? Explain using a ray diagram.

*Q8. When two plane mirrors are joined at right angles to one another, three images of an object can be seen. The image of the object formed by each mirror can serve as object for the other mirror. Where is the third image located? Explain using a ray diagram.
Q9. A light ray traveling in water $(n=1.33)$ passes from the water into a rectangular piece of glass ( $n=1.5$ ). Is the light ray bent toward the surface normal (the axis drawn perpendicular to the surface) of the glass or away from that axis? Explain.

Q10. Does either the speed or the velocity of light change when light passes from air into a glass block? Explain.
Q11. When we view an underwater object, is the image we see a real image or a virtual image? Explain.
Q12. A fish swimming in a pond looks up at an object lying a couple of feet above the surface of the water. Does this object appear to the fish to lie nearer to the surface or farther from the surface than its actual distance? Explain.
Q13. A light ray traveling in glass for which the critical angle is $42^{\circ}$ strikes a surface between the glass and air at an angle of $45^{\circ}$ to the surface normal. Is this ray refracted into the air at this surface? Explain.
Q14. Do light waves of different colors all travel at the same speed in glass? Explain.

Q15. Is reflection or refraction responsible for the separation of colors in a rainbow? Explain.
Q16. Can we see a rainbow looking eastward if it is raining in the early morning? Explain.
Q17. An object is located at a distance twice the focal length from a positive lens. Trace three rays from the top of the object to locate the image. Is the image real or virtual, erect or inverted?

Q18. Is it possible to form a virtual image with a positive (converging) lens? Explain.
Q19. An object is located at the left-side focal point of a negative lens. Trace three rays from the top of the object to locate the image. Is the image real or virtual, erect or inverted?
*Q20. Is there any position that an object could be placed in front of a negative (diverging) lens that will result in the formation of a real image? Explain.
Q21. Suppose that light rays approach a negative lens so that they are converging toward the focal point on the far side of the lens. Will these rays be diverging when they leave the lens? Explain.
Q22. Do rays traveling parallel to the axis of a concave mirror pass through the center of curvature of the mirror after they are reflected? Explain.

Q23. An object is located at the center of curvature of a concave mirror. Trace two rays from the top of the object to locate the image formed by the mirror. Is the image real or virtual, upright or inverted? Explain.
Q24. An object is located inside the focal point of a concave mirror. Will the image of the object be nearer or farther from the observer than the object itself? Explain.
Q25. Is there any distance at which an object can be located in front of a convex mirror that will produce a real image? Explain.
Q26. Why would you use a convex mirror rather than a concave or plane mirror for viewing activities in a store aisle? Explain.
*Q27. When a convex mirror is used as a side-view mirror on an automobile, where is the image located? Why does printing on the mirror warn you that vehicles may be closer than they appear to be when viewed in the mirror? Explain.

Q28. Does a nearsighted person have trouble seeing near objects? Explain.

Q29. Would you use a positive lens or a negative lens to correct the vision of a farsighted person? Explain.
Q30. Does each of the two lenses used in a microscope produce a magnification of the object being viewed? Explain.
Q31. Does each of the two lenses used in a telescope produce a magnification of the object being viewed? Explain.
Q32. Is it possible to produce an angular magnification of an object by simply bringing the object closer to your eye? Explain.
Q33. Is the objective lens of a microscope likely to have a longer focal length than that of the objective lens of a telescope? Explain.

Q34. What advantages might there be to using binoculars rather than an astronomical telescope for viewing distant objects on land? Explain.

## exercises

E1. A man with a height of 1.8 m stands 3.0 m in front of a plane mirror viewing his image. How tall is the image, and how far from the man is the image located?
E2. A fish lies 80 cm below the surface of a clear pond. If the index of refraction of water is assumed to be 1.33 and that of air is approximately 1 , how far below the surface does the fish appear to a person looking down from above?
E3. A rock appears to lie just 24 cm below the surface of a smooth stream when viewed from above the surface of the stream. Using the indices of refraction given in exercise 2, what is the actual distance of the rock below the surface?

E4. An insect is embedded inside a glass block ( $n=1.5$ ) so that it is located 2.4 cm below a plane surface of the block. How far from this surface does this insect appear to a person looking at the block?

E5. A positive lens has a focal length of 6 cm . An object is located 24 cm from the lens.
a. How far from the lens is the image?
b. Is the image real or virtual, erect or inverted?
c. Trace three rays from the top of the object to confirm your results.
E6. A positive lens has a focal length of 12 cm . An object is located at a distance of 3 cm from the lens.
a. How far from the lens is the image?
b. Is the image real or virtual, erect or inverted?
c. Trace three rays from the top of the object to confirm your results.
E7. A positive lens forms a real image of an object placed 8 cm to the left of the lens. The real image is found 16 cm to the right of the lens. What is the focal length of the lens?

E8. A negative lens has a focal length of -10 cm . An object is located 20 cm from the lens.
a. How far from the lens is the image?
b. Is the image real or virtual, erect or inverted?

E9. A magnifying glass with a focal length of +4 cm is placed 2 cm above a page of print.
a. At what distance from the lens is the image of the page?
b. What is the magnification of this image?

E10. A concave mirror has a focal length of 12 cm . An object is located 6 cm from the surface of the mirror.
a. How far from the mirror is the image of this object?
b. Is the image real or virtual, upright or inverted?

E11. A concave mirror has a focal length of 10 cm . An object is located 30 cm from the surface of the mirror.
a. How far from the mirror is the image of this object?
b. Is the image real or virtual, upright or inverted?
c. Trace three rays from the top of the object to confirm your numerical results.
E12. A convex mirror has a focal length of -10 cm . An object is located 10 cm from the surface of the mirror.
a. How far from the mirror is the image of this object?
b. Is the image real or virtual, upright or inverted?
c. Trace three rays from the top of the object to confirm your numerical results.
E13. A convex mirror used in a store aisle has a focal length of -60 cm . A person in the aisle is 3.0 m from the mirror.
a. How far from the mirror is the image of this object?
b. If the person is 1.8 m tall, how tall is the image viewed in the mirror?

E14. The objective lens of a microscope has a focal length of 0.4 cm . An object on the microscope slide is placed at a distance of 0.5 cm from the lens.
a. At what distance from the lens is the image formed by the objective lens?
b. What is the magnification of this image?

E15. The objective lens of a telescope has a focal length of 1.0 m . An object is located at a distance of 10 m from the lens.
a. At what distance from the objective lens is the image formed by this lens?
b. What is the magnification of this image?

E16. A telescope has an objective lens with a focal length of +40 cm and an eyepiece with a focal length of +2.5 cm . What is the angular magnification produced by this telescope?
E17. A telescope that produces an overall angular magnification of $20 \times$ uses an eyepiece lens with a focal length of 2.5 cm . What is the focal length of the objective lens?

## synthesis problems

SP1. A fish is viewed through the glass wall of a fish tank. The index of refraction of the glass is 1.5 and that of the water in the tank is 1.33 . The fish lies a distance of 6 cm behind the glass. Light rays coming from the fish are bent as they pass from the water to the glass and then again as they pass from the glass to air $(n=1)$. The glass is 0.4 cm thick.
a. Considering just the first interface between the water and the glass, how far behind the glass does the image of the fish lie? (This is an intermediate image formed by bending of light at just the first surface.)
b. Using this image as the object for the second interface between the glass and air, how far behind the front surface of the glass does this "object" lie?
c. Considering the bending of light at this second interface between the glass and air, how far behind the front surface of the glass does the fish appear to lie?
SP2. An object is located at the focal point of a positive lens with a focal length of 12 cm .
a. What is the image distance predicted by the objectimage distance formula?
b. Trace two rays to confirm the conclusion of part a.
c. Will the image be in focus in this situation? Explain.

SP3. An object with a height of 2.5 cm lies 10 cm in front of a lens with a focal length of 6 cm .
a. Using the object-image distance formula, calculate the image distance for this object.
b. What is the magnification of this image?
c. Trace three rays to confirm your conclusions of parts a and $b$.
d. Suppose that this image serves as the object for a second lens that has a focal length of +4 cm . The second lens is placed 6 cm beyond the image serving as its object. Where is the image formed by this second lens, and what is its magnification?
e. What is the overall magnification produced by this twolens system?

SP4. An object 2 cm tall is located 30 cm from a concave mirror with a focal length of 15 cm . Since the focal length is half the radius of the curvature, the object is located at the center of curvature of the mirror.
a. Using the object-image distance formula, find the location of the image.
b. Calculate the magnification of this image.
c. Is the image real or virtual, upright or inverted?
d. Trace two rays from the top of the object to confirm your results.
SP5. Suppose that a microscope has an objective lens with a focal length of 0.8 cm and an eyepiece lens with a focal length of 2.5 cm . The object is located 1.0 cm in front of the objective lens.
a. Calculate the position of the image formed by the objective lens.
b. What is the magnification of this image?
c. If the eyepiece lens is located 2 cm beyond the position of the image formed by the objective lens, where is the image formed by the eyepiece lens? (The image formed by the objective serves as the object for the eyepiece.)
d. What is the magnification of this image?
e. What is the overall magnification produced by this twolens system? (This is found by multiplying the magnifications produced by each lens.*)

[^2]
## home experiments and observations

HE1. Fill a clear glass almost to the top with water and insert various objects into the water.
a. Do the objects appear to be shorter than their actual length when viewed from above the glass?
b. Do the objects appear to be shorter than their actual length when viewed through the sides of the glass? What distortions do you notice when viewing the objects through the sides?
HE2. Locate two small plane mirrors like the ones often carried in a purse. Place the two mirrors next to each other so that they touch along one edge, making an angle of $90^{\circ}$ between the two mirrors. Place a small object like a paper clip in front of the two mirrors.
a. How many images do you see in the two mirrors when the angle between the mirrors is a right angle $\left(90^{\circ}\right)$ ?
b. As you decrease the angle between the two mirrors, describe what happens to the number of images that you can see.
c. Using the idea that each of the images formed can serve as an object for the other mirror, can you explain your observations? (Ray diagrams may be useful.)
HE3. If you have a magnifying (concave) mirror available, such as a shaving or makeup mirror, try moving the mirror slowly away from your face.
a. Describe the changes in the image of your face as the mirror is moved away from your face.
b. The image should become blurred and indistinct when your face is at the focal point of the mirror. Can you estimate the focal length of the mirror by finding the distance from your face where the image disappears?

HE4. The passenger-side side-view mirror on most cars is a convex mirror. (The warning that objects may be closer than they appear indicates that the mirror is convex.)
a. View some object of known height in the mirror. A friend will serve nicely. Estimate the height of the image viewed in the mirror. What is the approximate magnification produced by the mirror?
b. Using your binocular depth perception, estimate the distance behind the mirror that the image is located. (You first have to convince your brain that the image is behind the mirror.) Estimate also the distance of the object from the mirror. Using these values, calculate the focal length of the mirror. (It should be negative.)
HE5. If you have access to an overhead projector, examine the device carefully so that you can describe the optical system involved.
a. What optical elements (lenses or mirrors) are present?
b. What is the function of each of these elements? (Holding a white card or stiff paper at various places between the elements when the projector is in use may help you analyze their function.)
c. Can you produce a ray diagram showing how the rays coming from the object (the transparency being viewed) converge or diverge on their way to the screen?


[^0]:    * The law of refraction is stated quantitatively using the trigonometric function, the sine, which will only be familiar if you have some background in trigonometry. It is usually written in symbolic form as $n_{1} \sin \theta_{1}=n_{2} \sin \theta_{2}$, where $n_{1}$ and $n_{2}$ are the indices of refraction of the two media, and $\theta_{1}$ and $\theta_{2}$ are the angle of incidence and the angle of refraction (fig. 17.8).

[^1]:    *From the law of refraction, $\sin \theta_{c}=n_{2} / n_{1}$ where $n_{2}$ must be less than $n_{1}$ for the expression to be valid.

[^2]:    *This is not the usual way of calculating the magnifying power of a microscope. The standard method compares the size of the angle subtended at the eye with and without the use of the microscope (called angular magnification).

